Computer algebra independent integration tests

4-Trig-functions/4.5-Secant/4.5.1.4-d-tan-^n-a+b-sec-^m

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [365]. This is test number [120].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.73 (364)	% 0.27 (1)
Mathematica	% 93.42 (341)	% 6.58 (24)
Maple	% 90.68 (331)	% 9.32 (34)
Maxima	% 44.38 (162)	% 55.62 (203)
Fricas	% 66.03 (241)	% 33.97 (124)
Sympy	% 9.59 (35)	% 90.41 (330)
Giac	% 62.47 (228)	% 37.53 (137)

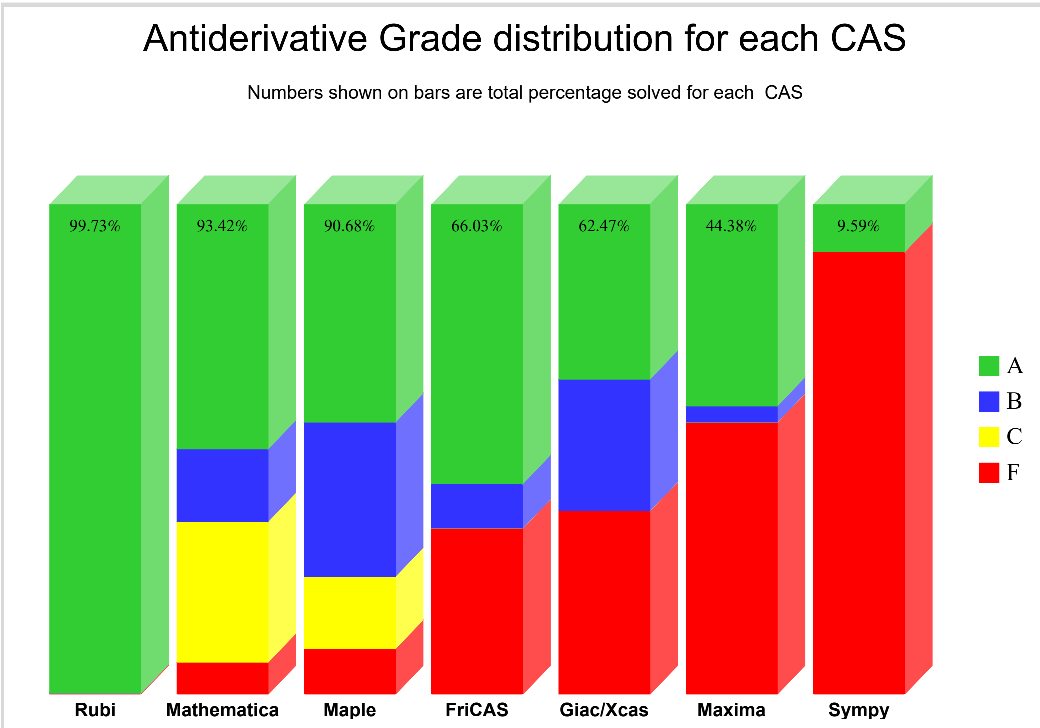
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

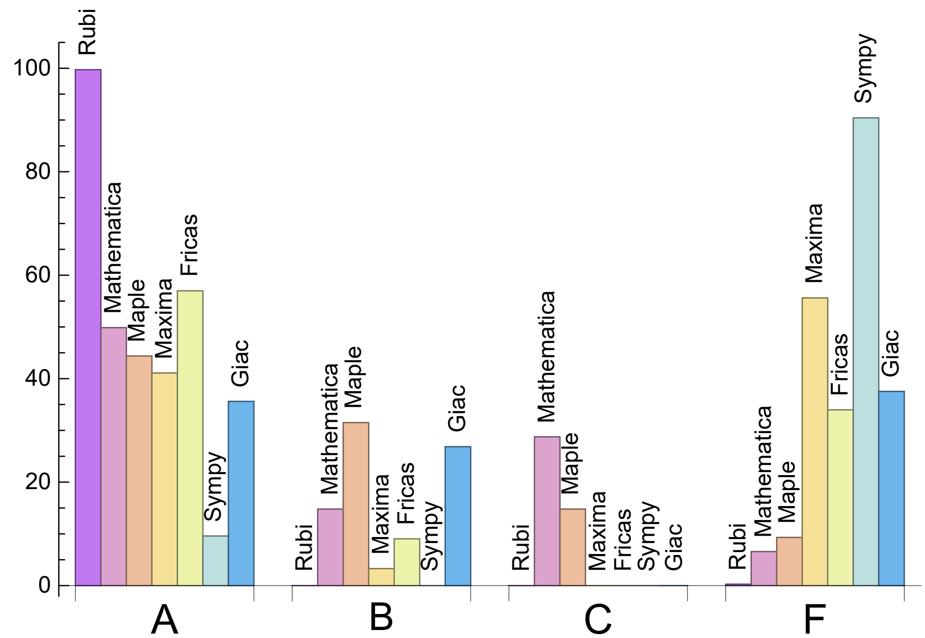
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.73	0.	0.	0.27
Mathematica	49.86	14.79	28.77	6.58
Maple	44.38	31.51	14.79	9.32
Maxima	41.1	3.29	0.	55.62
Fricas	56.99	9.04	0.	33.97
Sympy	9.59	0.	0.	90.41
Giac	35.62	26.85	0.	37.53

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.18	169.12	0.98	129.5	1.
Mathematica	4.54	580.71	3.33	137.	1.
Maple	0.17	612.82	2.9	226.	1.81
Maxima	1.25	143.92	1.42	124.5	1.28
Fricas	2.48	690.86	5.59	419.	3.78
Sympy	20.91	150.43	1.93	126.	1.48
Giac	5.01	271.13	2.42	213.	2.18

1.4 list of integrals that has no closed form antiderivative

{348, 349, 350, 351, 352, 358, 359, 360, 361, 362, 363, 364, 365}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {65, 107, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 131, 133, 134, 143, 144, 145, 146, 156, 157, 158, 169, 170, 177, 178, 179, 180, 181, 182, 189, 190, 191, 192, 193, 194, 201, 202, 203, 204, 205, 206, 207, 226, 227, 229, 230, 231, 232, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 286, 299, 312, 316, 317, 322, 330, 331, 334, 338, 339, 340, 341, 342, 347, 357}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

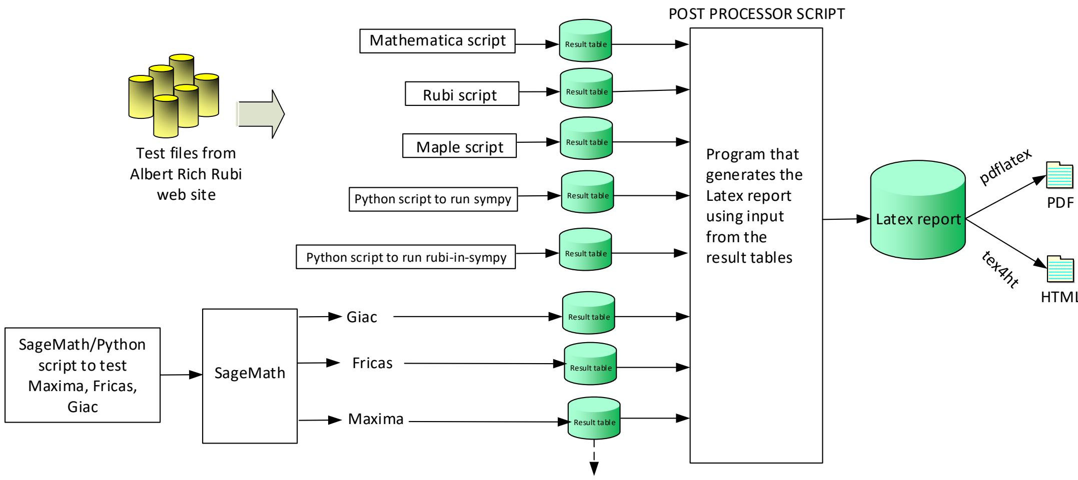
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365 }

B grade: { }

C grade: { }

F grade: { 347}

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 68, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 101, 103, 135, 136, 137, 138, 141, 142, 147, 148, 149, 150, 151, 153, 154, 155, 156, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 171, 172, 173, 179, 183, 184, 190, 195, 196, 211, 219, 220, 221, 222, 223, 224, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 271, 272, 273, 274, 275, 276, 277, 278, 279, 282, 283, 284, 285, 288, 289, 290, 291, 292, 296, 297, 301, 302, 303, 304, 309, 310, 311, 318, 319, 324, 325, 326, 333, 335, 336, 340, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365 }

B grade: { 29, 30, 31, 35, 36, 49, 50, 54, 64, 65, 66, 67, 69, 70, 79, 80, 81, 84, 85, 97, 98, 99, 100, 102, 201, 226, 227, 229, 230, 231, 232, 280, 281, 286, 287, 294, 295, 298, 299, 300, 307, 308, 320, 321, 322, 327, 328, 329, 330, 331, 337, 338, 339, 347 }

C grade: { 14, 15, 16, 17, 18, 32, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 131, 133, 134, 139, 140, 143, 144, 145, 146, 152, 157, 158, 169, 170, 174, 175, 176, 177, 178, 180, 181, 182, 185, 186, 187, 188, 189, 191, 192, 193, 194, 197, 198, 199, 200, 202, 203, 204, 205, 206, 207, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 267, 268, 269, 270, 293, 305, 306, 312, 313, 314, 315, 316, 317, 323, 334, 341, 342, 343 }

F grade: { 127, 128, 129, 130, 132, 208, 209, 210, 212, 213, 214, 215, 216, 217, 218, 225, 228, 250, 251, 252, 253, 254, 255, 332}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 56, 57, 58, 59, 60, 61, 62, 63, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102, 103, 137, 138, 149, 150, 161, 162, 173, 184, 185, 190, 197, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 297, 298, 300, 301, 302, 303, 304, 305, 306, 309, 310, 311, 320, 324, 328, 333, 337, 348, 349, 350, 351, 352, 358, 359, 360, 361, 362, 363, 364, 365 }

B grade: { 20, 22, 38, 52, 53, 54, 55, 64, 65, 66, 67, 79, 80, 81, 96, 97, 98, 135, 136, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 186, 187, 188, 189, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 256, 257, 258, 294, 295, 296, 299, 307, 308, 312, 313, 314, 315, 316, 317, 318, 319, 321, 322, 323, 325, 326, 327, 329, 330, 331, 332, 334, 335, 336, 338, 339, 340, 341, 342, 343 }

C grade: { 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255 }

F grade: { 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 100, 101, 102, 103, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 299, 300, 301, 302, 303, 304, 305, 348, 349, 350, 351, 352, 358, 359, 360, 361, 362, 363, 364, 365

B grade: { 64, 65, 66, 67, 79, 80, 81, 96, 97, 98, 99, 306 }

C grade: { }

F grade: { 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 294, 295, 296, 297, 298, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 135, 136, 137, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 153, 154, 155, 157, 158, 159, 160, 161, 162, 165, 166, 167, 169, 170, 171, 172, 177, 178, 179, 183, 184, 189, 190, 191, 195, 201, 202, 203, 207, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 308, 310, 318, 319, 320, 326, 327, 335, 336, 348, 349, 350, 351, 352, 358, 359, 360, 361, 362, 363, 364, 365 }

B grade: { 9, 13, 17, 18, 28, 46, 138, 150, 151, 152, 156, 163, 164, 168, 173, 185, 196, 197, 266, 293, 298, 305, 306, 307, 309, 311, 321, 322, 328, 329, 330, 337, 338 }

C grade: { }

F grade: { 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 174, 175, 176, 180, 181, 182, 186, 187, 188, 192, 193, 194, 198, 199, 200, 204, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 312, 313, 314, 315, 316, 317, 323, 324, 325, 331, 332, 333, 334, 339, 340, 341, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 19, 20, 21, 22, 23, 37, 38, 39, 40, 41, 60, 75, 90, 91, 256, 257, 258, 259, 271, 272, 273, 274, 275, 290, 349, 350, 351, 359, 363, 364 }

B grade: { }

C grade: { }

F grade: { 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 352, 353, 354, 355, 356, 357, 358, 360, 361, 362, 365 }

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A grade: { 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 60, 61, 62, 63, 64, 65, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 135, 136, 137, 138, 139, 140, 147, 148, 149, 150, 151, 152, 159, 160, 161, 162, 163, 164, 171, 175, 176, 180, 181, 182, 183, 186, 188, 192, 193, 194, 200, 204, 205, 206, 260, 267, 276, 279, 282, 291, 292, 297, 298, 309, 310, 311, 348, 349, 350, 351, 352, 358, 359, 360, 361, 362, 363, 364, 365 }

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C grade: { }

F grade: { 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 141, 142, 143, 144, 145, 146, 153, 154, 155, 156, 165, 166, 167, 168, 169, 177, 178, 179, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 300, 312, 313, 314, 315, 316, 317, 318, 319, 321, 322, 323, 324, 325, 326, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	134	273	157	362	184	396
normalized size	1	1.	0.89	1.81	1.04	2.4	1.22	2.62
time (sec)	N/A	0.072	0.458	0.052	1.08	1.09	53.389	17.811

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	106	216	127	284	148	333
normalized size	1	1.	0.9	1.83	1.08	2.41	1.25	2.82
time (sec)	N/A	0.062	0.442	0.045	1.166	1.	15.023	8.044

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	82	161	97	216	112	271
normalized size	1	1.	0.94	1.85	1.11	2.48	1.29	3.11
time (sec)	N/A	0.049	0.307	0.042	1.071	1.021	25.512	3.838

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	104	68	149	76	209
normalized size	1	1.	0.96	1.82	1.19	2.61	1.33	3.67
time (sec)	N/A	0.039	0.111	0.039	1.168	1.028	1.251	2.015

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	35	80	37	143
normalized size	1	1.	1.	1.	1.4	3.2	1.48	5.72
time (sec)	N/A	0.019	0.018	0.014	1.148	0.697	0.424	1.41

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	29	29	19	46	0	78
normalized size	1	1.	1.81	1.81	1.19	2.88	0.	4.88
time (sec)	N/A	0.02	0.025	0.042	1.144	0.848	0.	1.411

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	114	60	57	186	0	139
normalized size	1	1.	2.	1.05	1.	3.26	0.	2.44
time (sec)	N/A	0.044	0.803	0.075	1.184	0.808	0.	1.489

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	127	93	116	402	0	201
normalized size	1	1.	1.34	0.98	1.22	4.23	0.	2.12
time (sec)	N/A	0.064	0.539	0.073	1.195	0.907	0.	1.496

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	165	124	170	635	0	266
normalized size	1	1.	1.24	0.93	1.28	4.77	0.	2.
time (sec)	N/A	0.082	0.378	0.087	1.078	1.002	0.	1.629

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	115	227	221	466	0	235
normalized size	1	1.	0.89	1.76	1.71	3.61	0.	1.82
time (sec)	N/A	0.129	1.748	0.046	1.711	1.032	0.	12.009

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	95	178	181	377	0	197
normalized size	1	1.	0.93	1.75	1.77	3.7	0.	1.93
time (sec)	N/A	0.094	1.235	0.046	1.708	1.058	0.	4.619

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	75	127	138	302	0	159
normalized size	1	1.	1.03	1.74	1.89	4.14	0.	2.18
time (sec)	N/A	0.062	0.39	0.04	1.73	0.965	0.	2.562

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	60	78	88	234	0	119
normalized size	1	1.	1.33	1.73	1.96	5.2	0.	2.64
time (sec)	N/A	0.034	0.029	0.032	1.727	0.905	0.	1.758

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	43	35	42	82	0	35
normalized size	1	1.	1.65	1.35	1.62	3.15	0.	1.35
time (sec)	N/A	0.024	0.029	0.043	1.75	0.896	0.	1.325

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	62	86	80	177	0	76
normalized size	1	1.	1.13	1.56	1.45	3.22	0.	1.38
time (sec)	N/A	0.052	0.039	0.059	1.741	0.906	0.	1.46

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	79	129	107	356	0	112
normalized size	1	1.	0.94	1.54	1.27	4.24	0.	1.33
time (sec)	N/A	0.081	0.05	0.067	1.658	0.987	0.	1.37

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	92	162	135	551	0	153
normalized size	1	1.	0.83	1.46	1.22	4.96	0.	1.38
time (sec)	N/A	0.114	0.05	0.077	1.79	0.897	0.	1.381

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	111	205	161	745	0	189
normalized size	1	1.	0.79	1.46	1.15	5.32	0.	1.35
time (sec)	N/A	0.145	0.064	0.136	1.696	0.908	0.	1.512

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	140	327	201	424	314	462
normalized size	1	1.	0.73	1.7	1.05	2.21	1.64	2.41
time (sec)	N/A	0.099	0.508	0.053	1.14	1.079	109.844	10.821

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	110	264	149	312	252	394
normalized size	1	1.	0.83	2.	1.13	2.36	1.91	2.98
time (sec)	N/A	0.079	0.304	0.052	1.203	1.	35.879	16.048

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	125	203	131	266	189	327
normalized size	1	1.	1.04	1.69	1.09	2.22	1.58	2.72
time (sec)	N/A	0.073	0.409	0.05	1.229	0.951	11.612	3.863

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	83	140	78	163	126	259
normalized size	1	1.	1.28	2.15	1.2	2.51	1.94	3.98
time (sec)	N/A	0.056	0.182	0.044	1.143	1.185	3.544	2.071

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	51	46	58	127	60	192
normalized size	1	1.	1.06	0.96	1.21	2.65	1.25	4.
time (sec)	N/A	0.036	0.097	0.015	1.19	1.134	0.938	1.412

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	29	34	42	89	0	86
normalized size	1	1.	0.83	0.97	1.2	2.54	0.	2.46
time (sec)	N/A	0.04	0.038	0.05	1.117	1.032	0.	1.464

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	56	51	46	113	0	150
normalized size	1	1.	1.4	1.27	1.15	2.82	0.	3.75
time (sec)	N/A	0.05	0.066	0.066	1.121	1.196	0.	1.474

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	86	87	97	320	0	186
normalized size	1	1.	1.01	1.02	1.14	3.76	0.	2.19
time (sec)	N/A	0.067	0.267	0.08	1.104	1.135	0.	1.481

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	114	122	147	481	0	251
normalized size	1	1.	0.9	0.96	1.16	3.79	0.	1.98
time (sec)	N/A	0.087	0.227	0.081	1.174	0.897	0.	1.642

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	146	159	223	815	0	321
normalized size	1	1.	0.86	0.94	1.32	4.82	0.	1.9
time (sec)	N/A	0.111	0.332	0.095	1.198	0.97	0.	1.713

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	337	226	204	446	0	243
normalized size	1	1.	2.09	1.4	1.27	2.77	0.	1.51
time (sec)	N/A	0.179	1.459	0.051	1.772	0.913	0.	4.758

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	558	169	161	360	0	200
normalized size	1	1.	4.69	1.42	1.35	3.03	0.	1.68
time (sec)	N/A	0.139	5.403	0.046	1.766	1.03	0.	2.713

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	773	112	112	284	0	134
normalized size	1	1.	10.74	1.56	1.56	3.94	0.	1.86
time (sec)	N/A	0.109	6.31	0.038	1.832	0.966	0.	1.826

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	46	50	65	96	0	42
normalized size	1	1.	1.31	1.43	1.86	2.74	0.	1.2
time (sec)	N/A	0.072	0.038	0.047	1.669	0.898	0.	1.367

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	112	112	104	190	0	68
normalized size	1	1.	1.62	1.62	1.51	2.75	0.	0.99
time (sec)	N/A	0.111	0.252	0.068	1.733	0.94	0.	1.506

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	194	155	131	298	0	108
normalized size	1	1.	1.81	1.45	1.22	2.79	0.	1.01
time (sec)	N/A	0.128	0.776	0.067	1.787	0.852	0.	1.482

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	312	188	158	435	0	151
normalized size	1	1.	2.24	1.35	1.14	3.13	0.	1.09
time (sec)	N/A	0.141	1.073	0.078	1.767	1.077	0.	1.499

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	428	231	185	694	0	196
normalized size	1	1.	2.39	1.29	1.03	3.88	0.	1.09
time (sec)	N/A	0.155	1.88	0.087	2.109	0.948	0.	1.546

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	214	351	219	481	439	495
normalized size	1	1.	1.02	1.67	1.04	2.29	2.09	2.36
time (sec)	N/A	0.103	0.73	0.063	1.641	1.059	115.078	14.247

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	110	288	149	308	350	428
normalized size	1	1.	0.8	2.1	1.09	2.25	2.55	3.12
time (sec)	N/A	0.077	0.392	0.057	1.41	1.064	60.509	9.106

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	140	227	149	308	255	360
normalized size	1	1.	1.01	1.64	1.08	2.23	1.85	2.61
time (sec)	N/A	0.077	0.42	0.051	1.673	1.02	31.584	3.896

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	92	164	113	232	165	293
normalized size	1	1.	0.93	1.66	1.14	2.34	1.67	2.96
time (sec)	N/A	0.067	0.299	0.048	1.323	1.016	9.803	2.073

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	64	62	78	162	76	225
normalized size	1	1.	0.97	0.94	1.18	2.45	1.15	3.41
time (sec)	N/A	0.039	0.157	0.015	1.482	0.945	3.918	1.436

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	36	47	58	155	0	196
normalized size	1	1.	0.75	0.98	1.21	3.23	0.	4.08
time (sec)	N/A	0.046	0.088	0.046	1.502	1.198	0.	1.267

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	46	51	46	116	0	147
normalized size	1	1.	1.15	1.27	1.15	2.9	0.	3.68
time (sec)	N/A	0.05	0.127	0.07	1.428	1.231	0.	1.408

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	72	68	80	213	0	186
normalized size	1	1.	1.18	1.11	1.31	3.49	0.	3.05
time (sec)	N/A	0.059	0.17	0.077	1.373	1.02	0.	1.51

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	102	104	130	451	0	223
normalized size	1	1.	0.95	0.97	1.21	4.21	0.	2.08
time (sec)	N/A	0.076	0.669	0.087	1.217	1.196	0.	1.61

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	130	141	192	706	0	288
normalized size	1	1.	0.87	0.95	1.29	4.74	0.	1.93
time (sec)	N/A	0.098	0.335	0.096	1.614	1.226	0.	1.446

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	363	250	354	501	0	265
normalized size	1	1.	1.53	1.05	1.49	2.11	0.	1.12
time (sec)	N/A	0.299	2.066	0.056	1.752	1.312	0.	4.853

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	303	193	284	404	0	221
normalized size	1	1.	1.79	1.14	1.68	2.39	0.	1.31
time (sec)	N/A	0.224	1.404	0.049	1.682	1.238	0.	2.713

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	230	137	198	293	0	178
normalized size	1	1.	2.35	1.4	2.02	2.99	0.	1.82
time (sec)	N/A	0.158	0.835	0.041	1.898	1.188	0.	1.663

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	109	68	115	216	0	89
normalized size	1	1.	2.22	1.39	2.35	4.41	0.	1.82
time (sec)	N/A	0.097	0.236	0.05	1.777	1.2	0.	1.482

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	112	125	122	193	0	68
normalized size	1	1.	1.62	1.81	1.77	2.8	0.	0.99
time (sec)	N/A	0.132	0.233	0.065	1.792	1.115	0.	1.508

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	112	232	165	298	0	89
normalized size	1	1.	1.05	2.17	1.54	2.79	0.	0.83
time (sec)	N/A	0.164	0.663	0.075	1.76	1.106	0.	1.431

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	252	293	205	402	0	130
normalized size	1	1.	1.79	2.08	1.45	2.85	0.	0.92
time (sec)	N/A	0.178	0.987	0.08	1.678	1.138	0.	1.586

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	370	364	246	605	0	173
normalized size	1	1.	2.07	2.03	1.37	3.38	0.	0.97
time (sec)	N/A	0.199	1.37	0.086	1.694	1.172	0.	1.556

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	268	425	286	805	0	217
normalized size	1	1.	1.26	2.	1.34	3.78	0.	1.02
time (sec)	N/A	0.221	6.019	0.145	1.65	1.226	0.	1.589

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	137	125	122	266	0	331
normalized size	1	1.	1.01	0.93	0.9	1.97	0.	2.45
time (sec)	N/A	0.078	0.561	0.089	1.122	1.239	0.	17.637

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	103	93	95	201	0	271
normalized size	1	1.	1.06	0.96	0.98	2.07	0.	2.79
time (sec)	N/A	0.068	0.274	0.074	1.102	1.163	0.	8.726

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	65	62	68	142	0	212
normalized size	1	1.	0.98	0.94	1.03	2.15	0.	3.21
time (sec)	N/A	0.057	0.198	0.065	1.182	1.175	0.	3.779

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	21	30	38	78	0	150
normalized size	1	1.	0.75	1.07	1.36	2.79	0.	5.36
time (sec)	N/A	0.048	0.066	0.046	1.146	1.155	0.	1.979

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	19	33	23	49	41	42
normalized size	1	1.	1.12	1.94	1.35	2.88	2.41	2.47
time (sec)	N/A	0.026	0.018	0.02	1.153	1.112	4.074	1.321

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	67	54	63	182	0	116
normalized size	1	1.	1.1	0.89	1.03	2.98	0.	1.9
time (sec)	N/A	0.056	0.12	0.067	1.253	1.161	0.	1.414

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	107	90	123	389	0	212
normalized size	1	1.	1.04	0.87	1.19	3.78	0.	2.06
time (sec)	N/A	0.076	0.634	0.067	1.127	1.225	0.	1.53

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	135	126	176	610	0	285
normalized size	1	1.	0.93	0.87	1.21	4.21	0.	1.97
time (sec)	N/A	0.098	0.538	0.068	1.177	1.206	0.	1.413

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	301	312	444	354	0	201
normalized size	1	1.	2.87	2.97	4.23	3.37	0.	1.91
time (sec)	N/A	0.144	0.902	0.093	1.743	1.23	0.	12.276

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	893	228	333	288	0	166
normalized size	1	1.	11.45	2.92	4.27	3.69	0.	2.13
time (sec)	N/A	0.109	6.426	0.076	1.743	1.201	0.	5.199

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	241	144	220	224	0	130
normalized size	1	1.	4.92	2.94	4.49	4.57	0.	2.65
time (sec)	N/A	0.077	0.855	0.063	1.699	1.191	0.	2.513

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	60	59	105	93	0	68
normalized size	1	1.	2.86	2.81	5.	4.43	0.	3.24
time (sec)	N/A	0.05	0.09	0.056	1.708	1.172	0.	1.559

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	100	74	126	170	0	89
normalized size	1	1.	1.64	1.21	2.07	2.79	0.	1.46
time (sec)	N/A	0.097	0.768	0.056	1.736	1.086	0.	1.335

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	254	113	185	342	0	132
normalized size	1	1.	2.89	1.28	2.1	3.89	0.	1.5
time (sec)	N/A	0.127	0.81	0.068	1.714	1.131	0.	1.364

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	359	150	239	533	0	171
normalized size	1	1.	3.07	1.28	2.04	4.56	0.	1.46
time (sec)	N/A	0.162	1.072	0.067	1.698	1.169	0.	1.322

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	125	110	108	234	0	301
normalized size	1	1.	1.04	0.92	0.9	1.95	0.	2.51
time (sec)	N/A	0.074	0.518	0.084	1.15	1.225	0.	11.425

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	83	63	68	147	0	243
normalized size	1	1.	1.28	0.97	1.05	2.26	0.	3.74
time (sec)	N/A	0.058	0.202	0.075	1.13	1.189	0.	7.426

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	51	46	54	119	0	184
normalized size	1	1.	1.06	0.96	1.12	2.48	0.	3.83
time (sec)	N/A	0.05	0.114	0.058	1.175	1.204	0.	3.52

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	30	34	42	85	0	45
normalized size	1	1.	0.91	1.03	1.27	2.58	0.	1.36
time (sec)	N/A	0.045	0.06	0.072	1.122	1.198	0.	1.913

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	56	50	47	113	177	77
normalized size	1	1.	1.56	1.39	1.31	3.14	4.92	2.14
time (sec)	N/A	0.035	0.132	0.024	1.131	1.122	29.801	1.269

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	83	72	100	297	0	158
normalized size	1	1.	1.02	0.89	1.23	3.67	0.	1.95
time (sec)	N/A	0.061	0.186	0.076	1.191	1.199	0.	1.326

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	121	108	149	444	0	251
normalized size	1	1.	0.98	0.88	1.21	3.61	0.	2.04
time (sec)	N/A	0.087	0.37	0.077	1.149	1.211	0.	1.39

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	154	144	225	749	0	319
normalized size	1	1.	0.93	0.87	1.36	4.54	0.	1.93
time (sec)	N/A	0.111	0.805	0.075	1.157	1.229	0.	1.414

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	495	269	406	323	0	184
normalized size	1	1.	4.16	2.26	3.41	2.71	0.	1.55
time (sec)	N/A	0.19	5.589	0.089	1.756	1.192	0.	10.34

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	767	185	265	259	0	134
normalized size	1	1.	10.65	2.57	3.68	3.6	0.	1.86
time (sec)	N/A	0.149	6.294	0.076	1.797	1.306	0.	4.877

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	177	102	166	177	0	107
normalized size	1	1.	5.21	3.	4.88	5.21	0.	3.15
time (sec)	N/A	0.065	0.487	0.062	1.79	1.18	0.	2.487

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	35	42	37	66	99	0	39
normalized size	1	1.06	1.27	1.12	2.	3.	0.	1.18
time (sec)	N/A	0.112	0.022	0.069	1.766	1.126	0.	1.516

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	149	94	153	277	0	113
normalized size	1	1.	1.39	0.88	1.43	2.59	0.	1.06
time (sec)	N/A	0.174	1.34	0.062	1.74	1.1	0.	1.391

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	314	132	212	402	0	154
normalized size	1	1.	2.26	0.95	1.53	2.89	0.	1.11
time (sec)	N/A	0.191	0.986	0.074	1.978	1.262	0.	1.307

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	802	170	266	651	0	194
normalized size	1	1.	4.48	0.95	1.49	3.64	0.	1.08
time (sec)	N/A	0.208	6.571	0.074	1.742	1.262	0.	1.329

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	140	127	122	269	0	332
normalized size	1	1.	1.02	0.93	0.89	1.96	0.	2.42
time (sec)	N/A	0.077	0.316	0.097	1.086	1.249	0.	36.512

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	93	93	95	207	0	273
normalized size	1	1.	0.94	0.94	0.96	2.09	0.	2.76
time (sec)	N/A	0.067	0.385	0.084	1.158	1.269	0.	13.904

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	64	63	68	144	0	213
normalized size	1	1.	0.98	0.97	1.05	2.22	0.	3.28
time (sec)	N/A	0.057	0.184	0.075	1.077	1.185	0.	7.97

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	36	46	61	144	0	151
normalized size	1	1.	0.78	1.	1.33	3.13	0.	3.28
time (sec)	N/A	0.054	0.114	0.071	1.166	1.23	0.	3.677

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	33	51	49	112	457	76
normalized size	1	1.	0.94	1.46	1.4	3.2	13.06	2.17
time (sec)	N/A	0.051	0.062	0.092	1.063	1.116	27.452	1.846

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	79	68	81	204	411	117
normalized size	1	1.	1.41	1.21	1.45	3.64	7.34	2.09
time (sec)	N/A	0.041	0.13	0.027	1.12	1.106	27.845	1.326

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	97	90	132	419	0	193
normalized size	1	1.	0.96	0.89	1.31	4.15	0.	1.91
time (sec)	N/A	0.069	0.35	0.112	1.15	1.188	0.	1.331

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	140	126	197	652	0	286
normalized size	1	1.	0.98	0.88	1.38	4.56	0.	2.
time (sec)	N/A	0.097	0.634	0.088	1.186	1.2	0.	1.424

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	169	162	254	879	0	352
normalized size	1	1.	0.91	0.88	1.37	4.75	0.	1.9
time (sec)	N/A	0.124	1.182	0.082	1.147	1.295	0.	1.366

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	362	396	579	446	0	236
normalized size	1	1.	1.53	1.67	2.44	1.88	0.	1.
time (sec)	N/A	0.363	1.323	0.115	1.598	1.292	0.	69.781

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	303	312	463	362	0	201
normalized size	1	1.	1.79	1.85	2.74	2.14	0.	1.19
time (sec)	N/A	0.268	0.852	0.102	1.715	1.252	0.	42.399

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	230	228	347	265	0	166
normalized size	1	1.	2.32	2.3	3.51	2.68	0.	1.68
time (sec)	N/A	0.204	0.729	0.088	1.646	1.196	0.	13.369

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	241	144	231	234	0	131
normalized size	1	1.	3.65	2.18	3.5	3.55	0.	1.98
time (sec)	N/A	0.091	0.906	0.08	1.677	1.184	0.	5.852

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	48	117	76	132	225	0	85
normalized size	1	1.04	2.54	1.65	2.87	4.89	0.	1.85
time (sec)	N/A	0.141	0.259	0.082	1.706	1.151	0.	2.678

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	71	125	56	97	200	0	68
normalized size	1	1.18	2.08	0.93	1.62	3.33	0.	1.13
time (sec)	N/A	0.173	0.38	0.082	1.636	1.096	0.	1.654

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	252	113	180	377	0	134
normalized size	1	1.	1.76	0.79	1.26	2.64	0.	0.94
time (sec)	N/A	0.236	1.273	0.066	1.642	1.244	0.	1.48

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	366	151	239	566	0	177
normalized size	1	1.	2.07	0.85	1.35	3.2	0.	1.
time (sec)	N/A	0.253	1.173	0.079	1.616	1.488	0.	1.447

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	394	189	294	757	0	216
normalized size	1	1.	1.83	0.88	1.37	3.52	0.	1.
time (sec)	N/A	0.278	3.59	0.082	1.556	1.596	0.	1.568

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	186	1495	0	0	0	0
normalized size	1	1.	0.6	4.82	0.	0.	0.	0.
time (sec)	N/A	0.32	2.232	0.296	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	214	688	0	0	0	0
normalized size	1	1.	0.76	2.44	0.	0.	0.	0.
time (sec)	N/A	0.274	2.127	0.239	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	182	1405	0	0	0	0
normalized size	1	1.	0.67	5.17	0.	0.	0.	0.
time (sec)	N/A	0.244	1.507	0.28	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	244	244	220	284	0	0	0	0
normalized size	1	1.	0.9	1.16	0.	0.	0.	0.
time (sec)	N/A	0.213	1.683	0.243	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	196	1414	0	0	0	0
normalized size	1	1.	0.64	4.64	0.	0.	0.	0.
time (sec)	N/A	0.303	2.607	0.247	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	200	658	0	0	0	0
normalized size	1	1.	0.71	2.33	0.	0.	0.	0.
time (sec)	N/A	0.275	1.62	0.216	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	254	1427	0	0	0	0
normalized size	1	1.	0.73	4.12	0.	0.	0.	0.
time (sec)	N/A	0.365	2.472	0.269	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	366	366	117	1518	0	0	0	0
normalized size	1	1.	0.32	4.15	0.	0.	0.	0.
time (sec)	N/A	0.433	6.118	0.294	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	335	335	257	721	0	0	0	0
normalized size	1	1.	0.77	2.15	0.	0.	0.	0.
time (sec)	N/A	0.385	11.823	0.26	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	309	309	106	1480	0	0	0	0
normalized size	1	1.	0.34	4.79	0.	0.	0.	0.
time (sec)	N/A	0.336	1.271	0.263	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	278	278	220	655	0	0	0	0
normalized size	1	1.	0.79	2.36	0.	0.	0.	0.
time (sec)	N/A	0.303	2.502	0.463	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	310	310	238	1392	0	0	0	0
normalized size	1	1.	0.77	4.49	0.	0.	0.	0.
time (sec)	N/A	0.388	7.229	0.24	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	316	316	224	650	0	0	0	0
normalized size	1	1.	0.71	2.06	0.	0.	0.	0.
time (sec)	N/A	0.386	4.899	0.226	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	370	370	2820	1429	0	0	0	0
normalized size	1	1.	7.62	3.86	0.	0.	0.	0.
time (sec)	N/A	0.456	13.983	0.291	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	330	330	332	734	0	0	0	0
normalized size	1	1.	1.01	2.22	0.	0.	0.	0.
time (sec)	N/A	0.419	19.741	0.26	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	326	326	129	1505	0	0	0	0
normalized size	1	1.	0.4	4.62	0.	0.	0.	0.
time (sec)	N/A	0.394	12.155	0.248	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	295	295	271	698	0	0	0	0
normalized size	1	1.	0.92	2.37	0.	0.	0.	0.
time (sec)	N/A	0.353	50.033	0.259	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	285	285	105	1419	0	0	0	0
normalized size	1	1.	0.37	4.98	0.	0.	0.	0.
time (sec)	N/A	0.326	5.138	0.255	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	257	257	1211	319	0	0	0	0
normalized size	1	1.	4.71	1.24	0.	0.	0.	0.
time (sec)	N/A	0.292	12.594	0.226	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	315	315	2715	359	0	0	0	0
normalized size	1	1.	8.62	1.14	0.	0.	0.	0.
time (sec)	N/A	0.375	7.703	0.239	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	290	290	1253	1269	0	0	0	0
normalized size	1	1.	4.32	4.38	0.	0.	0.	0.
time (sec)	N/A	0.346	8.897	0.254	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	359	359	180	2113	0	0	0	0
normalized size	1	1.	0.5	5.89	0.	0.	0.	0.
time (sec)	N/A	0.452	12.861	0.245	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	328	328	1299	1896	0	0	0	0
normalized size	1	1.	3.96	5.78	0.	0.	0.	0.
time (sec)	N/A	0.417	9.178	0.257	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	372	372	0	1518	0	0	0	0
normalized size	1	1.	0.	4.08	0.	0.	0.	0.
time (sec)	N/A	0.492	13.149	0.281	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	339	339	0	721	0	0	0	0
normalized size	1	1.	0.	2.13	0.	0.	0.	0.
time (sec)	N/A	0.458	70.431	0.249	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	312	312	0	1504	0	0	0	0
normalized size	1	1.	0.	4.82	0.	0.	0.	0.
time (sec)	N/A	0.411	4.306	0.251	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	281	281	0	653	0	0	0	0
normalized size	1	1.	0.	2.32	0.	0.	0.	0.
time (sec)	N/A	0.378	3.871	0.259	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	310	310	812	360	0	0	0	0
normalized size	1	1.	2.62	1.16	0.	0.	0.	0.
time (sec)	N/A	0.465	6.672	0.251	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F(-1)	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	316	316	0	1267	0	0	0	0
normalized size	1	1.	0.	4.01	0.	0.	0.	0.
time (sec)	N/A	0.459	15.702	0.252	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	363	363	2792	2117	0	0	0	0
normalized size	1	1.	7.69	5.83	0.	0.	0.	0.
time (sec)	N/A	0.525	8.3	0.299	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	365	365	1281	1896	0	0	0	0
normalized size	1	1.	3.51	5.19	0.	0.	0.	0.
time (sec)	N/A	0.531	8.787	0.294	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	102	359	0	810	0	261
normalized size	1	1.	0.69	2.44	0.	5.51	0.	1.78
time (sec)	N/A	0.115	0.551	0.268	0.	2.429	0.	5.046

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	80	221	0	684	0	205
normalized size	1	1.	0.81	2.23	0.	6.91	0.	2.07
time (sec)	N/A	0.084	0.157	0.235	0.	1.885	0.	5.126

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	60	42	0	475	0	111
normalized size	1	1.	1.18	0.82	0.	9.31	0.	2.18
time (sec)	N/A	0.045	0.046	0.049	0.	1.885	0.	5.072

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	72	98	0	647	0	117
normalized size	1	1.	0.99	1.34	0.	8.86	0.	1.6
time (sec)	N/A	0.071	0.05	0.192	0.	1.702	0.	5.027

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	87	267	0	1118	0	189
normalized size	1	1.	0.66	2.04	0.	8.53	0.	1.44
time (sec)	N/A	0.118	0.274	0.26	0.	2.543	0.	5.073

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	102	407	0	1438	0	271
normalized size	1	1.	0.53	2.11	0.	7.45	0.	1.4
time (sec)	N/A	0.158	0.302	0.335	0.	2.639	0.	4.746

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	134	566	0	1007	0	0
normalized size	1	1.	0.6	2.55	0.	4.54	0.	0.
time (sec)	N/A	0.106	7.286	0.287	0.	1.892	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	110	317	0	890	0	0
normalized size	1	1.	0.69	1.98	0.	5.56	0.	0.
time (sec)	N/A	0.095	5.812	0.247	0.	1.801	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	226	210	0	753	0	0
normalized size	1	1.	2.35	2.19	0.	7.84	0.	0.
time (sec)	N/A	0.077	3.966	0.197	0.	1.7	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	5512	188	0	1133	0	0
normalized size	1	1.	50.57	1.72	0.	10.39	0.	0.
time (sec)	N/A	0.099	23.874	0.228	0.	2.463	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	196	196	5562	381	0	1449	0	0
normalized size	1	1.	28.38	1.94	0.	7.39	0.	0.
time (sec)	N/A	0.203	23.694	0.278	0.	2.607	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	280	280	5604	573	0	1781	0	0
normalized size	1	1.	20.01	2.05	0.	6.36	0.	0.
time (sec)	N/A	0.271	23.678	0.371	0.	2.867	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	112	429	0	921	0	298
normalized size	1	1.	0.66	2.54	0.	5.45	0.	1.76
time (sec)	N/A	0.136	0.489	0.237	0.	2.079	0.	4.877

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	92	291	0	782	0	238
normalized size	1	1.	0.76	2.4	0.	6.46	0.	1.97
time (sec)	N/A	0.104	0.248	0.204	0.	2.019	0.	4.986

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	70	57	0	626	0	163
normalized size	1	1.	0.96	0.78	0.	8.58	0.	2.23
time (sec)	N/A	0.055	0.137	0.031	0.	1.943	0.	4.696

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	72	101	0	644	0	128
normalized size	1	1.	0.99	1.38	0.	8.82	0.	1.75
time (sec)	N/A	0.079	0.057	0.141	0.	1.672	0.	4.71

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	99	258	0	1003	0	167
normalized size	1	1.	0.91	2.37	0.	9.2	0.	1.53
time (sec)	N/A	0.107	0.34	0.213	0.	1.765	0.	5.001

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	104	502	0	1555	0	230
normalized size	1	1.	0.61	2.94	0.	9.09	0.	1.35
time (sec)	N/A	0.143	0.273	0.283	0.	2.585	0.	5.044

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	147	656	0	1156	0	0
normalized size	1	1.	0.57	2.54	0.	4.48	0.	0.
time (sec)	N/A	0.122	8.414	0.259	0.	1.99	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	123	407	0	987	0	0
normalized size	1	1.	0.63	2.1	0.	5.09	0.	0.
time (sec)	N/A	0.11	6.464	0.222	0.	1.829	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	97	300	0	832	0	0
normalized size	1	1.	0.76	2.34	0.	6.5	0.	0.
time (sec)	N/A	0.093	5.536	0.183	0.	1.745	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	102	115	0	684	0	0
normalized size	1	1.	1.59	1.8	0.	10.69	0.	0.
time (sec)	N/A	0.071	0.363	0.197	0.	1.966	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	5552	372	0	1403	0	501
normalized size	1	1.	38.56	2.58	0.	9.74	0.	3.48
time (sec)	N/A	0.148	23.718	0.234	0.	2.563	0.	7.366

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	226	226	5592	720	0	1899	0	703
normalized size	1	1.	24.74	3.19	0.	8.4	0.	3.11
time (sec)	N/A	0.231	23.667	0.327	0.	2.738	0.	7.915

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	156	500	0	1056	0	332
normalized size	1	1.	0.81	2.59	0.	5.47	0.	1.72
time (sec)	N/A	0.146	0.735	0.316	0.	2.241	0.	4.859

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	102	362	0	873	0	266
normalized size	1	1.	0.7	2.5	0.	6.02	0.	1.83
time (sec)	N/A	0.118	0.583	0.225	0.	2.015	0.	4.819

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	82	74	0	729	0	203
normalized size	1	1.	0.85	0.76	0.	7.52	0.	2.09
time (sec)	N/A	0.065	0.207	0.029	0.	1.988	0.	4.608

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	83	124	0	782	0	159
normalized size	1	1.	0.87	1.31	0.	8.23	0.	1.67
time (sec)	N/A	0.089	0.091	0.149	0.	1.693	0.	4.644

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	115	248	0	1029	0	176
normalized size	1	1.	1.08	2.34	0.	9.71	0.	1.66
time (sec)	N/A	0.103	0.265	0.204	0.	1.771	0.	5.709

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	138	376	0	1319	0	198
normalized size	1	1.	0.94	2.56	0.	8.97	0.	1.35
time (sec)	N/A	0.127	1.295	0.262	0.	1.794	0.	6.688

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	173	747	0	1283	0	0
normalized size	1	1.	0.6	2.58	0.	4.42	0.	0.
time (sec)	N/A	0.128	9.977	0.286	0.	2.093	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	149	498	0	1095	0	0
normalized size	1	1.	0.67	2.22	0.	4.89	0.	0.
time (sec)	N/A	0.111	7.346	0.23	0.	1.929	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	125	391	0	945	0	0
normalized size	1	1.	0.78	2.44	0.	5.91	0.	0.
time (sec)	N/A	0.099	5.757	0.189	0.	1.827	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	124	192	0	693	0	0
normalized size	1	1.	1.88	2.91	0.	10.5	0.	0.
time (sec)	N/A	0.071	0.714	0.166	0.	1.995	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	81	214	0	888	0	0
normalized size	1	1.	0.84	2.23	0.	9.25	0.	0.
time (sec)	N/A	0.076	0.229	0.279	0.	1.999	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	176	176	5572	542	0	1708	0	657
normalized size	1	1.	31.66	3.08	0.	9.7	0.	3.73
time (sec)	N/A	0.184	23.75	0.285	0.	2.67	0.	10.62

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	88	293	0	759	0	257
normalized size	1	1.	0.7	2.33	0.	6.02	0.	2.04
time (sec)	N/A	0.101	0.184	0.239	0.	1.955	0.	12.256

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	66	155	0	625	0	201
normalized size	1	1.	0.85	1.99	0.	8.01	0.	2.58
time (sec)	N/A	0.074	0.083	0.207	0.	1.923	0.	10.243

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	44	26	0	359	0	74
normalized size	1	1.	1.42	0.84	0.	11.58	0.	2.39
time (sec)	N/A	0.039	0.037	0.04	0.	1.878	0.	9.692

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	57	259	0	0	0	203
normalized size	1	1.	0.62	2.82	0.	0.	0.	2.21
time (sec)	N/A	0.086	0.051	0.215	0.	0.	0.	9.354

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	90	504	0	0	0	309
normalized size	1	1.	0.59	3.32	0.	0.	0.	2.03
time (sec)	N/A	0.135	0.167	0.274	0.	0.	0.	9.823

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-2)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	102	746	0	0	0	400
normalized size	1	1.	0.48	3.49	0.	0.	0.	1.87
time (sec)	N/A	0.178	0.255	0.382	0.	0.	0.	10.182

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	189	189	469	480	0	957	0	0
normalized size	1	1.	2.48	2.54	0.	5.06	0.	0.
time (sec)	N/A	0.101	19.315	0.261	0.	1.82	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	125	125	238	231	0	832	0	0
normalized size	1	1.	1.9	1.85	0.	6.66	0.	0.
time (sec)	N/A	0.084	2.622	0.237	0.	1.763	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	119	116	0	620	0	0
normalized size	1	1.	1.89	1.84	0.	9.84	0.	0.
time (sec)	N/A	0.063	0.718	0.162	0.	1.671	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	165	165	5544	374	0	0	0	166
normalized size	1	1.	33.6	2.27	0.	0.	0.	1.01
time (sec)	N/A	0.141	23.722	0.232	0.	0.	0.	9.873

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	251	251	5584	722	0	0	0	346
normalized size	1	1.	22.25	2.88	0.	0.	0.	1.38
time (sec)	N/A	0.226	23.725	0.304	0.	0.	0.	9.984

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-2)	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	335	335	5628	1068	0	0	0	522
normalized size	1	1.	16.8	3.19	0.	0.	0.	1.56
time (sec)	N/A	0.322	23.598	0.292	0.	0.	0.	10.092

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	79	224	0	687	0	227
normalized size	1	1.	0.79	2.24	0.	6.87	0.	2.27
time (sec)	N/A	0.097	0.144	0.202	0.	1.944	0.	12.077

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	56	81	0	493	0	134
normalized size	1	1.	1.04	1.5	0.	9.13	0.	2.48
time (sec)	N/A	0.075	0.07	0.147	0.	1.834	0.	10.177

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	38	45	0	635	0	138
normalized size	1	1.	0.7	0.83	0.	11.76	0.	2.56
time (sec)	N/A	0.049	0.027	0.031	0.	1.874	0.	9.821

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	60	376	0	0	0	254
normalized size	1	1.	0.5	3.13	0.	0.	0.	2.12
time (sec)	N/A	0.107	0.059	0.194	0.	0.	0.	9.619

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	90	514	0	0	0	387
normalized size	1	1.	0.51	2.92	0.	0.	0.	2.2
time (sec)	N/A	0.162	0.14	0.265	0.	0.	0.	9.828

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-2)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	99	866	0	0	0	481
normalized size	1	1.	0.42	3.64	0.	0.	0.	2.02
time (sec)	N/A	0.205	0.265	0.276	0.	0.	0.	10.064

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	248	391	0	914	0	456
normalized size	1	1.	1.58	2.49	0.	5.82	0.	2.9
time (sec)	N/A	0.097	2.448	0.229	0.	1.825	0.	15.605

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	162	142	0	778	0	348
normalized size	1	1.	1.71	1.49	0.	8.19	0.	3.66
time (sec)	N/A	0.08	4.403	0.202	0.	1.787	0.	12.677

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	4752	142	0	784	0	302
normalized size	1	1.	55.91	1.67	0.	9.22	0.	3.55
time (sec)	N/A	0.086	23.453	0.127	0.	2.036	0.	12.545

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	215	215	5588	542	0	0	0	221
normalized size	1	1.	25.99	2.52	0.	0.	0.	1.03
time (sec)	N/A	0.195	23.586	0.238	0.	0.	0.	9.459

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-2)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	303	303	5630	732	0	0	0	390
normalized size	1	1.	18.58	2.42	0.	0.	0.	1.29
time (sec)	N/A	0.282	23.679	0.326	0.	0.	0.	10.069

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-2)	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	387	387	5672	1240	0	0	0	559
normalized size	1	1.	14.66	3.2	0.	0.	0.	1.44
time (sec)	N/A	0.368	23.623	0.379	0.	0.	0.	10.223

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	69	155	0	629	0	189
normalized size	1	1.	0.88	1.99	0.	8.06	0.	2.42
time (sec)	N/A	0.09	0.11	0.199	0.	1.909	0.	11.375

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	50	154	0	636	0	140
normalized size	1	1.	0.93	2.85	0.	11.78	0.	2.59
time (sec)	N/A	0.072	0.053	0.136	0.	1.92	0.	9.965

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	40	62	0	826	0	180
normalized size	1	1.	0.51	0.79	0.	10.59	0.	2.31
time (sec)	N/A	0.058	0.051	0.035	0.	1.928	0.	9.579

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	60	496	0	0	0	315
normalized size	1	1.	0.42	3.44	0.	0.	0.	2.19
time (sec)	N/A	0.125	0.062	0.224	0.	0.	0.	9.078

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	90	744	0	0	0	448
normalized size	1	1.	0.45	3.72	0.	0.	0.	2.24
time (sec)	N/A	0.175	0.181	0.318	0.	0.	0.	10.129

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-2)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	99	986	0	0	0	541
normalized size	1	1.	0.38	3.76	0.	0.	0.	2.06
time (sec)	N/A	0.227	0.291	0.342	0.	0.	0.	9.524

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	447	302	0	849	0	394
normalized size	1	1.	3.52	2.38	0.	6.69	0.	3.1
time (sec)	N/A	0.087	6.068	0.214	0.	2.03	0.	16.585

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	113	113	5501	327	0	1102	0	398
normalized size	1	1.	48.68	2.89	0.	9.75	0.	3.52
time (sec)	N/A	0.107	23.444	0.207	0.	2.906	0.	13.095

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	5531	370	0	1319	0	375
normalized size	1	1.	43.55	2.91	0.	10.39	0.	2.95
time (sec)	N/A	0.106	23.453	0.14	0.	3.285	0.	12.588

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	265	265	5614	714	0	0	0	277
normalized size	1	1.	21.18	2.69	0.	0.	0.	1.05
time (sec)	N/A	0.233	23.627	0.259	0.	0.	0.	9.283

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-2)	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	355	355	5656	1066	0	0	0	447
normalized size	1	1.	15.93	3.	0.	0.	0.	1.26
time (sec)	N/A	0.329	23.685	0.356	0.	0.	0.	10.126

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-2)	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	439	439	5698	1412	0	0	0	613
normalized size	1	1.	12.98	3.22	0.	0.	0.	1.4
time (sec)	N/A	0.42	23.71	0.444	0.	0.	0.	9.694

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	227	5594	724	0	1802	0	0
normalized size	1	1.28	31.6	4.09	0.	10.18	0.	0.
time (sec)	N/A	0.18	23.638	0.212	0.	3.592	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	125	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	1.259	0.735	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	243	243	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.229	2.58	180.	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	161	161	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.169	1.	0.532	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	105	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	0.702	0.68	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	130	130	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.159	0.473	0.684	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	169	169	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.272	0.859	0.359	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	252	252	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.341	1.47	0.451	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	3.446	0.255	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	7.357	0.279	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	2.214	0.283	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	9.738	0.251	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	87	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	0.418	0.439	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	72	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	0.145	0.309	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	49	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.048	0.244	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.042	0.317	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	57	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.041	0.33	0.	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	96	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	0.216	0.23	0.	0.	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	1.151	0.272	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	106	2419	0	0	0	0	0
normalized size	1	1.	22.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	14.911	0.21	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	2492	0	0	0	0	0
normalized size	1	1.	24.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	15.367	0.252	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	1.531	0.303	0.	0.	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	2072	0	0	0	0	0
normalized size	1	1.	18.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	18.543	0.282	0.	0.	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	238	0	0	0	0	0
normalized size	1	1.	2.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	2.086	0.28	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	229	0	0	0	0	0
normalized size	1	1.	2.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	1.377	0.288	0.	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	112	112	2164	0	0	0	0	0
normalized size	1	1.	19.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	15.55	0.256	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	185	658	0	0	0	0
normalized size	1	1.	0.58	2.06	0.	0.	0.	0.
time (sec)	N/A	0.264	3.68	0.304	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	191	1390	0	0	0	0
normalized size	1	1.	0.55	4.02	0.	0.	0.	0.
time (sec)	N/A	0.279	1.142	0.364	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	169	289	0	0	0	0
normalized size	1	1.	0.62	1.05	0.	0.	0.	0.
time (sec)	N/A	0.201	1.747	0.261	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	189	1429	0	0	0	0
normalized size	1	1.	0.63	4.78	0.	0.	0.	0.
time (sec)	N/A	0.233	1.578	0.313	0.	0.	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	224	688	0	0	0	0
normalized size	1	1.	0.7	2.15	0.	0.	0.	0.
time (sec)	N/A	0.251	2.415	0.258	0.	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	357	357	93	650	0	0	0	0
normalized size	1	1.	0.26	1.82	0.	0.	0.	0.
time (sec)	N/A	0.336	2.219	0.27	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	343	343	220	1392	0	0	0	0
normalized size	1	1.	0.64	4.06	0.	0.	0.	0.
time (sec)	N/A	0.329	5.062	0.28	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	311	311	118	653	0	0	0	0
normalized size	1	1.	0.38	2.1	0.	0.	0.	0.
time (sec)	N/A	0.287	1.747	0.289	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	339	339	221	1504	0	0	0	0
normalized size	1	1.	0.65	4.44	0.	0.	0.	0.
time (sec)	N/A	0.318	10.808	0.356	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	375	375	127	721	0	0	0	0
normalized size	1	1.	0.34	1.92	0.	0.	0.	0.
time (sec)	N/A	0.328	5.924	0.309	0.	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	405	405	316	2113	0	0	0	0
normalized size	1	1.	0.78	5.22	0.	0.	0.	0.
time (sec)	N/A	0.427	4.006	0.297	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	325	325	135	1269	0	0	0	0
normalized size	1	1.	0.42	3.9	0.	0.	0.	0.
time (sec)	N/A	0.325	1.581	0.301	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	347	347	249	359	0	0	0	0
normalized size	1	1.	0.72	1.03	0.	0.	0.	0.
time (sec)	N/A	0.357	3.5	0.303	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	290	290	112	323	0	0	0	0
normalized size	1	1.	0.39	1.11	0.	0.	0.	0.
time (sec)	N/A	0.289	8.212	0.259	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	325	325	194	1419	0	0	0	0
normalized size	1	1.	0.6	4.37	0.	0.	0.	0.
time (sec)	N/A	0.323	56.863	0.287	0.	0.	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	335	335	130	698	0	0	0	0
normalized size	1	1.	0.39	2.08	0.	0.	0.	0.
time (sec)	N/A	0.337	15.143	0.28	0.	0.	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	371	371	261	1505	0	0	0	0
normalized size	1	1.	0.7	4.06	0.	0.	0.	0.
time (sec)	N/A	0.368	19.291	0.28	0.	0.	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	413	413	0	2117	0	0	0	0
normalized size	1	1.	0.	5.13	0.	0.	0.	0.
time (sec)	N/A	0.427	12.977	0.315	0.	0.	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	359	359	0	1267	0	0	0	0
normalized size	1	1.	0.	3.53	0.	0.	0.	0.
time (sec)	N/A	0.389	9.47	0.277	0.	0.	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	355	355	0	367	0	0	0	0
normalized size	1	1.	0.	1.03	0.	0.	0.	0.
time (sec)	N/A	0.418	4.935	0.28	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	321	321	0	653	0	0	0	0
normalized size	1	1.	0.	2.03	0.	0.	0.	0.
time (sec)	N/A	0.372	4.756	0.3	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	357	357	0	1480	0	0	0	0
normalized size	1	1.	0.	4.15	0.	0.	0.	0.
time (sec)	N/A	0.394	52.015	0.268	0.	0.	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	389	389	0	721	0	0	0	0
normalized size	1	1.	0.	1.85	0.	0.	0.	0.
time (sec)	N/A	0.48	13.15	0.274	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	106	216	127	284	148	428
normalized size	1	1.	0.95	1.95	1.14	2.56	1.33	3.86
time (sec)	N/A	0.156	0.447	0.045	1.038	1.496	19.339	8.006

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	82	161	97	216	112	335
normalized size	1	1.	0.98	1.92	1.15	2.57	1.33	3.99
time (sec)	N/A	0.094	0.189	0.04	0.989	1.343	5.303	3.55

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	104	68	149	76	242
normalized size	1	1.	1.	1.89	1.24	2.71	1.38	4.4
time (sec)	N/A	0.066	0.156	0.037	0.983	1.095	1.27	1.753

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	35	80	37	144
normalized size	1	1.	1.	1.	1.4	3.2	1.48	5.76
time (sec)	N/A	0.037	0.014	0.016	0.992	0.771	0.983	1.202

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	60	35	46	113	0	82
normalized size	1	1.	1.4	0.81	1.07	2.63	0.	1.91
time (sec)	N/A	0.081	0.036	0.034	0.973	0.97	0.	1.285

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	114	85	84	254	0	230
normalized size	1	1.	1.58	1.18	1.17	3.53	0.	3.19
time (sec)	N/A	0.108	1.358	0.047	0.98	0.727	0.	1.275

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	166	134	134	440	0	359
normalized size	1	1.	1.63	1.31	1.31	4.31	0.	3.52
time (sec)	N/A	0.131	0.33	0.044	0.982	0.834	0.	1.408

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	216	185	180	630	0	483
normalized size	1	1.	1.66	1.42	1.38	4.85	0.	3.72
time (sec)	N/A	0.176	0.58	0.046	0.995	0.823	0.	1.331

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	103	178	181	377	0	308
normalized size	1	1.	1.01	1.75	1.77	3.7	0.	3.02
time (sec)	N/A	0.095	1.028	0.042	1.462	0.943	0.	5.017

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	79	127	138	302	0	232
normalized size	1	1.	1.08	1.74	1.89	4.14	0.	3.18
time (sec)	N/A	0.065	0.578	0.04	1.471	0.924	0.	2.325

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	60	78	88	234	0	155
normalized size	1	1.	1.33	1.73	1.96	5.2	0.	3.44
time (sec)	N/A	0.035	0.024	0.03	1.495	0.904	0.	1.511

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	43	35	42	82	0	70
normalized size	1	1.	1.65	1.35	1.62	3.15	0.	2.69
time (sec)	N/A	0.026	0.02	0.035	1.473	0.807	0.	1.153

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	62	86	80	213	0	151
normalized size	1	1.	1.13	1.56	1.45	3.87	0.	2.75
time (sec)	N/A	0.052	0.029	0.042	1.459	0.783	0.	1.314

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	79	129	107	343	0	230
normalized size	1	1.	0.94	1.54	1.27	4.08	0.	2.74
time (sec)	N/A	0.081	0.037	0.045	1.469	0.807	0.	1.287

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	92	162	135	477	0	304
normalized size	1	1.	0.83	1.46	1.22	4.3	0.	2.74
time (sec)	N/A	0.111	0.046	0.051	1.481	0.806	0.	1.48

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	217	173	317	235	483	314	660
normalized size	1	1.17	0.94	1.71	1.27	2.61	1.7	3.57
time (sec)	N/A	0.131	0.424	0.056	1.018	0.93	90.757	13.794

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	169	138	256	188	383	252	560
normalized size	1	1.13	0.93	1.72	1.26	2.57	1.69	3.76
time (sec)	N/A	0.111	0.331	0.052	1.036	0.983	25.294	7.908

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	131	105	197	146	293	189	460
normalized size	1	1.14	0.91	1.71	1.27	2.55	1.64	4.
time (sec)	N/A	0.091	0.257	0.051	0.982	0.804	9.82	3.562

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	74	136	101	205	126	360
normalized size	1	1.	0.85	1.56	1.16	2.36	1.45	4.14
time (sec)	N/A	0.069	0.436	0.048	0.999	0.97	4.028	1.933

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	42	45	57	127	60	258
normalized size	1	1.	0.89	0.96	1.21	2.7	1.28	5.49
time (sec)	N/A	0.034	0.057	0.015	0.975	0.845	0.682	1.202

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	53	53	84	184	0	136
normalized size	1	1.	0.87	0.87	1.38	3.02	0.	2.23
time (sec)	N/A	0.1	0.106	0.039	0.984	0.874	0.	1.385

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	82	108	97	275	0	282
normalized size	1	1.	0.89	1.17	1.05	2.99	0.	3.07
time (sec)	N/A	0.129	0.459	0.052	0.987	0.79	0.	1.353

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	148	169	165	504	0	486
normalized size	1	1.	1.17	1.34	1.31	4.	0.	3.86
time (sec)	N/A	0.159	3.225	0.053	1.064	0.99	0.	1.456

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	293	219	203	494	0	381
normalized size	1	1.	1.87	1.39	1.29	3.15	0.	2.43
time (sec)	N/A	0.198	1.355	0.051	1.463	0.834	0.	5.03

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	355	164	159	393	0	297
normalized size	1	1.	3.06	1.41	1.37	3.39	0.	2.56
time (sec)	N/A	0.152	0.938	0.049	1.531	0.809	0.	2.331

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	201	109	111	294	0	213
normalized size	1	1.	2.87	1.56	1.59	4.2	0.	3.04
time (sec)	N/A	0.114	1.161	0.04	1.481	0.943	0.	1.663

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	39	49	63	104	0	108
normalized size	1	1.	0.81	1.02	1.31	2.17	0.	2.25
time (sec)	N/A	0.075	0.366	0.038	1.464	0.852	0.	1.37

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	122	111	103	240	0	238
normalized size	1	1.	1.44	1.31	1.21	2.82	0.	2.8
time (sec)	N/A	0.115	0.46	0.047	1.48	0.832	0.	1.392

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	198	154	130	382	0	369
normalized size	1	1.	1.62	1.26	1.07	3.13	0.	3.02
time (sec)	N/A	0.135	0.592	0.05	1.499	0.781	0.	1.442

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	257	187	157	524	0	494
normalized size	1	1.	1.68	1.22	1.03	3.42	0.	3.23
time (sec)	N/A	0.148	0.815	0.055	1.532	0.952	0.	1.298

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	250	250	520	460	362	674	0	2267
normalized size	1	1.	2.08	1.84	1.45	2.7	0.	9.07
time (sec)	N/A	0.197	6.227	0.064	1.	1.133	0.	11.265

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	371	292	247	473	0	1299
normalized size	1	1.	2.18	1.72	1.45	2.78	0.	7.64
time (sec)	N/A	0.139	6.174	0.057	0.992	1.251	0.	7.652

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	108	163	149	305	0	636
normalized size	1	1.	1.	1.51	1.38	2.82	0.	5.89
time (sec)	N/A	0.096	0.369	0.052	0.946	1.127	0.	3.628

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	52	70	77	161	0	267
normalized size	1	1.	0.88	1.19	1.31	2.73	0.	4.53
time (sec)	N/A	0.071	0.121	0.041	0.972	0.93	0.	1.98

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	19	35	26	43	82	131
normalized size	1	1.	0.54	1.	0.74	1.23	2.34	3.74
time (sec)	N/A	0.032	0.037	0.02	0.964	0.928	6.83	1.35

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	70	80	92	190	0	180
normalized size	1	1.	0.74	0.85	0.98	2.02	0.	1.91
time (sec)	N/A	0.102	0.105	0.058	0.956	1.073	0.	1.29

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	141	167	194	587	0	346
normalized size	1	1.	0.9	1.06	1.24	3.74	0.	2.2
time (sec)	N/A	0.181	1.009	0.075	1.029	1.078	0.	1.451

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	625	308	390	1312	0	639
normalized size	1	1.	2.67	1.32	1.67	5.61	0.	2.73
time (sec)	N/A	0.295	6.242	0.076	1.014	1.889	0.	1.491

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	271	907	785	0	1423	0	639
normalized size	1	1.37	4.58	3.96	0.	7.19	0.	3.23
time (sec)	N/A	0.373	6.165	0.082	0.	2.826	0.	4.702

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	287	374	0	1048	0	324
normalized size	1	1.	2.28	2.97	0.	8.32	0.	2.57
time (sec)	N/A	0.343	2.148	0.072	0.	1.68	0.	2.504

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	115	153	0	599	0	189
normalized size	1	1.	1.51	2.01	0.	7.88	0.	2.49
time (sec)	N/A	0.185	0.146	0.054	0.	0.865	0.	1.587

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	135	147	123	0	815	0	190
normalized size	1	1.27	1.39	1.16	0.	7.69	0.	1.79
time (sec)	N/A	0.243	0.409	0.074	0.	0.969	0.	1.343

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	256	416	238	0	1623	0	386
normalized size	1	1.45	2.35	1.34	0.	9.17	0.	2.18
time (sec)	N/A	0.387	6.162	0.085	0.	0.974	0.	1.35

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	255	255	528	498	433	946	0	2290
normalized size	1	1.	2.07	1.95	1.7	3.71	0.	8.98
time (sec)	N/A	0.205	6.275	0.072	1.022	1.56	0.	10.865

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	383	324	306	690	0	0
normalized size	1	1.	2.14	1.81	1.71	3.85	0.	0.
time (sec)	N/A	0.146	6.198	0.069	0.985	1.396	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	187	192	201	485	0	767
normalized size	1	1.	1.55	1.59	1.66	4.01	0.	6.34
time (sec)	N/A	0.104	0.537	0.056	0.99	1.267	0.	3.704

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	62	93	100	230	0	423
normalized size	1	1.	0.84	1.26	1.35	3.11	0.	5.72
time (sec)	N/A	0.084	0.249	0.052	0.959	0.814	0.	1.982

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	54	55	113	0	321
normalized size	1	1.	1.	1.	1.02	2.09	0.	5.94
time (sec)	N/A	0.043	0.038	0.023	0.969	0.883	0.	1.312

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	189	141	192	525	0	409
normalized size	1	1.	1.37	1.02	1.39	3.8	0.	2.96
time (sec)	N/A	0.142	0.326	0.071	0.995	1.267	0.	1.33

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	351	226	409	1455	0	886
normalized size	1	1.	1.78	1.15	2.08	7.39	0.	4.5
time (sec)	N/A	0.229	2.054	0.084	1.024	1.516	0.	1.393

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	473	367	753	3028	0	1073
normalized size	1	1.	1.7	1.32	2.71	10.89	0.	3.86
time (sec)	N/A	0.371	3.035	0.09	1.111	2.645	0.	1.494

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	283	865	723	0	1893	0	555
normalized size	1	1.42	4.32	3.62	0.	9.46	0.	2.78
time (sec)	N/A	0.431	6.227	0.092	0.	2.396	0.	5.04

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	327	353	0	1374	0	397
normalized size	1	1.	2.18	2.35	0.	9.16	0.	2.65
time (sec)	N/A	0.326	1.575	0.075	0.	1.188	0.	2.601

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	80	120	0	845	0	194
normalized size	1	1.	0.94	1.41	0.	9.94	0.	2.28
time (sec)	N/A	0.146	0.255	0.065	0.	0.898	0.	1.493

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	209	255	0	1507	0	448
normalized size	1	1.	0.92	1.12	0.	6.64	0.	1.97
time (sec)	N/A	0.411	1.738	0.092	0.	0.881	0.	1.36

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	303	416	0	3191	0	657
normalized size	1	1.	0.84	1.16	0.	8.86	0.	1.82
time (sec)	N/A	0.567	2.454	0.099	0.	1.379	0.	1.384

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	761	761	1846	3747	0	0	0	0
normalized size	1	1.	2.43	4.92	0.	0.	0.	0.
time (sec)	N/A	1.175	26.212	0.288	0.	0.	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	740	740	202	1801	0	0	0	0
normalized size	1	1.	0.27	2.43	0.	0.	0.	0.
time (sec)	N/A	1.015	6.923	0.252	0.	0.	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	415	415	232	874	0	0	0	0
normalized size	1	1.	0.56	2.11	0.	0.	0.	0.
time (sec)	N/A	0.705	5.28	0.254	0.	0.	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	422	422	254	2313	0	0	0	0
normalized size	1	1.	0.6	5.48	0.	0.	0.	0.
time (sec)	N/A	0.564	12.325	0.232	0.	0.	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	863	863	1571	6426	0	0	0	0
normalized size	1	1.	1.82	7.45	0.	0.	0.	0.
time (sec)	N/A	1.25	27.113	0.251	0.	0.	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	836	836	2169	16178	0	0	0	0
normalized size	1	1.	2.59	19.35	0.	0.	0.	0.
time (sec)	N/A	1.063	23.659	0.459	0.	0.	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	254	3268	0	1048	0	0
normalized size	1	1.	1.5	19.34	0.	6.2	0.	0.
time (sec)	N/A	0.17	6.291	0.818	0.	3.736	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	194	2342	0	779	0	0
normalized size	1	1.	1.94	23.42	0.	7.79	0.	0.
time (sec)	N/A	0.113	6.217	0.497	0.	3.623	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	137	42	0	489	0	250
normalized size	1	1.	2.69	0.82	0.	9.59	0.	4.9
time (sec)	N/A	0.048	0.249	0.049	0.	3.003	0.	1.39

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	343	576	0	5389	0	0
normalized size	1	1.	3.24	5.43	0.	50.84	0.	0.
time (sec)	N/A	0.159	5.665	0.298	0.	3.979	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	215	215	937	2844	0	8409	0	0
normalized size	1	1.	4.36	13.23	0.	39.11	0.	0.
time (sec)	N/A	0.295	19.039	0.296	0.	11.789	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	692	1109	0	0	0	0
normalized size	1	1.	2.01	3.22	0.	0.	0.	0.
time (sec)	N/A	0.385	17.673	0.377	0.	0.	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	153	215	0	0	0	0
normalized size	1	1.	1.22	1.72	0.	0.	0.	0.
time (sec)	N/A	0.024	0.251	0.259	0.	0.	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	156	628	0	0	0	0
normalized size	1	1.	0.63	2.55	0.	0.	0.	0.
time (sec)	N/A	0.212	3.366	0.296	0.	0.	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	248	4997	0	945	0	0
normalized size	1	1.	1.68	33.76	0.	6.39	0.	0.
time (sec)	N/A	0.143	6.301	0.705	0.	4.081	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	194	3003	0	676	0	350
normalized size	1	1.	2.46	38.01	0.	8.56	0.	4.43
time (sec)	N/A	0.098	1.185	0.443	0.	3.931	0.	2.268

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	108	26	0	373	0	147
normalized size	1	1.	3.48	0.84	0.	12.03	0.	4.74
time (sec)	N/A	0.043	0.192	0.048	0.	2.883	0.	1.552

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	218	691	0	5951	0	0
normalized size	1	1.	2.06	6.52	0.	56.14	0.	0.
time (sec)	N/A	0.134	6.283	0.31	0.	11.286	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	260	260	1022	4203	0	10157	0	0
normalized size	1	1.	3.93	16.17	0.	39.07	0.	0.
time (sec)	N/A	0.263	6.903	0.409	0.	145.021	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	404	610	839	1780	0	0	0	0
normalized size	1	1.51	2.08	4.41	0.	0.	0.	0.
time (sec)	N/A	0.763	16.89	0.566	0.	0.	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	310	310	0	823	0	0	0	0
normalized size	1	1.	0.	2.65	0.	0.	0.	0.
time (sec)	N/A	0.248	0.	0.348	0.	0.	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	140	178	0	0	0	0
normalized size	1	1.	1.32	1.68	0.	0.	0.	0.
time (sec)	N/A	0.021	0.22	0.269	0.	0.	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	361	361	1198	1408	0	0	0	0
normalized size	1	1.	3.32	3.9	0.	0.	0.	0.
time (sec)	N/A	0.42	18.735	0.335	0.	0.	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	263	6612	0	1098	0	0
normalized size	1	1.	1.78	44.68	0.	7.42	0.	0.
time (sec)	N/A	0.172	6.379	0.848	0.	3.286	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	167	2830	0	763	0	0
normalized size	1	1.	1.9	32.16	0.	8.67	0.	0.
time (sec)	N/A	0.124	1.01	0.377	0.	2.913	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	128	45	0	664	0	0
normalized size	1	1.	2.37	0.83	0.	12.3	0.	0.
time (sec)	N/A	0.053	0.387	0.039	0.	2.292	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	142	142	1020	2766	0	9288	0	0
normalized size	1	1.	7.18	19.48	0.	65.41	0.	0.
time (sec)	N/A	0.197	6.93	0.263	0.	104.274	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	236	316	1114	10977	0	0	0	0
normalized size	1	1.34	4.72	46.51	0.	0.	0.	0.
time (sec)	N/A	0.385	7.161	0.76	0.	0.	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	530	907	864	1545	0	0	0	0
normalized size	1	1.71	1.63	2.92	0.	0.	0.	0.
time (sec)	N/A	1.298	17.139	0.455	0.	0.	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	344	344	5162	633	0	0	0	0
normalized size	1	1.	15.01	1.84	0.	0.	0.	0.
time (sec)	N/A	0.404	23.694	0.298	0.	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	347	347	1249	1209	0	0	0	0
normalized size	1	1.	3.6	3.48	0.	0.	0.	0.
time (sec)	N/A	0.319	6.149	0.287	0.	0.	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	664	663	2238	0	0	0	0
normalized size	1	1.48	1.48	4.98	0.	0.	0.	0.
time (sec)	N/A	0.953	13.432	0.3	0.	0.	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	238	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.272	3.366	180.	0.	0.	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	178	0	0	0	0	0
normalized size	1	1.	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.182	1.357	0.946	0.	0.	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	106	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	0.675	0.663	0.	0.	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	266	0	786	0	0	0	0	0
normalized size	1	0.	2.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	4.579	0.668	0.	0.	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	7.954	0.281	0.	0.	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.701	0.3	0.	0.	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	2.988	0.299	0.	0.	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	3.705	0.268	0.	0.	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	3.163	0.574	0.	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	298	0	0	0	0	0
normalized size	1	1.	1.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.201	2.926	0.402	0.	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	118	0	0	0	0	0
normalized size	1	1.	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	1.235	0.297	0.	0.	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	49	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.398	0.318	0.	0.	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	163	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.177	1.423	0.306	0.	0.	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	279	279	256	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.231	6.284	0.268	0.	0.	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	6.256	0.343	0.	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	236	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.33	3.359	0.263	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	3.561	0.25	0.	0.	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	5.714	0.322	0.	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	4.166	0.303	0.	0.	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	5.284	0.318	0.	0.	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	4.889	0.297	0.	0.	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	5.848	0.268	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [316] had the largest ratio of [0.92]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2	A	3	2	1.	19	0.105
3	A	3	2	1.	19	0.105
4	A	3	2	1.	19	0.105
5	A	3	2	1.	17	0.118
6	A	2	2	1.	17	0.118
7	A	3	2	1.	19	0.105
8	A	3	2	1.	19	0.105
9	A	3	2	1.	19	0.105
10	A	6	2	1.	19	0.105
11	A	5	2	1.	19	0.105
12	A	4	2	1.	19	0.105
13	A	3	2	1.	19	0.105
14	A	2	2	1.	19	0.105
15	A	3	2	1.	19	0.105
16	A	4	2	1.	19	0.105
17	A	5	2	1.	19	0.105
18	A	6	2	1.	19	0.105
19	A	3	2	1.	21	0.095
20	A	3	2	1.	21	0.095
21	A	3	2	1.	21	0.095
22	A	3	2	1.	21	0.095
23	A	3	2	1.	19	0.105
24	A	3	2	1.	19	0.105
25	A	3	2	1.	21	0.095
26	A	3	2	1.	21	0.095
27	A	3	2	1.	21	0.095
28	A	3	2	1.	21	0.095
29	A	12	7	1.	21	0.333
30	A	10	7	1.	21	0.333
31	A	8	7	1.	21	0.333
32	A	8	5	1.	21	0.238
33	A	9	6	1.	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
34	A	11	7	1.	21	0.333
35	A	12	7	1.	21	0.333
36	A	13	7	1.	21	0.333
37	A	3	2	1.	21	0.095
38	A	3	2	1.	21	0.095
39	A	3	2	1.	21	0.095
40	A	3	2	1.	21	0.095
41	A	3	2	1.	19	0.105
42	A	3	2	1.	19	0.105
43	A	3	2	1.	21	0.095
44	A	3	2	1.	21	0.095
45	A	3	2	1.	21	0.095
46	A	3	2	1.	21	0.095
47	A	17	8	1.	21	0.381
48	A	14	8	1.	21	0.381
49	A	11	8	1.	21	0.381
50	A	11	8	1.	21	0.381
51	A	11	6	1.	21	0.286
52	A	14	8	1.	21	0.381
53	A	15	8	1.	21	0.381
54	A	16	8	1.	21	0.381
55	A	17	8	1.	21	0.381
56	A	3	2	1.	21	0.095
57	A	3	2	1.	21	0.095
58	A	3	2	1.	21	0.095
59	A	3	2	1.	21	0.095
60	A	2	2	1.	19	0.105
61	A	3	2	1.	19	0.105
62	A	3	2	1.	21	0.095
63	A	3	2	1.	21	0.095
64	A	6	3	1.	21	0.143
65	A	5	3	1.	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
66	A	4	3	1.	21	0.143
67	A	3	2	1.	21	0.095
68	A	4	3	1.	21	0.143
69	A	5	3	1.	21	0.143
70	A	6	3	1.	21	0.143
71	A	3	2	1.	21	0.095
72	A	3	2	1.	21	0.095
73	A	3	2	1.	21	0.095
74	A	3	2	1.	21	0.095
75	A	3	2	1.	19	0.105
76	A	3	2	1.	19	0.105
77	A	3	2	1.	21	0.095
78	A	3	2	1.	21	0.095
79	A	11	8	1.	21	0.381
80	A	9	8	1.	21	0.381
81	A	5	5	1.	21	0.238
82	A	9	6	1.06	21	0.286
83	A	12	8	1.	21	0.381
84	A	13	8	1.	21	0.381
85	A	14	8	1.	21	0.381
86	A	3	2	1.	21	0.095
87	A	3	2	1.	21	0.095
88	A	3	2	1.	21	0.095
89	A	3	2	1.	21	0.095
90	A	3	2	1.	21	0.095
91	A	3	2	1.	19	0.105
92	A	3	2	1.	19	0.105
93	A	3	2	1.	21	0.095
94	A	3	2	1.	21	0.095
95	A	18	9	1.	21	0.429
96	A	15	9	1.	21	0.429
97	A	12	9	1.	21	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
98	A	6	6	1.	21	0.286
99	A	12	9	1.04	21	0.429
100	A	12	7	1.18	21	0.333
101	A	16	9	1.	21	0.429
102	A	17	9	1.	21	0.429
103	A	18	9	1.	21	0.429
104	A	17	14	1.	23	0.609
105	A	16	13	1.	23	0.565
106	A	16	13	1.	23	0.565
107	A	15	12	1.	23	0.522
108	A	17	14	1.	23	0.609
109	A	16	13	1.	23	0.565
110	A	18	14	1.	23	0.609
111	A	21	17	1.	25	0.68
112	A	20	16	1.	25	0.64
113	A	19	15	1.	25	0.6
114	A	18	14	1.	25	0.56
115	A	20	16	1.	25	0.64
116	A	20	16	1.	25	0.64
117	A	22	17	1.	25	0.68
118	A	18	14	1.	25	0.56
119	A	18	15	1.	25	0.6
120	A	17	14	1.	25	0.56
121	A	17	14	1.	25	0.56
122	A	16	13	1.	25	0.52
123	A	18	15	1.	25	0.6
124	A	17	14	1.	25	0.56
125	A	19	15	1.	25	0.6
126	A	18	14	1.	25	0.56
127	A	22	18	1.	25	0.72
128	A	21	17	1.	25	0.68
129	A	20	16	1.	25	0.64

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	19	15	1.	25	0.6
131	A	21	17	1.	25	0.68
132	A	21	17	1.	25	0.68
133	A	23	18	1.	25	0.72
134	A	23	17	1.	25	0.68
135	A	8	5	1.	23	0.217
136	A	6	5	1.	23	0.217
137	A	4	4	1.	21	0.19
138	A	6	4	1.	21	0.19
139	A	8	6	1.	23	0.261
140	A	10	7	1.	23	0.304
141	A	4	3	1.	23	0.13
142	A	4	3	1.	23	0.13
143	A	4	4	1.	23	0.174
144	A	5	4	1.	23	0.174
145	A	7	5	1.	23	0.217
146	A	9	6	1.	23	0.261
147	A	9	5	1.	23	0.217
148	A	7	5	1.	23	0.217
149	A	5	4	1.	21	0.19
150	A	6	4	1.	21	0.19
151	A	7	5	1.	23	0.217
152	A	9	7	1.	23	0.304
153	A	4	3	1.	23	0.13
154	A	4	3	1.	23	0.13
155	A	4	3	1.	23	0.13
156	A	3	3	1.	23	0.13
157	A	6	5	1.	23	0.217
158	A	8	5	1.	23	0.217
159	A	10	5	1.	23	0.217
160	A	8	5	1.	23	0.217
161	A	6	4	1.	21	0.19

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
162	A	7	5	1.	21	0.238
163	A	7	5	1.	23	0.217
164	A	8	6	1.	23	0.261
165	A	4	3	1.	23	0.13
166	A	4	3	1.	23	0.13
167	A	4	3	1.	23	0.13
168	A	3	3	1.	23	0.13
169	A	4	3	1.	23	0.13
170	A	7	5	1.	23	0.217
171	A	7	5	1.	23	0.217
172	A	5	5	1.	23	0.217
173	A	3	3	1.	21	0.143
174	A	7	5	1.	21	0.238
175	A	9	6	1.	23	0.261
176	A	11	7	1.	23	0.304
177	A	4	3	1.	23	0.13
178	A	5	4	1.	23	0.174
179	A	3	3	1.	23	0.13
180	A	6	5	1.	23	0.217
181	A	8	6	1.	23	0.261
182	A	10	6	1.	23	0.261
183	A	6	5	1.	23	0.217
184	A	4	4	1.	23	0.174
185	A	4	4	1.	21	0.19
186	A	8	6	1.	21	0.286
187	A	10	6	1.	23	0.261
188	A	12	7	1.	23	0.304
189	A	5	4	1.	23	0.174
190	A	4	3	1.	23	0.13
191	A	4	3	1.	23	0.13
192	A	7	6	1.	23	0.261
193	A	9	6	1.	23	0.261

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
194	A	11	6	1.	23	0.261
195	A	5	4	1.	23	0.174
196	A	4	4	1.	23	0.174
197	A	5	4	1.	21	0.19
198	A	9	6	1.	21	0.286
199	A	11	6	1.	23	0.261
200	A	13	7	1.	23	0.304
201	A	4	3	1.	23	0.13
202	A	5	4	1.	23	0.174
203	A	5	4	1.	23	0.174
204	A	8	6	1.	23	0.261
205	A	10	6	1.	23	0.261
206	A	12	6	1.	23	0.261
207	A	7	5	1.28	23	0.217
208	A	1	1	1.	23	0.043
209	A	8	6	1.	23	0.261
210	A	7	6	1.	23	0.261
211	A	4	4	1.	21	0.19
212	A	5	5	1.	23	0.217
213	A	8	7	1.	23	0.304
214	A	9	7	1.	23	0.304
215	A	1	1	1.	25	0.04
216	A	1	1	1.	25	0.04
217	A	1	1	1.	25	0.04
218	A	1	1	1.	25	0.04
219	A	4	3	1.	21	0.143
220	A	4	3	1.	21	0.143
221	A	3	3	1.	21	0.143
222	A	2	2	1.	19	0.105
223	A	4	4	1.	19	0.21
224	A	5	5	1.	21	0.238
225	A	1	1	1.	21	0.048

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
226	A	1	1	1.	21	0.048
227	A	1	1	1.	21	0.048
228	A	1	1	1.	21	0.048
229	A	1	1	1.	23	0.043
230	A	1	1	1.	23	0.043
231	A	1	1	1.	23	0.043
232	A	1	1	1.	23	0.043
233	A	17	14	1.	23	0.609
234	A	18	15	1.	23	0.652
235	A	16	13	1.	23	0.565
236	A	17	14	1.	23	0.609
237	A	17	14	1.	23	0.609
238	A	21	17	1.	25	0.68
239	A	21	17	1.	25	0.68
240	A	19	15	1.	25	0.6
241	A	20	16	1.	25	0.64
242	A	21	17	1.	25	0.68
243	A	20	16	1.	25	0.64
244	A	18	15	1.	25	0.6
245	A	19	16	1.	25	0.64
246	A	17	14	1.	25	0.56
247	A	18	15	1.	25	0.6
248	A	18	15	1.	25	0.6
249	A	19	16	1.	25	0.64
250	A	24	19	1.	25	0.76
251	A	22	18	1.	25	0.72
252	A	22	18	1.	25	0.72
253	A	20	16	1.	25	0.64
254	A	21	17	1.	25	0.68
255	A	22	18	1.	25	0.72
256	A	7	5	1.	19	0.263
257	A	6	5	1.	19	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
258	A	5	5	1.	19	0.263
259	A	4	4	1.	17	0.235
260	A	5	4	1.	17	0.235
261	A	6	5	1.	19	0.263
262	A	7	5	1.	19	0.263
263	A	8	5	1.	19	0.263
264	A	5	2	1.	19	0.105
265	A	4	2	1.	19	0.105
266	A	3	2	1.	19	0.105
267	A	2	2	1.	19	0.105
268	A	3	2	1.	19	0.105
269	A	4	2	1.	19	0.105
270	A	5	2	1.	19	0.105
271	A	3	2	1.17	21	0.095
272	A	3	2	1.13	21	0.095
273	A	3	2	1.14	21	0.095
274	A	3	2	1.	21	0.095
275	A	3	2	1.	19	0.105
276	A	3	2	1.	19	0.105
277	A	4	3	1.	21	0.143
278	A	5	4	1.	21	0.19
279	A	12	7	1.	21	0.333
280	A	10	7	1.	21	0.333
281	A	8	7	1.	21	0.333
282	A	8	5	1.	21	0.238
283	A	9	6	1.	21	0.286
284	A	11	7	1.	21	0.333
285	A	12	7	1.	21	0.333
286	A	3	2	1.	21	0.095
287	A	3	2	1.	21	0.095
288	A	3	2	1.	21	0.095
289	A	3	2	1.	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
290	A	4	4	1.	19	0.21
291	A	3	2	1.	19	0.105
292	A	3	2	1.	21	0.095
293	A	3	2	1.	21	0.095
294	A	15	8	1.37	21	0.381
295	A	6	6	1.	21	0.286
296	A	7	7	1.	21	0.333
297	A	9	8	1.27	21	0.381
298	A	15	8	1.45	21	0.381
299	A	3	2	1.	21	0.095
300	A	3	2	1.	21	0.095
301	A	3	2	1.	21	0.095
302	A	3	2	1.	21	0.095
303	A	3	2	1.	19	0.105
304	A	3	2	1.	19	0.105
305	A	3	2	1.	21	0.095
306	A	3	2	1.	21	0.095
307	A	16	10	1.42	21	0.476
308	A	6	6	1.	21	0.286
309	A	6	6	1.	21	0.286
310	A	11	7	1.	21	0.333
311	A	15	8	1.	21	0.381
312	A	38	22	1.	25	0.88
313	A	35	19	1.	25	0.76
314	A	21	16	1.	25	0.64
315	A	19	14	1.	25	0.56
316	A	39	23	1.	25	0.92
317	A	36	20	1.	25	0.8
318	A	5	4	1.	23	0.174
319	A	5	4	1.	23	0.174
320	A	4	4	1.	21	0.19
321	A	7	5	1.	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
322	A	13	9	1.	23	0.391
323	A	7	7	1.	23	0.304
324	A	1	1	1.	14	0.071
325	A	5	4	1.	23	0.174
326	A	5	4	1.	23	0.174
327	A	5	4	1.	23	0.174
328	A	3	3	1.	21	0.143
329	A	7	5	1.	21	0.238
330	A	11	6	1.	23	0.261
331	A	11	8	1.51	23	0.348
332	A	6	6	1.	23	0.261
333	A	1	1	1.	14	0.071
334	A	9	8	1.	23	0.348
335	A	5	4	1.	23	0.174
336	A	5	4	1.	23	0.174
337	A	4	4	1.	21	0.19
338	A	7	4	1.	21	0.19
339	A	11	5	1.34	23	0.217
340	A	17	11	1.71	23	0.478
341	A	7	7	1.	23	0.304
342	A	6	6	1.	14	0.429
343	A	14	11	1.48	23	0.478
344	A	8	6	1.	23	0.261
345	A	7	6	1.	23	0.261
346	A	4	4	1.	21	0.19
347	F	0	0	N/A	0	N/A
348	A	0	0	0.	0	0.
349	A	0	0	0.	0	0.
350	A	0	0	0.	0	0.
351	A	0	0	0.	0	0.
352	A	0	0	0.	0	0.
353	A	5	4	1.	21	0.19

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
354	A	4	4	1.	21	0.19
355	A	2	2	1.	19	0.105
356	A	8	5	1.	19	0.263
357	A	10	5	1.	21	0.238
358	A	0	0	0.	0	0.
359	A	0	0	0.	0	0.
360	A	0	0	0.	0	0.
361	A	0	0	0.	0	0.
362	A	0	0	0.	0	0.
363	A	0	0	0.	0	0.
364	A	0	0	0.	0	0.
365	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int (a + a\sec(c + dx))\tan^9(c + dx)\,dx$

Optimal. Leaf size=151

$$\frac{a\sec^9(c+dx)}{9d} + \frac{a\sec^8(c+dx)}{8d} - \frac{4a\sec^7(c+dx)}{7d} - \frac{2a\sec^6(c+dx)}{3d} + \frac{6a\sec^5(c+dx)}{5d} + \frac{3a\sec^4(c+dx)}{2d} - \frac{4a\sec^3(c+}{3d}$$

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d - (2*a*Sec[c + d*x]^2)/d -
(4*a*Sec[c + d*x]^3)/(3*d) + (3*a*Sec[c + d*x]^4)/(2*d) + (6*a*Sec[c + d*x]
^5)/(5*d) - (2*a*Sec[c + d*x]^6)/(3*d) - (4*a*Sec[c + d*x]^7)/(7*d) + (a*Se
c[c + d*x]^8)/(8*d) + (a*Sec[c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.0722948, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 88}

$$\frac{a\sec^9(c+dx)}{9d} + \frac{a\sec^8(c+dx)}{8d} - \frac{4a\sec^7(c+dx)}{7d} - \frac{2a\sec^6(c+dx)}{3d} + \frac{6a\sec^5(c+dx)}{5d} + \frac{3a\sec^4(c+dx)}{2d} - \frac{4a\sec^3(c+}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^9,x]
```

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d - (2*a*Sec[c + d*x]^2)/d -
(4*a*Sec[c + d*x]^3)/(3*d) + (3*a*Sec[c + d*x]^4)/(2*d) + (6*a*Sec[c + d*x]
^5)/(5*d) - (2*a*Sec[c + d*x]^6)/(3*d) - (4*a*Sec[c + d*x]^7)/(7*d) + (a*Se
c[c + d*x]^8)/(8*d) + (a*Sec[c + d*x]^9)/(9*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int (a + a\sec(c+dx))\tan^9(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^4(a+ax)^5}{x^{10}}\,dx, x, \cos(c+dx)\right)}{a^8 d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{a^9}{x^{10}} + \frac{a^9}{x^9} - \frac{4a^9}{x^8} - \frac{4a^9}{x^7} + \frac{6a^9}{x^6} + \frac{6a^9}{x^5} - \frac{4a^9}{x^4} - \frac{4a^9}{x^3} + \frac{a^9}{x^2} + \frac{a^9}{x}\right) dx, x, \cos(c +$$

$$= -\frac{a\log(\cos(c+dx))}{d} + \frac{a\sec(c+dx)}{d} - \frac{2a\sec^2(c+dx)}{d} - \frac{4a\sec^3(c+dx)}{3d} + \frac{3a\,\text{se}$$

Mathematica [A] time = 0.457898, size = 134, normalized size = 0.89

$$\frac{a\sec^9(c+dx)}{9d} - \frac{4a\sec^7(c+dx)}{7d} + \frac{6a\sec^5(c+dx)}{5d} - \frac{4a\sec^3(c+dx)}{3d} + \frac{a\sec(c+dx)}{d} - \frac{a\left(-3\tan^8(c+dx) + 4\tan^6(c+$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^9,x]
```

```
[Out] (a*Sec[c + d*x])/d - (4*a*Sec[c + d*x]^3)/(3*d) + (6*a*Sec[c + d*x]^5)/(5*d
) - (4*a*Sec[c + d*x]^7)/(7*d) + (a*Sec[c + d*x]^9)/(9*d) - (a*(24*Log[Cos[
c + d*x]] + 12*Tan[c + d*x]^2 - 6*Tan[c + d*x]^4 + 4*Tan[c + d*x]^6 - 3*Tan
[c + d*x]^8))/(24*d)
```

Maple [A] time = 0.052, size = 273, normalized size = 1.8

$$\frac{a(\tan(dx+c))^8}{8d} - \frac{a(\tan(dx+c))^6}{6d} + \frac{a(\tan(dx+c))^4}{4d} - \frac{a(\tan(dx+c))^2}{2d} - \frac{a\ln(\cos(dx+c))}{d} + \frac{a(\sin(dx+c))^{10}}{9d(\cos(dx+c))^9} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^9,x)
```

```
[Out] 1/8/d*a*tan(d*x+c)^8-1/6/d*a*tan(d*x+c)^6+1/4/d*a*tan(d*x+c)^4-1/2/d*a*tan(
d*x+c)^2-a*ln(cos(d*x+c))/d+1/9/d*a*sin(d*x+c)^10/cos(d*x+c)^9-1/63/d*a*sin
(d*x+c)^10/cos(d*x+c)^7+1/105/d*a*sin(d*x+c)^10/cos(d*x+c)^5-1/63/d*a*sin(d
*x+c)^10/cos(d*x+c)^3+1/9/d*a*sin(d*x+c)^10/cos(d*x+c)+128/315/d*a*cos(d*x+
c)+1/9/d*cos(d*x+c)*sin(d*x+c)^8*a+8/63/d*a*cos(d*x+c)*sin(d*x+c)^6+16/105/
d*a*cos(d*x+c)*sin(d*x+c)^4+64/315/d*a*cos(d*x+c)*sin(d*x+c)^2
```

Maxima [A] time = 1.08038, size = 157, normalized size = 1.04

$$-\frac{2520\,a\log(\cos(dx+c)) - \frac{2520\,a\cos(dx+c)^8 - 5040\,a\cos(dx+c)^7 - 3360\,a\cos(dx+c)^6 + 3780\,a\cos(dx+c)^5 + 3024\,a\cos(dx+c)^4 - 1680\,a\cos(dx+c)^3 - 14}{\cos(dx+c)^9}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -1/2520*(2520*a*log(cos(d*x + c)) - (2520*a*cos(d*x + c)^8 - 5040*a*cos(d*x
 + c)^7 - 3360*a*cos(d*x + c)^6 + 3780*a*cos(d*x + c)^5 + 3024*a*cos(d*x +
c)^4 - 1680*a*cos(d*x + c)^3 - 1440*a*cos(d*x + c)^2 + 315*a*cos(d*x + c) +
 280*a)/cos(d*x + c)^9)/d
```

Fricas [A] time = 1.08978, size = 362, normalized size = 2.4

$$-\frac{2520\,a\cos(dx+c)^9\log(-\cos(dx+c)) - 2520\,a\cos(dx+c)^8 + 5040\,a\cos(dx+c)^7 + 3360\,a\cos(dx+c)^6 - 3780\,a\,c}{2520\,d\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/2520*(2520*a*cos(d*x + c)^9*log(-cos(d*x + c)) - 2520*a*cos(d*x + c)^8 +
 5040*a*cos(d*x + c)^7 + 3360*a*cos(d*x + c)^6 - 3780*a*cos(d*x + c)^5 - 30
24*a*cos(d*x + c)^4 + 1680*a*cos(d*x + c)^3 + 1440*a*cos(d*x + c)^2 - 315*a
*cos(d*x + c) - 280*a)/(d*cos(d*x + c)^9)
```

Sympy [A] time = 53.3889, size = 184, normalized size = 1.22

$$\begin{cases} \frac{a\log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{a\tan^8(c+dx)\sec(c+dx)}{9d} + \frac{a\tan^8(c+dx)}{8d} - \frac{8a\tan^6(c+dx)\sec(c+dx)}{63d} - \frac{a\tan^6(c+dx)}{6d} + \frac{16a\tan^4(c+dx)\sec(c+dx)}{105d} + \frac{a\tan^4}{4} \\ x\left(a\sec(c)+a\right)\tan^9(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**9,x)
```

```
[Out] Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**8*sec(c + d*x
)/(9*d) + a*tan(c + d*x)**8/(8*d) - 8*a*tan(c + d*x)**6*sec(c + d*x)/(63*d)
 - a*tan(c + d*x)**6/(6*d) + 16*a*tan(c + d*x)**4*sec(c + d*x)/(105*d) + a*
tan(c + d*x)**4/(4*d) - 64*a*tan(c + d*x)**2*sec(c + d*x)/(315*d) - a*tan(c
 + d*x)**2/(2*d) + 128*a*sec(c + d*x)/(315*d), Ne(d, 0)), (x*(a*sec(c) + a)
*tan(c)**9, True))
```

Giac [B] time = 17.8114, size = 396, normalized size = 2.62

$$\frac{2520\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 2520\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{9177\,a+\frac{87633\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{375732\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{953988\,a(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^9,x, algorithm="giac")
```

```
[Out] 1/2520*(2520*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2520*
a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (9177*a + 87633*a*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 375732*a*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 + 953988*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 1594
782*a*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1336734*a*(cos(d*x + c) -
 1)^5/(cos(d*x + c) + 1)^5 + 781956*a*(cos(d*x + c) - 1)^6/(cos(d*x + c) +
1)^6 + 302004*a*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 69201*a*(cos(d*
```

```
x + c) - 1)^8/(cos(d*x + c) + 1)^8 + 7129*a*(cos(d*x + c) - 1)^9/(cos(d*x +
 c) + 1)^9)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^9)/d
```

3.2 $\int (a + a\sec(c + dx))\tan^7(c + dx)\,dx$

Optimal. Leaf size=118

$$\frac{a\sec^7(c+dx)}{7d}+\frac{a\sec^6(c+dx)}{6d}-\frac{3a\sec^5(c+dx)}{5d}-\frac{3a\sec^4(c+dx)}{4d}+\frac{a\sec^3(c+dx)}{d}+\frac{3a\sec^2(c+dx)}{2d}-\frac{a\sec(c+dx)}{d}$$

```
[Out] (a*Log[Cos[c + d*x]])/d - (a*Sec[c + d*x])/d + (3*a*Sec[c + d*x]^2)/(2*d) +
 (a*Sec[c + d*x]^3)/d - (3*a*Sec[c + d*x]^4)/(4*d) - (3*a*Sec[c + d*x]^5)/(
5*d) + (a*Sec[c + d*x]^6)/(6*d) + (a*Sec[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0619443, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 88}

$$\frac{a\sec^7(c+dx)}{7d}+\frac{a\sec^6(c+dx)}{6d}-\frac{3a\sec^5(c+dx)}{5d}-\frac{3a\sec^4(c+dx)}{4d}+\frac{a\sec^3(c+dx)}{d}+\frac{3a\sec^2(c+dx)}{2d}-\frac{a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^7,x]
```

```
[Out] (a*Log[Cos[c + d*x]])/d - (a*Sec[c + d*x])/d + (3*a*Sec[c + d*x]^2)/(2*d) +
 (a*Sec[c + d*x]^3)/d - (3*a*Sec[c + d*x]^4)/(4*d) - (3*a*Sec[c + d*x]^5)/(
5*d) + (a*Sec[c + d*x]^6)/(6*d) + (a*Sec[c + d*x]^7)/(7*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int (a+a\sec(c+dx))\tan^7(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^3(a+ax)^4}{x^8}\,dx,x,\cos(c+dx)\right)}{a^6d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^7}{x^8}+\frac{a^7}{x^7}-\frac{3a^7}{x^6}-\frac{3a^7}{x^5}+\frac{3a^7}{x^4}+\frac{3a^7}{x^3}-\frac{a^7}{x^2}-\frac{a^7}{x}\right)dx,x,\cos(c+dx)\right)}{a^6d}$$
$$= \frac{a\log(\cos(c+dx))}{d}-\frac{a\sec(c+dx)}{d}+\frac{3a\sec^2(c+dx)}{2d}+\frac{a\sec^3(c+dx)}{d}-\frac{3a\sec$$

Mathematica [A] time = 0.442353, size = 106, normalized size = 0.9

$$\frac{a\sec^7(c+dx)}{7d}-\frac{3a\sec^5(c+dx)}{5d}+\frac{a\sec^3(c+dx)}{d}-\frac{a\sec(c+dx)}{d}+\frac{a\left(2\tan^6(c+dx)-3\tan^4(c+dx)+6\tan^2(c+dx\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^7,x]
```

```
[Out] -((a*Sec[c + d*x])/d) + (a*Sec[c + d*x]^3)/d - (3*a*Sec[c + d*x]^5)/(5*d) +
 (a*Sec[c + d*x]^7)/(7*d) + (a*(12*Log[Cos[c + d*x]] + 6*Tan[c + d*x]^2 - 3
*Tan[c + d*x]^4 + 2*Tan[c + d*x]^6))/(12*d)
```

Maple [A] time = 0.045, size = 216, normalized size = 1.8

$$\frac{a(\tan(dx+c))^6}{6d}-\frac{a(\tan(dx+c))^4}{4d}+\frac{a(\tan(dx+c))^2}{2d}+\frac{a\ln(\cos(dx+c))}{d}+\frac{a(\sin(dx+c))^8}{7d(\cos(dx+c))^7}-\frac{a(\sin(dx+c))^8}{35d(\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^7,x)
```

```
[Out] 1/6/d*a*tan(d*x+c)^6-1/4/d*a*tan(d*x+c)^4+1/2/d*a*tan(d*x+c)^2+a*ln(cos(d*x
+c))/d+1/7/d*a*sin(d*x+c)^8/cos(d*x+c)^7-1/35/d*a*sin(d*x+c)^8/cos(d*x+c)^5
+1/35/d*a*sin(d*x+c)^8/cos(d*x+c)^3-1/7/d*a*sin(d*x+c)^8/cos(d*x+c)-16/35/d
*a*cos(d*x+c)-1/7/d*a*cos(d*x+c)*sin(d*x+c)^6-6/35/d*a*cos(d*x+c)*sin(d*x+c
)^4-8/35/d*a*cos(d*x+c)*sin(d*x+c)^2
```

Maxima [A] time = 1.16596, size = 127, normalized size = 1.08

$$\frac{420\,a\log\left(\cos\left(dx+c\right)\right)-\frac{420\,a\cos\left(dx+c\right)^{6}-630\,a\cos\left(dx+c\right)^{5}-420\,a\cos\left(dx+c\right)^{4}+315\,a\cos\left(dx+c\right)^{3}+252\,a\cos\left(dx+c\right)^{2}-70\,a\cos\left(dx+c\right)-60\,a}{\cos\left(dx+c\right)^{7}}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/420*(420*a*log(cos(d*x + c)) - (420*a*cos(d*x + c)^6 - 630*a*cos(d*x + c)
^5 - 420*a*cos(d*x + c)^4 + 315*a*cos(d*x + c)^3 + 252*a*cos(d*x + c)^2 - 7
0*a*cos(d*x + c) - 60*a)/cos(d*x + c)^7)/d
```

Fricas [A] time = 0.999816, size = 284, normalized size = 2.41

$$\frac{420\,a\cos\left(dx+c\right)^{7}\log\left(-\cos\left(dx+c\right)\right)-420\,a\cos\left(dx+c\right)^{6}+630\,a\cos\left(dx+c\right)^{5}+420\,a\cos\left(dx+c\right)^{4}-315\,a\cos\left(dx+\right.}{420\,d\cos\left(dx+c\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/420*(420*a*cos(d*x + c)^7*log(-cos(d*x + c)) - 420*a*cos(d*x + c)^6 + 630
*a*cos(d*x + c)^5 + 420*a*cos(d*x + c)^4 - 315*a*cos(d*x + c)^3 - 252*a*cos
(d*x + c)^2 + 70*a*cos(d*x + c) + 60*a)/(d*cos(d*x + c)^7)
```

Sympy [A] time = 15.0235, size = 148, normalized size = 1.25

$$\begin{cases}-\frac{a\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{a\tan^{6}\left(c+dx\right)\sec\left(c+dx\right)}{7d}+\frac{a\tan^{6}\left(c+dx\right)}{6d}-\frac{6a\tan^{4}\left(c+dx\right)\sec\left(c+dx\right)}{35d}-\frac{a\tan^{4}\left(c+dx\right)}{4d}+\frac{8a\tan^{2}\left(c+dx\right)\sec\left(c+dx\right)}{35d}+\frac{a\tan\right.}{} \\ x\left(a\sec\left(c\right)+a\right)\tan^{7}\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**7,x)
```

```
[Out] Piecewise((-a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**6*sec(c + d*
x)/(7*d) + a*tan(c + d*x)**6/(6*d) - 6*a*tan(c + d*x)**4*sec(c + d*x)/(35*d
```

```
) - a*tan(c + d*x)**4/(4*d) + 8*a*tan(c + d*x)**2*sec(c + d*x)/(35*d) + a*t
an(c + d*x)**2/(2*d) - 16*a*sec(c + d*x)/(35*d), Ne(d, 0)), (x*(a*sec(c) +
a)*tan(c)**7, True))
```

Giac [B] time = 8.04382, size = 333, normalized size = 2.82

$$-\frac{420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{1473\,a+\frac{11151\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{36813\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{69475\,a(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\cdots}{\left(\cdots\right.}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] -1/420*(420*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*a*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (1473*a + 11151*a*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + 36813*a*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2 + 69475*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*a*
(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 28749*a*(cos(d*x + c) - 1)^5/(c
os(d*x + c) + 1)^5 + 8463*a*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 108
9*a*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/((cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) + 1)^7)/d
```

3.3 $\int (a + a\sec(c + dx))\tan^5(c + dx)\,dx$

Optimal. Leaf size=87

$$\frac{a\sec^5(c+dx)}{5d}+\frac{a\sec^4(c+dx)}{4d}-\frac{2a\sec^3(c+dx)}{3d}-\frac{a\sec^2(c+dx)}{d}+\frac{a\sec(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d - (a*Sec[c + d*x]^2)/d - (2
*a*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*x]^4)/(4*d) + (a*Sec[c + d*x]^5)/(5
*d)
```

Rubi [A] time = 0.0491738, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 88}

$$\frac{a\sec^5(c+dx)}{5d}+\frac{a\sec^4(c+dx)}{4d}-\frac{2a\sec^3(c+dx)}{3d}-\frac{a\sec^2(c+dx)}{d}+\frac{a\sec(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^5,x]
```


```
[Out] -((a*Log[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d - (a*Sec[c + d*x]^2)/d - (2
*a*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*x]^4)/(4*d) + (a*Sec[c + d*x]^5)/(5
*d)
```


Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int (a+a\sec(c+dx))\tan^5(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^2(a+ax)^3}{x^6}\,dx, x, \cos(c+dx)\right)}{a^4d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^5}{x^6}+\frac{a^5}{x^5}-\frac{2a^5}{x^4}-\frac{2a^5}{x^3}+\frac{a^5}{x^2}+\frac{a^5}{x}\right)dx, x, \cos(c+dx)\right)}{a^4d}$$
$$= -\frac{a\log(\cos(c+dx))}{d}+\frac{a\sec(c+dx)}{d}-\frac{a\sec^2(c+dx)}{d}-\frac{2a\sec^3(c+dx)}{3d}+\frac{a\sec$$

Mathematica [A] time = 0.307393, size = 82, normalized size = 0.94

$$\frac{a\sec^5(c+dx)}{5d}-\frac{2a\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)}{d}-\frac{a\left(-\tan^4(c+dx)+2\tan^2(c+dx)+4\log(\cos(c+dx))\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] (a*Sec[c + d*x])/d - (2*a*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*x]^5)/(5*d)
- (a*(4*Log[Cos[c + d*x]] + 2*Tan[c + d*x]^2 - Tan[c + d*x]^4))/(4*d)
```

Maple [A] time = 0.042, size = 161, normalized size = 1.9

$$\frac{a(\tan(dx+c))^4}{4d}-\frac{a(\tan(dx+c))^2}{2d}-\frac{a\ln(\cos(dx+c))}{d}+\frac{a(\sin(dx+c))^6}{5d(\cos(dx+c))^5}-\frac{a(\sin(dx+c))^6}{15d(\cos(dx+c))^3}+\frac{a(\sin(dx+c))^6}{5d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^5,x)
```

```
[Out] 1/4/d*a*tan(d*x+c)^4-1/2/d*a*tan(d*x+c)^2-a*ln(cos(d*x+c))/d+1/5/d*a*sin(d*
x+c)^6/cos(d*x+c)^5-1/15/d*a*sin(d*x+c)^6/cos(d*x+c)^3+1/5/d*a*sin(d*x+c)^6
/cos(d*x+c)+8/15/d*a*cos(d*x+c)+1/5/d*a*cos(d*x+c)*sin(d*x+c)^4+4/15/d*a*co
s(d*x+c)*sin(d*x+c)^2
```

Maxima [A] time = 1.07069, size = 97, normalized size = 1.11

$$-\frac{60\,a\log(\cos(dx+c))-\frac{60\,a\cos(dx+c)^4-60\,a\cos(dx+c)^3-40\,a\cos(dx+c)^2+15\,a\cos(dx+c)+12\,a}{\cos(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(60*a*log(cos(d*x + c)) - (60*a*cos(d*x + c)^4 - 60*a*cos(d*x + c)^3
- 40*a*cos(d*x + c)^2 + 15*a*cos(d*x + c) + 12*a)/cos(d*x + c)^5)/d
```

Fricas [A] time = 1.02064, size = 216, normalized size = 2.48

$$-\frac{60\,a\cos\left(dx+c\right)^{5}\log\left(-\cos\left(dx+c\right)\right)-60\,a\cos\left(dx+c\right)^{4}+60\,a\cos\left(dx+c\right)^{3}+40\,a\cos\left(dx+c\right)^{2}-15\,a\cos\left(dx+c\right)-1}{60\,d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/60*(60*a*cos(d*x + c)^5*log(-cos(d*x + c)) - 60*a*cos(d*x + c)^4 + 60*a*
cos(d*x + c)^3 + 40*a*cos(d*x + c)^2 - 15*a*cos(d*x + c) - 12*a)/(d*cos(d*x
 + c)^5)
```

Sympy [A] time = 25.5118, size = 112, normalized size = 1.29

$$\begin{cases} \frac{a\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{a\tan^{4}\left(c+dx\right)\sec\left(c+dx\right)}{5d}+\frac{a\tan^{4}\left(c+dx\right)}{4d}-\frac{4a\tan^{2}\left(c+dx\right)\sec\left(c+dx\right)}{15d}-\frac{a\tan^{2}\left(c+dx\right)}{2d}+\frac{8a\sec\left(c+dx\right)}{15d} & \text{for}\: d\neq 0 \\ x\left(a\sec\left(c\right)+a\right)\tan^{5}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**5,x)
```

```
[Out] Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**4*sec(c + d*x
)/(5*d) + a*tan(c + d*x)**4/(4*d) - 4*a*tan(c + d*x)**2*sec(c + d*x)/(15*d)
 - a*tan(c + d*x)**2/(2*d) + 8*a*sec(c + d*x)/(15*d), Ne(d, 0)), (x*(a*sec(
c) + a)*tan(c)**5, True))
```

Giac [B] time = 3.83775, size = 271, normalized size = 3.11

$$\frac{60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{201\,a+\frac{1125\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2610\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{1970\,a(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{805\,a(\cos(\ldots}{(\cos(dx\ldots}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (201*a + 1125*a*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 2610*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 + 1970*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 805*a*(cos(d*x +
 c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*a*(cos(d*x + c) - 1)^5/(cos(d*x + c)
+ 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^5)/d
```

3.4 $\int (a + a \sec(c + dx)) \tan^3(c + dx) \, dx$

Optimal. Leaf size=57

$$\frac{a \sec^3(c + dx)}{3d} + \frac{a \sec^2(c + dx)}{2d} - \frac{a \sec(c + dx)}{d} + \frac{a \log(\cos(c + dx))}{d}$$

```
[Out] (a*Log[Cos[c + d*x]])/d - (a*Sec[c + d*x])/d + (a*Sec[c + d*x]^2)/(2*d) + (
a*Sec[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0391315, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 75}

$$\frac{a \sec^3(c + dx)}{3d} + \frac{a \sec^2(c + dx)}{2d} - \frac{a \sec(c + dx)}{d} + \frac{a \log(\cos(c + dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] (a*Log[Cos[c + d*x]])/d - (a*Sec[c + d*x])/d + (a*Sec[c + d*x]^2)/(2*d) + (
a*Sec[c + d*x]^3)/(3*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))\tan^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)(a+ax)^2}{x^4}\,dx,x,\cos(c+dx)\right)}{a^2d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^3}{x^4}+\frac{a^3}{x^3}-\frac{a^3}{x^2}-\frac{a^3}{x}\right)dx,x,\cos(c+dx)\right)}{a^2d}$$
$$= \frac{a\log(\cos(c+dx))}{d}-\frac{a\sec(c+dx)}{d}+\frac{a\sec^2(c+dx)}{2d}+\frac{a\sec^3(c+dx)}{3d}$$

Mathematica [A] time = 0.11106, size = 55, normalized size = 0.96

$$\frac{a\sec^3(c+dx)}{3d}-\frac{a\sec(c+dx)}{d}+\frac{a\left(\tan^2(c+dx)+2\log(\cos(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] -((a*Sec[c + d*x])/d) + (a*Sec[c + d*x]^3)/(3*d) + (a*(2*Log[Cos[c + d*x]]
+ Tan[c + d*x]^2))/(2*d)
```

Maple [A] time = 0.039, size = 104, normalized size = 1.8

$$\frac{a(\tan(dx+c))^2}{2d}+\frac{a\ln(\cos(dx+c))}{d}+\frac{a(\sin(dx+c))^4}{3d(\cos(dx+c))^3}-\frac{a(\sin(dx+c))^4}{3d\cos(dx+c)}-\frac{a\cos(dx+c)(\sin(dx+c))^2}{3d}-\frac{2a\cos}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^3,x)
```

```
[Out] 1/2/d*a*tan(d*x+c)^2+a*ln(cos(d*x+c))/d+1/3/d*a*sin(d*x+c)^4/cos(d*x+c)^3-1
/3/d*a*sin(d*x+c)^4/cos(d*x+c)-1/3/d*a*cos(d*x+c)*sin(d*x+c)^2-2/3/d*a*cos(
d*x+c)
```

Maxima [A] time = 1.16796, size = 68, normalized size = 1.19

$$\frac{6a\log(\cos(dx+c))-\frac{6a\cos(dx+c)^2-3a\cos(dx+c)-2a}{\cos(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(6*a*log(cos(d*x + c)) - (6*a*cos(d*x + c)^2 - 3*a*cos(d*x + c) - 2*a)/
cos(d*x + c)^3)/d
```

Fricas [A] time = 1.02806, size = 149, normalized size = 2.61

$$\frac{6\,a\cos\left(dx+c\right)^{3}\log\left(-\cos\left(dx+c\right)\right)-6\,a\cos\left(dx+c\right)^{2}+3\,a\cos\left(dx+c\right)+2\,a}{6\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*cos(d*x + c)^3*log(-cos(d*x + c)) - 6*a*cos(d*x + c)^2 + 3*a*cos(d
*x + c) + 2*a)/(d*cos(d*x + c)^3)
```

Sympy [A] time = 1.25095, size = 76, normalized size = 1.33

$$\begin{cases} -\frac{a\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{a\tan^{2}\left(c+dx\right)\sec\left(c+dx\right)}{3d}+\frac{a\tan^{2}\left(c+dx\right)}{2d}-\frac{2a\sec\left(c+dx\right)}{3d} & \text{for } d\neq 0 \\ x\left(a\sec\left(c\right)+a\right)\tan^{3}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**3,x)
```

```
[Out] Piecewise((-a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**2*sec(c + d*
x)/(3*d) + a*tan(c + d*x)**2/(2*d) - 2*a*sec(c + d*x)/(3*d), Ne(d, 0)), (x*
(a*sec(c) + a)*tan(c)**3, True))
```

Giac [B] time = 2.01544, size = 209, normalized size = 3.67

$$-\frac{6\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-6\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{19\,a+\frac{69\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{45\,a(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{11\,a(\cos(dx+c)-1)^{3}}{(\cos(dx+c)+1)^{3}}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/6*(6*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 6*a*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (19*a + 69*a*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 45*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
 11*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/((cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 1)^3)/d
```

3.5 $\int (a + a\sec(c + dx))\tan(c + dx)\,dx$

Optimal. Leaf size=25

$$\frac{a\sec(c + dx)}{d} - \frac{a\log(\cos(c + dx))}{d}$$

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d
```

Rubi [A] time = 0.0192775, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3879, 43}

$$\frac{a\sec(c + dx)}{d} - \frac{a\log(\cos(c + dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))\tan(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{a+ax}{x^2}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a}{x^2}+\frac{a}{x}\right)dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a\log(\cos(c+dx))}{d}+\frac{a\sec(c+dx)}{d}$$

Mathematica [A] time = 0.0178748, size = 25, normalized size = 1.

$$\frac{a\sec(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d
```

Maple [A] time = 0.014, size = 25, normalized size = 1.

$$\frac{a\ln(\sec(dx+c))}{d}+\frac{a\sec(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c),x)
```

```
[Out] 1/d*a*ln(sec(d*x+c))+a*sec(d*x+c)/d
```

Maxima [A] time = 1.14846, size = 35, normalized size = 1.4

$$-\frac{a\log(\cos(dx+c))-\frac{a}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*log(cos(d*x + c)) - a/cos(d*x + c))/d
```

Fricas [A] time = 0.697311, size = 80, normalized size = 3.2

$$-\frac{a\cos(dx+c)\log(-\cos(dx+c))-a}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -(a*cos(d*x + c)*log(-cos(d*x + c)) - a)/(d*cos(d*x + c))
```

Sympy [A] time = 0.423997, size = 37, normalized size = 1.48

$$\begin{cases} \frac{a\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{a\sec(c+dx)}{d} & \text{for } d\neq 0 \\ x\left(a\sec(c)+a\right)\tan(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c),x)
```

```
[Out] Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + a*sec(c + d*x)/d, Ne(d, 0)),
(x*(a*sec(c) + a)*tan(c), True))
```

Giac [B] time = 1.41025, size = 143, normalized size = 5.72

$$\frac{a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{3a+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c),x, algorithm="giac")
```

```
[Out] (a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - a*log(abs(-(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (3*a + a*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

3.6 $\int \cot(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=16

$$\frac{a\log(1-\cos(c+dx))}{d}$$

```
[Out] (a*Log[1 - Cos[c + d*x]])/d
```

Rubi [A] time = 0.0203078, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3879, 31}

$$\frac{a\log(1-\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Log[1 - Cos[c + d*x]])/d
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+a\sec(c+dx))\,dx &= -\frac{a^2\,\text{Subst}\left(\int \frac{1}{a-ax}\,dx, x, \cos(c+dx)\right)}{d}\\ &= \frac{a\log(1-\cos(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.024625, size = 29, normalized size = 1.81

$$\frac{2a\left(\log\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*a*(Log[Cos[(c + d*x)/2]] + Log[Tan[(c + d*x)/2]]))/d
```

Maple [A] time = 0.042, size = 29, normalized size = 1.8

$$\frac{a\ln\left(-1+\sec\left(dx+c\right)\right)}{d}-\frac{a\ln\left(\sec\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/d*a*ln(-1+sec(d*x+c))-1/d*a*ln(sec(d*x+c))
```

Maxima [A] time = 1.14439, size = 19, normalized size = 1.19

$$\frac{a\log\left(\cos\left(dx+c\right)-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] a*log(cos(d*x + c) - 1)/d
```

Fricas [A] time = 0.847773, size = 46, normalized size = 2.88

$$\frac{a\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] a*log(-1/2*cos(d*x + c) + 1/2)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\cot\left(c+dx\right)\sec\left(c+dx\right)dx+\int\cot\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cot(c + d*x)*sec(c + d*x), x) + Integral(cot(c + d*x), x))
```

Giac [B] time = 1.41107, size = 78, normalized size = 4.88

$$\frac{a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a*log(abs(-(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

3.7 $\int \cot^3(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=57

$$-\frac{a}{2d(1-\cos(c+dx))}-\frac{3a\log(1-\cos(c+dx))}{4d}-\frac{a\log(\cos(c+dx)+1)}{4d}$$

```
[Out] -a/(2*d*(1 - Cos[c + d*x])) - (3*a*Log[1 - Cos[c + d*x]])/(4*d) - (a*Log[1
+ Cos[c + d*x]])/(4*d)
```

Rubi [A] time = 0.0439983, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 88}

$$-\frac{a}{2d(1-\cos(c+dx))}-\frac{3a\log(1-\cos(c+dx))}{4d}-\frac{a\log(\cos(c+dx)+1)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] -a/(2*d*(1 - Cos[c + d*x])) - (3*a*Log[1 - Cos[c + d*x]])/(4*d) - (a*Log[1
+ Cos[c + d*x]])/(4*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cot^3(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a^4\,\text{Subst}\left(\int \frac{x^2}{(a-ax)^2(a+ax)}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^4\,\text{Subst}\left(\int \left(\frac{1}{2a^3(-1+x)^2}+\frac{3}{4a^3(-1+x)}+\frac{1}{4a^3(1+x)}\right)dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a}{2d(1-\cos(c+dx))}-\frac{3a\log(1-\cos(c+dx))}{4d}-\frac{a\log(1+\cos(c+dx))}{4d}$$

Mathematica [A] time = 0.803195, size = 114, normalized size = 2.

$$-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a\left(\cot^2(c+dx)+2\log\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] -(a*Csc[(c + d*x)/2]^2)/(8*d) + (a*Log[Cos[(c + d*x)/2]])/(2*d) - (a*Log[Si
n[(c + d*x)/2]])/(2*d) - (a*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[T
an[c + d*x]]))/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

Maple [A] time = 0.075, size = 60, normalized size = 1.1

$$-\frac{a\ln(1+\sec(dx+c))}{4d}-\frac{a}{2d(-1+\sec(dx+c))}-\frac{3a\ln(-1+\sec(dx+c))}{4d}+\frac{a\ln(\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c)),x)
```

```
[Out] -1/4/d*a*ln(1+sec(d*x+c))-1/2/d*a/(-1+sec(d*x+c))-3/4/d*a*ln(-1+sec(d*x+c))
+1/d*a*ln(sec(d*x+c))
```

Maxima [A] time = 1.18442, size = 57, normalized size = 1.

$$-\frac{a\log(\cos(dx+c)+1)+3a\log(\cos(dx+c)-1)-\frac{2a}{\cos(dx+c)-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(a*log(cos(d*x + c) + 1) + 3*a*log(cos(d*x + c) - 1) - 2*a/(cos(d*x +
c) - 1))/d
```

Fricas [A] time = 0.807956, size = 186, normalized size = 3.26

$$-\frac{(a\cos(dx+c)-a)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+3(a\cos(dx+c)-a)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-2a}{4(d\cos(dx+c)-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*((a*cos(d*x + c) - a)*log(1/2*cos(d*x + c) + 1/2) + 3*(a*cos(d*x + c)
- a)*log(-1/2*cos(d*x + c) + 1/2) - 2*a)/(d*cos(d*x + c) - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\cot^3(c+dx)\sec(c+dx)\,dx+\int\cot^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cot(c + d*x)**3*sec(c + d*x), x) + Integral(cot(c + d*x)**3, x)
)
```

Giac [B] time = 1.48934, size = 139, normalized size = 2.44

$$-\frac{3a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-4a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a+\frac{3a(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(3*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 4*a*log(abs(-
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a + 3*a*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/d
```

3.8 $\int \cot^5(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=95

$$\frac{3a}{4d(1-\cos(c+dx))}+\frac{a}{8d(\cos(c+dx)+1)}-\frac{a}{8d(1-\cos(c+dx))^2}+\frac{11a\log(1-\cos(c+dx))}{16d}+\frac{5a\log(\cos(c+dx)+1)}{16d}$$

```
[Out] -a/(8*d*(1 - Cos[c + d*x])^2) + (3*a)/(4*d*(1 - Cos[c + d*x])) + a/(8*d*(1
+ Cos[c + d*x])) + (11*a*Log[1 - Cos[c + d*x]])/(16*d) + (5*a*Log[1 + Cos[c
 + d*x]])/(16*d)
```

Rubi [A] time = 0.0643238, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 88}

$$\frac{3a}{4d(1-\cos(c+dx))}+\frac{a}{8d(\cos(c+dx)+1)}-\frac{a}{8d(1-\cos(c+dx))^2}+\frac{11a\log(1-\cos(c+dx))}{16d}+\frac{5a\log(\cos(c+dx)+1)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sec[c + d*x]),x]
```

```
[Out] -a/(8*d*(1 - Cos[c + d*x])^2) + (3*a)/(4*d*(1 - Cos[c + d*x])) + a/(8*d*(1
+ Cos[c + d*x])) + (11*a*Log[1 - Cos[c + d*x]])/(16*d) + (5*a*Log[1 + Cos[c
 + d*x]])/(16*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cot^5(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a^6\,\text{Subst}\left(\int \frac{x^4}{(a-ax)^3(a+ax)^2}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^6\,\text{Subst}\left(\int \left(-\frac{1}{4a^5(-1+x)^3} - \frac{3}{4a^5(-1+x)^2} - \frac{11}{16a^5(-1+x)} + \frac{1}{8a^5(1+x)^2} - \frac{5}{16a^5(1+x)}\right) dx, x,}{d}$$
$$= -\frac{a}{8d(1-\cos(c+dx))^2} + \frac{3a}{4d(1-\cos(c+dx))} + \frac{a}{8d(1+\cos(c+dx))} + \frac{11a\log(1}{}$$

Mathematica [A] time = 0.538715, size = 127, normalized size = 1.34

$$\frac{a\left(-16\cot^4(c+dx) + 32\cot^2(c+dx) - \csc^4\left(\frac{1}{2}(c+dx)\right) + 10\csc^2\left(\frac{1}{2}(c+dx)\right) + \sec^4\left(\frac{1}{2}(c+dx)\right) - 10\sec^2\left(\frac{1}{2}(c+dx)\right)\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*(32*Cot[c + d*x]^2 - 16*Cot[c + d*x]^4 + 10*Csc[(c + d*x)/2]^2 - Csc[(c
+ d*x)/2]^4 - 24*Log[Cos[(c + d*x)/2]] + 64*Log[Cos[c + d*x]] + 24*Log[Sin[
(c + d*x)/2]] + 64*Log[Tan[c + d*x]] - 10*Sec[(c + d*x)/2]^2 + Sec[(c + d*x
)/2]^4))/(64*d)
```

Maple [A] time = 0.073, size = 93, normalized size = 1.

$$-\frac{a}{8\,d\,(1+\sec(dx+c))} + \frac{5\,a\ln(1+\sec(dx+c))}{16\,d} - \frac{a}{8\,d\,(-1+\sec(dx+c))^2} + \frac{a}{2\,d\,(-1+\sec(dx+c))} + \frac{11\,a\ln(-1+\mathrm{se}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+a*sec(d*x+c)),x)
```

```
[Out] -1/8/d*a/(1+sec(d*x+c))+5/16/d*a*ln(1+sec(d*x+c))-1/8/d*a/(-1+sec(d*x+c))^2
+1/2/d*a/(-1+sec(d*x+c))+11/16/d*a*ln(-1+sec(d*x+c))-1/d*a*ln(sec(d*x+c))
```

Maxima [A] time = 1.19467, size = 116, normalized size = 1.22

$$\frac{5\,a\log\left(\cos\left(dx+c\right)+1\right)+11\,a\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(5\,a\cos\left(dx+c\right)^{2}+3\,a\cos\left(dx+c\right)-6\,a\right)}{\cos\left(dx+c\right)^{3}-\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(5*a*log(cos(d*x + c) + 1) + 11*a*log(cos(d*x + c) - 1) - 2*(5*a*cos(d
*x + c)^2 + 3*a*cos(d*x + c) - 6*a)/(cos(d*x + c)^3 - cos(d*x + c)^2 - cos(
d*x + c) + 1))/d
```

Fricas [A] time = 0.906858, size = 402, normalized size = 4.23

$$-\frac{10\,a\cos\left(dx+c\right)^{2}+6\,a\cos\left(dx+c\right)-5\left(a\cos\left(dx+c\right)^{3}-a\cos\left(dx+c\right)^{2}-a\cos\left(dx+c\right)+a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-}{16\left(d\cos\left(dx+c\right)^{3}-d\cos\left(dx+c\right)^{2}-d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(10*a*cos(d*x + c)^2 + 6*a*cos(d*x + c) - 5*(a*cos(d*x + c)^3 - a*cos
(d*x + c)^2 - a*cos(d*x + c) + a)*log(1/2*cos(d*x + c) + 1/2) - 11*(a*cos(d
*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*log(-1/2*cos(d*x + c) +
1/2) - 12*a)/(d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.49646, size = 201, normalized size = 2.12

$$\frac{22\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-32\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a+\frac{10\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{33\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{(\cos(dx+c)-1)^2}-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(22*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 32*a*log(abs
(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a + 10*a*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 33*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos
(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2 - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1))/d
```

3.9 $\int \cot^7(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=133

$$-\frac{15a}{16d(1-\cos(c+dx))}-\frac{a}{4d(\cos(c+dx)+1)}+\frac{9a}{32d(1-\cos(c+dx))^2}+\frac{a}{32d(\cos(c+dx)+1)^2}-\frac{a}{24d(1-\cos(c+dx))^3}-$$

```
[Out] -a/(24*d*(1 - Cos[c + d*x])^3) + (9*a)/(32*d*(1 - Cos[c + d*x])^2) - (15*a)
/(16*d*(1 - Cos[c + d*x])) + a/(32*d*(1 + Cos[c + d*x])^2) - a/(4*d*(1 + Co
s[c + d*x])) - (21*a*Log[1 - Cos[c + d*x]])/(32*d) - (11*a*Log[1 + Cos[c +
d*x]])/(32*d)
```

Rubi [A] time = 0.0823685, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 88}

$$-\frac{15a}{16d(1-\cos(c+dx))}-\frac{a}{4d(\cos(c+dx)+1)}+\frac{9a}{32d(1-\cos(c+dx))^2}+\frac{a}{32d(\cos(c+dx)+1)^2}-\frac{a}{24d(1-\cos(c+dx))^3}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*(a + a*Sec[c + d*x]),x]
```

```
[Out] -a/(24*d*(1 - Cos[c + d*x])^3) + (9*a)/(32*d*(1 - Cos[c + d*x])^2) - (15*a)
/(16*d*(1 - Cos[c + d*x])) + a/(32*d*(1 + Cos[c + d*x])^2) - a/(4*d*(1 + Co
s[c + d*x])) - (21*a*Log[1 - Cos[c + d*x]])/(32*d) - (11*a*Log[1 + Cos[c +
d*x]])/(32*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cot^7(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a^8\,\text{Subst}\left(\int \frac{x^6}{(a-ax)^4(a+ax)^3}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a^8\,\text{Subst}\left(\int \left(\frac{1}{8a^7(-1+x)^4} + \frac{9}{16a^7(-1+x)^3} + \frac{15}{16a^7(-1+x)^2} + \frac{21}{32a^7(-1+x)} + \frac{1}{16a^7(1+x)^3} - \frac{}{4a}\right.\right.}{d}$$

$$= -\frac{a}{24d(1-\cos(c+dx))^3} + \frac{9a}{32d(1-\cos(c+dx))^2} - \frac{15a}{16d(1-\cos(c+dx))} + \frac{}{32d(}$$

Mathematica [A] time = 0.378039, size = 165, normalized size = 1.24

$$-\frac{a\left(64\cot^6(c+dx) - 96\cot^4(c+dx) + 192\cot^2(c+dx) + \csc^6\left(\frac{1}{2}(c+dx)\right) - 12\csc^4\left(\frac{1}{2}(c+dx)\right) + 66\csc^2\left(\frac{1}{2}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + a*Sec[c + d*x]),x]
```

```
[Out] -(a*(192*Cot[c + d*x]^2 - 96*Cot[c + d*x]^4 + 64*Cot[c + d*x]^6 + 66*Csc[(c
 + d*x)/2]^2 - 12*Csc[(c + d*x)/2]^4 + Csc[(c + d*x)/2]^6 - 120*Log[Cos[(c
+ d*x)/2]] + 384*Log[Cos[c + d*x]] + 120*Log[Sin[(c + d*x)/2]] + 384*Log[Ta
n[c + d*x]] - 66*Sec[(c + d*x)/2]^2 + 12*Sec[(c + d*x)/2]^4 - Sec[(c + d*x)
/2]^6))/(384*d)
```

Maple [A] time = 0.087, size = 124, normalized size = 0.9

$$\frac{a}{32\,d\,(1+\sec(dx+c))^2} + \frac{3\,a}{16\,d\,(1+\sec(dx+c))} - \frac{11\,a\ln(1+\sec(dx+c))}{32\,d} - \frac{a}{24\,d\,(-1+\sec(dx+c))^3} + \frac{5\,a}{32\,d\,(-1+\text{se}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/32/d*a/(1+sec(d*x+c))^2+3/16/d*a/(1+sec(d*x+c))-11/32/d*a*ln(1+sec(d*x+c)
)-1/24/d*a/(-1+sec(d*x+c))^3+5/32/d*a/(-1+sec(d*x+c))^2-1/2/d*a/(-1+sec(d*x
+c))-21/32/d*a*ln(-1+sec(d*x+c))+1/d*a*ln(sec(d*x+c))
```

Maxima [A] time = 1.07772, size = 170, normalized size = 1.28

$$-\frac{33\,a\log\left(\cos\left(dx+c\right)+1\right)+63\,a\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(33\,a\cos\left(dx+c\right)^4+39\,a\cos\left(dx+c\right)^3-79\,a\cos\left(dx+c\right)^2-29\,a\cos\left(dx+c\right)+44\,a\right)}{\cos\left(dx+c\right)^5-\cos\left(dx+c\right)^4-2\,\cos\left(dx+c\right)^3+2\,\cos\left(dx+c\right)^2+\cos\left(dx+c\right)-1}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(33*a*log(cos(d*x + c) + 1) + 63*a*log(cos(d*x + c) - 1) - 2*(33*a*co
s(d*x + c)^4 + 39*a*cos(d*x + c)^3 - 79*a*cos(d*x + c)^2 - 29*a*cos(d*x + c
) + 44*a)/(cos(d*x + c)^5 - cos(d*x + c)^4 - 2*cos(d*x + c)^3 + 2*cos(d*x +
 c)^2 + cos(d*x + c) - 1))/d
```

Fricas [B] time = 1.00248, size = 635, normalized size = 4.77

$$\frac{66\,a\cos\left(dx+c\right)^4+78\,a\cos\left(dx+c\right)^3-158\,a\cos\left(dx+c\right)^2-58\,a\cos\left(dx+c\right)-33\left(a\cos\left(dx+c\right)^5-a\cos\left(dx+c\right)^4-2\,a\right.}{96\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(66*a*cos(d*x + c)^4 + 78*a*cos(d*x + c)^3 - 158*a*cos(d*x + c)^2 - 58
*a*cos(d*x + c) - 33*(a*cos(d*x + c)^5 - a*cos(d*x + c)^4 - 2*a*cos(d*x + c
)^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a)*log(1/2*cos(d*x + c) + 1/2)
- 63*(a*cos(d*x + c)^5 - a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 + 2*a*cos(d*
x + c)^2 + a*cos(d*x + c) - a)*log(-1/2*cos(d*x + c) + 1/2) + 88*a)/(d*cos(
d*x + c)^5 - d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 + d
*cos(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.62899, size = 266, normalized size = 2.

$$-\frac{252\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-384\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(2\,a+\frac{21\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{132\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{462\,a(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}\right)(\cos(dx+c)+1)^3}{(\cos(dx+c)-1)^3}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/384*(252*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 384*a*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (2*a + 21*a*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + 132*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^
2 + 462*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)^3/(
cos(d*x + c) - 1)^3 - 42*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/d
```

3.10 $\int (a + a\sec(c + dx)) \tan^8(c + dx)\, dx$

Optimal. Leaf size=129

$$\frac{35a\tanh^{-1}(\sin(c+dx))}{128d} + \frac{\tan^7(c+dx)(7a\sec(c+dx)+8a)}{56d} - \frac{\tan^5(c+dx)(35a\sec(c+dx)+48a)}{240d} + \frac{\tan^3(c+dx)(35a}{1}$$

```
[Out] a*x + (35*a*ArcTanh[Sin[c + d*x]])/(128*d) - ((128*a + 35*a*Sec[c + d*x])*T
an[c + d*x])/(128*d) + ((64*a + 35*a*Sec[c + d*x])*Tan[c + d*x]^3)/(192*d)
- ((48*a + 35*a*Sec[c + d*x])*Tan[c + d*x]^5)/(240*d) + ((8*a + 7*a*Sec[c +
 d*x])*Tan[c + d*x]^7)/(56*d)
```

Rubi [A] time = 0.128695, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3881, 3770}

$$\frac{35a\tanh^{-1}(\sin(c+dx))}{128d} + \frac{\tan^7(c+dx)(7a\sec(c+dx)+8a)}{56d} - \frac{\tan^5(c+dx)(35a\sec(c+dx)+48a)}{240d} + \frac{\tan^3(c+dx)(35a}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^8,x]
```

```
[Out] a*x + (35*a*ArcTanh[Sin[c + d*x]])/(128*d) - ((128*a + 35*a*Sec[c + d*x])*T
an[c + d*x])/(128*d) + ((64*a + 35*a*Sec[c + d*x])*Tan[c + d*x]^3)/(192*d)
- ((48*a + 35*a*Sec[c + d*x])*Tan[c + d*x]^5)/(240*d) + ((8*a + 7*a*Sec[c +
 d*x])*Tan[c + d*x]^7)/(56*d)
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))\tan^8(c+dx)\,dx = \frac{(8a+7a\sec(c+dx))\tan^7(c+dx)}{56d} - \frac{1}{8}\int (8a+7a\sec(c+dx))\tan^6(c+dx)\,dx$$
$$= -\frac{(48a+35a\sec(c+dx))\tan^5(c+dx)}{240d} + \frac{(8a+7a\sec(c+dx))\tan^7(c+dx)}{56d} +$$
$$= \frac{(64a+35a\sec(c+dx))\tan^3(c+dx)}{192d} - \frac{(48a+35a\sec(c+dx))\tan^5(c+dx)}{240d} +$$
$$= -\frac{(128a+35a\sec(c+dx))\tan(c+dx)}{128d} + \frac{(64a+35a\sec(c+dx))\tan^3(c+dx)}{192d}$$
$$= ax - \frac{(128a+35a\sec(c+dx))\tan(c+dx)}{128d} + \frac{(64a+35a\sec(c+dx))\tan^3(c+}{192d}$$
$$= ax + \frac{35a\tanh^{-1}(\sin(c+dx))}{128d} - \frac{(128a+35a\sec(c+dx))\tan(c+dx)}{128d} + \frac{(64a+}{}$$

Mathematica [A] time = 1.74826, size = 115, normalized size = 0.89

$$\frac{a\left(13440\tan^{-1}(\tan(c+dx)) + 3675\tanh^{-1}(\sin(c+dx)) - \frac{1}{32}(223232\cos(c+dx) + 75915\cos(2(c+dx)) + 147968\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^8,x]
```

```
[Out] (a*(13440*ArcTan[Tan[c + d*x]] + 3675*ArcTanh[Sin[c + d*x]] - ((18970 + 223
232*Cos[c + d*x] + 75915*Cos[2*(c + d*x)] + 147968*Cos[3*(c + d*x)] + 12950
*Cos[4*(c + d*x)] + 47616*Cos[5*(c + d*x)] + 9765*Cos[6*(c + d*x)] + 11264*
Cos[7*(c + d*x)])*Sec[c + d*x]^7*Tan[c + d*x])/32))/(13440*d)
```

Maple [A] time = 0.046, size = 227, normalized size = 1.8

$$\frac{a(\tan(dx+c))^7}{7d} - \frac{a(\tan(dx+c))^5}{5d} + \frac{a(\tan(dx+c))^3}{3d} - \frac{a\tan(dx+c)}{d} + ax + \frac{ac}{d} + \frac{a(\sin(dx+c))^9}{8d(\cos(dx+c))^8} - \frac{a(\sin(dx}{48d(\cos(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^8,x)
```

```
[Out] 1/7/d*a*tan(d*x+c)^7-1/5/d*a*tan(d*x+c)^5+1/3/d*a*tan(d*x+c)^3-1/d*a*tan(d*
x+c)+a*x+1/d*a*c+1/8/d*a*sin(d*x+c)^9/cos(d*x+c)^8-1/48/d*a*sin(d*x+c)^9/co
```

```
s(d*x+c)^6+1/64/d*a*sin(d*x+c)^9/cos(d*x+c)^4-5/128/d*a*sin(d*x+c)^9/cos(d*
x+c)^2-5/128/d*sin(d*x+c)^7*a-7/128/d*a*sin(d*x+c)^5-35/384/d*a*sin(d*x+c)^
3-35/128/d*a*sin(d*x+c)+35/128/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.7106, size = 221, normalized size = 1.71

$$\frac{256\left(15\,\tan\left(dx+c\right)^{7}-21\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+c\right)^{3}+105\,dx+105\,c-105\,\tan\left(dx+c\right)\right)a+35\,a\left(\frac{2\left(279\,\sin(dx+c)^{7}-\right.}{\sin(dx+c)^{8}-4}\right.}{26880\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] 1/26880*(256*(15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 1
05*d*x + 105*c - 105*tan(d*x + c))*a + 35*a*(2*(279*sin(d*x + c)^7 - 511*si
n(d*x + c)^5 + 385*sin(d*x + c)^3 - 105*sin(d*x + c))/(sin(d*x + c)^8 - 4*s
in(d*x + c)^6 + 6*sin(d*x + c)^4 - 4*sin(d*x + c)^2 + 1) + 105*log(sin(d*x
+ c) + 1) - 105*log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.03179, size = 466, normalized size = 3.61

$$\frac{26880\,adx\cos\left(dx+c\right)^{8}+3675\,a\cos\left(dx+c\right)^{8}\log\left(\sin\left(dx+c\right)+1\right)-3675\,a\cos\left(dx+c\right)^{8}\log\left(-\sin\left(dx+c\right)+1\right)-2\left(22\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/26880*(26880*a*d*x*cos(d*x + c)^8 + 3675*a*cos(d*x + c)^8*log(sin(d*x + c
) + 1) - 3675*a*cos(d*x + c)^8*log(-sin(d*x + c) + 1) - 2*(22528*a*cos(d*x
+ c)^7 + 9765*a*cos(d*x + c)^6 - 15616*a*cos(d*x + c)^5 - 11410*a*cos(d*x +
 c)^4 + 8448*a*cos(d*x + c)^3 + 7000*a*cos(d*x + c)^2 - 1920*a*cos(d*x + c)
 - 1680*a)*sin(d*x + c))/(d*cos(d*x + c)^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\tan^{8}\left(c+dx\right)\sec\left(c+dx\right)dx+\int\tan^{8}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**8,x)
```

```
[Out] a*(Integral(tan(c + d*x)**8*sec(c + d*x), x) + Integral(tan(c + d*x)**8, x)
)
```

Giac [A] time = 12.0087, size = 235, normalized size = 1.82

$$13440\,(dx+c)a + 3675\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 3675\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(9765\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15} - 83825\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^8,x, algorithm="giac")
```

```
[Out] 1/13440*(13440*(d*x + c)*a + 3675*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 36
75*a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(9765*a*tan(1/2*d*x + 1/2*c)^15
 - 83825*a*tan(1/2*d*x + 1/2*c)^13 + 321013*a*tan(1/2*d*x + 1/2*c)^11 - 724
649*a*tan(1/2*d*x + 1/2*c)^9 + 1078359*a*tan(1/2*d*x + 1/2*c)^7 - 508683*a*
tan(1/2*d*x + 1/2*c)^5 + 140175*a*tan(1/2*d*x + 1/2*c)^3 - 17115*a*tan(1/2*
d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^8)/d
```

3.11 $\int (a + a\sec(c + dx))\tan^6(c + dx)\,dx$

Optimal. Leaf size=102

$$-\frac{5a\tanh^{-1}(\sin(c+dx))}{16d}+\frac{\tan^5(c+dx)(5a\sec(c+dx)+6a)}{30d}-\frac{\tan^3(c+dx)(5a\sec(c+dx)+8a)}{24d}+\frac{\tan(c+dx)(5a\sec}{16d}$$

```
[Out] -(a*x) - (5*a*ArcTanh[Sin[c + d*x]])/(16*d) + ((16*a + 5*a*Sec[c + d*x])*Ta
n[c + d*x])/(16*d) - ((8*a + 5*a*Sec[c + d*x])*Tan[c + d*x]^3)/(24*d) + ((6
*a + 5*a*Sec[c + d*x])*Tan[c + d*x]^5)/(30*d)
```

Rubi [A] time = 0.0944652, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3881, 3770}

$$-\frac{5a\tanh^{-1}(\sin(c+dx))}{16d}+\frac{\tan^5(c+dx)(5a\sec(c+dx)+6a)}{30d}-\frac{\tan^3(c+dx)(5a\sec(c+dx)+8a)}{24d}+\frac{\tan(c+dx)(5a\sec}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^6,x]
```

```
[Out] -(a*x) - (5*a*ArcTanh[Sin[c + d*x]])/(16*d) + ((16*a + 5*a*Sec[c + d*x])*Ta
n[c + d*x])/(16*d) - ((8*a + 5*a*Sec[c + d*x])*Tan[c + d*x]^3)/(24*d) + ((6
*a + 5*a*Sec[c + d*x])*Tan[c + d*x]^5)/(30*d)
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))\tan^6(c+dx)\,dx = \frac{(6a+5a\sec(c+dx))\tan^5(c+dx)}{30d} - \frac{1}{6}\int (6a+5a\sec(c+dx))\tan^4(c+dx)\,dx$$
$$= -\frac{(8a+5a\sec(c+dx))\tan^3(c+dx)}{24d} + \frac{(6a+5a\sec(c+dx))\tan^5(c+dx)}{30d} + \frac{1}{24}$$
$$= \frac{(16a+5a\sec(c+dx))\tan(c+dx)}{16d} - \frac{(8a+5a\sec(c+dx))\tan^3(c+dx)}{24d} + \frac{(6a}{}$$
$$= -ax + \frac{(16a+5a\sec(c+dx))\tan(c+dx)}{16d} - \frac{(8a+5a\sec(c+dx))\tan^3(c+dx)}{24d}$$
$$= -ax - \frac{5a\tanh^{-1}(\sin(c+dx))}{16d} + \frac{(16a+5a\sec(c+dx))\tan(c+dx)}{16d} - \frac{(8a+5a}{}$$

Mathematica [A] time = 1.23455, size = 95, normalized size = 0.93

$$-\frac{a\left(240\tan^{-1}(\tan(c+dx)) + 75\tanh^{-1}(\sin(c+dx)) - \frac{1}{8}(1168\cos(c+dx) + 140\cos(2(c+dx)) + 568\cos(3(c+dx)) +\right.}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^6,x]
```

```
[Out] -(a*(240*ArcTan[Tan[c + d*x]] + 75*ArcTanh[Sin[c + d*x]] - ((295 + 1168*Cos
[c + d*x] + 140*Cos[2*(c + d*x)] + 568*Cos[3*(c + d*x)] + 165*Cos[4*(c + d*
x)] + 184*Cos[5*(c + d*x)])*Sec[c + d*x]^5*Tan[c + d*x])/8))/(240*d)
```

Maple [A] time = 0.046, size = 178, normalized size = 1.8

$$\frac{a(\tan(dx+c))^5}{5d} - \frac{a(\tan(dx+c))^3}{3d} + \frac{a\tan(dx+c)}{d} - ax - \frac{ac}{d} + \frac{a(\sin(dx+c))^7}{6d(\cos(dx+c))^6} - \frac{a(\sin(dx+c))^7}{24d(\cos(dx+c))^4} + \frac{a(\sin(}{16d(\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^6,x)
```

```
[Out] 1/5/d*a*tan(d*x+c)^5-1/3/d*a*tan(d*x+c)^3+1/d*a*tan(d*x+c)-a*x-1/d*a*c+1/6/
d*a*sin(d*x+c)^7/cos(d*x+c)^6-1/24/d*a*sin(d*x+c)^7/cos(d*x+c)^4+1/16/d*a*s
in(d*x+c)^7/cos(d*x+c)^2+1/16/d*a*sin(d*x+c)^5+5/48/d*a*sin(d*x+c)^3+5/16/d
*a*sin(d*x+c)-5/16/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.70835, size = 181, normalized size = 1.77

$$\frac{32\left(3\,\tan\left(dx+c\right)^5-5\,\tan\left(dx+c\right)^3-15\,dx-15\,c+15\,\tan\left(dx+c\right)\right)a-5\,a\left(\frac{2\left(33\,\sin(dx+c)^5-40\,\sin(dx+c)^3+15\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}+15\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x
 + c))*a - 5*a*(2*(33*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 15*sin(d*x + c))
/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) + 15*log(sin(d*
x + c) + 1) - 15*log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.05807, size = 377, normalized size = 3.7

$$-\frac{480\,adx\cos\left(dx+c\right)^6+75\,a\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-75\,a\cos\left(dx+c\right)^6\log\left(-\sin\left(dx+c\right)+1\right)-2\left(368\,a\cos\right.}{480\,d\cos\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/480*(480*a*d*x*cos(d*x + c)^6 + 75*a*cos(d*x + c)^6*log(sin(d*x + c) + 1
) - 75*a*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 2*(368*a*cos(d*x + c)^5 +
165*a*cos(d*x + c)^4 - 176*a*cos(d*x + c)^3 - 130*a*cos(d*x + c)^2 + 48*a*c
os(d*x + c) + 40*a)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\tan^6\left(c+dx\right)\sec\left(c+dx\right)dx+\int\tan^6\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**6,x)
```

```
[Out] a*(Integral(tan(c + d*x)**6*sec(c + d*x), x) + Integral(tan(c + d*x)**6, x)
)
```

Giac [A] time = 4.6186, size = 197, normalized size = 1.93

$$-\frac{240\,(dx+c)a + 75\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 75\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(165\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} - 1095\,a\tan\left(\frac{1}{2}\,dx \cdots\right.\right.}{\cdots}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] -1/240*(240*(d*x + c)*a + 75*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 75*a*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(165*a*tan(1/2*d*x + 1/2*c)^11 - 1095*
a*tan(1/2*d*x + 1/2*c)^9 + 3138*a*tan(1/2*d*x + 1/2*c)^7 - 5118*a*tan(1/2*d
*x + 1/2*c)^5 + 1945*a*tan(1/2*d*x + 1/2*c)^3 - 315*a*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.12 $\int (a + a\sec(c + dx))\tan^4(c + dx)\,dx$

Optimal. Leaf size=73

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{\tan^3(c+dx)(3a\sec(c+dx)+4a)}{12d} - \frac{\tan(c+dx)(3a\sec(c+dx)+8a)}{8d} + ax$$

```
[Out] a*x + (3*a*ArcTanh[Sin[c + d*x]])/(8*d) - ((8*a + 3*a*Sec[c + d*x])*Tan[c +
 d*x])/(8*d) + ((4*a + 3*a*Sec[c + d*x])*Tan[c + d*x]^3)/(12*d)
```

Rubi [A] time = 0.0619919, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3881, 3770}

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{\tan^3(c+dx)(3a\sec(c+dx)+4a)}{12d} - \frac{\tan(c+dx)(3a\sec(c+dx)+8a)}{8d} + ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] a*x + (3*a*ArcTanh[Sin[c + d*x]])/(8*d) - ((8*a + 3*a*Sec[c + d*x])*Tan[c +
 d*x])/(8*d) + ((4*a + 3*a*Sec[c + d*x])*Tan[c + d*x]^3)/(12*d)
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\sec(c+dx))\tan^4(c+dx)\,dx = \frac{(4a+3a\sec(c+dx))\tan^3(c+dx)}{12d} - \frac{1}{4}\int (4a+3a\sec(c+dx))\tan^2(c+dx)\,dx$$
$$= -\frac{(8a+3a\sec(c+dx))\tan(c+dx)}{8d} + \frac{(4a+3a\sec(c+dx))\tan^3(c+dx)}{12d} + \frac{1}{8}\int$$
$$= ax - \frac{(8a+3a\sec(c+dx))\tan(c+dx)}{8d} + \frac{(4a+3a\sec(c+dx))\tan^3(c+dx)}{12d} +$$
$$= ax + \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} - \frac{(8a+3a\sec(c+dx))\tan(c+dx)}{8d} + \frac{(4a+3a\,\mathrm{se}}{}$$

Mathematica [A] time = 0.390032, size = 75, normalized size = 1.03

$$\frac{a\left(24\tan^{-1}(\tan(c+dx)) + 9\tanh^{-1}(\sin(c+dx)) - \frac{1}{2}(32\cos(c+dx) + 15\cos(2(c+dx)) + 16\cos(3(c+dx)) + 3)\tan(c\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (a*(24*ArcTan[Tan[c + d*x]] + 9*ArcTanh[Sin[c + d*x]] - ((3 + 32*Cos[c + d*
x] + 15*Cos[2*(c + d*x)] + 16*Cos[3*(c + d*x)])*Sec[c + d*x]^3*Tan[c + d*x]
)/2))/(24*d)
```

Maple [A] time = 0.04, size = 127, normalized size = 1.7

$$\frac{a(\tan(dx+c))^3}{3d} - \frac{a\tan(dx+c)}{d} + ax + \frac{ac}{d} + \frac{a(\sin(dx+c))^5}{4d(\cos(dx+c))^4} - \frac{a(\sin(dx+c))^5}{8d(\cos(dx+c))^2} - \frac{a(\sin(dx+c))^3}{8d} - \frac{3a\sin(dx}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^4,x)
```

```
[Out] 1/3/d*a*tan(d*x+c)^3-1/d*a*tan(d*x+c)+a*x+1/d*a*c+1/4/d*a*sin(d*x+c)^5/cos(
d*x+c)^4-1/8/d*a*sin(d*x+c)^5/cos(d*x+c)^2-1/8/d*a*sin(d*x+c)^3-3/8/d*a*sin
(d*x+c)+3/8/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.72953, size = 138, normalized size = 1.89

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a+3\,a\left(\frac{2\left(5\,\sin(dx+c)^{3}-3\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx\right.\right.\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a + 3*a*(2*(5*sin(
d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) + 3*lo
g(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 0.965492, size = 302, normalized size = 4.14

$$\frac{48\,adx\cos\left(dx+c\right)^{4}+9\,a\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-9\,a\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)-2\left(32\,a\cos\left(dx+\right.\right.}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/48*(48*a*d*x*cos(d*x + c)^4 + 9*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) -
9*a*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 2*(32*a*cos(d*x + c)^3 + 15*a*c
os(d*x + c)^2 - 8*a*cos(d*x + c) - 6*a)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\tan^{4}\left(c+dx\right)\sec\left(c+dx\right)dx+\int\tan^{4}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**4,x)
```

```
[Out] a*(Integral(tan(c + d*x)**4*sec(c + d*x), x) + Integral(tan(c + d*x)**4, x)
)
```

Giac [A] time = 2.56151, size = 159, normalized size = 2.18

$$\frac{24\,(dx+c)a + 9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7} - 71\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5} + 13}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/24*(24*(d*x + c)*a + 9*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*a*log(abs
(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a*tan(1/2*d*x + 1/2*c)^7 - 71*a*tan(1/2
*d*x + 1/2*c)^5 + 137*a*tan(1/2*d*x + 1/2*c)^3 - 33*a*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.13 $\int (a + a\sec(c + dx))\tan^2(c + dx)\, dx$

Optimal. Leaf size=45

$$-\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)(a\sec(c+dx)+2a)}{2d}-ax$$

```
[Out] -(a*x) - (a*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*a + a*Sec[c + d*x])*Tan[c +
d*x])/(2*d)
```

Rubi [A] time = 0.0339243, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3881, 3770}

$$-\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)(a\sec(c+dx)+2a)}{2d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*x) - (a*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*a + a*Sec[c + d*x])*Tan[c +
d*x])/(2*d)
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))\tan^2(c+dx)\,dx = \frac{(2a+a\sec(c+dx))\tan(c+dx)}{2d} - \frac{1}{2}\int (2a+a\sec(c+dx))\,dx$$
$$= -ax + \frac{(2a+a\sec(c+dx))\tan(c+dx)}{2d} - \frac{1}{2}a\int \sec(c+dx)\,dx$$
$$= -ax - \frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{(2a+a\sec(c+dx))\tan(c+dx)}{2d}$$

Mathematica [A] time = 0.0293039, size = 60, normalized size = 1.33

$$-\frac{a\tan^{-1}(\tan(c+dx))}{d} + \frac{a\tan(c+dx)}{d} - \frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -((a*ArcTan[Tan[c + d*x]])/d) - (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c
+ d*x])/d + (a*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.032, size = 78, normalized size = 1.7

$$-ax + \frac{a\tan(dx+c)}{d} - \frac{ac}{d} + \frac{a(\sin(dx+c))^3}{2d(\cos(dx+c))^2} + \frac{a\sin(dx+c)}{2d} - \frac{a\ln(\sec(dx+c)+\tan(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^2,x)
```

```
[Out] -a*x+1/d*a*tan(d*x+c)-1/d*a*c+1/2/d*a*sin(d*x+c)^3/cos(d*x+c)^2+1/2/d*a*sin
(d*x+c)-1/2/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.72673, size = 88, normalized size = 1.96

$$-\frac{4(dx+c-\tan(dx+c))a + a\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} + \log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*(d*x + c - tan(d*x + c))*a + a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1)
 + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)))/d
```

Fricas [B] time = 0.904682, size = 234, normalized size = 5.2

$$-\frac{4\,adx\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)^{2}\log\left(\sin\left(dx+c\right)+1\right)-a\cos\left(dx+c\right)^{2}\log\left(-\sin\left(dx+c\right)+1\right)-2\left(2\,a\cos\left(dx+c\right)+\right.}{4\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*d*x*cos(d*x + c)^2 + a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - a*c
os(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(2*a*cos(d*x + c) + a)*sin(d*x + c
))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\tan^{2}\left(c+dx\right)\sec\left(c+dx\right)dx+\int\tan^{2}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**2,x)
```

```
[Out] a*(Integral(tan(c + d*x)**2*sec(c + d*x), x) + Integral(tan(c + d*x)**2, x)
)
```

Giac [B] time = 1.75812, size = 119, normalized size = 2.64

$$-\frac{2\left(dx+c\right)a+a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)*a + a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(
1/2*d*x + 1/2*c) - 1)) + 2*(a*tan(1/2*d*x + 1/2*c)^3 - 3*a*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.14 $\int \cot^2(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=26

$$-\frac{\cot(c+dx)(a\sec(c+dx)+a)}{d}-ax$$

```
[Out] -(a*x) - (Cot[c + d*x]*(a + a*Sec[c + d*x]))/d
```

Rubi [A] time = 0.0239717, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3882, 8}

$$-\frac{\cot(c+dx)(a\sec(c+dx)+a)}{d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

```
[Out] -(a*x) - (Cot[c + d*x]*(a + a*Sec[c + d*x]))/d
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)(a+a\sec(c+dx))\,dx &= -\frac{\cot(c+dx)(a+a\sec(c+dx))}{d}-\int a\,dx\\ &= -ax-\frac{\cot(c+dx)(a+a\sec(c+dx))}{d}\end{aligned}$$

Mathematica [C] time = 0.0287506, size = 43, normalized size = 1.65

$$-\frac{a\cot(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)}{d}-\frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Ta
n[c + d*x]^2])/d
```

Maple [A] time = 0.043, size = 35, normalized size = 1.4

$$\frac{1}{d}\left(a\left(-\cot\left(dx+c\right)-dx-c\right)-\frac{a}{\sin\left(dx+c\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/d*(a*(-cot(d*x+c)-d*x-c)-a/sin(d*x+c))
```

Maxima [A] time = 1.75011, size = 42, normalized size = 1.62

$$-\frac{\left(dx+c+\frac{1}{\tan(dx+c)}\right)a+\frac{a}{\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -((d*x + c + 1/tan(d*x + c))*a + a/sin(d*x + c))/d
```

Fricas [A] time = 0.895697, size = 82, normalized size = 3.15

$$-\frac{adx\sin\left(dx+c\right)+a\cos\left(dx+c\right)+a}{d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(a*d*x*sin(d*x + c) + a*cos(d*x + c) + a)/(d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \cot^2(c+dx)\sec(c+dx)\,dx + \int \cot^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cot(c + d*x)**2*sec(c + d*x), x) + Integral(cot(c + d*x)**2, x)
)
```

Giac [A] time = 1.3252, size = 35, normalized size = 1.35

$$-\frac{(dx+c)a + \frac{a}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)*a + a/tan(1/2*d*x + 1/2*c))/d
```

3.15 $\int \cot^4(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=55

$$-\frac{\cot^3(c+dx)(a\sec(c+dx)+a)}{3d}+\frac{\cot(c+dx)(2a\sec(c+dx)+3a)}{3d}+ax$$

```
[Out] a*x - (Cot[c + d*x]^3*(a + a*Sec[c + d*x]))/(3*d) + (Cot[c + d*x]*(3*a + 2*
a*Sec[c + d*x]))/(3*d)
```

Rubi [A] time = 0.0517449, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3882, 8}

$$-\frac{\cot^3(c+dx)(a\sec(c+dx)+a)}{3d}+\frac{\cot(c+dx)(2a\sec(c+dx)+3a)}{3d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

```
[Out] a*x - (Cot[c + d*x]^3*(a + a*Sec[c + d*x]))/(3*d) + (Cot[c + d*x]*(3*a + 2*
a*Sec[c + d*x]))/(3*d)
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sec(c+dx))\,dx = -\frac{\cot^3(c+dx)(a+a\sec(c+dx))}{3d} + \frac{1}{3}\int \cot^2(c+dx)(-3a-2a\sec(c+dx))\,dx$$
$$= -\frac{\cot^3(c+dx)(a+a\sec(c+dx))}{3d} + \frac{\cot(c+dx)(3a+2a\sec(c+dx))}{3d} + \frac{1}{3}\int 3a\,d$$
$$= ax - \frac{\cot^3(c+dx)(a+a\sec(c+dx))}{3d} + \frac{\cot(c+dx)(3a+2a\sec(c+dx))}{3d}$$

Mathematica [C] time = 0.0386997, size = 62, normalized size = 1.13

$$-\frac{a\cot^3(c+dx)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)}{3d} - \frac{a\csc^3(c+dx)}{3d} + \frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) - (a*Cot[c + d*x]^3*Hypergeom
etric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d)
```

Maple [A] time = 0.059, size = 86, normalized size = 1.6

$$\frac{1}{d}\left(a\left(-\frac{(\cot(dx+c))^3}{3} + \cot(dx+c) + dx + c\right) + a\left(-\frac{(\cos(dx+c))^4}{3(\sin(dx+c))^3} + \frac{(\cos(dx+c))^4}{3\sin(dx+c)} + \frac{\left(2+(\cos(dx+c))^2\right)\sin(dx}{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+a*(-1/3/sin(d*x+c)^3*cos(d*x+c)
^4+1/3/sin(d*x+c)*cos(d*x+c)^4+1/3*(2+cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 1.74058, size = 80, normalized size = 1.45

$$\frac{\left(3\,dx + 3\,c + \frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a + \frac{\left(3\,\sin(dx+c)^2-1\right)a}{\sin(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*((3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a + (3*sin(d*x +
 c)^2 - 1)*a/sin(d*x + c)^3)/d
```

Fricas [A] time = 0.905865, size = 177, normalized size = 3.22

$$\frac{4\,a\cos\left(dx+c\right)^{2}-a\cos\left(dx+c\right)+3\left(adx\cos\left(dx+c\right)-adx\right)\sin\left(dx+c\right)-2\,a}{3\left(d\cos\left(dx+c\right)-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(4*a*cos(d*x + c)^2 - a*cos(d*x + c) + 3*(a*d*x*cos(d*x + c) - a*d*x)*s
in(d*x + c) - 2*a)/((d*cos(d*x + c) - d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\cot^{4}\left(c+dx\right)\sec\left(c+dx\right)dx+\int\cot^{4}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cot(c + d*x)**4*sec(c + d*x), x) + Integral(cot(c + d*x)**4, x)
)
```

Giac [A] time = 1.45963, size = 76, normalized size = 1.38

$$\frac{12\left(dx+c\right)a-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*(d*x + c)*a - 3*a*tan(1/2*d*x + 1/2*c) + (12*a*tan(1/2*d*x + 1/2*c
)^2 - a)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.16 $\int \cot^6(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=84

$$-\frac{\cot^5(c+dx)(a\sec(c+dx)+a)}{5d}+\frac{\cot^3(c+dx)(4a\sec(c+dx)+5a)}{15d}-\frac{\cot(c+dx)(8a\sec(c+dx)+15a)}{15d}-ax$$

```
[Out] -(a*x) - (Cot[c + d*x]^5*(a + a*Sec[c + d*x]))/(5*d) + (Cot[c + d*x]^3*(5*a
 + 4*a*Sec[c + d*x]))/(15*d) - (Cot[c + d*x]*(15*a + 8*a*Sec[c + d*x]))/(15
*d)
```

Rubi [A] time = 0.0809355, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3882, 8}

$$-\frac{\cot^5(c+dx)(a\sec(c+dx)+a)}{5d}+\frac{\cot^3(c+dx)(4a\sec(c+dx)+5a)}{15d}-\frac{\cot(c+dx)(8a\sec(c+dx)+15a)}{15d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sec[c + d*x]),x]
```

```
[Out] -(a*x) - (Cot[c + d*x]^5*(a + a*Sec[c + d*x]))/(5*d) + (Cot[c + d*x]^3*(5*a
 + 4*a*Sec[c + d*x]))/(15*d) - (Cot[c + d*x]*(15*a + 8*a*Sec[c + d*x]))/(15
*d)
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^6(c+dx)(a+a\sec(c+dx))\,dx = -\frac{\cot^5(c+dx)(a+a\sec(c+dx))}{5d} + \frac{1}{5}\int \cot^4(c+dx)(-5a-4a\sec(c+dx))\,dx$$
$$= -\frac{\cot^5(c+dx)(a+a\sec(c+dx))}{5d} + \frac{\cot^3(c+dx)(5a+4a\sec(c+dx))}{15d} + \frac{1}{15}\int \text{co}$$
$$= -\frac{\cot^5(c+dx)(a+a\sec(c+dx))}{5d} + \frac{\cot^3(c+dx)(5a+4a\sec(c+dx))}{15d} - \text{cot}(c+$$
$$= -ax - \frac{\cot^5(c+dx)(a+a\sec(c+dx))}{5d} + \frac{\cot^3(c+dx)(5a+4a\sec(c+dx))}{15d} - \text{cot}$$

Mathematica [C] time = 0.0495894, size = 79, normalized size = 0.94

$$-\frac{a\cot^5(c+dx)\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(c+dx)\right)}{5d} - \frac{a\csc^5(c+dx)}{5d} + \frac{2a\csc^3(c+dx)}{3d} - \frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sec[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (2*a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d*x]^5)/(5*
d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])/(
5*d)
```

Maple [A] time = 0.067, size = 129, normalized size = 1.5

$$\frac{1}{d}\left(a\left(-\frac{(\cot(dx+c))^5}{5} + \frac{(\cot(dx+c))^3}{3} - \cot(dx+c) - dx - c\right) + a\left(-\frac{(\cos(dx+c))^6}{5\,(\sin(dx+c))^5} + \frac{(\cos(dx+c))^6}{15\,(\sin(dx+c))^3} - \frac{(\cos(dx}{5\,\sin(dx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+a*(-1/5/sin(d*
x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c)*cos(d*x+c
)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 1.65846, size = 107, normalized size = 1.27

$$-\frac{\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a+\frac{\left(15\,\sin(dx+c)^4-10\,\sin(dx+c)^2+3\right)a}{\sin(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/15*((15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x
+ c)^5)*a + (15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 + 3)*a/sin(d*x + c)^5)/d
```

Fricas [A] time = 0.986682, size = 356, normalized size = 4.24

$$-\frac{23\,a\cos\left(dx+c\right)^4-8\,a\cos\left(dx+c\right)^3-27\,a\cos\left(dx+c\right)^2+7\,a\cos\left(dx+c\right)+15\left(adx\cos\left(dx+c\right)^3-adx\cos\left(dx+c\right)^2\right.}{15\left(d\cos\left(dx+c\right)^3-d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(23*a*cos(d*x + c)^4 - 8*a*cos(d*x + c)^3 - 27*a*cos(d*x + c)^2 + 7*a
*cos(d*x + c) + 15*(a*d*x*cos(d*x + c)^3 - a*d*x*cos(d*x + c)^2 - a*d*x*cos
(d*x + c) + a*d*x)*sin(d*x + c) + 8*a)/((d*cos(d*x + c)^3 - d*cos(d*x + c)^
2 - d*cos(d*x + c) + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37042, size = 112, normalized size = 1.33

$$-\frac{5\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+240\,(dx+c)a-90\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{3\left(80\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/240*(5*a*tan(1/2*d*x + 1/2*c)^3 + 240*(d*x + c)*a - 90*a*tan(1/2*d*x + 1
/2*c) + 3*(80*a*tan(1/2*d*x + 1/2*c)^4 - 10*a*tan(1/2*d*x + 1/2*c)^2 + a)/t
an(1/2*d*x + 1/2*c)^5)/d
```

3.17 $\int \cot^8(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=111

$$-\frac{\cot^7(c+dx)(a\sec(c+dx)+a)}{7d}+\frac{\cot^5(c+dx)(6a\sec(c+dx)+7a)}{35d}-\frac{\cot^3(c+dx)(24a\sec(c+dx)+35a)}{105d}+\frac{\cot(c+$$

```
[Out] a*x - (Cot[c + d*x]^7*(a + a*Sec[c + d*x]))/(7*d) + (Cot[c + d*x]^5*(7*a +
6*a*Sec[c + d*x]))/(35*d) + (Cot[c + d*x]*(35*a + 16*a*Sec[c + d*x]))/(35*d
) - (Cot[c + d*x]^3*(35*a + 24*a*Sec[c + d*x]))/(105*d)
```

Rubi [A] time = 0.113559, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3882, 8}

$$-\frac{\cot^7(c+dx)(a\sec(c+dx)+a)}{7d}+\frac{\cot^5(c+dx)(6a\sec(c+dx)+7a)}{35d}-\frac{\cot^3(c+dx)(24a\sec(c+dx)+35a)}{105d}+\frac{\cot(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8*(a + a*Sec[c + d*x]),x]
```

```
[Out] a*x - (Cot[c + d*x]^7*(a + a*Sec[c + d*x]))/(7*d) + (Cot[c + d*x]^5*(7*a +
6*a*Sec[c + d*x]))/(35*d) + (Cot[c + d*x]*(35*a + 16*a*Sec[c + d*x]))/(35*d
) - (Cot[c + d*x]^3*(35*a + 24*a*Sec[c + d*x]))/(105*d)
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^8(c+dx)(a+a\sec(c+dx))\,dx = -\frac{\cot^7(c+dx)(a+a\sec(c+dx))}{7d} + \frac{1}{7}\int \cot^6(c+dx)(-7a-6a\sec(c+dx))\,dx$$
$$= -\frac{\cot^7(c+dx)(a+a\sec(c+dx))}{7d} + \frac{\cot^5(c+dx)(7a+6a\sec(c+dx))}{35d} + \frac{1}{35}\int \mathrm{co}$$
$$= -\frac{\cot^7(c+dx)(a+a\sec(c+dx))}{7d} + \frac{\cot^5(c+dx)(7a+6a\sec(c+dx))}{35d} - \frac{\cot^3(c+}{}$$
$$= -\frac{\cot^7(c+dx)(a+a\sec(c+dx))}{7d} + \frac{\cot^5(c+dx)(7a+6a\sec(c+dx))}{35d} + \frac{\cot(c+}{}$$
$$= ax - \frac{\cot^7(c+dx)(a+a\sec(c+dx))}{7d} + \frac{\cot^5(c+dx)(7a+6a\sec(c+dx))}{35d} + \frac{\cot(}{}$$

Mathematica [C] time = 0.0496859, size = 92, normalized size = 0.83

$$-\frac{a\cot^7(c+dx)\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},-\tan^2(c+dx)\right)}{7d} - \frac{a\csc^7(c+dx)}{7d} + \frac{3a\csc^5(c+dx)}{5d} - \frac{a\csc^3(c+dx)}{d} +$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/d + (3*a*Csc[c + d*x]^5)/(5*d) - (a
*Csc[c + d*x]^7)/(7*d) - (a*Cot[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/2,
 -Tan[c + d*x]^2])/(7*d)
```

Maple [A] time = 0.077, size = 162, normalized size = 1.5

$$\frac{1}{d}\left(a\left(-\frac{(\cot(dx+c))^7}{7} + \frac{(\cot(dx+c))^5}{5} - \frac{(\cot(dx+c))^3}{3} + \cot(dx+c) + dx + c\right) + a\left(-\frac{(\cos(dx+c))^8}{7(\sin(dx+c))^7} + \frac{(\cos(dx+}{35(\sin(dx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^8*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/7*cot(d*x+c)^7+1/5*cot(d*x+c)^5-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+
c)+a*(-1/7/sin(d*x+c)^7*cos(d*x+c)^8+1/35/sin(d*x+c)^5*cos(d*x+c)^8-1/35/si
n(d*x+c)^3*cos(d*x+c)^8+1/7/sin(d*x+c)*cos(d*x+c)^8+1/7*(16/5+cos(d*x+c)^6+
6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 1.78986, size = 135, normalized size = 1.22

$$\frac{\left(105\,dx+105\,c+\frac{105\,\tan(dx+c)^{6}-35\,\tan(dx+c)^{4}+21\,\tan(dx+c)^{2}-15}{\tan(dx+c)^{7}}\right)a+\frac{3\left(35\,\sin(dx+c)^{6}-35\,\sin(dx+c)^{4}+21\,\sin(dx+c)^{2}-5\right)a}{\sin(dx+c)^{7}}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/105*((105*d*x + 105*c + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan(
d*x + c)^2 - 15)/tan(d*x + c)^7)*a + 3*(35*sin(d*x + c)^6 - 35*sin(d*x + c)
^4 + 21*sin(d*x + c)^2 - 5)*a/sin(d*x + c)^7)/d
```

Fricas [B] time = 0.896661, size = 551, normalized size = 4.96

$$\frac{176\,a\cos\left(dx+c\right)^{6}-71\,a\cos\left(dx+c\right)^{5}-335\,a\cos\left(dx+c\right)^{4}+125\,a\cos\left(dx+c\right)^{3}+225\,a\cos\left(dx+c\right)^{2}-57\,a\cos\left(dx+\right.}{105\left(d\cos\left(dx+c\right)^{5}-d\cos\left(dx+c\right)^{4}-2\,d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/105*(176*a*cos(d*x + c)^6 - 71*a*cos(d*x + c)^5 - 335*a*cos(d*x + c)^4 +
125*a*cos(d*x + c)^3 + 225*a*cos(d*x + c)^2 - 57*a*cos(d*x + c) + 105*(a*d*
x*cos(d*x + c)^5 - a*d*x*cos(d*x + c)^4 - 2*a*d*x*cos(d*x + c)^3 + 2*a*d*x*
cos(d*x + c)^2 + a*d*x*cos(d*x + c) - a*d*x)*sin(d*x + c) - 48*a)/((d*cos(d
*x + c)^5 - d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 + d*
cos(d*x + c) - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**8*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38131, size = 153, normalized size = 1.38

$$-\frac{21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-280\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-6720\,(dx+c)a+3045\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{6720\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-1015\,a\tan\ldots}{\tan\ldots}}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6720*(21*a*tan(1/2*d*x + 1/2*c)^5 - 280*a*tan(1/2*d*x + 1/2*c)^3 - 6720*
(d*x + c)*a + 3045*a*tan(1/2*d*x + 1/2*c) - (6720*a*tan(1/2*d*x + 1/2*c)^6
- 1015*a*tan(1/2*d*x + 1/2*c)^4 + 168*a*tan(1/2*d*x + 1/2*c)^2 - 15*a)/tan(
1/2*d*x + 1/2*c)^7)/d
```

3.18 $\int \cot^{10}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=140

$$-\frac{\cot^9(c+dx)(a\sec(c+dx)+a)}{9d}+\frac{\cot^7(c+dx)(8a\sec(c+dx)+9a)}{63d}-\frac{\cot^5(c+dx)(16a\sec(c+dx)+21a)}{105d}+\frac{\cot^3(c+$$

```
[Out] -(a*x) - (Cot[c + d*x]^9*(a + a*Sec[c + d*x]))/(9*d) + (Cot[c + d*x]^7*(9*a
 + 8*a*Sec[c + d*x]))/(63*d) - (Cot[c + d*x]^5*(21*a + 16*a*Sec[c + d*x]))/
(105*d) + (Cot[c + d*x]^3*(105*a + 64*a*Sec[c + d*x]))/(315*d) - (Cot[c + d
*x]*(315*a + 128*a*Sec[c + d*x]))/(315*d)
```

Rubi [A] time = 0.144726, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3882, 8}

$$-\frac{\cot^9(c+dx)(a\sec(c+dx)+a)}{9d}+\frac{\cot^7(c+dx)(8a\sec(c+dx)+9a)}{63d}-\frac{\cot^5(c+dx)(16a\sec(c+dx)+21a)}{105d}+\frac{\cot^3(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^10*(a + a*Sec[c + d*x]),x]
```


```
[Out] -(a*x) - (Cot[c + d*x]^9*(a + a*Sec[c + d*x]))/(9*d) + (Cot[c + d*x]^7*(9*a
 + 8*a*Sec[c + d*x]))/(63*d) - (Cot[c + d*x]^5*(21*a + 16*a*Sec[c + d*x]))/
(105*d) + (Cot[c + d*x]^3*(105*a + 64*a*Sec[c + d*x]))/(315*d) - (Cot[c + d
*x]*(315*a + 128*a*Sec[c + d*x]))/(315*d)
```


Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))\,dx = -\frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d} + \frac{1}{9}\int \cot^8(c+dx)(-9a-8a\sec(c+dx))\,dx$$
$$= -\frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d} + \frac{\cot^7(c+dx)(9a+8a\sec(c+dx))}{63d} + \frac{1}{63}\int \mathrm{c}$$
$$= -\frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d} + \frac{\cot^7(c+dx)(9a+8a\sec(c+dx))}{63d} - \frac{\cot^5(c}{}$$
$$= -\frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d} + \frac{\cot^7(c+dx)(9a+8a\sec(c+dx))}{63d} - \frac{\cot^5(c}{}$$
$$= -\frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d} + \frac{\cot^7(c+dx)(9a+8a\sec(c+dx))}{63d} - \frac{\cot^5(c}{}$$
$$= -ax - \frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d} + \frac{\cot^7(c+dx)(9a+8a\sec(c+dx))}{63d} - \mathrm{co}$$

Mathematica [C] time = 0.0636214, size = 111, normalized size = 0.79

$$-\frac{a\cot^9(c+dx)\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},-\tan^2(c+dx)\right)}{9d} - \frac{a\csc^9(c+dx)}{9d} + \frac{4a\csc^7(c+dx)}{7d} - \frac{6a\csc^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^10*(a + a*Sec[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (4*a*Csc[c + d*x]^3)/(3*d) - (6*a*Csc[c + d*x]^5)/(
5*d) + (4*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(9*d) - (a*Cot[c + d
*x]^9*Hypergeometric2F1[-9/2, 1, -7/2, -Tan[c + d*x]^2])/(9*d)
```

Maple [A] time = 0.136, size = 205, normalized size = 1.5

$$\frac{1}{d}\left(a\left(-\frac{(\cot(dx+c))^9}{9} + \frac{(\cot(dx+c))^7}{7} - \frac{(\cot(dx+c))^5}{5} + \frac{(\cot(dx+c))^3}{3} - \cot(dx+c) - dx - c\right) + a\left(-\frac{(\cos(dx+c)}{9\,(\sin(dx+c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^10*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/9*cot(d*x+c)^9+1/7*cot(d*x+c)^7-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^
3-cot(d*x+c)-d*x-c)+a*(-1/9/sin(d*x+c)^9*cos(d*x+c)^10+1/63/sin(d*x+c)^7*co
```

```
s(d*x+c)^10-1/105/sin(d*x+c)^5*cos(d*x+c)^10+1/63/sin(d*x+c)^3*cos(d*x+c)^1
0-1/9/sin(d*x+c)*cos(d*x+c)^10-1/9*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48
/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 1.69648, size = 161, normalized size = 1.15

$$-\frac{\left(315\,dx+315\,c+\frac{315\,\tan(dx+c)^{8}-105\,\tan(dx+c)^{6}+63\,\tan(dx+c)^{4}-45\,\tan(dx+c)^{2}+35}{\tan(dx+c)^{9}}\right)a+\frac{\left(315\,\sin(dx+c)^{8}-420\,\sin(dx+c)^{6}+378\,\sin(dx+c)^{4}-18}{\sin(dx+c)^{9}}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/315*((315*d*x + 315*c + (315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 + 63*ta
n(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)/tan(d*x + c)^9)*a + (315*sin(d*x + c
)^8 - 420*sin(d*x + c)^6 + 378*sin(d*x + c)^4 - 180*sin(d*x + c)^2 + 35)*a/
sin(d*x + c)^9)/d
```

Fricas [B] time = 0.907772, size = 745, normalized size = 5.32

$$-\frac{563\,a\cos\left(dx+c\right)^{8}-248\,a\cos\left(dx+c\right)^{7}-1498\,a\cos\left(dx+c\right)^{6}+658\,a\cos\left(dx+c\right)^{5}+1610\,a\cos\left(dx+c\right)^{4}-602\,a\cos}{315\left(d\cos\left(dx+c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/315*(563*a*cos(d*x + c)^8 - 248*a*cos(d*x + c)^7 - 1498*a*cos(d*x + c)^6
 + 658*a*cos(d*x + c)^5 + 1610*a*cos(d*x + c)^4 - 602*a*cos(d*x + c)^3 - 76
3*a*cos(d*x + c)^2 + 187*a*cos(d*x + c) + 315*(a*d*x*cos(d*x + c)^7 - a*d*x
*cos(d*x + c)^6 - 3*a*d*x*cos(d*x + c)^5 + 3*a*d*x*cos(d*x + c)^4 + 3*a*d*x
*cos(d*x + c)^3 - 3*a*d*x*cos(d*x + c)^2 - a*d*x*cos(d*x + c) + a*d*x)*sin(
d*x + c) + 128*a)/((d*cos(d*x + c)^7 - d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^
5 + 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^3 - 3*d*cos(d*x + c)^2 - d*cos(d*
x + c) + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**10*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.51179, size = 189, normalized size = 1.35

$$-\frac{45\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-630\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4830\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+80640\,(dx+c)a-40950\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/80640*(45*a*tan(1/2*d*x + 1/2*c)^7 - 630*a*tan(1/2*d*x + 1/2*c)^5 + 4830
*a*tan(1/2*d*x + 1/2*c)^3 + 80640*(d*x + c)*a - 40950*a*tan(1/2*d*x + 1/2*c
) + (80640*a*tan(1/2*d*x + 1/2*c)^8 - 13650*a*tan(1/2*d*x + 1/2*c)^6 + 2898
*a*tan(1/2*d*x + 1/2*c)^4 - 450*a*tan(1/2*d*x + 1/2*c)^2 + 35*a)/tan(1/2*d*
x + 1/2*c)^9)/d
```

3.19 $\int (a+a\sec(c+dx))^2\tan^9(c+dx)\,dx$

Optimal. Leaf size=192

$$\frac{a^2\sec^{10}(c+dx)}{10d}+\frac{2a^2\sec^9(c+dx)}{9d}-\frac{3a^2\sec^8(c+dx)}{8d}-\frac{8a^2\sec^7(c+dx)}{7d}+\frac{a^2\sec^6(c+dx)}{3d}+\frac{12a^2\sec^5(c+dx)}{5d}+\frac{a^2 \mathrm{s}}{}$$

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a^2*Sec[c + d*x])/d - (3*a^2*Sec[c + d*x]
^2)/(2*d) - (8*a^2*Sec[c + d*x]^3)/(3*d) + (a^2*Sec[c + d*x]^4)/(2*d) + (12
*a^2*Sec[c + d*x]^5)/(5*d) + (a^2*Sec[c + d*x]^6)/(3*d) - (8*a^2*Sec[c + d*
x]^7)/(7*d) - (3*a^2*Sec[c + d*x]^8)/(8*d) + (2*a^2*Sec[c + d*x]^9)/(9*d) +
 (a^2*Sec[c + d*x]^10)/(10*d)
```

Rubi [A] time = 0.0988049, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{a^2\sec^{10}(c+dx)}{10d}+\frac{2a^2\sec^9(c+dx)}{9d}-\frac{3a^2\sec^8(c+dx)}{8d}-\frac{8a^2\sec^7(c+dx)}{7d}+\frac{a^2\sec^6(c+dx)}{3d}+\frac{12a^2\sec^5(c+dx)}{5d}+\frac{a^2 \mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^9,x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a^2*Sec[c + d*x])/d - (3*a^2*Sec[c + d*x]
^2)/(2*d) - (8*a^2*Sec[c + d*x]^3)/(3*d) + (a^2*Sec[c + d*x]^4)/(2*d) + (12
*a^2*Sec[c + d*x]^5)/(5*d) + (a^2*Sec[c + d*x]^6)/(3*d) - (8*a^2*Sec[c + d*
x]^7)/(7*d) - (3*a^2*Sec[c + d*x]^8)/(8*d) + (2*a^2*Sec[c + d*x]^9)/(9*d) +
 (a^2*Sec[c + d*x]^10)/(10*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int (a + a\sec(c+dx))^2 \tan^9(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^4(a+ax)^6}{x^{11}}\,dx, x, \cos(c+dx)\right)}{a^8 d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^{10}}{x^{11}} + \frac{2a^{10}}{x^{10}} - \frac{3a^{10}}{x^9} - \frac{8a^{10}}{x^8} + \frac{2a^{10}}{x^7} + \frac{12a^{10}}{x^6} + \frac{2a^{10}}{x^5} - \frac{8a^{10}}{x^4} - \frac{3a^{10}}{x^3} + \frac{2a^{10}}{x^2} + \right.\right.}{a^8 d}$$
$$= -\frac{a^2\log(\cos(c+dx))}{d} + \frac{2a^2\sec(c+dx)}{d} - \frac{3a^2\sec^2(c+dx)}{2d} - \frac{8a^2\sec^3(c+dx)}{3d} +$$

Mathematica [A] time = 0.507658, size = 140, normalized size = 0.73

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-252\sec^{10}(c+dx) - 560\sec^9(c+dx) + 945\sec^8(c+dx) + 2880\sec^7(c+dx) - \right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^9,x]
```

```
[Out] -(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(2520*Log[Cos[c + d*x]] - 504
0*Sec[c + d*x] + 3780*Sec[c + d*x]^2 + 6720*Sec[c + d*x]^3 - 1260*Sec[c + d
*x]^4 - 6048*Sec[c + d*x]^5 - 840*Sec[c + d*x]^6 + 2880*Sec[c + d*x]^7 + 94
5*Sec[c + d*x]^8 - 560*Sec[c + d*x]^9 - 252*Sec[c + d*x]^10))/(10080*d)
```

Maple [A] time = 0.053, size = 327, normalized size = 1.7

$$\frac{a^2(\tan(dx+c))^8}{8d} - \frac{a^2(\tan(dx+c))^6}{6d} + \frac{a^2(\tan(dx+c))^4}{4d} - \frac{a^2(\tan(dx+c))^2}{2d} - \frac{a^2\ln(\cos(dx+c))}{d} + \frac{2a^2(\sin(dx+c}{9d(\cos(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^9,x)
```

```
[Out] 1/8/d*a^2*tan(d*x+c)^8-1/6/d*a^2*tan(d*x+c)^6+1/4/d*a^2*tan(d*x+c)^4-1/2/d*
a^2*tan(d*x+c)^2-a^2*ln(cos(d*x+c))/d+2/9/d*a^2*sin(d*x+c)^10/cos(d*x+c)^9-
```

```
2/63/d*a^2*sin(d*x+c)^10/cos(d*x+c)^7+2/105/d*a^2*sin(d*x+c)^10/cos(d*x+c)^
5-2/63/d*a^2*sin(d*x+c)^10/cos(d*x+c)^3+2/9/d*a^2*sin(d*x+c)^10/cos(d*x+c)+
256/315/d*a^2*cos(d*x+c)+2/9/d*a^2*cos(d*x+c)*sin(d*x+c)^8+16/63/d*a^2*cos(
d*x+c)*sin(d*x+c)^6+32/105/d*a^2*cos(d*x+c)*sin(d*x+c)^4+128/315/d*a^2*cos(
d*x+c)*sin(d*x+c)^2+1/10/d*a^2*sin(d*x+c)^10/cos(d*x+c)^10
```

Maxima [A] time = 1.14016, size = 201, normalized size = 1.05

$$-\frac{2520\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{5040\,a^2\cos(dx+c)^9-3780\,a^2\cos(dx+c)^8-6720\,a^2\cos(dx+c)^7+1260\,a^2\cos(dx+c)^6+6048\,a^2\cos(dx+c)^5+840\,a^2\cos(dx+}{\cos(dx+c)^{10}}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -1/2520*(2520*a^2*log(cos(d*x + c)) - (5040*a^2*cos(d*x + c)^9 - 3780*a^2*c
os(d*x + c)^8 - 6720*a^2*cos(d*x + c)^7 + 1260*a^2*cos(d*x + c)^6 + 6048*a^
2*cos(d*x + c)^5 + 840*a^2*cos(d*x + c)^4 - 2880*a^2*cos(d*x + c)^3 - 945*a
^2*cos(d*x + c)^2 + 560*a^2*cos(d*x + c) + 252*a^2)/cos(d*x + c)^10)/d
```

Fricas [A] time = 1.07857, size = 424, normalized size = 2.21

$$-\frac{2520\,a^2\cos\left(dx+c\right)^{10}\log\left(-\cos\left(dx+c\right)\right)-5040\,a^2\cos\left(dx+c\right)^9+3780\,a^2\cos\left(dx+c\right)^8+6720\,a^2\cos\left(dx+c\right)^7-12}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/2520*(2520*a^2*cos(d*x + c)^10*log(-cos(d*x + c)) - 5040*a^2*cos(d*x + c
)^9 + 3780*a^2*cos(d*x + c)^8 + 6720*a^2*cos(d*x + c)^7 - 1260*a^2*cos(d*x
+ c)^6 - 6048*a^2*cos(d*x + c)^5 - 840*a^2*cos(d*x + c)^4 + 2880*a^2*cos(d*
x + c)^3 + 945*a^2*cos(d*x + c)^2 - 560*a^2*cos(d*x + c) - 252*a^2)/(d*cos(
d*x + c)^10)
```

Sympy [A] time = 109.844, size = 314, normalized size = 1.64

$$\begin{cases} \frac{a^2\log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{a^2\tan^8(c+dx)\sec^2(c+dx)}{10d} + \frac{2a^2\tan^8(c+dx)\sec(c+dx)}{9d} + \frac{a^2\tan^8(c+dx)}{8d} - \frac{a^2\tan^6(c+dx)\sec^2(c+dx)}{10d} - \frac{16a^2\tan^6(c+dx)\mathrm{se}}{63d} \\ x\left(a\sec(c)+a\right)^2\tan^9(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**9,x)
```

```
[Out] Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**8*sec(c
 + d*x)**2/(10*d) + 2*a**2*tan(c + d*x)**8*sec(c + d*x)/(9*d) + a**2*tan(c
+ d*x)**8/(8*d) - a**2*tan(c + d*x)**6*sec(c + d*x)**2/(10*d) - 16*a**2*tan
(c + d*x)**6*sec(c + d*x)/(63*d) - a**2*tan(c + d*x)**6/(6*d) + a**2*tan(c
+ d*x)**4*sec(c + d*x)**2/(10*d) + 32*a**2*tan(c + d*x)**4*sec(c + d*x)/(10
5*d) + a**2*tan(c + d*x)**4/(4*d) - a**2*tan(c + d*x)**2*sec(c + d*x)**2/(1
0*d) - 128*a**2*tan(c + d*x)**2*sec(c + d*x)/(315*d) - a**2*tan(c + d*x)**2
/(2*d) + a**2*sec(c + d*x)**2/(10*d) + 256*a**2*sec(c + d*x)/(315*d), Ne(d,
 0)), (x*(a*sec(c) + a)**2*tan(c)**9, True))
```

Giac [A] time = 10.8213, size = 462, normalized size = 2.41

$$\frac{2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{11477\,a^2 + \frac{119810\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{566865\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{1605720\,a^2(\cos}{(\cos(dx+c}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="giac")
```

```
[Out] 1/2520*(2520*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 252
0*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (11477*a^2 + 1
19810*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 566865*a^2*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2 + 1605720*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c
) + 1)^3 + 3031770*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 2995020*
a^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 2171610*a^2*(cos(d*x + c) -
 1)^6/(cos(d*x + c) + 1)^6 + 1114200*a^2*(cos(d*x + c) - 1)^7/(cos(d*x + c)
 + 1)^7 + 382545*a^2*(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8 + 78850*a^2*
(cos(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9 + 7381*a^2*(cos(d*x + c) - 1)^10/
(cos(d*x + c) + 1)^10)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^10)/d
```

3.20 $\int (a+a\sec(c+dx))^2\tan^7(c+dx)\,dx$

Optimal. Leaf size=132

$$\frac{a^2\sec^8(c+dx)}{8d}+\frac{2a^2\sec^7(c+dx)}{7d}-\frac{a^2\sec^6(c+dx)}{3d}-\frac{6a^2\sec^5(c+dx)}{5d}+\frac{2a^2\sec^3(c+dx)}{d}+\frac{a^2\sec^2(c+dx)}{d}-\frac{2a^2\mathrm{se}}{}$$

```
[Out] (a^2*Log[Cos[c + d*x]])/d - (2*a^2*Sec[c + d*x])/d + (a^2*Sec[c + d*x]^2)/d
 + (2*a^2*Sec[c + d*x]^3)/d - (6*a^2*Sec[c + d*x]^5)/(5*d) - (a^2*Sec[c + d
*x]^6)/(3*d) + (2*a^2*Sec[c + d*x]^7)/(7*d) + (a^2*Sec[c + d*x]^8)/(8*d)
```

Rubi [A] time = 0.0792304, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{a^2\sec^8(c+dx)}{8d}+\frac{2a^2\sec^7(c+dx)}{7d}-\frac{a^2\sec^6(c+dx)}{3d}-\frac{6a^2\sec^5(c+dx)}{5d}+\frac{2a^2\sec^3(c+dx)}{d}+\frac{a^2\sec^2(c+dx)}{d}-\frac{2a^2\mathrm{se}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^7,x]
```

```
[Out] (a^2*Log[Cos[c + d*x]])/d - (2*a^2*Sec[c + d*x])/d + (a^2*Sec[c + d*x]^2)/d
 + (2*a^2*Sec[c + d*x]^3)/d - (6*a^2*Sec[c + d*x]^5)/(5*d) - (a^2*Sec[c + d
*x]^6)/(3*d) + (2*a^2*Sec[c + d*x]^7)/(7*d) + (a^2*Sec[c + d*x]^8)/(8*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\tan^7(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^3(a+ax)^5}{x^9}\,dx, x, \cos(c+dx)\right)}{a^6 d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^8}{x^9}+\frac{2a^8}{x^8}-\frac{2a^8}{x^7}-\frac{6a^8}{x^6}+\frac{6a^8}{x^4}+\frac{2a^8}{x^3}-\frac{2a^8}{x^2}-\frac{a^8}{x}\right)dx, x, \cos(c+dx)\right)}{a^6 d}$$
$$= \frac{a^2\log(\cos(c+dx))}{d}-\frac{2a^2\sec(c+dx)}{d}+\frac{a^2\sec^2(c+dx)}{d}+\frac{2a^2\sec^3(c+dx)}{d}-\frac{6}{}$$

Mathematica [A] time = 0.303636, size = 110, normalized size = 0.83

$$\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(105\sec^8(c+dx)+240\sec^7(c+dx)-280\sec^6(c+dx)-1008\sec^5(c+dx)+1680\right.}{3360d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^7,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(840*Log[Cos[c + d*x]] - 1680*
Sec[c + d*x] + 840*Sec[c + d*x]^2 + 1680*Sec[c + d*x]^3 - 1008*Sec[c + d*x]
^5 - 280*Sec[c + d*x]^6 + 240*Sec[c + d*x]^7 + 105*Sec[c + d*x]^8))/(3360*d
)
```

Maple [B] time = 0.052, size = 264, normalized size = 2.

$$\frac{a^2(\tan(dx+c))^6}{6d}-\frac{a^2(\tan(dx+c))^4}{4d}+\frac{a^2(\tan(dx+c))^2}{2d}+\frac{a^2\ln(\cos(dx+c))}{d}+\frac{2a^2(\sin(dx+c))^8}{7d(\cos(dx+c))^7}-\frac{2a^2(\sin(dx+}{35d(\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^7,x)
```

```
[Out] 1/6/d*a^2*tan(d*x+c)^6-1/4/d*a^2*tan(d*x+c)^4+1/2/d*a^2*tan(d*x+c)^2+a^2*ln
(cos(d*x+c))/d+2/7/d*a^2*sin(d*x+c)^8/cos(d*x+c)^7-2/35/d*a^2*sin(d*x+c)^8/
cos(d*x+c)^5+2/35/d*a^2*sin(d*x+c)^8/cos(d*x+c)^3-2/7/d*a^2*sin(d*x+c)^8/co
s(d*x+c)-32/35/d*a^2*cos(d*x+c)-2/7/d*a^2*cos(d*x+c)*sin(d*x+c)^6-12/35/d*a
^2*cos(d*x+c)*sin(d*x+c)^4-16/35/d*a^2*cos(d*x+c)*sin(d*x+c)^2+1/8/d*a^2*si
n(d*x+c)^8/cos(d*x+c)^8
```

Maxima [A] time = 1.20336, size = 149, normalized size = 1.13

$$\frac{840\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{1680\,a^2\cos(dx+c)^7-840\,a^2\cos(dx+c)^6-1680\,a^2\cos(dx+c)^5+1008\,a^2\cos(dx+c)^3+280\,a^2\cos(dx+c)^2-240\,a^2\cos(dx+c)-105}{\cos(dx+c)^8}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/840*(840*a^2*log(cos(d*x + c)) - (1680*a^2*cos(d*x + c)^7 - 840*a^2*cos(d
*x + c)^6 - 1680*a^2*cos(d*x + c)^5 + 1008*a^2*cos(d*x + c)^3 + 280*a^2*cos
(d*x + c)^2 - 240*a^2*cos(d*x + c) - 105*a^2)/cos(d*x + c)^8)/d
```

Fricas [A] time = 1.00015, size = 312, normalized size = 2.36

$$\frac{840\,a^2\cos\left(dx+c\right)^8\log\left(-\cos\left(dx+c\right)\right)-1680\,a^2\cos\left(dx+c\right)^7+840\,a^2\cos\left(dx+c\right)^6+1680\,a^2\cos\left(dx+c\right)^5-1008\,a^2}{840\,d\cos\left(dx+c\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/840*(840*a^2*cos(d*x + c)^8*log(-cos(d*x + c)) - 1680*a^2*cos(d*x + c)^7
+ 840*a^2*cos(d*x + c)^6 + 1680*a^2*cos(d*x + c)^5 - 1008*a^2*cos(d*x + c)^
3 - 280*a^2*cos(d*x + c)^2 + 240*a^2*cos(d*x + c) + 105*a^2)/(d*cos(d*x + c
)^8)
```

Sympy [A] time = 35.8794, size = 252, normalized size = 1.91

$$\begin{cases}-\frac{a^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^2\tan^6\left(c+dx\right)\sec^2\left(c+dx\right)}{8d}+\frac{2a^2\tan^6\left(c+dx\right)\sec\left(c+dx\right)}{7d}+\frac{a^2\tan^6\left(c+dx\right)}{6d}-\frac{a^2\tan^4\left(c+dx\right)\sec^2\left(c+dx\right)}{8d}-\frac{12a^2\tan^4\left(c+d\right.}{35}\\ x\left(a\sec\left(c\right)+a\right)^2\tan^7\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**7,x)
```

```
[Out] Piecewise((-a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**6*sec(
c + d*x)**2/(8*d) + 2*a**2*tan(c + d*x)**6*sec(c + d*x)/(7*d) + a**2*tan(c
+ d*x)**6/(6*d) - a**2*tan(c + d*x)**4*sec(c + d*x)**2/(8*d) - 12*a**2*tan(
c + d*x)**4*sec(c + d*x)/(35*d) - a**2*tan(c + d*x)**4/(4*d) + a**2*tan(c +
 d*x)**2*sec(c + d*x)**2/(8*d) + 16*a**2*tan(c + d*x)**2*sec(c + d*x)/(35*d
) + a**2*tan(c + d*x)**2/(2*d) - a**2*sec(c + d*x)**2/(8*d) - 32*a**2*sec(c
 + d*x)/(35*d), Ne(d, 0)), (x*(a*sec(c) + a)**2*tan(c)**7, True))
```

Giac [B] time = 16.0484, size = 394, normalized size = 2.98

$$-\frac{840\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-840\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{3819\,a^2+\frac{32232\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{120372\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{261464\,a^2(\cos(dx+c}{(\cos(dx+c)+1)}}{}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] -1/840*(840*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 840*
a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (3819*a^2 + 3223
2*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 120372*a^2*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 + 261464*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1
)^3 + 258370*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 175448*a^2*(co
s(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 77364*a^2*(cos(d*x + c) - 1)^6/(co
s(d*x + c) + 1)^6 + 19944*a^2*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 2
283*a^2*(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8)/((cos(d*x + c) - 1)/(cos
(d*x + c) + 1) + 1)^8)/d
```

3.21 $\int (a+a\sec(c+dx))^2\tan^5(c+dx)\,dx$

Optimal. Leaf size=120

$$\frac{a^2\sec^6(c+dx)}{6d}+\frac{2a^2\sec^5(c+dx)}{5d}-\frac{a^2\sec^4(c+dx)}{4d}-\frac{4a^2\sec^3(c+dx)}{3d}-\frac{a^2\sec^2(c+dx)}{2d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\log(c}{}$$

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a^2*Sec[c + d*x])/d - (a^2*Sec[c + d*x]^2
)/(2*d) - (4*a^2*Sec[c + d*x]^3)/(3*d) - (a^2*Sec[c + d*x]^4)/(4*d) + (2*a^
2*Sec[c + d*x]^5)/(5*d) + (a^2*Sec[c + d*x]^6)/(6*d)
```

Rubi [A] time = 0.072974, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{a^2\sec^6(c+dx)}{6d}+\frac{2a^2\sec^5(c+dx)}{5d}-\frac{a^2\sec^4(c+dx)}{4d}-\frac{4a^2\sec^3(c+dx)}{3d}-\frac{a^2\sec^2(c+dx)}{2d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\log(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a^2*Sec[c + d*x])/d - (a^2*Sec[c + d*x]^2
)/(2*d) - (4*a^2*Sec[c + d*x]^3)/(3*d) - (a^2*Sec[c + d*x]^4)/(4*d) + (2*a^
2*Sec[c + d*x]^5)/(5*d) + (a^2*Sec[c + d*x]^6)/(6*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\tan^5(c+dx)\,dx = -\frac{\text{Subst}\left(\int\frac{(a-ax)^2(a+ax)^4}{x^7}\,dx,x,\cos(c+dx)\right)}{a^4d}$$
$$= -\frac{\text{Subst}\left(\int\left(\frac{a^6}{x^7}+\frac{2a^6}{x^6}-\frac{a^6}{x^5}-\frac{4a^6}{x^4}-\frac{a^6}{x^3}+\frac{2a^6}{x^2}+\frac{a^6}{x}\right)dx,x,\cos(c+dx)\right)}{a^4d}$$
$$= -\frac{a^2\log(\cos(c+dx))}{d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\sec^2(c+dx)}{2d}-\frac{4a^2\sec^3(c+dx)}{3d}-$$

Mathematica [A] time = 0.409315, size = 125, normalized size = 1.04

$$\frac{a^2\sec^6(c+dx)(312\cos(c+dx)-5(-28\cos(3(c+dx))+6\cos(4(c+dx))-12\cos(5(c+dx))+18\cos(4(c+dx))\log(\cos}{480a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] (a^2*(312*Cos[c + d*x] - 5*(14 - 28*Cos[3*(c + d*x)] + 6*Cos[4*(c + d*x)] -
 12*Cos[5*(c + d*x)] + 30*Log[Cos[c + d*x]] + 18*Cos[4*(c + d*x)]*Log[Cos[c
 + d*x]] + 3*Cos[6*(c + d*x)]*Log[Cos[c + d*x]] + 9*Cos[2*(c + d*x)]*(4 + 5
*Log[Cos[c + d*x]])))*Sec[c + d*x]^6)/(480*d)
```

Maple [A] time = 0.05, size = 203, normalized size = 1.7

$$\frac{a^2(\tan(dx+c))^4}{4d}-\frac{a^2(\tan(dx+c))^2}{2d}-\frac{a^2\ln(\cos(dx+c))}{d}+\frac{2a^2(\sin(dx+c))^6}{5d(\cos(dx+c))^5}-\frac{2a^2(\sin(dx+c))^6}{15d(\cos(dx+c))^3}+\frac{2a^2(\sin(dx+}{5d\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^5,x)
```

```
[Out] 1/4/d*a^2*tan(d*x+c)^4-1/2/d*a^2*tan(d*x+c)^2-a^2*ln(cos(d*x+c))/d+2/5/d*a^
2*sin(d*x+c)^6/cos(d*x+c)^5-2/15/d*a^2*sin(d*x+c)^6/cos(d*x+c)^3+2/5/d*a^2*
sin(d*x+c)^6/cos(d*x+c)+16/15/d*a^2*cos(d*x+c)+2/5/d*a^2*cos(d*x+c)*sin(d*x
+c)^4+8/15/d*a^2*cos(d*x+c)*sin(d*x+c)^2+1/6/d*a^2*sin(d*x+c)^6/cos(d*x+c)^
6
```

Maxima [A] time = 1.22873, size = 131, normalized size = 1.09

$$-\frac{60\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{120\,a^2\cos(dx+c)^5-30\,a^2\cos(dx+c)^4-80\,a^2\cos(dx+c)^3-15\,a^2\cos(dx+c)^2+24\,a^2\cos(dx+c)+10\,a^2}{\cos(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(60*a^2*log(cos(d*x + c)) - (120*a^2*cos(d*x + c)^5 - 30*a^2*cos(d*x
+ c)^4 - 80*a^2*cos(d*x + c)^3 - 15*a^2*cos(d*x + c)^2 + 24*a^2*cos(d*x + c
) + 10*a^2)/cos(d*x + c)^6)/d
```

Fricas [A] time = 0.951059, size = 266, normalized size = 2.22

$$-\frac{60\,a^2\cos\left(dx+c\right)^6\log\left(-\cos\left(dx+c\right)\right)-120\,a^2\cos\left(dx+c\right)^5+30\,a^2\cos\left(dx+c\right)^4+80\,a^2\cos\left(dx+c\right)^3+15\,a^2\cos\left(dx\right.}{60\,d\cos\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/60*(60*a^2*cos(d*x + c)^6*log(-cos(d*x + c)) - 120*a^2*cos(d*x + c)^5 +
30*a^2*cos(d*x + c)^4 + 80*a^2*cos(d*x + c)^3 + 15*a^2*cos(d*x + c)^2 - 24*
a^2*cos(d*x + c) - 10*a^2)/(d*cos(d*x + c)^6)
```

Sympy [A] time = 11.612, size = 189, normalized size = 1.58

$$\begin{cases}\frac{a^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^2\tan^4\left(c+dx\right)\sec^2\left(c+dx\right)}{6d}+\frac{2a^2\tan^4\left(c+dx\right)\sec\left(c+dx\right)}{5d}+\frac{a^2\tan^4\left(c+dx\right)}{4d}-\frac{a^2\tan^2\left(c+dx\right)\sec^2\left(c+dx\right)}{6d}-\frac{8a^2\tan^2\left(c+dx\right)\mathrm{s}}{15d}\\ x\left(a\sec\left(c\right)+a\right)^2\tan^5\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**5,x)
```

```
[Out] Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**4*sec(c
 + d*x)**2/(6*d) + 2*a**2*tan(c + d*x)**4*sec(c + d*x)/(5*d) + a**2*tan(c +
```

```
d*x)**4/(4*d) - a**2*tan(c + d*x)**2*sec(c + d*x)**2/(6*d) - 8*a**2*tan(c
+ d*x)**2*sec(c + d*x)/(15*d) - a**2*tan(c + d*x)**2/(2*d) + a**2*sec(c + d
*x)**2/(6*d) + 16*a**2*sec(c + d*x)/(15*d), Ne(d, 0)), (x*(a*sec(c) + a)**2
*tan(c)**5, True))
```

Giac [B] time = 3.86297, size = 327, normalized size = 2.72

$$\frac{60\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-60\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{275\,a^2+\frac{1770\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4845\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{4780\,a^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{2925\cdots}{(\cdots}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^2*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (275*a^2 + 1770*a^2*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4845*a^2*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 + 4780*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 2925
*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1002*a^2*(cos(d*x + c) - 1
)^5/(cos(d*x + c) + 1)^5 + 147*a^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^
6)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^6)/d
```

3.22 $\int (a+a\sec(c+dx))^2\tan^3(c+dx)\,dx$

Optimal. Leaf size=65

$$\frac{a^2\sec^4(c+dx)}{4d}+\frac{2a^2\sec^3(c+dx)}{3d}-\frac{2a^2\sec(c+dx)}{d}+\frac{a^2\log(\cos(c+dx))}{d}$$

```
[Out] (a^2*Log[Cos[c + d*x]])/d - (2*a^2*Sec[c + d*x])/d + (2*a^2*Sec[c + d*x]^3)
/(3*d) + (a^2*Sec[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.0558754, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 75}

$$\frac{a^2\sec^4(c+dx)}{4d}+\frac{2a^2\sec^3(c+dx)}{3d}-\frac{2a^2\sec(c+dx)}{d}+\frac{a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (a^2*Log[Cos[c + d*x]])/d - (2*a^2*Sec[c + d*x])/d + (2*a^2*Sec[c + d*x]^3)
/(3*d) + (a^2*Sec[c + d*x]^4)/(4*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\tan^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)(a+ax)^3}{x^5}\,dx, x, \cos(c+dx)\right)}{a^2d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^4}{x^5}+\frac{2a^4}{x^4}-\frac{2a^4}{x^2}-\frac{a^4}{x}\right)dx, x, \cos(c+dx)\right)}{a^2d}$$
$$= \frac{a^2\log(\cos(c+dx))}{d}-\frac{2a^2\sec(c+dx)}{d}+\frac{2a^2\sec^3(c+dx)}{3d}+\frac{a^2\sec^4(c+dx)}{4d}$$

Mathematica [A] time = 0.182215, size = 83, normalized size = 1.28

$$\frac{a^2\sec^4(c+dx)(3(-4\cos(3(c+dx))+4\cos(2(c+dx))\log(\cos(c+dx))+\cos(4(c+dx))\log(\cos(c+dx))+3\log(\cos(c+\ldots}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (a^2*(-20*Cos[c + d*x] + 3*(2 - 4*Cos[3*(c + d*x)] + 3*Log[Cos[c + d*x]] +
4*Cos[2*(c + d*x)]*Log[Cos[c + d*x]] + Cos[4*(c + d*x)]*Log[Cos[c + d*x]]))
*Sec[c + d*x]^4)/(24*d)
```

Maple [B] time = 0.044, size = 140, normalized size = 2.2

$$\frac{a^2(\tan(dx+c))^2}{2d}+\frac{a^2\ln(\cos(dx+c))}{d}+\frac{2a^2(\sin(dx+c))^4}{3d(\cos(dx+c))^3}-\frac{2a^2(\sin(dx+c))^4}{3d\cos(dx+c)}-\frac{2a^2\cos(dx+c)(\sin(dx+c))^2}{3d}-\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^3,x)
```

```
[Out] 1/2/d*a^2*tan(d*x+c)^2+a^2*ln(cos(d*x+c))/d+2/3/d*a^2*sin(d*x+c)^4/cos(d*x+
c)^3-2/3/d*a^2*sin(d*x+c)^4/cos(d*x+c)-2/3/d*a^2*cos(d*x+c)*sin(d*x+c)^2-4/
3/d*a^2*cos(d*x+c)+1/4/d*a^2*sin(d*x+c)^4/cos(d*x+c)^4
```

Maxima [A] time = 1.14314, size = 78, normalized size = 1.2

$$\frac{12\,a^2\log(\cos(dx+c))-\frac{24\,a^2\cos(dx+c)^3-8\,a^2\cos(dx+c)-3\,a^2}{\cos(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(12*a^2*log(cos(d*x + c)) - (24*a^2*cos(d*x + c)^3 - 8*a^2*cos(d*x + c
) - 3*a^2)/cos(d*x + c)^4)/d
```

Fricas [A] time = 1.18463, size = 163, normalized size = 2.51

$$\frac{12\,a^2\cos\left(dx+c\right)^4\log\left(-\cos\left(dx+c\right)\right)-24\,a^2\cos\left(dx+c\right)^3+8\,a^2\cos\left(dx+c\right)+3\,a^2}{12\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(12*a^2*cos(d*x + c)^4*log(-cos(d*x + c)) - 24*a^2*cos(d*x + c)^3 + 8*
a^2*cos(d*x + c) + 3*a^2)/(d*cos(d*x + c)^4)
```

Sympy [A] time = 3.54419, size = 126, normalized size = 1.94

$$\begin{cases} -\frac{a^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^2\tan^2\left(c+dx\right)\sec^2\left(c+dx\right)}{4d}+\frac{2a^2\tan^2\left(c+dx\right)\sec\left(c+dx\right)}{3d}+\frac{a^2\tan^2\left(c+dx\right)}{2d}-\frac{a^2\sec^2\left(c+dx\right)}{4d}-\frac{4a^2\sec\left(c+dx\right)}{3d} & \text{for } d \neq \\ x\left(a\sec\left(c\right)+a\right)^2\tan^3\left(c\right) & \text{otherw} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**3,x)
```

```
[Out] Piecewise((-a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**2*sec(
c + d*x)**2/(4*d) + 2*a**2*tan(c + d*x)**2*sec(c + d*x)/(3*d) + a**2*tan(c
+ d*x)**2/(2*d) - a**2*sec(c + d*x)**2/(4*d) - 4*a**2*sec(c + d*x)/(3*d), N
e(d, 0)), (x*(a*sec(c) + a)**2*tan(c)**3, True))
```

Giac [B] time = 2.0711, size = 259, normalized size = 3.98

$$-\frac{12\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-12\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{57\,a^2+\frac{252\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{246\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{124\,a^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{25\,a^2(\cos}{(\cos}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/12*(12*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 12*a^2
*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (57*a^2 + 252*a^2*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 246*a^2*(cos(d*x + c) - 1)^2/(cos(d*
x + c) + 1)^2 + 124*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 25*a^2*
(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/((cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 1)^4)/d
```

3.23 $\int (a+a\sec(c+dx))^2\tan(c+dx)\,dx$

Optimal. Leaf size=48

$$\frac{a^2\sec^2(c+dx)}{2d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\log(\cos(c+dx))}{d}$$

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a^2*Sec[c + d*x])/d + (a^2*Sec[c + d*x]^2
)/(2*d)
```

Rubi [A] time = 0.0356408, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 43}

$$\frac{a^2\sec^2(c+dx)}{2d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a^2*Sec[c + d*x])/d + (a^2*Sec[c + d*x]^2
)/(2*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\tan(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a+ax)^2}{x^3}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^2}{x^3}+\frac{2a^2}{x^2}+\frac{a^2}{x}\right)dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^2\log(\cos(c+dx))}{d}+\frac{2a^2\sec(c+dx)}{d}+\frac{a^2\sec^2(c+dx)}{2d}$$

Mathematica [A] time = 0.0972948, size = 51, normalized size = 1.06

$$-\frac{a^2\sec^2(c+dx)(-4\cos(c+dx)+\cos(2(c+dx))\log(\cos(c+dx))+\log(\cos(c+dx))-1)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -(a^2*(-1 - 4*Cos[c + d*x] + Log[Cos[c + d*x]] + Cos[2*(c + d*x)]*Log[Cos[c
 + d*x]])*Sec[c + d*x]^2)/(2*d)
```

Maple [A] time = 0.015, size = 46, normalized size = 1.

$$\frac{a^2(\sec(dx+c))^2}{2d}+2\frac{a^2\sec(dx+c)}{d}+\frac{a^2\ln(\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c),x)
```

```
[Out] 1/2*a^2*sec(d*x+c)^2/d+2*a^2*sec(d*x+c)/d+1/d*a^2*ln(sec(d*x+c))
```

Maxima [A] time = 1.19007, size = 58, normalized size = 1.21

$$-\frac{2a^2\log(\cos(dx+c))-\frac{4a^2\cos(dx+c)+a^2}{\cos(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a^2*log(cos(d*x + c)) - (4*a^2*cos(d*x + c) + a^2)/cos(d*x + c)^2)/
d
```

Fricas [A] time = 1.13372, size = 127, normalized size = 2.65

$$-\frac{2\,a^2\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)-4\,a^2\cos\left(dx+c\right)-a^2}{2\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*cos(d*x + c)^2*log(-cos(d*x + c)) - 4*a^2*cos(d*x + c) - a^2)/(
d*cos(d*x + c)^2)
```

Sympy [A] time = 0.93767, size = 60, normalized size = 1.25

$$\begin{cases} \frac{a^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^2\sec^2\left(c+dx\right)}{2d}+\frac{2a^2\sec\left(c+dx\right)}{d} & \text{for } d\neq 0\\ x\left(a\sec\left(c\right)+a\right)^2\tan\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c),x)
```

```
[Out] Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*sec(c + d*x)**2/(2*d)
 + 2*a**2*sec(c + d*x)/d, Ne(d, 0)), (x*(a*sec(c) + a)**2*tan(c), True))
```

Giac [B] time = 1.41181, size = 192, normalized size = 4.

$$\frac{2\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-2\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{11\,a^2+\frac{10\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2*a^2*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (11*a^2 + 10*a^2*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

3.24 $\int \cot(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=35

$$\frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{a^2\log(\cos(c+dx))}{d}$$

```
[Out] (2*a^2*Log[1 - Cos[c + d*x]])/d - (a^2*Log[Cos[c + d*x]])/d
```

Rubi [A] time = 0.0404139, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 72}

$$\frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*Log[1 - Cos[c + d*x]])/d - (a^2*Log[Cos[c + d*x]])/d
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \cot(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2\,\mathrm{Subst}\left(\int \frac{a+ax}{x(a-ax)}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^2\,\mathrm{Subst}\left(\int \left(-\frac{2}{-1+x}+\frac{1}{x}\right)dx, x, \cos(c+dx)\right)}{d}$$
$$= \frac{2a^2\log(1-\cos(c+dx))}{d} - \frac{a^2\log(\cos(c+dx))}{d}$$

Mathematica [A] time = 0.0378019, size = 29, normalized size = 0.83

$$-\frac{a^2\left(\log(\cos(c+dx)) - 4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x])^2,x]

[Out] -((a^2*(Log[Cos[c + d*x]] - 4*Log[Sin[(c + d*x)/2]]))/d)
```

Maple [A] time = 0.05, size = 34, normalized size = 1.

$$2\,\frac{a^2\ln(-1+\sec(dx+c))}{d} - \frac{a^2\ln(\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c))^2,x)

[Out] 2/d*a^2*ln(-1+sec(d*x+c))-1/d*a^2*ln(sec(d*x+c))
```

Maxima [A] time = 1.11702, size = 42, normalized size = 1.2

$$\frac{2\,a^2\log(\cos(dx+c)-1) - a^2\log(\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (2*a^2*log(cos(d*x + c) - 1) - a^2*log(cos(d*x + c)))/d
```

Fricas [A] time = 1.03161, size = 89, normalized size = 2.54

$$-\frac{a^{2} \log \left(-\cos \left(dx+c\right)\right)-2 \, a^{2} \log \left(-\frac{1}{2} \, \cos \left(dx+c\right)+\frac{1}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*log(-cos(d*x + c)) - 2*a^2*log(-1/2*cos(d*x + c) + 1/2))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{2} \left(\int 2 \cot \left(c+dx\right) \sec \left(c+dx\right)\, dx+\int \cot \left(c+dx\right) \sec^{2}\left(c+dx\right)\, dx+\int \cot \left(c+dx\right)\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cot(c + d*x)*sec(c + d*x), x) + Integral(cot(c + d*x)*sec(
c + d*x)**2, x) + Integral(cot(c + d*x), x))
```

Giac [A] time = 1.46376, size = 86, normalized size = 2.46

$$\frac{2 \, a^{2} \log \left(\frac{\left|-\cos \left(dx+c\right)+1\right|}{\left|\cos \left(dx+c\right)+1\right|}\right)-a^{2} \log \left(\left|\frac{\left(\cos \left(dx+c\right)-1\right)^{2}}{\left(\cos \left(dx+c\right)+1\right)^{2}}-1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a^2*log(abs((cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1)))/d
```

3.25 $\int \cot^3(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=40

$$-\frac{a^2}{d(1-\cos(c+dx))}-\frac{a^2\log(1-\cos(c+dx))}{d}$$

```
[Out] -(a^2/(d*(1 - Cos[c + d*x]))) - (a^2*Log[1 - Cos[c + d*x]])/d
```

Rubi [A] time = 0.0496429, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 43}

$$-\frac{a^2}{d(1-\cos(c+dx))}-\frac{a^2\log(1-\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2/(d*(1 - Cos[c + d*x]))) - (a^2*Log[1 - Cos[c + d*x]])/d
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^4\,\text{Subst}\left(\int \frac{x}{(a-ax)^2}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^4\,\text{Subst}\left(\int \left(\frac{1}{a^2(-1+x)^2}+\frac{1}{a^2(-1+x)}\right)dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^2}{d(1-\cos(c+dx))} - \frac{a^2\log(1-\cos(c+dx))}{d}$$

Mathematica [A] time = 0.0662764, size = 56, normalized size = 1.4

$$\frac{a^2\csc^2\left(\frac{1}{2}(c+dx)\right)\left(-2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\cos(c+dx)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-1\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Csc[(c + d*x)/2]^2*(-1 - 2*Log[Sin[(c + d*x)/2]] + 2*Cos[c + d*x]*Log[
Sin[(c + d*x)/2]]))/(2*d)
```

Maple [A] time = 0.066, size = 51, normalized size = 1.3

$$-\frac{a^2}{d(-1+\sec(dx+c))} - \frac{a^2\ln(-1+\sec(dx+c))}{d} + \frac{a^2\ln(\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/d*a^2/(-1+sec(d*x+c))-1/d*a^2*ln(-1+sec(d*x+c))+1/d*a^2*ln(sec(d*x+c))
```

Maxima [A] time = 1.12142, size = 46, normalized size = 1.15

$$-\frac{a^2\log(\cos(dx+c)-1) - \frac{a^2}{\cos(dx+c)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(a^2*log(cos(d*x + c) - 1) - a^2/(cos(d*x + c) - 1))/d
```

Fricas [A] time = 1.19576, size = 113, normalized size = 2.82

$$\frac{a^2 - \left(a^2 \cos\left(dx+c\right) - a^2\right) \log\left(-\frac{1}{2} \cos\left(dx+c\right) + \frac{1}{2}\right)}{d \cos\left(dx+c\right) - d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a^2 - (a^2*cos(d*x + c) - a^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x +
c) - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2 \left(\int 2 \cot^3\left(c+dx\right) \sec\left(c+dx\right) dx + \int \cot^3\left(c+dx\right) \sec^2\left(c+dx\right) dx + \int \cot^3\left(c+dx\right) dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cot(c + d*x)**3*sec(c + d*x), x) + Integral(cot(c + d*x)**
3*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**3, x))
```

Giac [B] time = 1.47374, size = 150, normalized size = 3.75

$$-\frac{2\,a^2 \log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 2\,a^2 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - \frac{\left(a^2+\frac{2\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 2*a^2*log(a
bs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a^2 + 2*a^2*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/d
```

3.26 $\int \cot^5(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=85

$$\frac{5a^2}{4d(1-\cos(c+dx))}-\frac{a^2}{4d(1-\cos(c+dx))^2}+\frac{7a^2\log(1-\cos(c+dx))}{8d}+\frac{a^2\log(\cos(c+dx)+1)}{8d}$$

```
[Out] -a^2/(4*d*(1 - Cos[c + d*x])^2) + (5*a^2)/(4*d*(1 - Cos[c + d*x])) + (7*a^2
*Log[1 - Cos[c + d*x]])/(8*d) + (a^2*Log[1 + Cos[c + d*x]])/(8*d)
```

Rubi [A] time = 0.0669472, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{5a^2}{4d(1-\cos(c+dx))}-\frac{a^2}{4d(1-\cos(c+dx))^2}+\frac{7a^2\log(1-\cos(c+dx))}{8d}+\frac{a^2\log(\cos(c+dx)+1)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -a^2/(4*d*(1 - Cos[c + d*x])^2) + (5*a^2)/(4*d*(1 - Cos[c + d*x])) + (7*a^2
*Log[1 - Cos[c + d*x]])/(8*d) + (a^2*Log[1 + Cos[c + d*x]])/(8*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^6\,\text{Subst}\left(\int \frac{x^3}{(a-ax)^3(a+ax)}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^6\,\text{Subst}\left(\int \left(-\frac{1}{2a^4(-1+x)^3}-\frac{5}{4a^4(-1+x)^2}-\frac{7}{8a^4(-1+x)}-\frac{1}{8a^4(1+x)}\right)dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^2}{4d(1-\cos(c+dx))^2}+\frac{5a^2}{4d(1-\cos(c+dx))}+\frac{7a^2\log(1-\cos(c+dx))}{8d}+\frac{a^2\,\text{lo}}{}$$

Mathematica [A] time = 0.267052, size = 86, normalized size = 1.01

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\csc^4\left(\frac{1}{2}(c+dx)\right)-10\csc^2\left(\frac{1}{2}(c+dx)\right)-4\left(7\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(\right.\right.\right.\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*(1 + Cos[c + d*x])^2*(-10*Csc[(c + d*x)/2]^2 + Csc[(c + d*x)/2]^4 - 4
*(Log[Cos[(c + d*x)/2]] + 7*Log[Sin[(c + d*x)/2]]))*Sec[(c + d*x)/2]^4)/(64
*d)
```

Maple [A] time = 0.08, size = 87, normalized size = 1.

$$\frac{a^2\ln(1+\sec(dx+c))}{8d}-\frac{a^2}{4d(-1+\sec(dx+c))^2}+\frac{3a^2}{4d(-1+\sec(dx+c))}+\frac{7a^2\ln(-1+\sec(dx+c))}{8d}-\frac{a^2\ln(\sec(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/8/d*a^2*ln(1+sec(d*x+c))-1/4/d*a^2/(-1+sec(d*x+c))^2+3/4/d*a^2/(-1+sec(d*
x+c))+7/8/d*a^2*ln(-1+sec(d*x+c))-1/d*a^2*ln(sec(d*x+c))
```

Maxima [A] time = 1.10416, size = 97, normalized size = 1.14

$$\frac{a^2\log(\cos(dx+c)+1)+7a^2\log(\cos(dx+c)-1)-\frac{2\left(5a^2\cos(dx+c)-4a^2\right)}{\cos(dx+c)^2-2\cos(dx+c)+1}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(a^2*log(cos(d*x + c) + 1) + 7*a^2*log(cos(d*x + c) - 1) - 2*(5*a^2*cos
(d*x + c) - 4*a^2)/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1))/d
```

Fricas [A] time = 1.13477, size = 320, normalized size = 3.76

$$-\frac{10\,a^2\cos\left(dx+c\right)-8\,a^2-\left(a^2\cos\left(dx+c\right)^2-2\,a^2\cos\left(dx+c\right)+a^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-7\left(a^2\cos\left(dx+c\right)^2-2\right.}{8\left(d\cos\left(dx+c\right)^2-2\,d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(10*a^2*cos(d*x + c) - 8*a^2 - (a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c
) + a^2)*log(1/2*cos(d*x + c) + 1/2) - 7*(a^2*cos(d*x + c)^2 - 2*a^2*cos(d*
x + c) + a^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - 2*d*cos(d*x
 + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.48089, size = 186, normalized size = 2.19

$$\frac{14\,a^2\log\left(\frac{\left|-\cos(dx+c)+1\right|}{\left|\cos(dx+c)+1\right|}\right)-16\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a^2+\frac{8\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{21\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{(\cos(dx+c)-1)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(14*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 16*a^2*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a^2 + 8*a^2*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) + 21*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2)/d
```

3.27 $\int \cot^7(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=127

$$-\frac{23a^2}{16d(1-\cos(c+dx))}-\frac{a^2}{16d(\cos(c+dx)+1)}+\frac{a^2}{2d(1-\cos(c+dx))^2}-\frac{a^2}{12d(1-\cos(c+dx))^3}-\frac{13a^2\log(1-\cos(c+d}{16d}$$

```
[Out] -a^2/(12*d*(1 - Cos[c + d*x])^3) + a^2/(2*d*(1 - Cos[c + d*x])^2) - (23*a^2
)/(16*d*(1 - Cos[c + d*x])) - a^2/(16*d*(1 + Cos[c + d*x])) - (13*a^2*Log[1
 - Cos[c + d*x]])/(16*d) - (3*a^2*Log[1 + Cos[c + d*x]])/(16*d)
```

Rubi [A] time = 0.0865423, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$-\frac{23a^2}{16d(1-\cos(c+dx))}-\frac{a^2}{16d(\cos(c+dx)+1)}+\frac{a^2}{2d(1-\cos(c+dx))^2}-\frac{a^2}{12d(1-\cos(c+dx))^3}-\frac{13a^2\log(1-\cos(c+d}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*(a + a*Sec[c + d*x])^2,x]
```



```
[Out] -a^2/(12*d*(1 - Cos[c + d*x])^3) + a^2/(2*d*(1 - Cos[c + d*x])^2) - (23*a^2
)/(16*d*(1 - Cos[c + d*x])) - a^2/(16*d*(1 + Cos[c + d*x])) - (13*a^2*Log[1
 - Cos[c + d*x]])/(16*d) - (3*a^2*Log[1 + Cos[c + d*x]])/(16*d)
```


Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^8\,\text{Subst}\left(\int \frac{x^5}{(a-ax)^4(a+ax)^2}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^8\,\text{Subst}\left(\int \left(\frac{1}{4a^6(-1+x)^4} + \frac{1}{a^6(-1+x)^3} + \frac{23}{16a^6(-1+x)^2} + \frac{13}{16a^6(-1+x)} - \frac{1}{16a^6(1+x)^2} + \frac{3}{16a^6(\cdots}\right.\right.}{d}$$
$$= -\frac{a^2}{12d(1-\cos(c+dx))^3} + \frac{a^2}{2d(1-\cos(c+dx))^2} - \frac{23a^2}{16d(1-\cos(c+dx))} - \frac{\cdots}{16d(1\cdots}$$

Mathematica [A] time = 0.227062, size = 114, normalized size = 0.9

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\csc^6\left(\frac{1}{2}(c+dx)\right) - 12\csc^4\left(\frac{1}{2}(c+dx)\right) + 69\csc^2\left(\frac{1}{2}(c+dx)\right) + 3\left(\sec^2\left(\frac{1}{2}(c+dx)\cdots\right.\right.\right.}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(69*Csc[(c + d*x)/2]^2 - 12*C
sc[(c + d*x)/2]^4 + Csc[(c + d*x)/2]^6 + 3*(12*Log[Cos[(c + d*x)/2]] + 52*L
og[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^2)))/(384*d)
```

Maple [A] time = 0.081, size = 122, normalized size = 1.

$$\frac{a^2}{16\,d\,(1+\sec(dx+c))} - \frac{3\,a^2\ln(1+\sec(dx+c))}{16\,d} - \frac{a^2}{12\,d\,(-1+\sec(dx+c))^3} + \frac{a^2}{4\,d\,(-1+\sec(dx+c))^2} - \frac{11\,a^2}{16\,d\,(-1+\sec(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/16/d*a^2/(1+sec(d*x+c))-3/16/d*a^2*ln(1+sec(d*x+c))-1/12/d*a^2/(-1+sec(d*
x+c))^3+1/4/d*a^2/(-1+sec(d*x+c))^2-11/16/d*a^2/(-1+sec(d*x+c))-13/16/d*a^2
*ln(-1+sec(d*x+c))+1/d*a^2*ln(sec(d*x+c))
```

Maxima [A] time = 1.17354, size = 147, normalized size = 1.16

$$-\frac{9\,a^2\log\left(\cos\left(dx+c\right)+1\right)+39\,a^2\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(33\,a^2\cos(dx+c)^3-18\,a^2\cos(dx+c)^2-37\,a^2\cos(dx+c)+26\,a^2\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^3+2\,\cos(dx+c)-1}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/48*(9*a^2*log(cos(d*x + c) + 1) + 39*a^2*log(cos(d*x + c) - 1) - 2*(33*a
^2*cos(d*x + c)^3 - 18*a^2*cos(d*x + c)^2 - 37*a^2*cos(d*x + c) + 26*a^2)/(
cos(d*x + c)^4 - 2*cos(d*x + c)^3 + 2*cos(d*x + c) - 1))/d
```

Fricas [A] time = 0.896504, size = 481, normalized size = 3.79

$$\frac{66\,a^2\cos\left(dx+c\right)^3-36\,a^2\cos\left(dx+c\right)^2-74\,a^2\cos\left(dx+c\right)+52\,a^2-9\left(a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right)^3+2\,a^2\cos}{48\left(d\cos\left(dx+c\right)^4-2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(66*a^2*cos(d*x + c)^3 - 36*a^2*cos(d*x + c)^2 - 74*a^2*cos(d*x + c) +
 52*a^2 - 9*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^3 + 2*a^2*cos(d*x + c)
 - a^2)*log(1/2*cos(d*x + c) + 1/2) - 39*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*
x + c)^3 + 2*a^2*cos(d*x + c) - a^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d
*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.6418, size = 251, normalized size = 1.98

$$-\frac{78\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-96\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{3\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{\left(a^2+\frac{9\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{48\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{143\,a^2(\cos(dx+c)-}{(\cos(dx+c)+1)^3}\right.}{(\cos(dx+c)-1)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/96*(78*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 96*a^2*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 3*a^2*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) - (a^2 + 9*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ 48*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 143*a^2*(cos(d*x + c)
- 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)^3/(cos(d*x + c) - 1)^3)/d
```

3.28 $\int \cot^9(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=169

$$\frac{51a^2}{32d(1-\cos(c+dx))}+\frac{9a^2}{64d(\cos(c+dx)+1)}-\frac{3a^2}{4d(1-\cos(c+dx))^2}-\frac{a^2}{64d(\cos(c+dx)+1)^2}+\frac{11a^2}{48d(1-\cos(c+dx))^3}$$

```
[Out] -a^2/(32*d*(1 - Cos[c + d*x])^4) + (11*a^2)/(48*d*(1 - Cos[c + d*x])^3) - (
3*a^2)/(4*d*(1 - Cos[c + d*x])^2) + (51*a^2)/(32*d*(1 - Cos[c + d*x])) - a^
2/(64*d*(1 + Cos[c + d*x])^2) + (9*a^2)/(64*d*(1 + Cos[c + d*x])) + (99*a^2
*Log[1 - Cos[c + d*x]])/(128*d) + (29*a^2*Log[1 + Cos[c + d*x]])/(128*d)
```

Rubi [A] time = 0.111059, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{51a^2}{32d(1-\cos(c+dx))}+\frac{9a^2}{64d(\cos(c+dx)+1)}-\frac{3a^2}{4d(1-\cos(c+dx))^2}-\frac{a^2}{64d(\cos(c+dx)+1)^2}+\frac{11a^2}{48d(1-\cos(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^9*(a + a*Sec[c + d*x])^2,x]
```


```
[Out] -a^2/(32*d*(1 - Cos[c + d*x])^4) + (11*a^2)/(48*d*(1 - Cos[c + d*x])^3) - (
3*a^2)/(4*d*(1 - Cos[c + d*x])^2) + (51*a^2)/(32*d*(1 - Cos[c + d*x])) - a^
2/(64*d*(1 + Cos[c + d*x])^2) + (9*a^2)/(64*d*(1 + Cos[c + d*x])) + (99*a^2
*Log[1 - Cos[c + d*x]])/(128*d) + (29*a^2*Log[1 + Cos[c + d*x]])/(128*d)
```


Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^{10}\,\mathrm{Subst}\left(\int \frac{x^7}{(a-ax)^5(a+ax)^3}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^{10}\,\mathrm{Subst}\left(\int \left(-\frac{1}{8a^8(-1+x)^5} - \frac{11}{16a^8(-1+x)^4} - \frac{3}{2a^8(-1+x)^3} - \frac{51}{32a^8(-1+x)^2} - \frac{99}{128a^8(-1+x)} - \right.\right.}{d}$$
$$= -\frac{a^2}{32d(1-\cos(c+dx))^4} + \frac{11a^2}{48d(1-\cos(c+dx))^3} - \frac{3a^2}{4d(1-\cos(c+dx))^2} + \frac{}{32d(}$$

Mathematica [A] time = 0.331827, size = 146, normalized size = 0.86

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(3\csc^8\left(\frac{1}{2}(c+dx)\right) - 44\csc^6\left(\frac{1}{2}(c+dx)\right) + 288\csc^4\left(\frac{1}{2}(c+dx)\right) - 1224\csc^2\left(\frac{1}{2}(c\right.\right.}{6144d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^9*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(-1224*Csc[(c + d*x)/2]^2 + 2
88*Csc[(c + d*x)/2]^4 - 44*Csc[(c + d*x)/2]^6 + 3*Csc[(c + d*x)/2]^8 - 6*(1
16*Log[Cos[(c + d*x)/2]] + 396*Log[Sin[(c + d*x)/2]] + 18*Sec[(c + d*x)/2]^
2 - Sec[(c + d*x)/2]^4)))/(6144*d)
```

Maple [A] time = 0.095, size = 159, normalized size = 0.9

$$-\frac{a^2}{64\,d\,(1+\sec(dx+c))^2} - \frac{7\,a^2}{64\,d\,(1+\sec(dx+c))} + \frac{29\,a^2\ln(1+\sec(dx+c))}{128\,d} - \frac{a^2}{32\,d\,(-1+\sec(dx+c))^4} + \frac{5}{48\,d\,(-1+\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^9*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/64/d*a^2/(1+sec(d*x+c))^2-7/64/d*a^2/(1+sec(d*x+c))+29/128/d*a^2*ln(1+se
c(d*x+c))-1/32/d*a^2/(-1+sec(d*x+c))^4+5/48/d*a^2/(-1+sec(d*x+c))^3-1/4/d*a
```

```
^2/(-1+sec(d*x+c))^2+21/32/d*a^2/(-1+sec(d*x+c))+99/128/d*a^2*ln(-1+sec(d*x
+c))-1/d*a^2*ln(sec(d*x+c))
```

Maxima [A] time = 1.19769, size = 223, normalized size = 1.32

$$\frac{87\,a^2\log\left(\cos\left(dx+c\right)+1\right)+297\,a^2\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(279\,a^2\cos(dx+c)^5-78\,a^2\cos(dx+c)^4-634\,a^2\cos(dx+c)^3+338\,a^2\cos(dx+c)^2\cdots\right.}{\cos(dx+c)^6-2\cos(dx+c)^5-\cos(dx+c)^4+4\cos(dx+c)^3-\cos(dx+c)^2\cdots}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/384*(87*a^2*log(cos(d*x + c) + 1) + 297*a^2*log(cos(d*x + c) - 1) - 2*(27
9*a^2*cos(d*x + c)^5 - 78*a^2*cos(d*x + c)^4 - 634*a^2*cos(d*x + c)^3 + 338
*a^2*cos(d*x + c)^2 + 343*a^2*cos(d*x + c) - 224*a^2)/(cos(d*x + c)^6 - 2*c
os(d*x + c)^5 - cos(d*x + c)^4 + 4*cos(d*x + c)^3 - cos(d*x + c)^2 - 2*cos(
d*x + c) + 1))/d
```

Fricas [B] time = 0.969955, size = 815, normalized size = 4.82

$$-\frac{558\,a^2\cos\left(dx+c\right)^5-156\,a^2\cos\left(dx+c\right)^4-1268\,a^2\cos\left(dx+c\right)^3+676\,a^2\cos\left(dx+c\right)^2+686\,a^2\cos\left(dx+c\right)-448\,a^2\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/384*(558*a^2*cos(d*x + c)^5 - 156*a^2*cos(d*x + c)^4 - 1268*a^2*cos(d*x
+ c)^3 + 676*a^2*cos(d*x + c)^2 + 686*a^2*cos(d*x + c) - 448*a^2 - 87*(a^2*
cos(d*x + c)^6 - 2*a^2*cos(d*x + c)^5 - a^2*cos(d*x + c)^4 + 4*a^2*cos(d*x
+ c)^3 - a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c) + a^2)*log(1/2*cos(d*x + c
) + 1/2) - 297*(a^2*cos(d*x + c)^6 - 2*a^2*cos(d*x + c)^5 - a^2*cos(d*x + c
)^4 + 4*a^2*cos(d*x + c)^3 - a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c) + a^2)
*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^6 - 2*d*cos(d*x + c)^5 - d*c
os(d*x + c)^4 + 4*d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - 2*d*cos(d*x + c) +
d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**9*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.71284, size = 321, normalized size = 1.9

$$\frac{1188\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-1536\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{96\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{6\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{\left(3\,a^2+\frac{32\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{174\,a\cdots}{(c\cdots}\right.}{\cdots}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1536*(1188*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 1536*a
^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 96*a^2*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) - 6*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2 - (3*a^2 + 32*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 174*a^2*(cos(d
*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 768*a^2*(cos(d*x + c) - 1)^3/(cos(d*x
 + c) + 1)^3 + 2475*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)*(cos(d*x
 + c) + 1)^4/(cos(d*x + c) - 1)^4)/d
```

3.29 $\int (a+a\sec(c+dx))^2\tan^6(c+dx)\,dx$

Optimal. Leaf size=161

$$\frac{a^2\tan^7(c+dx)}{7d}+\frac{a^2\tan^5(c+dx)}{5d}-\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}-\frac{5a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\tan^5(c+dx)\sec}{3d}$$

```
[Out] -(a^2*x) - (5*a^2*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*Tan[c + d*x])/d + (5*
a^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (a^2*Tan[c + d*x]^3)/(3*d) - (5*a^2*
Sec[c + d*x]*Tan[c + d*x]^3)/(12*d) + (a^2*Tan[c + d*x]^5)/(5*d) + (a^2*Sec
[c + d*x]*Tan[c + d*x]^5)/(3*d) + (a^2*Tan[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.178566, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3886, 3473, 8, 2611, 3770, 2607, 30}

$$\frac{a^2\tan^7(c+dx)}{7d}+\frac{a^2\tan^5(c+dx)}{5d}-\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}-\frac{5a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\tan^5(c+dx)\sec}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^6,x]
```

```
[Out] -(a^2*x) - (5*a^2*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*Tan[c + d*x])/d + (5*
a^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (a^2*Tan[c + d*x]^3)/(3*d) - (5*a^2*
Sec[c + d*x]*Tan[c + d*x]^3)/(12*d) + (a^2*Tan[c + d*x]^5)/(5*d) + (a^2*Sec
[c + d*x]*Tan[c + d*x]^5)/(3*d) + (a^2*Tan[c + d*x]^7)/(7*d)
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sec(c+dx))^2\tan^6(c+dx)\,dx &= \int \left(a^2\tan^6(c+dx)+2a^2\sec(c+dx)\tan^6(c+dx)+a^2\sec^2(c+dx)\tan^6(c+dx\right.\\
&= a^2\int \tan^6(c+dx)\,dx+a^2\int \sec^2(c+dx)\tan^6(c+dx)\,dx+\left(2a^2\right)\int \sec(c+dx\\
&= \frac{a^2\tan^5(c+dx)}{5d}+\frac{a^2\sec(c+dx)\tan^5(c+dx)}{3d}-a^2\int \tan^4(c+dx)\,dx-\frac{1}{3}\left(5a^2\right.\\
&= -\frac{a^2\tan^3(c+dx)}{3d}-\frac{5a^2\sec(c+dx)\tan^3(c+dx)}{12d}+\frac{a^2\tan^5(c+dx)}{5d}+\frac{a^2\sec(c-}{}\\
&= \frac{a^2\tan(c+dx)}{d}+\frac{5a^2\sec(c+dx)\tan(c+dx)}{8d}-\frac{a^2\tan^3(c+dx)}{3d}-\frac{5a^2\sec(c+d}{1}\\
&= -a^2x-\frac{5a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\tan(c+dx)}{d}+\frac{5a^2\sec(c+dx)\tan(c+dx}{8d}
\end{aligned}$$

Mathematica [B] time = 1.45914, size = 337, normalized size = 2.09

$$\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\sec^7(c+dx)\left(33600\cos^7(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\text{s}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^6,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*Sec[c + d*x]^7*(33600*Cos[c +
d*x]^7*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]]) + Sec[c]*(-14700*d*x*Cos[d*x] - 14700*d*x*Cos[2*c + d*x]
- 8820*d*x*Cos[2*c + 3*d*x] - 8820*d*x*Cos[4*c + 3*d*x] - 2940*d*x*Cos[4*c
+ 5*d*x] - 2940*d*x*Cos[6*c + 5*d*x] - 420*d*x*Cos[6*c + 7*d*x] - 420*d*x*C
os[8*c + 7*d*x] + 24640*Sin[d*x] - 16240*Sin[2*c + d*x] + 2975*Sin[c + 2*d*
x] + 2975*Sin[3*c + 2*d*x] + 14448*Sin[2*c + 3*d*x] - 10080*Sin[4*c + 3*d*x
] + 980*Sin[3*c + 4*d*x] + 980*Sin[5*c + 4*d*x] + 6496*Sin[4*c + 5*d*x] - 1
680*Sin[6*c + 5*d*x] + 1155*Sin[5*c + 6*d*x] + 1155*Sin[7*c + 6*d*x] + 1168
*Sin[6*c + 7*d*x])))/(215040*d)
```

Maple [A] time = 0.051, size = 226, normalized size = 1.4

$$\frac{a^2(\tan(dx+c))^5}{5d}-\frac{a^2(\tan(dx+c))^3}{3d}+\frac{a^2\tan(dx+c)}{d}-a^2x-\frac{a^2c}{d}+\frac{a^2(\sin(dx+c))^7}{3d(\cos(dx+c))^6}-\frac{a^2(\sin(dx+c))^7}{12d(\cos(dx+c))^4}+\frac{a^2(}{8d(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^6,x)
```

```
[Out] 1/5*a^2*tan(d*x+c)^5/d-1/3*a^2*tan(d*x+c)^3/d+a^2*tan(d*x+c)/d-a^2*x-1/d*a^
2*c+1/3/d*a^2*sin(d*x+c)^7/cos(d*x+c)^6-1/12/d*a^2*sin(d*x+c)^7/cos(d*x+c)^
4+1/8/d*a^2*sin(d*x+c)^7/cos(d*x+c)^2+1/8/d*a^2*sin(d*x+c)^5+5/24/d*a^2*sin
(d*x+c)^3+5/8/d*a^2*sin(d*x+c)-5/8/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/7/d*a^
2*sin(d*x+c)^7/cos(d*x+c)^7
```

Maxima [A] time = 1.77152, size = 204, normalized size = 1.27

$$\frac{240a^2\tan(dx+c)^7+112\left(3\tan(dx+c)^5-5\tan(dx+c)^3-15dx-15c+15\tan(dx+c)\right)a^2-35a^2\left(\frac{2\left(33\sin(dx+c)^5-\right.}{\sin(dx+c)^6-3\text{ s}}\right.}{1680d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/1680*(240*a^2*tan(d*x + c)^7 + 112*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 -
 15*d*x - 15*c + 15*tan(d*x + c))*a^2 - 35*a^2*(2*(33*sin(d*x + c)^5 - 40*s
in(d*x + c)^3 + 15*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin
(d*x + c)^2 - 1) + 15*log(sin(d*x + c) + 1) - 15*log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 0.913171, size = 446, normalized size = 2.77

$$-\frac{1680\,a^2dx\cos\left(dx+c\right)^7+525\,a^2\cos\left(dx+c\right)^7\log\left(\sin\left(dx+c\right)+1\right)-525\,a^2\cos\left(dx+c\right)^7\log\left(-\sin\left(dx+c\right)+1\right)-2\left(11\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/1680*(1680*a^2*d*x*cos(d*x + c)^7 + 525*a^2*cos(d*x + c)^7*log(sin(d*x +
 c) + 1) - 525*a^2*cos(d*x + c)^7*log(-sin(d*x + c) + 1) - 2*(1168*a^2*cos(
d*x + c)^6 + 1155*a^2*cos(d*x + c)^5 - 256*a^2*cos(d*x + c)^4 - 910*a^2*cos
(d*x + c)^3 - 192*a^2*cos(d*x + c)^2 + 280*a^2*cos(d*x + c) + 120*a^2)*sin(
d*x + c))/(d*cos(d*x + c)^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int 2\tan^6\left(c+dx\right)\sec\left(c+dx\right)dx+\int\tan^6\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\tan^6\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**6,x)
```

```
[Out] a**2*(Integral(2*tan(c + d*x)**6*sec(c + d*x), x) + Integral(tan(c + d*x)**
6*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**6, x))
```

Giac [A] time = 4.75821, size = 243, normalized size = 1.51

$$-\frac{840\,(dx+c)a^2+525\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-525\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(315\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-2660\,a^2}{}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] -1/840*(840*(d*x + c)*a^2 + 525*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 52
5*a^2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(315*a^2*tan(1/2*d*x + 1/2*c)^
13 - 2660*a^2*tan(1/2*d*x + 1/2*c)^11 + 9863*a^2*tan(1/2*d*x + 1/2*c)^9 - 2
1216*a^2*tan(1/2*d*x + 1/2*c)^7 + 29673*a^2*tan(1/2*d*x + 1/2*c)^5 - 9660*a
^2*tan(1/2*d*x + 1/2*c)^3 + 1365*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1
/2*c)^2 - 1)^7)/d
```

3.30 $\int (a + a\sec(c + dx))^2 \tan^4(c + dx)\, dx$

Optimal. Leaf size=119

$$\frac{a^2\tan^5(c+dx)}{5d}+\frac{a^2\tan^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+\frac{3a^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^2\tan^3(c+dx)\sec(c+dx)}{2d}-\frac{3a^2\tan(}{}$$

```
[Out] a^2*x + (3*a^2*ArcTanh[Sin[c + d*x]])/(4*d) - (a^2*Tan[c + d*x])/d - (3*a^2
*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (a^2*Tan[c + d*x]^3)/(3*d) + (a^2*Sec[c
 + d*x]*Tan[c + d*x]^3)/(2*d) + (a^2*Tan[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.139138, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3886, 3473, 8, 2611, 3770, 2607, 30}

$$\frac{a^2\tan^5(c+dx)}{5d}+\frac{a^2\tan^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+\frac{3a^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^2\tan^3(c+dx)\sec(c+dx)}{2d}-\frac{3a^2\tan(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^4,x]
```


```
[Out] a^2*x + (3*a^2*ArcTanh[Sin[c + d*x]])/(4*d) - (a^2*Tan[c + d*x])/d - (3*a^2
*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (a^2*Tan[c + d*x]^3)/(3*d) + (a^2*Sec[c
 + d*x]*Tan[c + d*x]^3)/(2*d) + (a^2*Tan[c + d*x]^5)/(5*d)
```


Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sec(c+dx))^2\tan^4(c+dx)\,dx &= \int \left(a^2\tan^4(c+dx)+2a^2\sec(c+dx)\tan^4(c+dx)+a^2\sec^2(c+dx)\tan^4(c+
\\
&= a^2\int \tan^4(c+dx)\,dx+a^2\int \sec^2(c+dx)\tan^4(c+dx)\,dx+\left(2a^2\right)\int \sec(c+
\\
&= \frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\sec(c+dx)\tan^3(c+dx)}{2d}-a^2\int \tan^2(c+dx)\,dx-\frac{1}{2}\left(3a
\\
&= -\frac{a^2\tan(c+dx)}{d}-\frac{3a^2\sec(c+dx)\tan(c+dx)}{4d}+\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\sec(c+
\\
&= a^2x+\frac{3a^2\tanh^{-1}(\sin(c+dx))}{4d}-\frac{a^2\tan(c+dx)}{d}-\frac{3a^2\sec(c+dx)\tan(c+dx)}{4d}
\end{aligned}$$

Mathematica [B] time = 5.40286, size = 558, normalized size = 4.69

$$\frac{1}{960}a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{\cos\left(\frac{c}{2}\right)\left(\frac{151}{\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{36}{\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\right.}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(240*x - (180*Log[Cos[(c + d*x
)/2] - Sin[(c + d*x)/2]])/d + (180*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
)/d - ((293*Cos[(d*x)/2] + 333*Cos[2*c + (3*d*x)/2] + 287*Cos[2*c + (5*d*x)
/2] + 67*Cos[4*c + (7*d*x)/2] + 68*Cos[4*c + (9*d*x)/2])*Sec[c]*Sec[c + d*x
]^5*Sin[(d*x)/2])/(2*d) - (24*Sin[c/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c +
d*x)/2] - Sin[(c + d*x)/2])^4) + (149*Sin[c/2])/(d*(Cos[c/2] - Sin[c/2])*(C
os[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - (24*Sin[c/2])/(d*(Cos[c/2] + Sin[c
/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4) + (149*Sin[c/2])/(d*(Cos[c/2]
 + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (Cos[c/2]*(36/((Cos
[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4) - 151/((Cos[c/2]
 - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - 36/((Cos[c/2] + Sin
[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4) + 151/((Cos[c/2] + Sin[c/2]
)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)))/d))/960
```

Maple [A] time = 0.046, size = 169, normalized size = 1.4

$$\frac{a^2(\tan(dx+c))^3}{3d}-\frac{a^2\tan(dx+c)}{d}+a^2x+\frac{a^2c}{d}+\frac{a^2(\sin(dx+c))^5}{2d(\cos(dx+c))^4}-\frac{a^2(\sin(dx+c))^5}{4d(\cos(dx+c))^2}-\frac{a^2(\sin(dx+c))^3}{4d}-\frac{3a^2\sin}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^4,x)
```

```
[Out] 1/3*a^2*tan(d*x+c)^3/d-a^2*tan(d*x+c)/d+a^2*x+1/d*a^2*c+1/2/d*a^2*sin(d*x+c
)^5/cos(d*x+c)^4-1/4/d*a^2*sin(d*x+c)^5/cos(d*x+c)^2-1/4/d*a^2*sin(d*x+c)^3
-3/4/d*a^2*sin(d*x+c)+3/4/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/5/d*a^2*sin(d*x
+c)^5/cos(d*x+c)^5
```

Maxima [A] time = 1.76646, size = 161, normalized size = 1.35

$$\frac{24\,a^2\tan\left(dx+c\right)^5+40\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a^2+15\,a^2\left(\frac{2\left(5\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}+3\,\log\left(\sin\left(dx\right.\right.\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/120*(24*a^2*tan(d*x + c)^5 + 40*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x
 + c))*a^2 + 15*a^2*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4
- 2*sin(d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)
))/d
```

Fricas [A] time = 1.02987, size = 360, normalized size = 3.03

$$\frac{120\,a^2dx\cos\left(dx+c\right)^5+45\,a^2\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-45\,a^2\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)-2\left(68\,a^2\right.}{120\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/120*(120*a^2*d*x*cos(d*x + c)^5 + 45*a^2*cos(d*x + c)^5*log(sin(d*x + c)
+ 1) - 45*a^2*cos(d*x + c)^5*log(-sin(d*x + c) + 1) - 2*(68*a^2*cos(d*x + c
)^4 + 75*a^2*cos(d*x + c)^3 + 4*a^2*cos(d*x + c)^2 - 30*a^2*cos(d*x + c) -
12*a^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int 2\tan^4\left(c+dx\right)\sec\left(c+dx\right)dx+\int\tan^4\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\tan^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**4,x)
```

```
[Out] a**2*(Integral(2*tan(c + d*x)**4*sec(c + d*x), x) + Integral(tan(c + d*x)**
4*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**4, x))
```

Giac [A] time = 2.71275, size = 200, normalized size = 1.68

$$\frac{60\,(dx+c)a^2 + 45\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 45\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 110\,a^2\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/60*(60*(d*x + c)*a^2 + 45*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*a^2
*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a^2*tan(1/2*d*x + 1/2*c)^9 - 11
0*a^2*tan(1/2*d*x + 1/2*c)^7 + 328*a^2*tan(1/2*d*x + 1/2*c)^5 - 530*a^2*tan
(1/2*d*x + 1/2*c)^3 + 105*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
 - 1)^5)/d
```

3.31 $\int (a+a\sec(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=72

$$\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}-\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{d}-a^2x$$

```
[Out] -(a^2*x) - (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d + (a^2*Sec[
c + d*x]*Tan[c + d*x])/d + (a^2*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.109061, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3886, 3473, 8, 2611, 3770, 2607, 30}

$$\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}-\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{d}-a^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -(a^2*x) - (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d + (a^2*Sec[
c + d*x]*Tan[c + d*x])/d + (a^2*Tan[c + d*x]^3)/(3*d)
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\tan^2(c+dx)\,dx = \int \left(a^2\tan^2(c+dx)+2a^2\sec(c+dx)\tan^2(c+dx)+a^2\sec^2(c+dx)\tan^2(c+dx\right.$$

$$= a^2\int \tan^2(c+dx)\,dx + a^2\int \sec^2(c+dx)\tan^2(c+dx)\,dx + \left(2a^2\right)\int \sec(c+dx$$

$$= \frac{a^2\tan(c+dx)}{d} + \frac{a^2\sec(c+dx)\tan(c+dx)}{d} - a^2\int 1\,dx - a^2\int \sec(c+dx)\,dx$$

$$= -a^2x - \frac{a^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2\tan(c+dx)}{d} + \frac{a^2\sec(c+dx)\tan(c+dx)}{d} +$$

Mathematica [B] time = 6.30994, size = 773, normalized size = 10.74

$$-\frac{1}{4}x\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^2 + \frac{\sin\left(\frac{dx}{2}\right)\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^2}{6d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)} + \frac{\sin\left(\frac{dx}{2}\right.}{6d\left(\sin\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -(x*Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/4 + (Cos[c
+ d*x]^2*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^4*
(a + a*Sec[c + d*x])^2)/(4*d) - (Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)/2] + Si
n[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(4*d) + (Cos
[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(24*d
*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + (Cos[
c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(7*Cos[c/2] - 5*Sin[
c/2]))/(48*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]
)^2) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x
)/2])/(6*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]))
 + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2]
)/(24*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3)
+ (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(-7*Cos[c/2]
- 5*Sin[c/2]))/(48*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 +
(d*x)/2])^2) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*
Sin[(d*x)/2])/(6*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d
*x)/2]))
```

Maple [A] time = 0.038, size = 112, normalized size = 1.6

$$-a^2x + \frac{a^2\tan(dx+c)}{d} - \frac{a^2c}{d} + \frac{a^2(\sin(dx+c))^3}{d(\cos(dx+c))^2} + \frac{a^2\sin(dx+c)}{d} - \frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{a^2(\sin(dx+}{3d(\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^2,x)
```

```
[Out] -a^2*x+a^2*tan(d*x+c)/d-1/d*a^2*c+1/d*a^2*sin(d*x+c)^3/cos(d*x+c)^2+1/d*a^2
*sin(d*x+c)-1/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/3/d*a^2*sin(d*x+c)^3/cos(d*
x+c)^3
```

Maxima [A] time = 1.8317, size = 112, normalized size = 1.56

$$\frac{2a^2\tan(dx+c)^3 - 6(dx+c-\tan(dx+c))a^2 - 3a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} + \log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a^2*tan(d*x + c)^3 - 6*(d*x + c - tan(d*x + c))*a^2 - 3*a^2*(2*sin(d
*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
)))/d
```

Fricas [A] time = 0.966053, size = 284, normalized size = 3.94

$$-\frac{6\,a^2dx\cos\left(dx+c\right)^3+3\,a^2\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,a^2\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)-2\left(2\,a^2\cos\left(a\right.\right.}{6\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*a^2*d*x*cos(d*x + c)^3 + 3*a^2*cos(d*x + c)^3*log(sin(d*x + c) + 1)
 - 3*a^2*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(2*a^2*cos(d*x + c)^2 +
3*a^2*cos(d*x + c) + a^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int 2\tan^2\left(c+dx\right)\sec\left(c+dx\right)dx+\int\tan^2\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\tan^2\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**2,x)
```

```
[Out] a**2*(Integral(2*tan(c + d*x)**2*sec(c + d*x), x) + Integral(tan(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**2, x))
```

Giac [A] time = 1.82555, size = 134, normalized size = 1.86

$$-\frac{3\left(dx+c\right)a^2+3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{4\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)*a^2 + 3*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) - 4*(a^2*tan(1/2*d*x + 1/2*c)^3 - 3*a^2*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.32 $\int \cot^2(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=35

$$-\frac{2a^2\cot(c+dx)}{d}-\frac{2a^2\csc(c+dx)}{d}+a^2(-x)$$

```
[Out] -(a^2*x) - (2*a^2*Cot[c + d*x])/d - (2*a^2*Csc[c + d*x])/d
```

Rubi [A] time = 0.0721307, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3886, 3473, 8, 2606, 3767}

$$-\frac{2a^2\cot(c+dx)}{d}-\frac{2a^2\csc(c+dx)}{d}+a^2(-x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*x) - (2*a^2*Cot[c + d*x])/d - (2*a^2*Csc[c + d*x])/d
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)(a+a\sec(c+dx))^2\,dx &= \int \left(a^2\cot^2(c+dx)+2a^2\cot(c+dx)\csc(c+dx)+a^2\csc^2(c+dx)\right)\,dx\\ &= a^2\int \cot^2(c+dx)\,dx+a^2\int \csc^2(c+dx)\,dx+\left(2a^2\right)\int \cot(c+dx)\csc(c+dx\\ &= -\frac{a^2\cot(c+dx)}{d}-a^2\int 1\,dx-\frac{a^2\,\mathrm{Subst}(\int 1\,dx,x,\cot(c+dx))}{d}-\frac{\left(2a^2\right)\mathrm{Subst}}{}\\ &= -a^2x-\frac{2a^2\cot(c+dx)}{d}-\frac{2a^2\csc(c+dx)}{d}\end{aligned}$$

Mathematica [C] time = 0.0382712, size = 46, normalized size = 1.31

$$-\frac{2a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*a^2*Cot[c/2 + (d*x)/2]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c/2 + (d*x)
/2]^2])/d
```

Maple [A] time = 0.047, size = 50, normalized size = 1.4

$$\frac{1}{d}\left(a^2\left(-\cot(dx+c)-dx-c\right)-2\,\frac{a^2}{\sin(dx+c)}-a^2\cot(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-cot(d*x+c)-d*x-c)-2*a^2/sin(d*x+c)-a^2*cot(d*x+c))
```

Maxima [A] time = 1.66891, size = 65, normalized size = 1.86

$$-\frac{\left(dx + c + \frac{1}{\tan(dx+c)}\right)a^2 + \frac{2\,a^2}{\sin(dx+c)} + \frac{a^2}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((d*x + c + 1/tan(d*x + c))*a^2 + 2*a^2/sin(d*x + c) + a^2/tan(d*x + c))/d
```

Fricas [A] time = 0.898445, size = 96, normalized size = 2.74

$$-\frac{a^2 dx \sin(dx + c) + 2\,a^2 \cos(dx + c) + 2\,a^2}{d \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*d*x*sin(d*x + c) + 2*a^2*cos(d*x + c) + 2*a^2)/(d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int 2\cot^2(c + dx)\sec(c + dx)\,dx + \int \cot^2(c + dx)\sec^2(c + dx)\,dx + \int \cot^2(c + dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cot(c + d*x)**2*sec(c + d*x), x) + Integral(cot(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**2, x))
```

Giac [A] time = 1.36719, size = 42, normalized size = 1.2

$$-\frac{(dx+c)a^2 + \frac{2\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -((d*x + c)*a^2 + 2*a^2/tan(1/2*d*x + 1/2*c))/d
```

3.33 $\int \cot^4(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=69

$$-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^3(c+dx)}{3d}+\frac{2a^2\csc(c+dx)}{d}+a^2x$$

```
[Out] a^2*x + (a^2*Cot[c + d*x])/d - (2*a^2*Cot[c + d*x]^3)/(3*d) + (2*a^2*Csc[c
+ d*x])/d - (2*a^2*Csc[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.110513, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3886, 3473, 8, 2606, 2607, 30}

$$-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^3(c+dx)}{3d}+\frac{2a^2\csc(c+dx)}{d}+a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] a^2*x + (a^2*Cot[c + d*x])/d - (2*a^2*Cot[c + d*x]^3)/(3*d) + (2*a^2*Csc[c
+ d*x])/d - (2*a^2*Csc[c + d*x]^3)/(3*d)
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cot^4(c+dx)(a+a\sec(c+dx))^2\,dx &= \int \left(a^2\cot^4(c+dx)+2a^2\cot^3(c+dx)\csc(c+dx)+a^2\cot^2(c+dx)\csc^2(c+d\right.\\
&= a^2\int \cot^4(c+dx)\,dx+a^2\int \cot^2(c+dx)\csc^2(c+dx)\,dx+\left(2a^2\right)\int \cot^3(c+\\
&= -\frac{a^2\cot^3(c+dx)}{3d}-a^2\int \cot^2(c+dx)\,dx+\frac{a^2\,\mathrm{Subst}\left(\int x^2\,dx,x,-\cot(c+dx)\right)}{d}\\
&= \frac{a^2\cot(c+dx)}{d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{2a^2\csc(c+dx)}{d}-\frac{2a^2\csc^3(c+dx)}{3d}+a^2\int\\
&= a^2x+\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{2a^2\csc(c+dx)}{d}-\frac{2a^2\csc^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.25178, size = 112, normalized size = 1.62

$$\frac{a^2\csc\left(\frac{c}{2}\right)\csc^3\left(\frac{1}{2}(c+dx)\right)\left(-12\sin\left(c+\frac{dx}{2}\right)+10\sin\left(c+\frac{3dx}{2}\right)-9dx\cos\left(c+\frac{dx}{2}\right)-3dx\cos\left(c+\frac{3dx}{2}\right)+3dx\cos\left(2c+\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Csc[c/2]*Csc[(c + d*x)/2]^3*(9*d*x*Cos[(d*x)/2] - 9*d*x*Cos[c + (d*x)/
2] - 3*d*x*Cos[c + (3*d*x)/2] + 3*d*x*Cos[2*c + (3*d*x)/2] - 18*Sin[(d*x)/2
```

```
] - 12*Sin[c + (d*x)/2] + 10*Sin[c + (3*d*x)/2]))/(24*d)
```

Maple [A] time = 0.068, size = 112, normalized size = 1.6

$$\frac{1}{d}\left(a^2\left(-\frac{(\cot(dx+c))^3}{3}+\cot(dx+c)+dx+c\right)+2a^2\left(-1/3\frac{(\cos(dx+c))^4}{(\sin(dx+c))^3}+1/3\frac{(\cos(dx+c))^4}{\sin(dx+c)}+1/3\left(2+(\cos(dx+c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+2*a^2*(-1/3/sin(d*x+c)^3*cos(
d*x+c)^4+1/3/sin(d*x+c)*cos(d*x+c)^4+1/3*(2+cos(d*x+c)^2)*sin(d*x+c))-1/3*a
^2/sin(d*x+c)^3*cos(d*x+c)^3)
```

Maxima [A] time = 1.73252, size = 104, normalized size = 1.51

$$\frac{\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2+\frac{2\left(3\,\sin(dx+c)^2-1\right)a^2}{\sin(dx+c)^3}-\frac{a^2}{\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 + 2*(3*sin(d
*x + c)^2 - 1)*a^2/sin(d*x + c)^3 - a^2/tan(d*x + c)^3)/d
```

Fricas [A] time = 0.940468, size = 190, normalized size = 2.75

$$\frac{5\,a^2\cos(dx+c)^2+a^2\cos(dx+c)-4\,a^2+3\left(a^2dx\cos(dx+c)-a^2dx\right)\sin(dx+c)}{3\,(d\cos(dx+c)-d)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(5*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c) - 4*a^2 + 3*(a^2*d*x*cos(d*x +
 c) - a^2*d*x)*sin(d*x + c))/((d*cos(d*x + c) - d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int 2\cot^4(c+dx)\sec(c+dx)\,dx+\int \cot^4(c+dx)\sec^2(c+dx)\,dx+\int \cot^4(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cot(c + d*x)**4*sec(c + d*x), x) + Integral(cot(c + d*x)**
4*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**4, x))
```

Giac [A] time = 1.50626, size = 68, normalized size = 0.99

$$\frac{6\,(dx+c)a^2+\frac{9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*a^2 + (9*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2)/tan(1/2*d*x + 1
/2*c)^3)/d
```

3.34 $\int \cot^6(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=107

$$-\frac{2a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^5(c+dx)}{5d}+\frac{4a^2\csc^3(c+dx)}{3d}-\frac{2a^2\csc(c+dx)}{d}-a^2x$$

```
[Out] -(a^2*x) - (a^2*Cot[c + d*x])/d + (a^2*Cot[c + d*x]^3)/(3*d) - (2*a^2*Cot[c
 + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x])/d + (4*a^2*Csc[c + d*x]^3)/(3*d) -
(2*a^2*Csc[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.127637, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3886, 3473, 8, 2606, 194, 2607, 30}

$$-\frac{2a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^5(c+dx)}{5d}+\frac{4a^2\csc^3(c+dx)}{3d}-\frac{2a^2\csc(c+dx)}{d}-a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*x) - (a^2*Cot[c + d*x])/d + (a^2*Cot[c + d*x]^3)/(3*d) - (2*a^2*Cot[c
 + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x])/d + (4*a^2*Csc[c + d*x]^3)/(3*d) -
(2*a^2*Csc[c + d*x]^5)/(5*d)
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \cot^6(c+dx)(a+a\sec(c+dx))^2\,dx &= \int \left(a^2\cot^6(c+dx)+2a^2\cot^5(c+dx)\csc(c+dx)+a^2\cot^4(c+dx)\csc^2(c+d\right.\\
&= a^2\int \cot^6(c+dx)\,dx+a^2\int \cot^4(c+dx)\csc^2(c+dx)\,dx+\left(2a^2\right)\int \cot^5(c+\\
&= -\frac{a^2\cot^5(c+dx)}{5d}-a^2\int \cot^4(c+dx)\,dx+\frac{a^2\,\mathrm{Subst}\left(\int x^4\,dx,x,-\cot(c+dx)\right)}{d}\\
&= \frac{a^2\cot^3(c+dx)}{3d}-\frac{2a^2\cot^5(c+dx)}{5d}+a^2\int \cot^2(c+dx)\,dx-\frac{\left(2a^2\right)\mathrm{Subst}\left(\int \left(\right.\right.}{}\\
&= -\frac{a^2\cot(c+dx)}{d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{2a^2\csc(c+dx)}{d}+\frac{4a^2}{}\\
&= -a^2x-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{2a^2\csc(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.776342, size = 194, normalized size = 1.81

$$\frac{a^2 \csc\left(\frac{c}{2}\right) \sec\left(\frac{c}{2}\right) \csc^5\left(\frac{1}{2}(c+dx)\right) \sec\left(\frac{1}{2}(c+dx)\right) (445 \sin(c+dx) - 356 \sin(2(c+dx)) + 89 \sin(3(c+dx)) + 240 \sin(2c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Csc[c/2]*Csc[(c + d*x)/2]^5*Sec[c/2]*Sec[(c + d*x)/2]*(-150*d*x*Cos[d*
x] + 150*d*x*Cos[2*c + d*x] + 120*d*x*Cos[c + 2*d*x] - 120*d*x*Cos[3*c + 2*
d*x] - 30*d*x*Cos[2*c + 3*d*x] + 30*d*x*Cos[4*c + 3*d*x] - 80*Sin[c] + 280*
Sin[d*x] + 445*Sin[c + d*x] - 356*Sin[2*(c + d*x)] + 89*Sin[3*(c + d*x)] +
240*Sin[2*c + d*x] - 296*Sin[c + 2*d*x] - 120*Sin[3*c + 2*d*x] + 104*Sin[2*
c + 3*d*x]))/(3840*d)
```

Maple [A] time = 0.067, size = 155, normalized size = 1.5

$$\frac{1}{d}\left(a^2\left(-\frac{(\cot(dx+c))^5}{5} + \frac{(\cot(dx+c))^3}{3} - \cot(dx+c) - dx - c\right) + 2a^2\left(-1/5\frac{(\cos(dx+c))^6}{(\sin(dx+c))^5} + 1/15\frac{(\cos(dx+c))^6}{(\sin(dx+c))^3} - 1/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+2*a^2*(-1/5/
sin(d*x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c)*cos
(d*x+c)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-1/5*a^2/sin(d
*x+c)^5*cos(d*x+c)^5)
```

Maxima [A] time = 1.78702, size = 131, normalized size = 1.22

$$-\frac{\left(15\,dx + 15\,c + \frac{15\,\tan(dx+c)^4 - 5\,\tan(dx+c)^2 + 3}{\tan(dx+c)^5}\right)a^2 + \frac{2\left(15\,\sin(dx+c)^4 - 10\,\sin(dx+c)^2 + 3\right)a^2}{\sin(dx+c)^5} + \frac{3\,a^2}{\tan(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/15*((15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x
+ c)^5)*a^2 + 2*(15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 + 3)*a^2/sin(d*x + c
)^5 + 3*a^2/tan(d*x + c)^5)/d
```

Fricas [A] time = 0.852108, size = 298, normalized size = 2.79

$$-\frac{26\,a^2\cos\left(dx+c\right)^3-22\,a^2\cos\left(dx+c\right)^2-17\,a^2\cos\left(dx+c\right)+16\,a^2+15\left(a^2dx\cos\left(dx+c\right)^2-2\,a^2dx\cos\left(dx+c\right)+a^2}{15\left(d\cos\left(dx+c\right)^2-2\,d\cos\left(dx+c\right)+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(26*a^2*cos(d*x + c)^3 - 22*a^2*cos(d*x + c)^2 - 17*a^2*cos(d*x + c)
+ 16*a^2 + 15*(a^2*d*x*cos(d*x + c)^2 - 2*a^2*d*x*cos(d*x + c) + a^2*d*x)*s
in(d*x + c))/((d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.48166, size = 108, normalized size = 1.01

$$-\frac{120\left(dx+c\right)a^2-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{165\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-25\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/120*(120*(d*x + c)*a^2 - 15*a^2*tan(1/2*d*x + 1/2*c) + (165*a^2*tan(1/2*
d*x + 1/2*c)^4 - 25*a^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2)/tan(1/2*d*x + 1/2*c
)^5)/d
```

3.35 $\int \cot^8(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=139

$$-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^7(c+dx)}{7d}+\frac{6a^2\csc^5(c+dx)}{5d}-2a^2\,\mathrm{c}$$

```
[Out] a^2*x + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^3)/(3*d) + (a^2*Cot[c + d*
x]^5)/(5*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) + (2*a^2*Csc[c + d*x])/d - (2*a^
2*Csc[c + d*x]^3)/d + (6*a^2*Csc[c + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x]^7)
/(7*d)
```

Rubi [A] time = 0.140545, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3886, 3473, 8, 2606, 194, 2607, 30}

$$-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^7(c+dx)}{7d}+\frac{6a^2\csc^5(c+dx)}{5d}-2a^2\,\mathrm{c}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] a^2*x + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^3)/(3*d) + (a^2*Cot[c + d*
x]^5)/(5*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) + (2*a^2*Csc[c + d*x])/d - (2*a^
2*Csc[c + d*x]^3)/d + (6*a^2*Csc[c + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x]^7)
/(7*d)
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^2\,dx = \int \left(a^2\cot^8(c+dx)+2a^2\cot^7(c+dx)\csc(c+dx)+a^2\cot^6(c+dx)\csc^2(c+d\right.$$
$$= a^2\int \cot^8(c+dx)\,dx + a^2\int \cot^6(c+dx)\csc^2(c+dx)\,dx + \left(2a^2\right)\int \cot^7(c+$$
$$= -\frac{a^2\cot^7(c+dx)}{7d} - a^2\int \cot^6(c+dx)\,dx + \frac{a^2\,\mathrm{Subst}\left(\int x^6\,dx, x, -\cot(c+dx)\right)}{d}$$
$$= \frac{a^2\cot^5(c+dx)}{5d} - \frac{2a^2\cot^7(c+dx)}{7d} + a^2\int \cot^4(c+dx)\,dx - \frac{\left(2a^2\right)\mathrm{Subst}\left(\int \left(\right.\right.}{}$$
$$= -\frac{a^2\cot^3(c+dx)}{3d} + \frac{a^2\cot^5(c+dx)}{5d} - \frac{2a^2\cot^7(c+dx)}{7d} + \frac{2a^2\csc(c+dx)}{d} - \frac{2a}{}$$
$$= \frac{a^2\cot(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d} + \frac{a^2\cot^5(c+dx)}{5d} - \frac{2a^2\cot^7(c+dx)}{7d} + \frac{2a^2\,c}{}$$
$$= a^2x + \frac{a^2\cot(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d} + \frac{a^2\cot^5(c+dx)}{5d} - \frac{2a^2\cot^7(c+dx)}{7d} +$$

Mathematica [B] time = 1.07294, size = 312, normalized size = 2.24

$$\frac{a^2\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\csc^7\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)(-16002\sin(c+dx)+9144\sin(2(c+dx))+3429\sin(3(c+dx))-4}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Csc[c/2]*Csc[(c + d*x)/2]^7*Sec[c/2]*Sec[(c + d*x)/2]^3*(5880*d*x*Cos[
d*x] - 5880*d*x*Cos[2*c + d*x] - 3360*d*x*Cos[c + 2*d*x] + 3360*d*x*Cos[3*c
 + 2*d*x] - 1260*d*x*Cos[2*c + 3*d*x] + 1260*d*x*Cos[4*c + 3*d*x] + 1680*d*
x*Cos[3*c + 4*d*x] - 1680*d*x*Cos[5*c + 4*d*x] - 420*d*x*Cos[4*c + 5*d*x] +
 420*d*x*Cos[6*c + 5*d*x] + 4032*Sin[c] - 9632*Sin[d*x] - 16002*Sin[c + d*x
] + 9144*Sin[2*(c + d*x)] + 3429*Sin[3*(c + d*x)] - 4572*Sin[4*(c + d*x)] +
 1143*Sin[5*(c + d*x)] - 11760*Sin[2*c + d*x] + 8864*Sin[c + 2*d*x] + 3360*
Sin[3*c + 2*d*x] + 2064*Sin[2*c + 3*d*x] + 2520*Sin[4*c + 3*d*x] - 4432*Sin
[3*c + 4*d*x] - 1680*Sin[5*c + 4*d*x] + 1528*Sin[4*c + 5*d*x]))/(860160*d)
```

Maple [A] time = 0.078, size = 188, normalized size = 1.4

$$\frac{1}{d}\left(a^2\left(-\frac{(\cot(dx+c))^7}{7}+\frac{(\cot(dx+c))^5}{5}-\frac{(\cot(dx+c))^3}{3}+\cot(dx+c)+dx+c\right)+2a^2\left(-1/7\frac{(\cos(dx+c))^8}{(\sin(dx+c))^7}+1/35\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^8*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/7*cot(d*x+c)^7+1/5*cot(d*x+c)^5-1/3*cot(d*x+c)^3+cot(d*x+c)+d*
x+c)+2*a^2*(-1/7/sin(d*x+c)^7*cos(d*x+c)^8+1/35/sin(d*x+c)^5*cos(d*x+c)^8-1
/35/sin(d*x+c)^3*cos(d*x+c)^8+1/7/sin(d*x+c)*cos(d*x+c)^8+1/7*(16/5+cos(d*x
+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-1/7*a^2/sin(d*x+c)^7*c
os(d*x+c)^7)
```

Maxima [A] time = 1.76686, size = 158, normalized size = 1.14

$$\frac{\left(105\,dx+105\,c+\frac{105\,\tan(dx+c)^{6}-35\,\tan(dx+c)^{4}+21\,\tan(dx+c)^{2}-15}{\tan(dx+c)^{7}}\right)a^{2}+\frac{6\left(35\,\sin(dx+c)^{6}-35\,\sin(dx+c)^{4}+21\,\sin(dx+c)^{2}-5\right)a^{2}}{\sin(dx+c)^{7}}-\frac{15\,a^{2}}{\tan(dx+c)^{7}}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*((105*d*x + 105*c + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan(
d*x + c)^2 - 15)/tan(d*x + c)^7)*a^2 + 6*(35*sin(d*x + c)^6 - 35*sin(d*x +
c)^4 + 21*sin(d*x + c)^2 - 5)*a^2/sin(d*x + c)^7 - 15*a^2/tan(d*x + c)^7)/d
```

Fricas [A] time = 1.07674, size = 435, normalized size = 3.13

$$\frac{191\,a^{2}\cos\left(dx+c\right)^{5}-172\,a^{2}\cos\left(dx+c\right)^{4}-253\,a^{2}\cos\left(dx+c\right)^{3}+258\,a^{2}\cos\left(dx+c\right)^{2}+87\,a^{2}\cos\left(dx+c\right)-96\,a^{2}+105}{105\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{3}+2\,d\cos\left(dx+c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(191*a^2*cos(d*x + c)^5 - 172*a^2*cos(d*x + c)^4 - 253*a^2*cos(d*x +
c)^3 + 258*a^2*cos(d*x + c)^2 + 87*a^2*cos(d*x + c) - 96*a^2 + 105*(a^2*d*x
*cos(d*x + c)^4 - 2*a^2*d*x*cos(d*x + c)^3 + 2*a^2*d*x*cos(d*x + c) - a^2*d
*x)*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c
) - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**8*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.49927, size = 151, normalized size = 1.09

$$\frac{35\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3360\,(dx+c)a^{2}-735\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{4410\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-770\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+147\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-15\,a^{2}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}}}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3360*(35*a^2*tan(1/2*d*x + 1/2*c)^3 + 3360*(d*x + c)*a^2 - 735*a^2*tan(1/
2*d*x + 1/2*c) + (4410*a^2*tan(1/2*d*x + 1/2*c)^6 - 770*a^2*tan(1/2*d*x + 1
/2*c)^4 + 147*a^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2)/tan(1/2*d*x + 1/2*c)^7)/
d
```

3.36 $\int \cot^{10}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=179

$$-\frac{2a^2\cot^9(c+dx)}{9d}+\frac{a^2\cot^7(c+dx)}{7d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^9(c+dx)}{9d}+\frac{8a^2\csc^7}{7}$$

```
[Out] -(a^2*x) - (a^2*Cot[c + d*x])/d + (a^2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c +
 d*x]^5)/(5*d) + (a^2*Cot[c + d*x]^7)/(7*d) - (2*a^2*Cot[c + d*x]^9)/(9*d)
- (2*a^2*Csc[c + d*x])/d + (8*a^2*Csc[c + d*x]^3)/(3*d) - (12*a^2*Csc[c + d
*x]^5)/(5*d) + (8*a^2*Csc[c + d*x]^7)/(7*d) - (2*a^2*Csc[c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.154976, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3886, 3473, 8, 2606, 194, 2607, 30}

$$-\frac{2a^2\cot^9(c+dx)}{9d}+\frac{a^2\cot^7(c+dx)}{7d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^9(c+dx)}{9d}+\frac{8a^2\csc^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^10*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*x) - (a^2*Cot[c + d*x])/d + (a^2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c +
 d*x]^5)/(5*d) + (a^2*Cot[c + d*x]^7)/(7*d) - (2*a^2*Cot[c + d*x]^9)/(9*d)
- (2*a^2*Csc[c + d*x])/d + (8*a^2*Csc[c + d*x]^3)/(3*d) - (12*a^2*Csc[c + d
*x]^5)/(5*d) + (8*a^2*Csc[c + d*x]^7)/(7*d) - (2*a^2*Csc[c + d*x]^9)/(9*d)
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^2\,dx = \int \left(a^2\cot^{10}(c+dx)+2a^2\cot^9(c+dx)\csc(c+dx)+a^2\cot^8(c+dx)\csc^2(c+d\right.$$
$$= a^2\int \cot^{10}(c+dx)\,dx + a^2\int \cot^8(c+dx)\csc^2(c+dx)\,dx + \left(2a^2\right)\int \cot^9(c+$$
$$= -\frac{a^2\cot^9(c+dx)}{9d} - a^2\int \cot^8(c+dx)\,dx + \frac{a^2\,\mathrm{Subst}\left(\int x^8\,dx, x, -\cot(c+dx)\right)}{d}$$
$$= \frac{a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9(c+dx)}{9d} + a^2\int \cot^6(c+dx)\,dx - \frac{\left(2a^2\right)\mathrm{Subst}\left(\int (1\right.}{}$$
$$= -\frac{a^2\cot^5(c+dx)}{5d} + \frac{a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9(c+dx)}{9d} - \frac{2a^2\csc(c+dx)}{d} + \frac{8a^2}{}$$
$$= \frac{a^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^5(c+dx)}{5d} + \frac{a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9(c+dx)}{9d} - \frac{2a^2\,c}{}$$
$$= -\frac{a^2\cot(c+dx)}{d} + \frac{a^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^5(c+dx)}{5d} + \frac{a^2\cot^7(c+dx)}{7d} - \frac{2a^2\,c}{}$$
$$= -a^2x - \frac{a^2\cot(c+dx)}{d} + \frac{a^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^5(c+dx)}{5d} + \frac{a^2\cot^7(c+dx)}{7d} -$$

Mathematica [B] time = 1.87963, size = 428, normalized size = 2.39

$$-\frac{a^2\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\csc^9\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)(-1152405\sin(c+dx)+512180\sin(2(c+dx))+486571\sin(3(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^10*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*Csc[c/2]*Csc[(c + d*x)/2]^9*Sec[c/2]*Sec[(c + d*x)/2]^5*(453600*d*x*C
os[d*x] - 453600*d*x*Cos[2*c + d*x] - 201600*d*x*Cos[c + 2*d*x] + 201600*d*
x*Cos[3*c + 2*d*x] - 191520*d*x*Cos[2*c + 3*d*x] + 191520*d*x*Cos[4*c + 3*d
*x] + 161280*d*x*Cos[3*c + 4*d*x] - 161280*d*x*Cos[5*c + 4*d*x] + 10080*d*x
*Cos[4*c + 5*d*x] - 10080*d*x*Cos[6*c + 5*d*x] - 40320*d*x*Cos[5*c + 6*d*x]
 + 40320*d*x*Cos[7*c + 6*d*x] + 10080*d*x*Cos[6*c + 7*d*x] - 10080*d*x*Cos[
8*c + 7*d*x] + 259584*Sin[c] - 897024*Sin[d*x] - 1152405*Sin[c + d*x] + 512
180*Sin[2*(c + d*x)] + 486571*Sin[3*(c + d*x)] - 409744*Sin[4*(c + d*x)] -
25609*Sin[5*(c + d*x)] + 102436*Sin[6*(c + d*x)] - 25609*Sin[7*(c + d*x)] -
 825216*Sin[2*c + d*x] + 622976*Sin[c + 2*d*x] + 142464*Sin[3*c + 2*d*x] +
297088*Sin[2*c + 3*d*x] + 430080*Sin[4*c + 3*d*x] - 424192*Sin[3*c + 4*d*x]
 - 188160*Sin[5*c + 4*d*x] + 2048*Sin[4*c + 5*d*x] - 40320*Sin[6*c + 5*d*x]
 + 112768*Sin[5*c + 6*d*x] + 40320*Sin[7*c + 6*d*x] - 38272*Sin[6*c + 7*d*x
]))/(330301440*d)
```

Maple [A] time = 0.087, size = 231, normalized size = 1.3

$$\frac{1}{d}\left(a^2\left(-\frac{(\cot(dx+c))^9}{9}+\frac{(\cot(dx+c))^7}{7}-\frac{(\cot(dx+c))^5}{5}+\frac{(\cot(dx+c))^3}{3}-\cot(dx+c)-dx-c\right)+2a^2\left(-1/9\frac{(\cos}{(\sin}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^10*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/9*cot(d*x+c)^9+1/7*cot(d*x+c)^7-1/5*cot(d*x+c)^5+1/3*cot(d*x+c
)^3-cot(d*x+c)-d*x-c)+2*a^2*(-1/9/sin(d*x+c)^9*cos(d*x+c)^10+1/63/sin(d*x+c
)^7*cos(d*x+c)^10-1/105/sin(d*x+c)^5*cos(d*x+c)^10+1/63/sin(d*x+c)^3*cos(d*
x+c)^10-1/9/sin(d*x+c)*cos(d*x+c)^10-1/9*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c
)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))-1/9*a^2/sin(d*x+c)^9
*cos(d*x+c)^9)
```

Maxima [A] time = 2.10935, size = 185, normalized size = 1.03

$$-\frac{\left(315\,dx+315\,c+\frac{315\,\tan(dx+c)^8-105\,\tan(dx+c)^6+63\,\tan(dx+c)^4-45\,\tan(dx+c)^2+35}{\tan(dx+c)^9}\right)a^2+\frac{2\left(315\,\sin(dx+c)^8-420\,\sin(dx+c)^6+378\,\sin(dx+c)^4-\right.}{\sin(dx+c)^9}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/315*((315*d*x + 315*c + (315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 + 63*ta
n(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)/tan(d*x + c)^9)*a^2 + 2*(315*sin(d*x
 + c)^8 - 420*sin(d*x + c)^6 + 378*sin(d*x + c)^4 - 180*sin(d*x + c)^2 + 35
)*a^2/sin(d*x + c)^9 + 35*a^2/tan(d*x + c)^9)/d
```

Fricas [A] time = 0.947558, size = 694, normalized size = 3.88

$$-\frac{598\,a^2\cos(dx+c)^7-566\,a^2\cos(dx+c)^6-1212\,a^2\cos(dx+c)^5+1310\,a^2\cos(dx+c)^4+860\,a^2\cos(dx+c)^3-1014}{315\left(d\cos(dx+c)^6-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/315*(598*a^2*cos(d*x + c)^7 - 566*a^2*cos(d*x + c)^6 - 1212*a^2*cos(d*x
+ c)^5 + 1310*a^2*cos(d*x + c)^4 + 860*a^2*cos(d*x + c)^3 - 1014*a^2*cos(d*
x + c)^2 - 197*a^2*cos(d*x + c) + 256*a^2 + 315*(a^2*d*x*cos(d*x + c)^6 - 2
*a^2*d*x*cos(d*x + c)^5 - a^2*d*x*cos(d*x + c)^4 + 4*a^2*d*x*cos(d*x + c)^3
 - a^2*d*x*cos(d*x + c)^2 - 2*a^2*d*x*cos(d*x + c) + a^2*d*x)*sin(d*x + c))
/((d*cos(d*x + c)^6 - 2*d*cos(d*x + c)^5 - d*cos(d*x + c)^4 + 4*d*cos(d*x +
 c)^3 - d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**10*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.54551, size = 196, normalized size = 1.09

$$\frac{63\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-945\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-40320\,(dx+c)a^2+11655\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{51345\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-9}{}}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/40320*(63*a^2*tan(1/2*d*x + 1/2*c)^5 - 945*a^2*tan(1/2*d*x + 1/2*c)^3 - 4
0320*(d*x + c)*a^2 + 11655*a^2*tan(1/2*d*x + 1/2*c) - (51345*a^2*tan(1/2*d*
x + 1/2*c)^8 - 9765*a^2*tan(1/2*d*x + 1/2*c)^6 + 2331*a^2*tan(1/2*d*x + 1/2
*c)^4 - 405*a^2*tan(1/2*d*x + 1/2*c)^2 + 35*a^2)/tan(1/2*d*x + 1/2*c)^9)/d
```

3.37 $\int (a + a\sec(c + dx))^3 \tan^9(c + dx)\, dx$

Optimal. Leaf size=210

$$\frac{a^3\sec^{11}(c+dx)}{11d} + \frac{3a^3\sec^{10}(c+dx)}{10d} - \frac{a^3\sec^9(c+dx)}{9d} - \frac{11a^3\sec^8(c+dx)}{8d} - \frac{6a^3\sec^7(c+dx)}{7d} + \frac{7a^3\sec^6(c+dx)}{3d} + 14$$

```
[Out] -((a^3*Log[Cos[c + d*x]])/d) + (3*a^3*Sec[c + d*x])/d - (a^3*Sec[c + d*x]^2
)/(2*d) - (11*a^3*Sec[c + d*x]^3)/(3*d) - (3*a^3*Sec[c + d*x]^4)/(2*d) + (1
4*a^3*Sec[c + d*x]^5)/(5*d) + (7*a^3*Sec[c + d*x]^6)/(3*d) - (6*a^3*Sec[c +
 d*x]^7)/(7*d) - (11*a^3*Sec[c + d*x]^8)/(8*d) - (a^3*Sec[c + d*x]^9)/(9*d)
 + (3*a^3*Sec[c + d*x]^10)/(10*d) + (a^3*Sec[c + d*x]^11)/(11*d)
```

Rubi [A] time = 0.10263, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{a^3\sec^{11}(c+dx)}{11d} + \frac{3a^3\sec^{10}(c+dx)}{10d} - \frac{a^3\sec^9(c+dx)}{9d} - \frac{11a^3\sec^8(c+dx)}{8d} - \frac{6a^3\sec^7(c+dx)}{7d} + \frac{7a^3\sec^6(c+dx)}{3d} + 14$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^9,x]
```



```
[Out] -((a^3*Log[Cos[c + d*x]])/d) + (3*a^3*Sec[c + d*x])/d - (a^3*Sec[c + d*x]^2
)/(2*d) - (11*a^3*Sec[c + d*x]^3)/(3*d) - (3*a^3*Sec[c + d*x]^4)/(2*d) + (1
4*a^3*Sec[c + d*x]^5)/(5*d) + (7*a^3*Sec[c + d*x]^6)/(3*d) - (6*a^3*Sec[c +
 d*x]^7)/(7*d) - (11*a^3*Sec[c + d*x]^8)/(8*d) - (a^3*Sec[c + d*x]^9)/(9*d)
 + (3*a^3*Sec[c + d*x]^10)/(10*d) + (a^3*Sec[c + d*x]^11)/(11*d)
```


Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\tan^9(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^4(a+ax)^7}{x^{12}}\,dx,x,\cos(c+dx)\right)}{a^8d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^{11}}{x^{12}}+\frac{3a^{11}}{x^{11}}-\frac{a^{11}}{x^{10}}-\frac{11a^{11}}{x^9}-\frac{6a^{11}}{x^8}+\frac{14a^{11}}{x^7}+\frac{14a^{11}}{x^6}-\frac{6a^{11}}{x^5}-\frac{11a^{11}}{x^4}-\frac{a^{11}}{x^3}+\right.}{a^8d}$$
$$= -\frac{a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}-\frac{a^3\sec^2(c+dx)}{2d}-\frac{11a^3\sec^3(c+dx)}{3d}-$$

Mathematica [A] time = 0.729802, size = 214, normalized size = 1.02

$$-\frac{a^3\sec^{11}(c+dx)(-1613260\cos(2(c+dx))+960960\cos(3(c+dx))-1131504\cos(4(c+dx))+314160\cos(5(c+dx))-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^9,x]
```

```
[Out] -(a^3*(-1151740 - 1613260*Cos[2*(c + d*x)] + 960960*Cos[3*(c + d*x)] - 1131
504*Cos[4*(c + d*x)] + 314160*Cos[5*(c + d*x)] - 432894*Cos[6*(c + d*x)] +
145530*Cos[7*(c + d*x)] - 106260*Cos[8*(c + d*x)] + 6930*Cos[9*(c + d*x)] -
 20790*Cos[10*(c + d*x)] + 1143450*Cos[3*(c + d*x)]*Log[Cos[c + d*x]] + 571
725*Cos[5*(c + d*x)]*Log[Cos[c + d*x]] + 190575*Cos[7*(c + d*x)]*Log[Cos[c
+ d*x]] + 38115*Cos[9*(c + d*x)]*Log[Cos[c + d*x]] + 3465*Cos[11*(c + d*x)]
*Log[Cos[c + d*x]] + 462*Cos[c + d*x]*(2606 + 3465*Log[Cos[c + d*x]]))*Sec[
c + d*x]^11)/(3548160*d)
```

Maple [A] time = 0.063, size = 351, normalized size = 1.7

$$\frac{4352\,a^3\cos(dx+c)}{3465\,d}+\frac{a^3(\sin(dx+c))^{10}}{11\,d(\cos(dx+c))^{11}}+\frac{3\,a^3(\sin(dx+c))^{10}}{10\,d(\cos(dx+c))^{10}}+\frac{34\,a^3(\sin(dx+c))^{10}}{99\,d(\cos(dx+c))^9}-\frac{34\,a^3(\sin(dx+c))^{10}}{693\,d(\cos(dx+c))^7}+\frac{34}{115}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^9,x)
```

```
[Out] 4352/3465*a^3*cos(d*x+c)/d+1/11/d*a^3*sin(d*x+c)^10/cos(d*x+c)^11+3/10/d*a^
3*sin(d*x+c)^10/cos(d*x+c)^10+34/99/d*a^3*sin(d*x+c)^10/cos(d*x+c)^9-34/693
/d*a^3*sin(d*x+c)^10/cos(d*x+c)^7+34/1155/d*a^3*sin(d*x+c)^10/cos(d*x+c)^5-
34/693/d*a^3*sin(d*x+c)^10/cos(d*x+c)^3+34/99/d*a^3*sin(d*x+c)^10/cos(d*x+c
)+34/99/d*a^3*cos(d*x+c)*sin(d*x+c)^8+272/693/d*a^3*cos(d*x+c)*sin(d*x+c)^6
+544/1155/d*a^3*cos(d*x+c)*sin(d*x+c)^4+2176/3465/d*a^3*cos(d*x+c)*sin(d*x+
c)^2+1/8/d*a^3*tan(d*x+c)^8-1/6/d*a^3*tan(d*x+c)^6+1/4/d*a^3*tan(d*x+c)^4-1
/2/d*a^3*tan(d*x+c)^2-a^3*ln(cos(d*x+c))/d
```

Maxima [A] time = 1.64076, size = 219, normalized size = 1.04

$$-\frac{27720\,a^3\log\left(\cos\left(dx+c\right)\right)-\frac{83160\,a^3\cos(dx+c)^{10}-13860\,a^3\cos(dx+c)^9-101640\,a^3\cos(dx+c)^8-41580\,a^3\cos(dx+c)^7+77616\,a^3\cos(dx+c)^6+64680}{\cos(dx+c)}}{27720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -1/27720*(27720*a^3*log(cos(d*x + c)) - (83160*a^3*cos(d*x + c)^10 - 13860*
a^3*cos(d*x + c)^9 - 101640*a^3*cos(d*x + c)^8 - 41580*a^3*cos(d*x + c)^7 +
 77616*a^3*cos(d*x + c)^6 + 64680*a^3*cos(d*x + c)^5 - 23760*a^3*cos(d*x +
c)^4 - 38115*a^3*cos(d*x + c)^3 - 3080*a^3*cos(d*x + c)^2 + 8316*a^3*cos(d*
x + c) + 2520*a^3)/cos(d*x + c)^11)/d
```

Fricas [A] time = 1.05857, size = 481, normalized size = 2.29

$$-\frac{27720\,a^3\cos\left(dx+c\right)^{11}\log\left(-\cos\left(dx+c\right)\right)-83160\,a^3\cos\left(dx+c\right)^{10}+13860\,a^3\cos\left(dx+c\right)^9+101640\,a^3\cos\left(dx+c\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/27720*(27720*a^3*cos(d*x + c)^11*log(-cos(d*x + c)) - 83160*a^3*cos(d*x
+ c)^10 + 13860*a^3*cos(d*x + c)^9 + 101640*a^3*cos(d*x + c)^8 + 41580*a^3*
cos(d*x + c)^7 - 77616*a^3*cos(d*x + c)^6 - 64680*a^3*cos(d*x + c)^5 + 2376
0*a^3*cos(d*x + c)^4 + 38115*a^3*cos(d*x + c)^3 + 3080*a^3*cos(d*x + c)^2 -
 8316*a^3*cos(d*x + c) - 2520*a^3)/(d*cos(d*x + c)^11)
```

Sympy [A] time = 115.078, size = 439, normalized size = 2.09

$$\begin{cases} \frac{a^3 \log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{a^3 \tan^8(c+dx)\sec^3(c+dx)}{11d} + \frac{3a^3 \tan^8(c+dx)\sec^2(c+dx)}{10d} + \frac{a^3 \tan^8(c+dx)\sec(c+dx)}{3d} + \frac{a^3 \tan^8(c+dx)}{8d} - \frac{8a^3 \tan^6(c+dx)\sec}{99d} \\ x\left(a\sec(c)+a\right)^3 \tan^9(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**9,x)
```

```
[Out] Piecewise((a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*tan(c + d*x)**8*sec(c
 + d*x)**3/(11*d) + 3*a**3*tan(c + d*x)**8*sec(c + d*x)**2/(10*d) + a**3*ta
n(c + d*x)**8*sec(c + d*x)/(3*d) + a**3*tan(c + d*x)**8/(8*d) - 8*a**3*tan(
c + d*x)**6*sec(c + d*x)**3/(99*d) - 3*a**3*tan(c + d*x)**6*sec(c + d*x)**2
/(10*d) - 8*a**3*tan(c + d*x)**6*sec(c + d*x)/(21*d) - a**3*tan(c + d*x)**6
/(6*d) + 16*a**3*tan(c + d*x)**4*sec(c + d*x)**3/(231*d) + 3*a**3*tan(c + d
*x)**4*sec(c + d*x)**2/(10*d) + 16*a**3*tan(c + d*x)**4*sec(c + d*x)/(35*d)
 + a**3*tan(c + d*x)**4/(4*d) - 64*a**3*tan(c + d*x)**2*sec(c + d*x)**3/(11
55*d) - 3*a**3*tan(c + d*x)**2*sec(c + d*x)**2/(10*d) - 64*a**3*tan(c + d*x
)**2*sec(c + d*x)/(105*d) - a**3*tan(c + d*x)**2/(2*d) + 128*a**3*sec(c + d
*x)**3/(3465*d) + 3*a**3*sec(c + d*x)**2/(10*d) + 128*a**3*sec(c + d*x)/(10
5*d), Ne(d, 0)), (x*(a*sec(c) + a)**3*tan(c)**9, True))
```

Giac [A] time = 14.2469, size = 495, normalized size = 2.36

$$\frac{27720\,a^3 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 27720\,a^3 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{153343\,a^3 + \frac{1742213\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{9043705\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{2836927}{(c}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^9,x, algorithm="giac")
```

```
[Out] 1/27720*(27720*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2
7720*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (153343*a^3
 + 1742213*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9043705*a^3*(cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2 + 28369275*a^3*(cos(d*x + c) - 1)^3/(cos(
d*x + c) + 1)^3 + 59954070*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 +
67458930*a^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 57997170*a^3*(cos(
```

```
d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 36975510*a^3*(cos(d*x + c) - 1)^7/(c
os(d*x + c) + 1)^7 + 16879995*a^3*(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8
 + 5213945*a^3*(cos(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9 + 976261*a^3*(cos(
d*x + c) - 1)^10/(cos(d*x + c) + 1)^10 + 83711*a^3*(cos(d*x + c) - 1)^11/(c
os(d*x + c) + 1)^11)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^11)/d
```

3.38 $\int (a + a\sec(c + dx))^3 \tan^7(c + dx)\, dx$

Optimal. Leaf size=137

$$\frac{a^3\sec^9(c+dx)}{9d}+\frac{3a^3\sec^8(c+dx)}{8d}-\frac{4a^3\sec^6(c+dx)}{3d}-\frac{6a^3\sec^5(c+dx)}{5d}+\frac{3a^3\sec^4(c+dx)}{2d}+\frac{8a^3\sec^3(c+dx)}{3d}-3a^3\,\mathrm{se}$$

```
[Out] (a^3*Log[Cos[c + d*x]])/d - (3*a^3*Sec[c + d*x])/d + (8*a^3*Sec[c + d*x]^3)
/(3*d) + (3*a^3*Sec[c + d*x]^4)/(2*d) - (6*a^3*Sec[c + d*x]^5)/(5*d) - (4*a
^3*Sec[c + d*x]^6)/(3*d) + (3*a^3*Sec[c + d*x]^8)/(8*d) + (a^3*Sec[c + d*x]
^9)/(9*d)
```

Rubi [A] time = 0.0771652, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{a^3\sec^9(c+dx)}{9d}+\frac{3a^3\sec^8(c+dx)}{8d}-\frac{4a^3\sec^6(c+dx)}{3d}-\frac{6a^3\sec^5(c+dx)}{5d}+\frac{3a^3\sec^4(c+dx)}{2d}+\frac{8a^3\sec^3(c+dx)}{3d}-3a^3\,\mathrm{se}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^7,x]
```

```
[Out] (a^3*Log[Cos[c + d*x]])/d - (3*a^3*Sec[c + d*x])/d + (8*a^3*Sec[c + d*x]^3)
/(3*d) + (3*a^3*Sec[c + d*x]^4)/(2*d) - (6*a^3*Sec[c + d*x]^5)/(5*d) - (4*a
^3*Sec[c + d*x]^6)/(3*d) + (3*a^3*Sec[c + d*x]^8)/(8*d) + (a^3*Sec[c + d*x]
^9)/(9*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\tan^7(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^3(a+ax)^6}{x^{10}}\,dx,x,\cos(c+dx)\right)}{a^6d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^9}{x^{10}}+\frac{3a^9}{x^9}-\frac{8a^9}{x^7}-\frac{6a^9}{x^6}+\frac{6a^9}{x^5}+\frac{8a^9}{x^4}-\frac{3a^9}{x^2}-\frac{a^9}{x}\right)dx,x,\cos(c+dx)\right)}{a^6d}$$
$$= \frac{a^3\log(\cos(c+dx))}{d}-\frac{3a^3\sec(c+dx)}{d}+\frac{8a^3\sec^3(c+dx)}{3d}+\frac{3a^3\sec^4(c+dx)}{2d}$$

Mathematica [A] time = 0.392007, size = 110, normalized size = 0.8

$$\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(40\sec^9(c+dx)+135\sec^8(c+dx)-480\sec^6(c+dx)-432\sec^5(c+dx)+540\text{s}\right.}{2880d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^7,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(360*Log[Cos[c + d*x]] - 1080*
Sec[c + d*x] + 960*Sec[c + d*x]^3 + 540*Sec[c + d*x]^4 - 432*Sec[c + d*x]^5
 - 480*Sec[c + d*x]^6 + 135*Sec[c + d*x]^8 + 40*Sec[c + d*x]^9))/(2880*d)
```

Maple [B] time = 0.057, size = 288, normalized size = 2.1

$$\frac{a^3(\tan(dx+c))^6}{6d}-\frac{a^3(\tan(dx+c))^4}{4d}+\frac{a^3(\tan(dx+c))^2}{2d}+\frac{a^3\ln(\cos(dx+c))}{d}+\frac{4a^3(\sin(dx+c))^8}{9d(\cos(dx+c))^7}-\frac{4a^3(\sin(dx}{45d(\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^7,x)
```

```
[Out] 1/6/d*a^3*tan(d*x+c)^6-1/4/d*a^3*tan(d*x+c)^4+1/2/d*a^3*tan(d*x+c)^2+a^3*ln
(cos(d*x+c))/d+4/9/d*a^3*sin(d*x+c)^8/cos(d*x+c)^7-4/45/d*a^3*sin(d*x+c)^8/
cos(d*x+c)^5+4/45/d*a^3*sin(d*x+c)^8/cos(d*x+c)^3-4/9/d*a^3*sin(d*x+c)^8/co
s(d*x+c)-64/45*a^3*cos(d*x+c)/d-4/9/d*a^3*cos(d*x+c)*sin(d*x+c)^6-8/15/d*a^
3*cos(d*x+c)*sin(d*x+c)^4-32/45/d*a^3*cos(d*x+c)*sin(d*x+c)^2+3/8/d*a^3*sin
(d*x+c)^8/cos(d*x+c)^8+1/9/d*a^3*sin(d*x+c)^8/cos(d*x+c)^9
```

Maxima [A] time = 1.40997, size = 149, normalized size = 1.09

$$\frac{360\,a^3\log\left(\cos\left(dx+c\right)\right)-\frac{1080\,a^3\cos(dx+c)^8-960\,a^3\cos(dx+c)^6-540\,a^3\cos(dx+c)^5+432\,a^3\cos(dx+c)^4+480\,a^3\cos(dx+c)^3-135\,a^3\cos(dx+c)-40\,a^3}{\cos(dx+c)^9}}{360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/360*(360*a^3*log(cos(d*x + c)) - (1080*a^3*cos(d*x + c)^8 - 960*a^3*cos(d
*x + c)^6 - 540*a^3*cos(d*x + c)^5 + 432*a^3*cos(d*x + c)^4 + 480*a^3*cos(d
*x + c)^3 - 135*a^3*cos(d*x + c) - 40*a^3)/cos(d*x + c)^9)/d
```

Fricas [A] time = 1.06437, size = 308, normalized size = 2.25

$$\frac{360\,a^3\cos\left(dx+c\right)^9\log\left(-\cos\left(dx+c\right)\right)-1080\,a^3\cos\left(dx+c\right)^8+960\,a^3\cos\left(dx+c\right)^6+540\,a^3\cos\left(dx+c\right)^5-432\,a^3\cos}{360\,d\cos\left(dx+c\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/360*(360*a^3*cos(d*x + c)^9*log(-cos(d*x + c)) - 1080*a^3*cos(d*x + c)^8
+ 960*a^3*cos(d*x + c)^6 + 540*a^3*cos(d*x + c)^5 - 432*a^3*cos(d*x + c)^4
- 480*a^3*cos(d*x + c)^3 + 135*a^3*cos(d*x + c) + 40*a^3)/(d*cos(d*x + c)^9
)
```

Sympy [A] time = 60.509, size = 350, normalized size = 2.55

$$\begin{cases} -\frac{a^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^3\tan^6\left(c+dx\right)\sec^3\left(c+dx\right)}{9d}+\frac{3a^3\tan^6\left(c+dx\right)\sec^2\left(c+dx\right)}{8d}+\frac{3a^3\tan^6\left(c+dx\right)\sec\left(c+dx\right)}{7d}+\frac{a^3\tan^6\left(c+dx\right)}{6d}-\frac{2a^3\tan^4\left(c+dx\right)}{21d} \\ x\left(a\sec\left(c\right)+a\right)^3\tan^7\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**7,x)
```

```
[Out] Piecewise((-a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*tan(c + d*x)**6*sec(
c + d*x)**3/(9*d) + 3*a**3*tan(c + d*x)**6*sec(c + d*x)**2/(8*d) + 3*a**3*t
an(c + d*x)**6*sec(c + d*x)/(7*d) + a**3*tan(c + d*x)**6/(6*d) - 2*a**3*tan
(c + d*x)**4*sec(c + d*x)**3/(21*d) - 3*a**3*tan(c + d*x)**4*sec(c + d*x)**
2/(8*d) - 18*a**3*tan(c + d*x)**4*sec(c + d*x)/(35*d) - a**3*tan(c + d*x)**
4/(4*d) + 8*a**3*tan(c + d*x)**2*sec(c + d*x)**3/(105*d) + 3*a**3*tan(c + d
*x)**2*sec(c + d*x)**2/(8*d) + 24*a**3*tan(c + d*x)**2*sec(c + d*x)/(35*d)
+ a**3*tan(c + d*x)**2/(2*d) - 16*a**3*sec(c + d*x)**3/(315*d) - 3*a**3*sec
(c + d*x)**2/(8*d) - 48*a**3*sec(c + d*x)/(35*d), Ne(d, 0)), (x*(a*sec(c) +
 a)**3*tan(c)**7, True))
```

Giac [B] time = 9.10637, size = 428, normalized size = 3.12

$$-\frac{2520\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-2520\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{14297\,a^3+\frac{133713\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{560052\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{1384068\,a^3\cdots}{(\cos\cdots}}{\cdots}}{252\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] -1/2520*(2520*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 25
20*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (14297*a^3 +
133713*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 560052*a^3*(cos(d*x + c)
 - 1)^2/(cos(d*x + c) + 1)^2 + 1384068*a^3*(cos(d*x + c) - 1)^3/(cos(d*x +
c) + 1)^3 + 1594782*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1336734
*a^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 781956*a^3*(cos(d*x + c) -
 1)^6/(cos(d*x + c) + 1)^6 + 302004*a^3*(cos(d*x + c) - 1)^7/(cos(d*x + c)
+ 1)^7 + 69201*a^3*(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8 + 7129*a^3*(co
s(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9)/((cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + 1)^9)/d
```

3.39 $\int (a + a\sec(c + dx))^3 \tan^5(c + dx)\, dx$

Optimal. Leaf size=138

$$\frac{a^3 \sec^7(c+dx)}{7d} + \frac{a^3 \sec^6(c+dx)}{2d} + \frac{a^3 \sec^5(c+dx)}{5d} - \frac{5a^3 \sec^4(c+dx)}{4d} - \frac{5a^3 \sec^3(c+dx)}{3d} + \frac{a^3 \sec^2(c+dx)}{2d} + \frac{3a^3 \sec(c}{d}$$

```
[Out] -((a^3*Log[Cos[c + d*x]])/d) + (3*a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2
)/(2*d) - (5*a^3*Sec[c + d*x]^3)/(3*d) - (5*a^3*Sec[c + d*x]^4)/(4*d) + (a^
3*Sec[c + d*x]^5)/(5*d) + (a^3*Sec[c + d*x]^6)/(2*d) + (a^3*Sec[c + d*x]^7)
/(7*d)
```

Rubi [A] time = 0.0772046, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{a^3 \sec^7(c+dx)}{7d} + \frac{a^3 \sec^6(c+dx)}{2d} + \frac{a^3 \sec^5(c+dx)}{5d} - \frac{5a^3 \sec^4(c+dx)}{4d} - \frac{5a^3 \sec^3(c+dx)}{3d} + \frac{a^3 \sec^2(c+dx)}{2d} + \frac{3a^3 \sec(c}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^5,x]
```


```
[Out] -((a^3*Log[Cos[c + d*x]])/d) + (3*a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2
)/(2*d) - (5*a^3*Sec[c + d*x]^3)/(3*d) - (5*a^3*Sec[c + d*x]^4)/(4*d) + (a^
3*Sec[c + d*x]^5)/(5*d) + (a^3*Sec[c + d*x]^6)/(2*d) + (a^3*Sec[c + d*x]^7)
/(7*d)
```


Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\tan^5(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^2(a+ax)^5}{x^8}\,dx, x, \cos(c+dx)\right)}{a^4 d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^7}{x^8}+\frac{3a^7}{x^7}+\frac{a^7}{x^6}-\frac{5a^7}{x^5}-\frac{5a^7}{x^4}+\frac{a^7}{x^3}+\frac{3a^7}{x^2}+\frac{a^7}{x}\right)dx, x, \cos(c+dx)\right)}{a^4 d}$$
$$= -\frac{a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}-\frac{5a^3\sec^3(c+dx)}{3d}$$

Mathematica [A] time = 0.419514, size = 140, normalized size = 1.01

$$-\frac{a^3\sec^7(c+dx)(-4522\cos(2(c+dx))+1050\cos(3(c+dx))-2380\cos(4(c+dx))-210\cos(5(c+dx))-630\cos(6(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^5,x]
```

```
[Out] -(a^3*(-3732 - 4522*Cos[2*(c + d*x)] + 1050*Cos[3*(c + d*x)] - 2380*Cos[4*(
c + d*x)] - 210*Cos[5*(c + d*x)] - 630*Cos[6*(c + d*x)] + 2205*Cos[3*(c + d
*x)]*Log[Cos[c + d*x]] + 735*Cos[5*(c + d*x)]*Log[Cos[c + d*x]] + 105*Cos[7
*(c + d*x)]*Log[Cos[c + d*x]] + 105*Cos[c + d*x]*(8 + 35*Log[Cos[c + d*x]])
)*Sec[c + d*x]^7)/(6720*d)
```

Maple [A] time = 0.051, size = 227, normalized size = 1.6

$$\frac{a^3(\tan(dx+c))^4}{4d}-\frac{a^3(\tan(dx+c))^2}{2d}-\frac{a^3\ln(\cos(dx+c))}{d}+\frac{22a^3(\sin(dx+c))^6}{35d(\cos(dx+c))^5}-\frac{22a^3(\sin(dx+c))^6}{105d(\cos(dx+c))^3}+\frac{22a^3(\sin}{35d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^5,x)
```

```
[Out] 1/4/d*a^3*tan(d*x+c)^4-1/2/d*a^3*tan(d*x+c)^2-a^3*ln(cos(d*x+c))/d+22/35/d*
a^3*sin(d*x+c)^6/cos(d*x+c)^5-22/105/d*a^3*sin(d*x+c)^6/cos(d*x+c)^3+22/35/
d*a^3*sin(d*x+c)^6/cos(d*x+c)+176/105*a^3*cos(d*x+c)/d+22/35/d*a^3*cos(d*x+
c)*sin(d*x+c)^4+88/105/d*a^3*cos(d*x+c)*sin(d*x+c)^2+1/2/d*a^3*sin(d*x+c)^6
```

```
/cos(d*x+c)^6+1/7/d*a^3*sin(d*x+c)^6/cos(d*x+c)^7
```

Maxima [A] time = 1.67266, size = 149, normalized size = 1.08

$$-\frac{420\,a^3\log\left(\cos\left(dx+c\right)\right)-\frac{1260\,a^3\cos(dx+c)^6+210\,a^3\cos(dx+c)^5-700\,a^3\cos(dx+c)^4-525\,a^3\cos(dx+c)^3+84\,a^3\cos(dx+c)^2+210\,a^3\cos(dx+c)+60\,a^3}{\cos(dx+c)^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/420*(420*a^3*log(cos(d*x + c)) - (1260*a^3*cos(d*x + c)^6 + 210*a^3*cos(
d*x + c)^5 - 700*a^3*cos(d*x + c)^4 - 525*a^3*cos(d*x + c)^3 + 84*a^3*cos(d
*x + c)^2 + 210*a^3*cos(d*x + c) + 60*a^3)/cos(d*x + c)^7)/d
```

Fricas [A] time = 1.02049, size = 308, normalized size = 2.23

$$-\frac{420\,a^3\cos\left(dx+c\right)^7\log\left(-\cos\left(dx+c\right)\right)-1260\,a^3\cos\left(dx+c\right)^6-210\,a^3\cos\left(dx+c\right)^5+700\,a^3\cos\left(dx+c\right)^4+525\,a^3\cos}{420\,d\cos\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/420*(420*a^3*cos(d*x + c)^7*log(-cos(d*x + c)) - 1260*a^3*cos(d*x + c)^6
 - 210*a^3*cos(d*x + c)^5 + 700*a^3*cos(d*x + c)^4 + 525*a^3*cos(d*x + c)^3
 - 84*a^3*cos(d*x + c)^2 - 210*a^3*cos(d*x + c) - 60*a^3)/(d*cos(d*x + c)^7
)
```

Sympy [A] time = 31.5841, size = 255, normalized size = 1.85

$$\begin{cases}\frac{a^3\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{a^3\tan^4(c+dx)\sec^3(c+dx)}{7d}+\frac{a^3\tan^4(c+dx)\sec^2(c+dx)}{2d}+\frac{3a^3\tan^4(c+dx)\sec(c+dx)}{5d}+\frac{a^3\tan^4(c+dx)}{4d}-\frac{4a^3\tan^2(c+dx)\sec}{35d}\\ x\left(a\sec\left(c\right)+a\right)^3\tan^5\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**5,x)
```

```
[Out] Piecewise((a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*tan(c + d*x)**4*sec(c
 + d*x)**3/(7*d) + a**3*tan(c + d*x)**4*sec(c + d*x)**2/(2*d) + 3*a**3*tan(
c + d*x)**4*sec(c + d*x)/(5*d) + a**3*tan(c + d*x)**4/(4*d) - 4*a**3*tan(c
+ d*x)**2*sec(c + d*x)**3/(35*d) - a**3*tan(c + d*x)**2*sec(c + d*x)**2/(2*
d) - 4*a**3*tan(c + d*x)**2*sec(c + d*x)/(5*d) - a**3*tan(c + d*x)**2/(2*d)
 + 8*a**3*sec(c + d*x)**3/(105*d) + a**3*sec(c + d*x)**2/(2*d) + 8*a**3*sec
(c + d*x)/(5*d), Ne(d, 0)), (x*(a*sec(c) + a)**3*tan(c)**5, True))
```

Giac [B] time = 3.89641, size = 360, normalized size = 2.61

$$\frac{420\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-420\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{2497\,a^3+\frac{18319\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{58317\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{69475\,a^3(\cos(dx+c)-}{(\cos(dx+c)+1)^3}}{}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/420*(420*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*a
^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (2497*a^3 + 18319
*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 58317*a^3*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 + 69475*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3
 + 56035*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 28749*a^3*(cos(d*x
 + c) - 1)^5/(cos(d*x + c) + 1)^5 + 8463*a^3*(cos(d*x + c) - 1)^6/(cos(d*x
+ c) + 1)^6 + 1089*a^3*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/((cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 1)^7)/d
```

3.40 $\int (a + a \sec(c + dx))^3 \tan^3(c + dx)\, dx$

Optimal. Leaf size=99

$$\frac{a^3\sec^5(c+dx)}{5d}+\frac{3a^3\sec^4(c+dx)}{4d}+\frac{2a^3\sec^3(c+dx)}{3d}-\frac{a^3\sec^2(c+dx)}{d}-\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\log(\cos(c+dx))}{d}$$

```
[Out] (a^3*Log[Cos[c + d*x]])/d - (3*a^3*Sec[c + d*x])/d - (a^3*Sec[c + d*x]^2)/d
 + (2*a^3*Sec[c + d*x]^3)/(3*d) + (3*a^3*Sec[c + d*x]^4)/(4*d) + (a^3*Sec[c
 + d*x]^5)/(5*d)
```

Rubi [A] time = 0.066601, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 75}

$$\frac{a^3\sec^5(c+dx)}{5d}+\frac{3a^3\sec^4(c+dx)}{4d}+\frac{2a^3\sec^3(c+dx)}{3d}-\frac{a^3\sec^2(c+dx)}{d}-\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] (a^3*Log[Cos[c + d*x]])/d - (3*a^3*Sec[c + d*x])/d - (a^3*Sec[c + d*x]^2)/d
 + (2*a^3*Sec[c + d*x]^3)/(3*d) + (3*a^3*Sec[c + d*x]^4)/(4*d) + (a^3*Sec[c
 + d*x]^5)/(5*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\tan^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)(a+ax)^4}{x^6}\,dx, x, \cos(c+dx)\right)}{a^2d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^5}{x^6}+\frac{3a^5}{x^5}+\frac{2a^5}{x^4}-\frac{2a^5}{x^3}-\frac{3a^5}{x^2}-\frac{a^5}{x}\right)dx, x, \cos(c+dx)\right)}{a^2d}$$
$$= \frac{a^3\log(\cos(c+dx))}{d} - \frac{3a^3\sec(c+dx)}{d} - \frac{a^3\sec^2(c+dx)}{d} + \frac{2a^3\sec^3(c+dx)}{3d} +$$

Mathematica [A] time = 0.299458, size = 92, normalized size = 0.93

$$-\frac{a^3\sec^5(c+dx)(280\cos(2(c+dx))+90\cos(4(c+dx))+\cos(3(c+dx))(60-75\log(\cos(c+dx)))-150\cos(c+dx)\,\text{lo}}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] -(a^3*(142 + 280*Cos[2*(c + d*x)] + 90*Cos[4*(c + d*x)] + Cos[3*(c + d*x)]*
(60 - 75*Log[Cos[c + d*x]]) - 150*Cos[c + d*x]*Log[Cos[c + d*x]] - 15*Cos[5
*(c + d*x)]*Log[Cos[c + d*x]])*Sec[c + d*x]^5)/(240*d)
```

Maple [A] time = 0.048, size = 164, normalized size = 1.7

$$\frac{a^3(\tan(dx+c))^2}{2d} + \frac{a^3\ln(\cos(dx+c))}{d} + \frac{16a^3(\sin(dx+c))^4}{15d(\cos(dx+c))^3} - \frac{16a^3(\sin(dx+c))^4}{15d\cos(dx+c)} - \frac{16a^3\cos(dx+c)(\sin(dx+c))}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^3,x)
```

```
[Out] 1/2/d*a^3*tan(d*x+c)^2+a^3*ln(cos(d*x+c))/d+16/15/d*a^3*sin(d*x+c)^4/cos(d*
x+c)^3-16/15/d*a^3*sin(d*x+c)^4/cos(d*x+c)-16/15/d*a^3*cos(d*x+c)*sin(d*x+c
)^2-32/15*a^3*cos(d*x+c)/d+3/4/d*a^3*sin(d*x+c)^4/cos(d*x+c)^4+1/5/d*a^3*si
n(d*x+c)^4/cos(d*x+c)^5
```

Maxima [A] time = 1.32302, size = 113, normalized size = 1.14

$$\frac{60\,a^3\log\left(\cos\left(dx+c\right)\right)-\frac{180\,a^3\cos(dx+c)^4+60\,a^3\cos(dx+c)^3-40\,a^3\cos(dx+c)^2-45\,a^3\cos(dx+c)-12\,a^3}{\cos(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/60*(60*a^3*log(cos(d*x + c)) - (180*a^3*cos(d*x + c)^4 + 60*a^3*cos(d*x +
 c)^3 - 40*a^3*cos(d*x + c)^2 - 45*a^3*cos(d*x + c) - 12*a^3)/cos(d*x + c)^
5)/d
```

Fricas [A] time = 1.01575, size = 232, normalized size = 2.34

$$\frac{60\,a^3\cos\left(dx+c\right)^5\log\left(-\cos\left(dx+c\right)\right)-180\,a^3\cos\left(dx+c\right)^4-60\,a^3\cos\left(dx+c\right)^3+40\,a^3\cos\left(dx+c\right)^2+45\,a^3\cos\left(dx+\right.}{60\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/60*(60*a^3*cos(d*x + c)^5*log(-cos(d*x + c)) - 180*a^3*cos(d*x + c)^4 - 6
0*a^3*cos(d*x + c)^3 + 40*a^3*cos(d*x + c)^2 + 45*a^3*cos(d*x + c) + 12*a^3
)/(d*cos(d*x + c)^5)
```

Sympy [A] time = 9.80314, size = 165, normalized size = 1.67

$$\begin{cases}-\frac{a^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^3\tan^2\left(c+dx\right)\sec^3\left(c+dx\right)}{5d}+\frac{3a^3\tan^2\left(c+dx\right)\sec^2\left(c+dx\right)}{4d}+\frac{a^3\tan^2\left(c+dx\right)\sec\left(c+dx\right)}{d}+\frac{a^3\tan^2\left(c+dx\right)}{2d}-\frac{2a^3\sec^3\left(c+dx\right)}{15d} \\ x\left(a\sec\left(c\right)+a\right)^3\tan^3\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**3,x)
```

```
[Out] Piecewise((-a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*tan(c + d*x)**2*sec(
c + d*x)**3/(5*d) + 3*a**3*tan(c + d*x)**2*sec(c + d*x)**2/(4*d) + a**3*tan
```

```
(c + d*x)**2*sec(c + d*x)/d + a**3*tan(c + d*x)**2/(2*d) - 2*a**3*sec(c + d
*x)**3/(15*d) - 3*a**3*sec(c + d*x)**2/(4*d) - 2*a**3*sec(c + d*x)/d, Ne(d,
 0)), (x*(a*sec(c) + a)**3*tan(c)**3, True))
```

Giac [B] time = 2.07299, size = 293, normalized size = 2.96

$$-\frac{60\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-60\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{393\,a^3+\frac{2085\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2610\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{1970\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/60*(60*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^3
*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (393*a^3 + 2085*a^3
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2610*a^3*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2 + 1970*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 805
*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*a^3*(cos(d*x + c) - 1)
^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^5)/d
```

3.41 $\int (a + a\sec(c + dx))^3 \tan(c + dx)\, dx$

Optimal. Leaf size=66

$$\frac{a^3 \sec^3(c+dx)}{3d} + \frac{3a^3 \sec^2(c+dx)}{2d} + \frac{3a^3 \sec(c+dx)}{d} - \frac{a^3 \log(\cos(c+dx))}{d}$$

```
[Out] -((a^3*Log[Cos[c + d*x]])/d) + (3*a^3*Sec[c + d*x])/d + (3*a^3*Sec[c + d*x]
^2)/(2*d) + (a^3*Sec[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0388226, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 43}

$$\frac{a^3 \sec^3(c+dx)}{3d} + \frac{3a^3 \sec^2(c+dx)}{2d} + \frac{3a^3 \sec(c+dx)}{d} - \frac{a^3 \log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] -((a^3*Log[Cos[c + d*x]])/d) + (3*a^3*Sec[c + d*x])/d + (3*a^3*Sec[c + d*x]
^2)/(2*d) + (a^3*Sec[c + d*x]^3)/(3*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\tan(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a+ax)^3}{x^4}\,dx,x,\cos(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^3}{x^4}+\frac{3a^3}{x^3}+\frac{3a^3}{x^2}+\frac{a^3}{x}\right)dx,x,\cos(c+dx)\right)}{d}$$
$$= -\frac{a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{3a^3\sec^2(c+dx)}{2d}+\frac{a^3\sec^3(c+dx)}{3d}$$

Mathematica [A] time = 0.156663, size = 64, normalized size = 0.97

$$-\frac{a^3\sec^3(c+dx)(-18\cos(2(c+dx))+9\cos(c+dx)(\log(\cos(c+dx))-2)+3\cos(3(c+dx))\log(\cos(c+dx))-22)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] -(a^3*(-22 - 18*Cos[2*(c + d*x)] + 9*Cos[c + d*x]*(-2 + Log[Cos[c + d*x]])
+ 3*Cos[3*(c + d*x)]*Log[Cos[c + d*x]])*Sec[c + d*x]^3)/(12*d)
```

Maple [A] time = 0.015, size = 62, normalized size = 0.9

$$\frac{a^3(\sec(dx+c))^3}{3d}+\frac{3a^3(\sec(dx+c))^2}{2d}+3\frac{a^3\sec(dx+c)}{d}+\frac{a^3\ln(\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c),x)
```

```
[Out] 1/3*a^3*sec(d*x+c)^3/d+3/2*a^3*sec(d*x+c)^2/d+3*a^3*sec(d*x+c)/d+1/d*a^3*ln
(sec(d*x+c))
```

Maxima [A] time = 1.48249, size = 78, normalized size = 1.18

$$-\frac{6a^3\log(\cos(dx+c))-\frac{18a^3\cos(dx+c)^2+9a^3\cos(dx+c)+2a^3}{\cos(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/6*(6*a^3*log(cos(d*x + c)) - (18*a^3*cos(d*x + c)^2 + 9*a^3*cos(d*x + c)
 + 2*a^3)/cos(d*x + c)^3)/d
```

Fricas [A] time = 0.945276, size = 162, normalized size = 2.45

$$-\frac{6\,a^3\cos\left(dx+c\right)^3\log\left(-\cos\left(dx+c\right)\right)-18\,a^3\cos\left(dx+c\right)^2-9\,a^3\cos\left(dx+c\right)-2\,a^3}{6\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/6*(6*a^3*cos(d*x + c)^3*log(-cos(d*x + c)) - 18*a^3*cos(d*x + c)^2 - 9*a
^3*cos(d*x + c) - 2*a^3)/(d*cos(d*x + c)^3)
```

Sympy [A] time = 3.91752, size = 76, normalized size = 1.15

$$\begin{cases}\frac{a^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^3\sec^3\left(c+dx\right)}{3d}+\frac{3a^3\sec^2\left(c+dx\right)}{2d}+\frac{3a^3\sec\left(c+dx\right)}{d} & \text{for } d\neq 0\\ x\left(a\sec\left(c\right)+a\right)^3\tan\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c),x)
```

```
[Out] Piecewise((a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*sec(c + d*x)**3/(3*d)
 + 3*a**3*sec(c + d*x)**2/(2*d) + 3*a**3*sec(c + d*x)/d, Ne(d, 0)), (x*(a*s
ec(c) + a)**3*tan(c), True))
```

Giac [B] time = 1.43613, size = 225, normalized size = 3.41

$$\frac{6\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-6\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{51\,a^3+\frac{69\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{45\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{11\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c),x, algorithm="giac")
```

```
[Out] 1/6*(6*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 6*a^3*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (51*a^3 + 69*a^3*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 45*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 + 11*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/((cos(d*x + c)
- 1)/(cos(d*x + c) + 1) + 1)^3)/d
```

3.42 $\int \cot(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=48

$$\frac{a^3\sec(c+dx)}{d}+\frac{4a^3\log(1-\cos(c+dx))}{d}-\frac{3a^3\log(\cos(c+dx))}{d}$$

```
[Out] (4*a^3*Log[1 - Cos[c + d*x]])/d - (3*a^3*Log[Cos[c + d*x]])/d + (a^3*Sec[c
+ d*x])/d
```

Rubi [A] time = 0.0460482, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 88}

$$\frac{a^3\sec(c+dx)}{d}+\frac{4a^3\log(1-\cos(c+dx))}{d}-\frac{3a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (4*a^3*Log[1 - Cos[c + d*x]])/d - (3*a^3*Log[Cos[c + d*x]])/d + (a^3*Sec[c
+ d*x])/d
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cot(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^2\operatorname{Subst}\left(\int \frac{(a+ax)^2}{x^2(a-ax)}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^2\operatorname{Subst}\left(\int \left(-\frac{4a}{-1+x}+\frac{a}{x^2}+\frac{3a}{x}\right)dx, x, \cos(c+dx)\right)}{d}$$
$$= \frac{4a^3\log(1-\cos(c+dx))}{d} - \frac{3a^3\log(\cos(c+dx))}{d} + \frac{a^3\sec(c+dx)}{d}$$

Mathematica [A] time = 0.0876936, size = 36, normalized size = 0.75

$$\frac{a^3\left(\sec(c+dx)+8\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-3\log(\cos(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(-3*Log[Cos[c + d*x]] + 8*Log[Sin[(c + d*x)/2]] + Sec[c + d*x]))/d
```

Maple [A] time = 0.046, size = 47, normalized size = 1.

$$\frac{a^3\sec(dx+c)}{d}+4\,\frac{a^3\ln(-1+\sec(dx+c))}{d}-\frac{a^3\ln(\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c))^3,x)
```

```
[Out] a^3*sec(d*x+c)/d+4/d*a^3*ln(-1+sec(d*x+c))-1/d*a^3*ln(sec(d*x+c))
```

Maxima [A] time = 1.50157, size = 58, normalized size = 1.21

$$\frac{4\,a^3\log(\cos(dx+c)-1)-3\,a^3\log(\cos(dx+c))+\frac{a^3}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (4*a^3*log(cos(d*x + c) - 1) - 3*a^3*log(cos(d*x + c)) + a^3/cos(d*x + c))/
d
```

Fricas [A] time = 1.19775, size = 155, normalized size = 3.23

$$-\frac{3\,a^3\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)-4\,a^3\cos\left(dx+c\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-a^3}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(3*a^3*cos(d*x + c)*log(-cos(d*x + c)) - 4*a^3*cos(d*x + c)*log(-1/2*cos(d
*x + c) + 1/2) - a^3)/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int 3\cot\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3\cot\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\cot\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int\cot\left(c+dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*cot(c + d*x)*sec(c + d*x), x) + Integral(3*cot(c + d*x)*se
c(c + d*x)**2, x) + Integral(cot(c + d*x)*sec(c + d*x)**3, x) + Integral(co
t(c + d*x), x))
```

Giac [B] time = 1.26674, size = 196, normalized size = 4.08

$$\frac{4\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-3\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{5\,a^3+\frac{3\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (4*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a^3*log(abs(-(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 3*a^3*log(abs(-(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) - 1)) + (5*a^3 + 3*a^3*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

3.43 $\int \cot^3(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=40

$$-\frac{2a^3}{d(1-\cos(c+dx))}-\frac{a^3\log(1-\cos(c+dx))}{d}$$

```
[Out] (-2*a^3)/(d*(1 - Cos[c + d*x])) - (a^3*Log[1 - Cos[c + d*x]])/d
```

Rubi [A] time = 0.0503526, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 43}

$$-\frac{2a^3}{d(1-\cos(c+dx))}-\frac{a^3\log(1-\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-2*a^3)/(d*(1 - Cos[c + d*x])) - (a^3*Log[1 - Cos[c + d*x]])/d
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^4 \operatorname{Subst}\left(\int \frac{a+ax}{(a-ax)^2}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^4 \operatorname{Subst}\left(\int \left(\frac{2}{a(-1+x)^2}+\frac{1}{a(-1+x)}\right)dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{2a^3}{d(1-\cos(c+dx))} - \frac{a^3\log(1-\cos(c+dx))}{d}$$

Mathematica [A] time = 0.127466, size = 46, normalized size = 1.15

$$-\frac{a^3\left(\cot^2\left(\frac{1}{2}(c+dx)\right)+2\left(\log\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -((a^3*(Cot[(c + d*x)/2]^2 + 2*(Log[Cos[(c + d*x)/2]] + Log[Tan[(c + d*x)/2
]])))/d)
```

Maple [A] time = 0.07, size = 51, normalized size = 1.3

$$-2\,\frac{a^3}{d\,(-1+\sec(dx+c))} - \frac{a^3\ln(-1+\sec(dx+c))}{d} + \frac{a^3\ln(\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -2/d*a^3/(-1+sec(d*x+c))-1/d*a^3*ln(-1+sec(d*x+c))+1/d*a^3*ln(sec(d*x+c))
```

Maxima [A] time = 1.42799, size = 46, normalized size = 1.15

$$-\frac{a^3\log(\cos(dx+c)-1) - \frac{2\,a^3}{\cos(dx+c)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(a^3*log(cos(d*x + c) - 1) - 2*a^3/(cos(d*x + c) - 1))/d
```

Fricas [A] time = 1.23091, size = 116, normalized size = 2.9

$$\frac{2\,a^3 - \left(a^3\cos\left(dx+c\right) - a^3\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right) + \frac{1}{2}\right)}{d\cos\left(dx+c\right) - d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (2*a^3 - (a^3*cos(d*x + c) - a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x
+ c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.40767, size = 147, normalized size = 3.68

$$-\frac{a^3\log\left(\frac{\left|-\cos(dx+c)+1\right|}{\left|\cos(dx+c)+1\right|}\right) - a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - \frac{\left(a^3+\frac{a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a^3*log(abs(-(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a^3 + a^3*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/d
```

3.44 $\int \cot^5(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=61

$$\frac{2a^3}{d(1-\cos(c+dx))}-\frac{a^3}{2d(1-\cos(c+dx))^2}+\frac{a^3\log(1-\cos(c+dx))}{d}$$

```
[Out] -a^3/(2*d*(1 - Cos[c + d*x])^2) + (2*a^3)/(d*(1 - Cos[c + d*x])) + (a^3*Log
[1 - Cos[c + d*x]])/d
```

Rubi [A] time = 0.058681, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 43}

$$\frac{2a^3}{d(1-\cos(c+dx))}-\frac{a^3}{2d(1-\cos(c+dx))^2}+\frac{a^3\log(1-\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```


```
[Out] -a^3/(2*d*(1 - Cos[c + d*x])^2) + (2*a^3)/(d*(1 - Cos[c + d*x])) + (a^3*Log
[1 - Cos[c + d*x]])/d
```


Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^6\operatorname{Subst}\left(\int \frac{x^2}{(a-ax)^3}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^6\operatorname{Subst}\left(\int \left(-\frac{1}{a^3(-1+x)^3}-\frac{2}{a^3(-1+x)^2}-\frac{1}{a^3(-1+x)}\right)dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^3}{2d(1-\cos(c+dx))^2}+\frac{2a^3}{d(1-\cos(c+dx))}+\frac{a^3\log(1-\cos(c+dx))}{d}$$

Mathematica [A] time = 0.170142, size = 72, normalized size = 1.18

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\csc^4\left(\frac{1}{2}(c+dx)\right)-8\csc^2\left(\frac{1}{2}(c+dx)\right)-16\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(a^3*(1 + Cos[c + d*x])^3*(-8*Csc[(c + d*x)/2]^2 + Csc[(c + d*x)/2]^4 - 16
*Log[Sin[(c + d*x)/2]])*Sec[(c + d*x)/2]^6)/(64*d)
```

Maple [A] time = 0.077, size = 68, normalized size = 1.1

$$\frac{a^3}{d(-1+\sec(dx+c))}-\frac{a^3}{2d(-1+\sec(dx+c))^2}+\frac{a^3\ln(-1+\sec(dx+c))}{d}-\frac{a^3\ln(\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/d*a^3/(-1+sec(d*x+c))-1/2/d*a^3/(-1+sec(d*x+c))^2+1/d*a^3*ln(-1+sec(d*x+c
))-1/d*a^3*ln(sec(d*x+c))
```

Maxima [A] time = 1.37319, size = 80, normalized size = 1.31

$$\frac{2a^3\log(\cos(dx+c)-1)-\frac{4a^3\cos(dx+c)-3a^3}{\cos(dx+c)^2-2\cos(dx+c)+1}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^3*log(cos(d*x + c) - 1) - (4*a^3*cos(d*x + c) - 3*a^3)/(cos(d*x +
c)^2 - 2*cos(d*x + c) + 1))/d
```

Fricas [A] time = 1.02003, size = 213, normalized size = 3.49

$$-\frac{4\,a^{3}\cos\left(dx+c\right)-3\,a^{3}-2\,\left(a^{3}\cos\left(dx+c\right)^{2}-2\,a^{3}\cos\left(dx+c\right)+a^{3}\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,\left(d\cos\left(dx+c\right)^{2}-2\,d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^3*cos(d*x + c) - 3*a^3 - 2*(a^3*cos(d*x + c)^2 - 2*a^3*cos(d*x +
c) + a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - 2*d*cos(d*x + c
) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.51033, size = 186, normalized size = 3.05

$$\frac{8\,a^{3}\log\left(\frac{\left|-\cos(dx+c)+1\right|}{\left|\cos(dx+c)+1\right|}\right)-8\,a^{3}\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a^{3}+\frac{6\,a^{3}(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{12\,a^{3}(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}\right)(\cos(dx+c)+1)^{2}}{(\cos(dx+c)-1)^{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 8*a^3*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a^3 + 6*a^3*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 12*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2)/d
```

3.45 $\int \cot^7(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=107

$$-\frac{17a^3}{8d(1-\cos(c+dx))}+\frac{7a^3}{8d(1-\cos(c+dx))^2}-\frac{a^3}{6d(1-\cos(c+dx))^3}-\frac{15a^3\log(1-\cos(c+dx))}{16d}-\frac{a^3\log(\cos(c+dx)+1)}{16d}$$

```
[Out] -a^3/(6*d*(1 - Cos[c + d*x])^3) + (7*a^3)/(8*d*(1 - Cos[c + d*x])^2) - (17*
a^3)/(8*d*(1 - Cos[c + d*x])) - (15*a^3*Log[1 - Cos[c + d*x]])/(16*d) - (a^
3*Log[1 + Cos[c + d*x]])/(16*d)
```

Rubi [A] time = 0.0763585, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$-\frac{17a^3}{8d(1-\cos(c+dx))}+\frac{7a^3}{8d(1-\cos(c+dx))^2}-\frac{a^3}{6d(1-\cos(c+dx))^3}-\frac{15a^3\log(1-\cos(c+dx))}{16d}-\frac{a^3\log(\cos(c+dx)+1)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -a^3/(6*d*(1 - Cos[c + d*x])^3) + (7*a^3)/(8*d*(1 - Cos[c + d*x])^2) - (17*
a^3)/(8*d*(1 - Cos[c + d*x])) - (15*a^3*Log[1 - Cos[c + d*x]])/(16*d) - (a^
3*Log[1 + Cos[c + d*x]])/(16*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^8\,\text{Subst}\left(\int \frac{x^4}{(a-ax)^4(a+ax)}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a^8\,\text{Subst}\left(\int\left(\frac{1}{2a^5(-1+x)^4}+\frac{7}{4a^5(-1+x)^3}+\frac{17}{8a^5(-1+x)^2}+\frac{15}{16a^5(-1+x)}+\frac{1}{16a^5(1+x)}\right)dx, x\right.}{d}$$

$$= -\frac{a^3}{6d(1-\cos(c+dx))^3}+\frac{7a^3}{8d(1-\cos(c+dx))^2}-\frac{17a^3}{8d(1-\cos(c+dx))}-\frac{15a^3\,\text{lo}}{}$$

Mathematica [A] time = 0.669128, size = 102, normalized size = 0.95

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(2\csc^6\left(\frac{1}{2}(c+dx)\right)-21\csc^4\left(\frac{1}{2}(c+dx)\right)+102\csc^2\left(\frac{1}{2}(c+dx)\right)+12\left(15\log\left(\text{si}\right.\right.\right.}{768d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(a^3*(1 + Cos[c + d*x])^3*(102*Csc[(c + d*x)/2]^2 - 21*Csc[(c + d*x)/2]^4
+ 2*Csc[(c + d*x)/2]^6 + 12*(Log[Cos[(c + d*x)/2]] + 15*Log[Sin[(c + d*x)/2
]]))*Sec[(c + d*x)/2]^6)/(768*d)
```

Maple [A] time = 0.087, size = 104, normalized size = 1.

$$-\frac{a^3\ln(1+\sec(dx+c))}{16d}-\frac{a^3}{6d(-1+\sec(dx+c))^3}+\frac{3a^3}{8d(-1+\sec(dx+c))^2}-\frac{7a^3}{8d(-1+\sec(dx+c))}-\frac{15a^3\ln(-1+}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/16/d*a^3*ln(1+sec(d*x+c))-1/6/d*a^3/(-1+sec(d*x+c))^3+3/8/d*a^3/(-1+sec(
d*x+c))^2-7/8/d*a^3/(-1+sec(d*x+c))-15/16/d*a^3*ln(-1+sec(d*x+c))+1/d*a^3*l
n(sec(d*x+c))
```

Maxima [A] time = 1.2166, size = 130, normalized size = 1.21

$$-\frac{3\,a^3\log\left(\cos\left(dx+c\right)+1\right)+45\,a^3\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(51\,a^3\cos(dx+c)^2-81\,a^3\cos(dx+c)+34\,a^3\right)}{\cos(dx+c)^3-3\,\cos(dx+c)^2+3\,\cos(dx+c)-1}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/48*(3*a^3*log(cos(d*x + c) + 1) + 45*a^3*log(cos(d*x + c) - 1) - 2*(51*a
^3*cos(d*x + c)^2 - 81*a^3*cos(d*x + c) + 34*a^3)/(cos(d*x + c)^3 - 3*cos(d
*x + c)^2 + 3*cos(d*x + c) - 1))/d
```

Fricas [A] time = 1.19566, size = 451, normalized size = 4.21

$$\frac{102\,a^3\cos\left(dx+c\right)^2-162\,a^3\cos\left(dx+c\right)+68\,a^3-3\left(a^3\cos\left(dx+c\right)^3-3\,a^3\cos\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)-a^3\right)\log\left(\frac{1}{2}\right.}{48\left(d\cos\left(dx+c\right)^3-3\,d\cos\left(dx+c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(102*a^3*cos(d*x + c)^2 - 162*a^3*cos(d*x + c) + 68*a^3 - 3*(a^3*cos(d
*x + c)^3 - 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) - a^3)*log(1/2*cos(d*
x + c) + 1/2) - 45*(a^3*cos(d*x + c)^3 - 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d
*x + c) - a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^3 - 3*d*cos(d*
x + c)^2 + 3*d*cos(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

<u>Giac</u> [A] time = 1.61034, size = 223, normalized size = 2.08

$$-\frac{90\,a^{3}\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-96\,a^{3}\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(2\,a^{3}+\frac{15\,a^{3}(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{66\,a^{3}(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{165\,a^{3}(\cos(dx+c)-1)^{3}}{(\cos(dx+c)+1)^{3}}\right)(\cos(dx+c)+1}{(\cos(dx+c)-1)^{3}}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/96*(90*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 96*a^3*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (2*a^3 + 15*a^3*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 66*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 + 165*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c)
+ 1)^3/(cos(d*x + c) - 1)^3)/d
```

3.46 $\int \cot^9(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=149

$$\frac{9a^3}{4d(1-\cos(c+dx))}+\frac{a^3}{32d(\cos(c+dx)+1)}-\frac{39a^3}{32d(1-\cos(c+dx))^2}+\frac{5a^3}{12d(1-\cos(c+dx))^3}-\frac{a^3}{16d(1-\cos(c+dx))^4}+$$

```
[Out] -a^3/(16*d*(1 - Cos[c + d*x])^4) + (5*a^3)/(12*d*(1 - Cos[c + d*x])^3) - (3
9*a^3)/(32*d*(1 - Cos[c + d*x])^2) + (9*a^3)/(4*d*(1 - Cos[c + d*x])) + a^3
/(32*d*(1 + Cos[c + d*x])) + (57*a^3*Log[1 - Cos[c + d*x]])/(64*d) + (7*a^3
*Log[1 + Cos[c + d*x]])/(64*d)
```

Rubi [A] time = 0.097923, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{9a^3}{4d(1-\cos(c+dx))}+\frac{a^3}{32d(\cos(c+dx)+1)}-\frac{39a^3}{32d(1-\cos(c+dx))^2}+\frac{5a^3}{12d(1-\cos(c+dx))^3}-\frac{a^3}{16d(1-\cos(c+dx))^4}+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^9*(a + a*Sec[c + d*x])^3,x]
```



```
[Out] -a^3/(16*d*(1 - Cos[c + d*x])^4) + (5*a^3)/(12*d*(1 - Cos[c + d*x])^3) - (3
9*a^3)/(32*d*(1 - Cos[c + d*x])^2) + (9*a^3)/(4*d*(1 - Cos[c + d*x])) + a^3
/(32*d*(1 + Cos[c + d*x])) + (57*a^3*Log[1 - Cos[c + d*x]])/(64*d) + (7*a^3
*Log[1 + Cos[c + d*x]])/(64*d)
```


Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^{10}\,\mathrm{Subst}\left(\int \frac{x^6}{(a-ax)^5(a+ax)^2}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^{10}\,\mathrm{Subst}\left(\int \left(-\frac{1}{4a^7(-1+x)^5} - \frac{5}{4a^7(-1+x)^4} - \frac{39}{16a^7(-1+x)^3} - \frac{9}{4a^7(-1+x)^2} - \frac{57}{64a^7(-1+x)} + \right.\right.}{d}$$
$$= -\frac{a^3}{16d(1-\cos(c+dx))^4} + \frac{5a^3}{12d(1-\cos(c+dx))^3} - \frac{39a^3}{32d(1-\cos(c+dx))^2} + \frac{}{4d}$$

Mathematica [A] time = 0.334718, size = 130, normalized size = 0.87

$$\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-3\csc^8\left(\frac{1}{2}(c+dx)\right)+40\csc^6\left(\frac{1}{2}(c+dx)\right)-234\csc^4\left(\frac{1}{2}(c+dx)\right)+864\csc^2\left(\frac{1}{2}(c\right.\right.}{6144d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^9*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(864*Csc[(c + d*x)/2]^2 - 234*
Csc[(c + d*x)/2]^4 + 40*Csc[(c + d*x)/2]^6 - 3*Csc[(c + d*x)/2]^8 + 12*(14*
Log[Cos[(c + d*x)/2]] + 114*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^2)))/(
6144*d)
```

Maple [A] time = 0.096, size = 141, normalized size = 1.

$$-\frac{a^3}{32\,d\,(1+\sec(dx+c))} + \frac{7\,a^3\ln(1+\sec(dx+c))}{64\,d} - \frac{a^3}{16\,d\,(-1+\sec(dx+c))^4} + \frac{a^3}{6\,d\,(-1+\sec(dx+c))^3} - \frac{11}{32\,d\,(-1+\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^9*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/32/d*a^3/(1+sec(d*x+c))+7/64/d*a^3*ln(1+sec(d*x+c))-1/16/d*a^3/(-1+sec(d
*x+c))^4+1/6/d*a^3/(-1+sec(d*x+c))^3-11/32/d*a^3/(-1+sec(d*x+c))^2+13/16/d*
```

```
a^3/(-1+sec(d*x+c))+57/64/d*a^3*ln(-1+sec(d*x+c))-1/d*a^3*ln(sec(d*x+c))
```

Maxima [A] time = 1.6136, size = 192, normalized size = 1.29

$$\frac{21\,a^3\log\left(\cos\left(dx+c\right)+1\right)+171\,a^3\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(213\,a^3\cos(dx+c)^4-303\,a^3\cos(dx+c)^3-95\,a^3\cos(dx+c)^2+333\,a^3\cos(dx+c)-136}{\cos(dx+c)^5-3\,\cos(dx+c)^4+2\,\cos(dx+c)^3+2\,\cos(dx+c)^2-3\,\cos(dx+c)+1}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/192*(21*a^3*log(cos(d*x + c) + 1) + 171*a^3*log(cos(d*x + c) - 1) - 2*(21
3*a^3*cos(d*x + c)^4 - 303*a^3*cos(d*x + c)^3 - 95*a^3*cos(d*x + c)^2 + 333
*a^3*cos(d*x + c) - 136*a^3)/(cos(d*x + c)^5 - 3*cos(d*x + c)^4 + 2*cos(d*x
 + c)^3 + 2*cos(d*x + c)^2 - 3*cos(d*x + c) + 1))/d
```

Fricas [B] time = 1.22619, size = 706, normalized size = 4.74

$$-\frac{426\,a^3\cos\left(dx+c\right)^4-606\,a^3\cos\left(dx+c\right)^3-190\,a^3\cos\left(dx+c\right)^2+666\,a^3\cos\left(dx+c\right)-272\,a^3-21\left(a^3\cos\left(dx+c\right)^5-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/192*(426*a^3*cos(d*x + c)^4 - 606*a^3*cos(d*x + c)^3 - 190*a^3*cos(d*x +
 c)^2 + 666*a^3*cos(d*x + c) - 272*a^3 - 21*(a^3*cos(d*x + c)^5 - 3*a^3*cos
(d*x + c)^4 + 2*a^3*cos(d*x + c)^3 + 2*a^3*cos(d*x + c)^2 - 3*a^3*cos(d*x +
 c) + a^3)*log(1/2*cos(d*x + c) + 1/2) - 171*(a^3*cos(d*x + c)^5 - 3*a^3*co
s(d*x + c)^4 + 2*a^3*cos(d*x + c)^3 + 2*a^3*cos(d*x + c)^2 - 3*a^3*cos(d*x
+ c) + a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^5 - 3*d*cos(d*x +
 c)^4 + 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 - 3*d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**9*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.44596, size = 288, normalized size = 1.93

$$\frac{684\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-768\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{12\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{\left(3\,a^3+\frac{28\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{132\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{504\,a^3(\cos\cdots}{(\cos\cdots}\cdots\right.}{(\cos(dx+\cdots}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/768*(684*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 768*a^3*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 12*a^3*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) - (3*a^3 + 28*a^3*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) + 132*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 504*a^3*(cos(d*
x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 1425*a^3*(cos(d*x + c) - 1)^4/(cos(d*x
 + c) + 1)^4)*(cos(d*x + c) + 1)^4/(cos(d*x + c) - 1)^4)/d
```

3.47 $\int (a + a\sec(c + dx))^3 \tan^6(c + dx)\,dx$

Optimal. Leaf size=237

$$\frac{3a^3\tan^7(c+dx)}{7d} + \frac{a^3\tan^5(c+dx)}{5d} - \frac{a^3\tan^3(c+dx)}{3d} + \frac{a^3\tan(c+dx)}{d} - \frac{125a^3\tanh^{-1}(\sin(c+dx))}{128d} + \frac{a^3\tan^5(c+dx)\,\mathrm{s}}{8d}$$

```
[Out] -(a^3*x) - (125*a^3*ArcTanh[Sin[c + d*x]])/(128*d) + (a^3*Tan[c + d*x])/d +
 (115*a^3*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (5*a^3*Sec[c + d*x]^3*Tan[c
+ d*x])/(64*d) - (a^3*Tan[c + d*x]^3)/(3*d) - (5*a^3*Sec[c + d*x]*Tan[c + d
*x]^3)/(8*d) - (5*a^3*Sec[c + d*x]^3*Tan[c + d*x]^3)/(48*d) + (a^3*Tan[c +
d*x]^5)/(5*d) + (a^3*Sec[c + d*x]*Tan[c + d*x]^5)/(2*d) + (a^3*Sec[c + d*x]
^3*Tan[c + d*x]^5)/(8*d) + (3*a^3*Tan[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.299389, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3886, 3473, 8, 2611, 3770, 2607, 30, 3768}

$$\frac{3a^3\tan^7(c+dx)}{7d} + \frac{a^3\tan^5(c+dx)}{5d} - \frac{a^3\tan^3(c+dx)}{3d} + \frac{a^3\tan(c+dx)}{d} - \frac{125a^3\tanh^{-1}(\sin(c+dx))}{128d} + \frac{a^3\tan^5(c+dx)\,\mathrm{s}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^6,x]
```


```
[Out] -(a^3*x) - (125*a^3*ArcTanh[Sin[c + d*x]])/(128*d) + (a^3*Tan[c + d*x])/d +
 (115*a^3*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (5*a^3*Sec[c + d*x]^3*Tan[c
+ d*x])/(64*d) - (a^3*Tan[c + d*x]^3)/(3*d) - (5*a^3*Sec[c + d*x]*Tan[c + d
*x]^3)/(8*d) - (5*a^3*Sec[c + d*x]^3*Tan[c + d*x]^3)/(48*d) + (a^3*Tan[c +
d*x]^5)/(5*d) + (a^3*Sec[c + d*x]*Tan[c + d*x]^5)/(2*d) + (a^3*Sec[c + d*x]
^3*Tan[c + d*x]^5)/(8*d) + (3*a^3*Tan[c + d*x]^7)/(7*d)
```


Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (a + a\sec(c+dx))^3 \tan^6(c+dx)\,dx &= \int \left(a^3\tan^6(c+dx) + 3a^3\sec(c+dx)\tan^6(c+dx) + 3a^3\sec^2(c+dx)\tan^6(c+ \right.\\
&= a^3\int \tan^6(c+dx)\,dx + a^3\int \sec^3(c+dx)\tan^6(c+dx)\,dx + \left(3a^3\right)\int \sec(c+dx\\
&= \frac{a^3\tan^5(c+dx)}{5d} + \frac{a^3\sec(c+dx)\tan^5(c+dx)}{2d} + \frac{a^3\sec^3(c+dx)\tan^5(c+dx)}{8d} -\\
&= -\frac{a^3\tan^3(c+dx)}{3d} - \frac{5a^3\sec(c+dx)\tan^3(c+dx)}{8d} - \frac{5a^3\sec^3(c+dx)\tan^3(c+dx}{48d}\\
&= \frac{a^3\tan(c+dx)}{d} + \frac{15a^3\sec(c+dx)\tan(c+dx)}{16d} + \frac{5a^3\sec^3(c+dx)\tan(c+dx)}{64d} -\\
&= -a^3x - \frac{15a^3\tanh^{-1}(\sin(c+dx))}{16d} + \frac{a^3\tan(c+dx)}{d} + \frac{115a^3\sec(c+dx)\tan(c+}{128d}\\
&= -a^3x - \frac{125a^3\tanh^{-1}(\sin(c+dx))}{128d} + \frac{a^3\tan(c+dx)}{d} + \frac{115a^3\sec(c+dx)\tan(c-}{128d}\end{aligned}$$

Mathematica [A] time = 2.06622, size = 363, normalized size = 1.53

$$\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^8(c+dx)\left(1680000\cos^8(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^6,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^8*(1680000*Cos[c
+ d*x]^8*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]]) - Sec[c]*(470400*d*x*Cos[c] + 376320*d*x*Cos[c + 2*d*x]
 + 376320*d*x*Cos[3*c + 2*d*x] + 188160*d*x*Cos[3*c + 4*d*x] + 188160*d*x*C
os[5*c + 4*d*x] + 53760*d*x*Cos[5*c + 6*d*x] + 53760*d*x*Cos[7*c + 6*d*x] +
 6720*d*x*Cos[7*c + 8*d*x] + 6720*d*x*Cos[9*c + 8*d*x] + 519680*Sin[c] - 13
3175*Sin[d*x] - 133175*Sin[2*c + d*x] - 544768*Sin[c + 2*d*x] + 286720*Sin[
3*c + 2*d*x] - 63595*Sin[2*c + 3*d*x] - 63595*Sin[4*c + 3*d*x] - 254464*Sin
[3*c + 4*d*x] + 161280*Sin[5*c + 4*d*x] - 65135*Sin[4*c + 5*d*x] - 65135*Si
n[6*c + 5*d*x] - 118784*Sin[5*c + 6*d*x] - 27195*Sin[6*c + 7*d*x] - 27195*S
in[8*c + 7*d*x] - 14848*Sin[7*c + 8*d*x])))/(13762560*d)
```

Maple [A] time = 0.056, size = 250, normalized size = 1.1

$$\frac{a^3(\tan(dx+c))^5}{5d} - \frac{a^3(\tan(dx+c))^3}{3d} + \frac{a^3\tan(dx+c)}{d} - a^3x - \frac{a^3c}{d} + \frac{25a^3(\sin(dx+c))^7}{48d(\cos(dx+c))^6} - \frac{25a^3(\sin(dx+c))^7}{192d(\cos(dx+c))^4} + \frac{25}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^6,x)
```

```
[Out] 1/5*a^3*tan(d*x+c)^5/d-1/3*a^3*tan(d*x+c)^3/d+a^3*tan(d*x+c)/d-a^3*x-1/d*a^
3*c+25/48/d*a^3*sin(d*x+c)^7/cos(d*x+c)^6-25/192/d*a^3*sin(d*x+c)^7/cos(d*x
+c)^4+25/128/d*a^3*sin(d*x+c)^7/cos(d*x+c)^2+25/128/d*a^3*sin(d*x+c)^5+125/
384/d*a^3*sin(d*x+c)^3+125/128/d*a^3*sin(d*x+c)-125/128/d*a^3*ln(sec(d*x+c)
+tan(d*x+c))+3/7/d*a^3*sin(d*x+c)^7/cos(d*x+c)^7+1/8/d*a^3*sin(d*x+c)^7/cos
(d*x+c)^8
```

Maxima [A] time = 1.75189, size = 354, normalized size = 1.49

$$\frac{11520\,a^3\tan\left(dx+c\right)^7+1792\left(3\,\tan\left(dx+c\right)^5-5\,\tan\left(dx+c\right)^3-15\,dx-15\,c+15\,\tan\left(dx+c\right)\right)a^3+35\,a^3\left(\frac{2\left(15\,\sin(dx+\cdots}{\sin(dx+c)\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/26880*(11520*a^3*tan(d*x + c)^7 + 1792*(3*tan(d*x + c)^5 - 5*tan(d*x + c)
^3 - 15*d*x - 15*c + 15*tan(d*x + c))*a^3 + 35*a^3*(2*(15*sin(d*x + c)^7 +
73*sin(d*x + c)^5 - 55*sin(d*x + c)^3 + 15*sin(d*x + c))/(sin(d*x + c)^8 -
4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 - 4*sin(d*x + c)^2 + 1) - 15*log(sin(d*
x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 840*a^3*(2*(33*sin(d*x + c)^5 - 4
0*sin(d*x + c)^3 + 15*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*
sin(d*x + c)^2 - 1) + 15*log(sin(d*x + c) + 1) - 15*log(sin(d*x + c) - 1)))
/d
```

Fricas [A] time = 1.312, size = 501, normalized size = 2.11

$$-\frac{26880\,a^3dx\cos\left(dx+c\right)^8+13125\,a^3\cos\left(dx+c\right)^8\log\left(\sin\left(dx+c\right)+1\right)-13125\,a^3\cos\left(dx+c\right)^8\log\left(-\sin\left(dx+c\right)+1\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/26880*(26880*a^3*d*x*cos(d*x + c)^8 + 13125*a^3*cos(d*x + c)^8*log(sin(d
*x + c) + 1) - 13125*a^3*cos(d*x + c)^8*log(-sin(d*x + c) + 1) - 2*(14848*a
^3*cos(d*x + c)^7 + 27195*a^3*cos(d*x + c)^6 + 7424*a^3*cos(d*x + c)^5 - 17
710*a^3*cos(d*x + c)^4 - 14592*a^3*cos(d*x + c)^3 + 1960*a^3*cos(d*x + c)^2
 + 5760*a^3*cos(d*x + c) + 1680*a^3)*sin(d*x + c))/(d*cos(d*x + c)^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int 3\tan^6(c+dx)\sec(c+dx)\,dx+\int 3\tan^6(c+dx)\sec^2(c+dx)\,dx+\int \tan^6(c+dx)\sec^3(c+dx)\,dx+\int \tan^6(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**6,x)
```

```
[Out] a**3*(Integral(3*tan(c + d*x)**6*sec(c + d*x), x) + Integral(3*tan(c + d*x)
**6*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**6*sec(c + d*x)**3, x) + In
tegral(tan(c + d*x)**6, x))
```

Giac [A] time = 4.85306, size = 265, normalized size = 1.12

$$-\frac{13440\,(dx+c)a^3+13125\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-13125\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(315\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}-}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] -1/13440*(13440*(d*x + c)*a^3 + 13125*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)
) - 13125*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(315*a^3*tan(1/2*d*x +
 1/2*c)^15 - 11375*a^3*tan(1/2*d*x + 1/2*c)^13 + 79723*a^3*tan(1/2*d*x + 1/
2*c)^11 - 269879*a^3*tan(1/2*d*x + 1/2*c)^9 + 550089*a^3*tan(1/2*d*x + 1/2*
c)^7 - 749973*a^3*tan(1/2*d*x + 1/2*c)^5 + 212625*a^3*tan(1/2*d*x + 1/2*c)^
3 - 26565*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^8)/d
```

3.48 $\int (a+a\sec(c+dx))^3\tan^4(c+dx)\,dx$

Optimal. Leaf size=169

$$\frac{3a^3\tan^5(c+dx)}{5d}+\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan(c+dx)}{d}+\frac{19a^3\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^3\tan^3(c+dx)\sec^3(c+dx)}{6d}-a^3\,t$$

```
[Out] a^3*x + (19*a^3*ArcTanh[Sin[c + d*x]])/(16*d) - (a^3*Tan[c + d*x])/d - (17*
a^3*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (a^3*Sec[c + d*x]^3*Tan[c + d*x])/(
8*d) + (a^3*Tan[c + d*x]^3)/(3*d) + (3*a^3*Sec[c + d*x]*Tan[c + d*x]^3)/(4*
d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x]^3)/(6*d) + (3*a^3*Tan[c + d*x]^5)/(5*
d)
```

Rubi [A] time = 0.224364, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3886, 3473, 8, 2611, 3770, 2607, 30, 3768}

$$\frac{3a^3\tan^5(c+dx)}{5d}+\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan(c+dx)}{d}+\frac{19a^3\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^3\tan^3(c+dx)\sec^3(c+dx)}{6d}-a^3\,t$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^4,x]
```


```
[Out] a^3*x + (19*a^3*ArcTanh[Sin[c + d*x]])/(16*d) - (a^3*Tan[c + d*x])/d - (17*
a^3*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (a^3*Sec[c + d*x]^3*Tan[c + d*x])/(
8*d) + (a^3*Tan[c + d*x]^3)/(3*d) + (3*a^3*Sec[c + d*x]*Tan[c + d*x]^3)/(4*
d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x]^3)/(6*d) + (3*a^3*Tan[c + d*x]^5)/(5*
d)
```


Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\tan^4(c+dx)\,dx = \int \left(a^3\tan^4(c+dx)+3a^3\sec(c+dx)\tan^4(c+dx)+3a^3\sec^2(c+dx)\tan^4(c+\right.$$

$$= a^3\int \tan^4(c+dx)\,dx + a^3\int \sec^3(c+dx)\tan^4(c+dx)\,dx + \left(3a^3\right)\int \sec(c+d$$

$$= \frac{a^3\tan^3(c+dx)}{3d} + \frac{3a^3\sec(c+dx)\tan^3(c+dx)}{4d} + \frac{a^3\sec^3(c+dx)\tan^3(c+dx)}{6d}$$

$$= -\frac{a^3\tan(c+dx)}{d} - \frac{9a^3\sec(c+dx)\tan(c+dx)}{8d} - \frac{a^3\sec^3(c+dx)\tan(c+dx)}{8d}$$

$$= a^3x + \frac{9a^3\tanh^{-1}(\sin(c+dx))}{8d} - \frac{a^3\tan(c+dx)}{d} - \frac{17a^3\sec(c+dx)\tan(c+dx)}{16d}$$

$$= a^3x + \frac{19a^3\tanh^{-1}(\sin(c+dx))}{16d} - \frac{a^3\tan(c+dx)}{d} - \frac{17a^3\sec(c+dx)\tan(c+dx)}{16d}$$

Mathematica [A] time = 1.40425, size = 303, normalized size = 1.79

$$\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\left(\sec(c)(210\sin(2c+dx)-1440\sin(c+2dx)+1200\sin(3c+2dx)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^6*(-9120*Cos[c +
d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]]) + Sec[c]*(2400*d*x*Cos[c] + 1800*d*x*Cos[c + 2*d*x] + 180
0*d*x*Cos[3*c + 2*d*x] + 720*d*x*Cos[3*c + 4*d*x] + 720*d*x*Cos[5*c + 4*d*x
] + 120*d*x*Cos[5*c + 6*d*x] + 120*d*x*Cos[7*c + 6*d*x] + 1760*Sin[c] + 210
*Sin[d*x] + 210*Sin[2*c + d*x] - 1440*Sin[c + 2*d*x] + 1200*Sin[3*c + 2*d*x
] - 865*Sin[2*c + 3*d*x] - 865*Sin[4*c + 3*d*x] - 1296*Sin[3*c + 4*d*x] - 2
40*Sin[5*c + 4*d*x] - 435*Sin[4*c + 5*d*x] - 435*Sin[6*c + 5*d*x] - 176*Sin
[5*c + 6*d*x])))/(61440*d)
```

Maple [A] time = 0.049, size = 193, normalized size = 1.1

$$\frac{a^3(\tan(dx+c))^3}{3d} - \frac{a^3\tan(dx+c)}{d} + a^3x + \frac{a^3c}{d} + \frac{19a^3(\sin(dx+c))^5}{24d(\cos(dx+c))^4} - \frac{19a^3(\sin(dx+c))^5}{48d(\cos(dx+c))^2} - \frac{19a^3(\sin(dx+c))^3}{48d} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^4,x)
```

```
[Out] 1/3*a^3*tan(d*x+c)^3/d-a^3*tan(d*x+c)/d+a^3*x+1/d*a^3*c+19/24/d*a^3*sin(d*x
+c)^5/cos(d*x+c)^4-19/48/d*a^3*sin(d*x+c)^5/cos(d*x+c)^2-19/48/d*a^3*sin(d*
x+c)^3-19/16/d*a^3*sin(d*x+c)+19/16/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/5/d*a
^3*sin(d*x+c)^5/cos(d*x+c)^5+1/6/d*a^3*sin(d*x+c)^5/cos(d*x+c)^6
```

Maxima [A] time = 1.68206, size = 284, normalized size = 1.68

$$288\,a^3\tan(dx+c)^5 + 160\left(\tan(dx+c)^3 + 3\,dx + 3\,c - 3\,\tan(dx+c)\right)a^3 - 5\,a^3\left(\frac{2\left(3\,\sin(dx+c)^5 + 8\,\sin(dx+c)^3 - 3\,\sin(dx+c)\right)}{\sin(dx+c)^6 - 3\,\sin(dx+c)^4 + 3\,\sin(dx+c)^2 - 1} - 3\,\mathrm{l}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/480*(288*a^3*tan(d*x + c)^5 + 160*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d
*x + c))*a^3 - 5*a^3*(2*(3*sin(d*x + c)^5 + 8*sin(d*x + c)^3 - 3*sin(d*x +
c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(
d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) + 90*a^3*(2*(5*sin(d*x + c)^3 - 3*
sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) + 3*log(sin(d*x + c)
+ 1) - 3*log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.23788, size = 404, normalized size = 2.39

$$\frac{480\,a^3dx\cos(dx+c)^6 + 285\,a^3\cos(dx+c)^6\log(\sin(dx+c)+1) - 285\,a^3\cos(dx+c)^6\log(-\sin(dx+c)+1) - 2\left(176\ldots\right.}{480\,d\,\mathrm{co}\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/480*(480*a^3*d*x*cos(d*x + c)^6 + 285*a^3*cos(d*x + c)^6*log(sin(d*x + c)
 + 1) - 285*a^3*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 2*(176*a^3*cos(d*x
+ c)^5 + 435*a^3*cos(d*x + c)^4 + 208*a^3*cos(d*x + c)^3 - 110*a^3*cos(d*x
+ c)^2 - 144*a^3*cos(d*x + c) - 40*a^3)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int 3\tan^4(c+dx)\sec(c+dx)\,dx+\int 3\tan^4(c+dx)\sec^2(c+dx)\,dx+\int \tan^4(c+dx)\sec^3(c+dx)\,dx+\int \tan^4($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**4,x)
```

```
[Out] a**3*(Integral(3*tan(c + d*x)**4*sec(c + d*x), x) + Integral(3*tan(c + d*x)
**4*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**4*sec(c + d*x)**3, x) + In
tegral(tan(c + d*x)**4, x))
```

Giac [A] time = 2.71284, size = 221, normalized size = 1.31

$$\frac{240\,(dx+c)a^3+285\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-285\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-95\,a^3\tan(}{}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/240*(240*(d*x + c)*a^3 + 285*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 285
*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(45*a^3*tan(1/2*d*x + 1/2*c)^11
 - 95*a^3*tan(1/2*d*x + 1/2*c)^9 - 366*a^3*tan(1/2*d*x + 1/2*c)^7 + 1746*a^
3*tan(1/2*d*x + 1/2*c)^5 - 3135*a^3*tan(1/2*d*x + 1/2*c)^3 + 525*a^3*tan(1/
2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.49 $\int (a + a\sec(c + dx))^3 \tan^2(c + dx)\,dx$

Optimal. Leaf size=98

$$\frac{a^3\tan^3(c+dx)}{d} + \frac{a^3\tan(c+dx)}{d} - \frac{13a^3\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3\tan(c+dx)\sec^3(c+dx)}{4d} + \frac{11a^3\tan(c+dx)\sec(c+}{8d}$$

```
[Out] -(a^3*x) - (13*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*Tan[c + d*x])/d + (1
1*a^3*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x])/
(4*d) + (a^3*Tan[c + d*x]^3)/d
```

Rubi [A] time = 0.158117, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3886, 3473, 8, 2611, 3770, 2607, 30, 3768}

$$\frac{a^3\tan^3(c+dx)}{d} + \frac{a^3\tan(c+dx)}{d} - \frac{13a^3\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3\tan(c+dx)\sec^3(c+dx)}{4d} + \frac{11a^3\tan(c+dx)\sec(c+}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^2,x]
```


```
[Out] -(a^3*x) - (13*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*Tan[c + d*x])/d + (1
1*a^3*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x])/
(4*d) + (a^3*Tan[c + d*x]^3)/d
```


Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\tan^2(c+dx)\,dx = \int \left(a^3\tan^2(c+dx)+3a^3\sec(c+dx)\tan^2(c+dx)+3a^3\sec^2(c+dx)\tan^2(c+
$$

$$= a^3\int \tan^2(c+dx)\,dx + a^3\int \sec^3(c+dx)\tan^2(c+dx)\,dx + \left(3a^3\right)\int \sec(c+dx
$$

$$= \frac{a^3\tan(c+dx)}{d} + \frac{3a^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{a^3\sec^3(c+dx)\tan(c+dx)}{4d} - \frac{1}{4}
$$

$$= -a^3x - \frac{3a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^3\tan(c+dx)}{d} + \frac{11a^3\sec(c+dx)\tan(c+dx}{8d}
$$

$$= -a^3x - \frac{13a^3\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3\tan(c+dx)}{d} + \frac{11a^3\sec(c+dx)\tan(c+}{8d}
$$

Mathematica [B] time = 0.834652, size = 230, normalized size = 2.35

$$-\frac{a^3\sec^4(c+dx)\left(-38\sin(c+dx)-32\sin(2(c+dx))-22\sin(3(c+dx))-39\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] -(a^3*Sec[c + d*x]^4*(24*d*x - 39*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
+ 39*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4*Cos[2*(c + d*x)]*(8*d*x -
 13*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 13*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]]) + Cos[4*(c + d*x)]*(8*d*x - 13*Log[Cos[(c + d*x)/2] - Sin[
(c + d*x)/2]] + 13*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 38*Sin[c + d
*x] - 32*Sin[2*(c + d*x)] - 22*Sin[3*(c + d*x)]))/(64*d)
```

Maple [A] time = 0.041, size = 137, normalized size = 1.4

$$-a^3x + \frac{a^3\tan(dx+c)}{d} - \frac{a^3c}{d} + \frac{13a^3(\sin(dx+c))^3}{8d(\cos(dx+c))^2} + \frac{13a^3\sin(dx+c)}{8d} - \frac{13a^3\ln(\sec(dx+c)+\tan(dx+c))}{8d} + \frac{a^3(\sin}{d(\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^2,x)
```

```
[Out] -a^3*x+a^3*tan(d*x+c)/d-1/d*a^3*c+13/8/d*a^3*sin(d*x+c)^3/cos(d*x+c)^2+13/8
/d*a^3*sin(d*x+c)-13/8/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^3*sin(d*x+c)^3
```

```
/cos(d*x+c)^3+1/4/d*a^3*sin(d*x+c)^3/cos(d*x+c)^4
```

Maxima [A] time = 1.89841, size = 198, normalized size = 2.02

$$\frac{16\,a^3\tan\left(dx+c\right)^3-16\left(dx+c-\tan\left(dx+c\right)\right)a^3+a^3\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)\right.\right.}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^3*tan(d*x + c)^3 - 16*(d*x + c - tan(d*x + c))*a^3 + a^3*(2*(sin
(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*a^3*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.18766, size = 293, normalized size = 2.99

$$-\frac{16\,a^3dx\cos\left(dx+c\right)^4+13\,a^3\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-13\,a^3\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)-2\left(11\,a^3\right.}{16\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(16*a^3*d*x*cos(d*x + c)^4 + 13*a^3*cos(d*x + c)^4*log(sin(d*x + c) +
 1) - 13*a^3*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 2*(11*a^3*cos(d*x + c)
^2 + 8*a^3*cos(d*x + c) + 2*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int 3\tan^2\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3\tan^2\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\tan^2\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int\tan^2\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**2,x)
```

```
[Out] a**3*(Integral(3*tan(c + d*x)**2*sec(c + d*x), x) + Integral(3*tan(c + d*x)
**2*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**2*sec(c + d*x)**3, x) + In
tegral(tan(c + d*x)**2, x))
```

Giac [A] time = 1.66335, size = 178, normalized size = 1.82

$$-\frac{8\,(dx+c)a^3+13\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-13\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-13\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{\left(\tan\left(\frac{1}{2}\,d\right.\right.}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*(8*(d*x + c)*a^3 + 13*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 13*a^3*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(5*a^3*tan(1/2*d*x + 1/2*c)^7 - 13*a
^3*tan(1/2*d*x + 1/2*c)^5 + 3*a^3*tan(1/2*d*x + 1/2*c)^3 + 21*a^3*tan(1/2*d
*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.50 $\int \cot^2(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=49

$$-\frac{4a^3\cot(c+dx)}{d}-\frac{4a^3\csc(c+dx)}{d}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+a^3(-x)$$

```
[Out] -(a^3*x) + (a^3*ArcTanh[Sin[c + d*x]])/d - (4*a^3*Cot[c + d*x])/d - (4*a^3*
Csc[c + d*x])/d
```

Rubi [A] time = 0.0970836, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3886, 3473, 8, 2606, 3767, 2621, 321, 207}

$$-\frac{4a^3\cot(c+dx)}{d}-\frac{4a^3\csc(c+dx)}{d}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+a^3(-x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```


```
[Out] -(a^3*x) + (a^3*ArcTanh[Sin[c + d*x]])/d - (4*a^3*Cot[c + d*x])/d - (4*a^3*
Csc[c + d*x])/d
```


Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\cot^2(c+dx)+3a^3\cot(c+dx)\csc(c+dx)+3a^3\csc^2(c+dx)+a^3\csc^2(\right.$$
$$= a^3\int \cot^2(c+dx)\,dx + a^3\int \csc^2(c+dx)\sec(c+dx)\,dx + \left(3a^3\right)\int \cot(c+dx$$
$$= -\frac{a^3\cot(c+dx)}{d} - a^3\int 1\,dx - \frac{a^3\operatorname{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} - \frac{\left(3a^3\right)\mathrm{S}}{}$$
$$= -a^3x - \frac{4a^3\cot(c+dx)}{d} - \frac{4a^3\csc(c+dx)}{d} - \frac{a^3\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+d\right.}{d}$$
$$= -a^3x + \frac{a^3\tanh^{-1}(\sin(c+dx))}{d} - \frac{4a^3\cot(c+dx)}{d} - \frac{4a^3\csc(c+dx)}{d}$$

Mathematica [B] time = 0.235522, size = 109, normalized size = 2.22

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-4\csc\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\csc\left(\frac{1}{2}(c+dx)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(d*x + Log[Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 4*Csc[c/2]*
Csc[(c + d*x)/2]*Sin[(d*x)/2]))/(8*d)
```

Maple [A] time = 0.05, size = 68, normalized size = 1.4

$$-a^3x - 4\,\frac{a^3\cot(dx+c)}{d} - \frac{a^3c}{d} - 4\,\frac{a^3}{d\sin(dx+c)} + \frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -a^3*x-4*a^3*cot(d*x+c)/d-1/d*a^3*c-4/d*a^3/sin(d*x+c)+1/d*a^3*ln(sec(d*x+c
)+tan(d*x+c))
```

Maxima [A] time = 1.77699, size = 115, normalized size = 2.35

$$-\frac{2\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^3+a^3\left(\frac{2}{\sin(dx+c)}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{6\,a^3}{\sin(dx+c)}+\frac{6\,a^3}{\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c + 1/tan(d*x + c))*a^3 + a^3*(2/sin(d*x + c) - log(sin(d*x
+ c) + 1) + log(sin(d*x + c) - 1)) + 6*a^3/sin(d*x + c) + 6*a^3/tan(d*x + c
))/d
```

Fricas [A] time = 1.19995, size = 216, normalized size = 4.41

$$-\frac{2\,a^3dx\sin\left(dx+c\right)-a^3\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+a^3\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+8\,a^3\cos\left(dx+c\right)+8}{2\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*d*x*sin(d*x + c) - a^3*log(sin(d*x + c) + 1)*sin(d*x + c) + a^3
*log(-sin(d*x + c) + 1)*sin(d*x + c) + 8*a^3*cos(d*x + c) + 8*a^3)/(d*sin(d
*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int 3\cot^2\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3\cot^2\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int \cot^2\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int \cot^2\left(c+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*cot(c + d*x)**2*sec(c + d*x), x) + Integral(3*cot(c + d*x)
**2*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**2*sec(c + d*x)**3, x) + In
tegral(cot(c + d*x)**2, x))
```

Giac [A] time = 1.48206, size = 89, normalized size = 1.82

$$-\frac{(dx+c)a^3 - a^3 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) + a^3 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{4\,a^3}{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((d*x + c)*a^3 - a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + a^3*log(abs(tan(
1/2*d*x + 1/2*c) - 1)) + 4*a^3/tan(1/2*d*x + 1/2*c))/d
```

3.51 $\int \cot^4(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=69

$$-\frac{4a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^3(c+dx)}{3d}+\frac{3a^3\csc(c+dx)}{d}+a^3x$$

```
[Out] a^3*x + (a^3*Cot[c + d*x])/d - (4*a^3*Cot[c + d*x]^3)/(3*d) + (3*a^3*Csc[c
+ d*x])/d - (4*a^3*Csc[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.132014, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3886, 3473, 8, 2606, 2607, 30}

$$-\frac{4a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^3(c+dx)}{3d}+\frac{3a^3\csc(c+dx)}{d}+a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] a^3*x + (a^3*Cot[c + d*x])/d - (4*a^3*Cot[c + d*x]^3)/(3*d) + (3*a^3*Csc[c
+ d*x])/d - (4*a^3*Csc[c + d*x]^3)/(3*d)
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\cot^4(c+dx)+3a^3\cot^3(c+dx)\csc(c+dx)+3a^3\cot^2(c+dx)\csc^2(c+\right.$$
$$= a^3\int \cot^4(c+dx)\,dx + a^3\int \cot(c+dx)\csc^3(c+dx)\,dx + \left(3a^3\right)\int \cot^3(c+d$$
$$= -\frac{a^3\cot^3(c+dx)}{3d} - a^3\int \cot^2(c+dx)\,dx - \frac{a^3\operatorname{Subst}\left(\int x^2\,dx, x, \csc(c+dx)\right)}{d}$$
$$= \frac{a^3\cot(c+dx)}{d} - \frac{4a^3\cot^3(c+dx)}{3d} + \frac{3a^3\csc(c+dx)}{d} - \frac{4a^3\csc^3(c+dx)}{3d} + a^3$$
$$= a^3x + \frac{a^3\cot(c+dx)}{d} - \frac{4a^3\cot^3(c+dx)}{3d} + \frac{3a^3\csc(c+dx)}{d} - \frac{4a^3\csc^3(c+dx)}{3d}$$

Mathematica [A] time = 0.233152, size = 112, normalized size = 1.62

$$\frac{a^3\csc\left(\frac{c}{2}\right)\csc^3\left(\frac{1}{2}(c+dx)\right)\left(-18\sin\left(c+\frac{dx}{2}\right)+14\sin\left(c+\frac{3dx}{2}\right)-9dx\cos\left(c+\frac{dx}{2}\right)-3dx\cos\left(c+\frac{3dx}{2}\right)+3dx\cos\left(2c+\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*Csc[c/2]*Csc[(c + d*x)/2]^3*(9*d*x*Cos[(d*x)/2] - 9*d*x*Cos[c + (d*x)/
2] - 3*d*x*Cos[c + (3*d*x)/2] + 3*d*x*Cos[2*c + (3*d*x)/2] - 24*Sin[(d*x)/2
```

```
] - 18*Sin[c + (d*x)/2] + 14*Sin[c + (3*d*x)/2]))/(24*d)
```

Maple [A] time = 0.065, size = 125, normalized size = 1.8

$$\frac{1}{d}\left(a^3\left(-\frac{(\cot(dx+c))^3}{3}+\cot(dx+c)+dx+c\right)+3a^3\left(-1/3\,\frac{(\cos(dx+c))^4}{(\sin(dx+c))^3}+1/3\,\frac{(\cos(dx+c))^4}{\sin(dx+c)}+1/3\,\left(2+(\cos(dx+c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+3*a^3*(-1/3/sin(d*x+c)^3*cos(
d*x+c)^4+1/3/sin(d*x+c)*cos(d*x+c)^4+1/3*(2+cos(d*x+c)^2)*sin(d*x+c))-a^3/s
in(d*x+c)^3*cos(d*x+c)^3-1/3*a^3/sin(d*x+c)^3)
```

Maxima [A] time = 1.79234, size = 122, normalized size = 1.77

$$\frac{\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^3+\frac{3\left(3\,\sin(dx+c)^2-1\right)a^3}{\sin(dx+c)^3}-\frac{a^3}{\sin(dx+c)^3}-\frac{3\,a^3}{\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*((3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^3 + 3*(3*sin(d
*x + c)^2 - 1)*a^3/sin(d*x + c)^3 - a^3/sin(d*x + c)^3 - 3*a^3/tan(d*x + c)
^3)/d
```

Fricas [A] time = 1.11453, size = 193, normalized size = 2.8

$$\frac{7\,a^3\cos(dx+c)^2+2\,a^3\cos(dx+c)-5\,a^3+3\left(a^3dx\cos(dx+c)-a^3dx\right)\sin(dx+c)}{3\,(d\cos(dx+c)-d)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3*(7*a^3*cos(d*x + c)^2 + 2*a^3*cos(d*x + c) - 5*a^3 + 3*(a^3*d*x*cos(d*x
 + c) - a^3*d*x)*sin(d*x + c))/((d*cos(d*x + c) - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.50818, size = 68, normalized size = 0.99

$$\frac{3\,(dx+c)a^3 + \frac{6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^3 + (6*a^3*tan(1/2*d*x + 1/2*c)^2 - a^3)/tan(1/2*d*x + 1
/2*c)^3)/d
```

3.52 $\int \cot^6(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=107

$$-\frac{4a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^5(c+dx)}{5d}+\frac{7a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc(c+dx)}{d}-a^3x$$

```
[Out] -(a^3*x) - (a^3*Cot[c + d*x])/d + (a^3*Cot[c + d*x]^3)/(3*d) - (4*a^3*Cot[c
 + d*x]^5)/(5*d) - (3*a^3*Csc[c + d*x])/d + (7*a^3*Csc[c + d*x]^3)/(3*d) -
(4*a^3*Csc[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.164053, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3886, 3473, 8, 2606, 194, 2607, 30, 14}

$$-\frac{4a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^5(c+dx)}{5d}+\frac{7a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc(c+dx)}{d}-a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```


```
[Out] -(a^3*x) - (a^3*Cot[c + d*x])/d + (a^3*Cot[c + d*x]^3)/(3*d) - (4*a^3*Cot[c
 + d*x]^5)/(5*d) - (3*a^3*Csc[c + d*x])/d + (7*a^3*Csc[c + d*x]^3)/(3*d) -
(4*a^3*Csc[c + d*x]^5)/(5*d)
```


Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\cot^6(c+dx)+3a^3\cot^5(c+dx)\csc(c+dx)+3a^3\cot^4(c+dx)\csc^2(c+dx\right.$$
$$= a^3\int \cot^6(c+dx)\,dx + a^3\int \cot^3(c+dx)\csc^3(c+dx)\,dx + \left(3a^3\right)\int \cot^5(c+dx$$
$$= -\frac{a^3\cot^5(c+dx)}{5d} - a^3\int \cot^4(c+dx)\,dx - \frac{a^3\,\mathrm{Subst}\left(\int x^2\left(-1+x^2\right)dx, x, \csc(c\right.}{d}$$
$$= \frac{a^3\cot^3(c+dx)}{3d} - \frac{4a^3\cot^5(c+dx)}{5d} + a^3\int \cot^2(c+dx)\,dx - \frac{a^3\,\mathrm{Subst}\left(\int\left(-x^2\right.\right.}{}$$
$$= -\frac{a^3\cot(c+dx)}{d} + \frac{a^3\cot^3(c+dx)}{3d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{3a^3\csc(c+dx)}{d} + \frac{7a^3\,c}{}$$
$$= -a^3x - \frac{a^3\cot(c+dx)}{d} + \frac{a^3\cot^3(c+dx)}{3d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{3a^3\csc(c+dx)}{d} +$$

Mathematica [A] time = 0.662737, size = 112, normalized size = 1.05

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\cot\left(\frac{c}{2}\right)(13\cos(c+dx)-10)\csc^4\left(\frac{1}{2}(c+dx)\right)+\csc\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)(51\cos(c+dx)-\right.}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(60*d*x + (-10 + 13*Cos[c + d
*x])*Cot[c/2]*Csc[(c + d*x)/2]^4 + (-38 + 51*Cos[c + d*x] - 16*Cos[2*(c + d
*x)])*Csc[c/2]*Csc[(c + d*x)/2]^5*Sin[(d*x)/2]))/(480*d)
```

Maple [B] time = 0.075, size = 232, normalized size = 2.2

$$\frac{1}{d}\left(a^3\left(-\frac{(\cot(dx+c))^5}{5}+\frac{(\cot(dx+c))^3}{3}-\cot(dx+c)-dx-c\right)+3a^3\left(-1/5\frac{(\cos(dx+c))^6}{(\sin(dx+c))^5}+1/15\frac{(\cos(dx+c))^6}{(\sin(dx+c))^3}-1/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+3*a^3*(-1/5/
sin(d*x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c)*cos
```

```
(d*x+c)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-3/5*a^3/sin(d
*x+c)^5*cos(d*x+c)^5+a^3*(-1/5/sin(d*x+c)^5*cos(d*x+c)^4-1/15/sin(d*x+c)^3*
cos(d*x+c)^4+1/15/sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))
)
```

Maxima [A] time = 1.76012, size = 165, normalized size = 1.54

$$-\frac{\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^3+\frac{3\left(15\,\sin(dx+c)^4-10\,\sin(dx+c)^2+3\right)a^3}{\sin(dx+c)^5}-\frac{\left(5\,\sin(dx+c)^2-3\right)a^3}{\sin(dx+c)^5}+\frac{9\,a^3}{\tan(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/15*((15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x
+ c)^5)*a^3 + 3*(15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 + 3)*a^3/sin(d*x + c
)^5 - (5*sin(d*x + c)^2 - 3)*a^3/sin(d*x + c)^5 + 9*a^3/tan(d*x + c)^5)/d
```

Fricas [A] time = 1.1057, size = 298, normalized size = 2.79

$$-\frac{32\,a^3\cos(dx+c)^3-19\,a^3\cos(dx+c)^2-29\,a^3\cos(dx+c)+22\,a^3+15\left(a^3dx\cos(dx+c)^2-2\,a^3dx\cos(dx+c)+a^3\right.}{15\left(d\cos(dx+c)^2-2\,d\cos(dx+c)+d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(32*a^3*cos(d*x + c)^3 - 19*a^3*cos(d*x + c)^2 - 29*a^3*cos(d*x + c)
+ 22*a^3 + 15*(a^3*d*x*cos(d*x + c)^2 - 2*a^3*d*x*cos(d*x + c) + a^3*d*x)*s
in(d*x + c))/((d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43085, size = 89, normalized size = 0.83

$$-\frac{60\,(dx+c)a^3+\frac{105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-20\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/60*(60*(d*x + c)*a^3 + (105*a^3*tan(1/2*d*x + 1/2*c)^4 - 20*a^3*tan(1/2*
d*x + 1/2*c)^2 + 3*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.53 $\int \cot^8(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=141

$$-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^7(c+dx)}{7d}+\frac{11a^3\csc^5(c+dx)}{5d}-\frac{10a^3}{}$$

```
[Out] a^3*x + (a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d) + (a^3*Cot[c + d*
x]^5)/(5*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) + (3*a^3*Csc[c + d*x])/d - (10*a
^3*Csc[c + d*x]^3)/(3*d) + (11*a^3*Csc[c + d*x]^5)/(5*d) - (4*a^3*Csc[c + d
*x]^7)/(7*d)
```

Rubi [A] time = 0.178499, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3886, 3473, 8, 2606, 194, 2607, 30, 270}

$$-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^7(c+dx)}{7d}+\frac{11a^3\csc^5(c+dx)}{5d}-\frac{10a^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8*(a + a*Sec[c + d*x])^3,x]
```


```
[Out] a^3*x + (a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d) + (a^3*Cot[c + d*
x]^5)/(5*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) + (3*a^3*Csc[c + d*x])/d - (10*a
^3*Csc[c + d*x]^3)/(3*d) + (11*a^3*Csc[c + d*x]^5)/(5*d) - (4*a^3*Csc[c + d
*x]^7)/(7*d)
```


Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\cot^8(c+dx)+3a^3\cot^7(c+dx)\csc(c+dx)+3a^3\cot^6(c+dx)\csc^2(c+\right.$$
$$= a^3\int \cot^8(c+dx)\,dx + a^3\int \cot^5(c+dx)\csc^3(c+dx)\,dx + \left(3a^3\right)\int \cot^7(c+$$
$$= -\frac{a^3\cot^7(c+dx)}{7d} - a^3\int \cot^6(c+dx)\,dx - \frac{a^3\,\text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx, x, \text{cs}\right.}{d}$$
$$= \frac{a^3\cot^5(c+dx)}{5d} - \frac{4a^3\cot^7(c+dx)}{7d} + a^3\int \cot^4(c+dx)\,dx - \frac{a^3\,\text{Subst}\left(\int\left(x^2\right.\right.}{}$$
$$= -\frac{a^3\cot^3(c+dx)}{3d} + \frac{a^3\cot^5(c+dx)}{5d} - \frac{4a^3\cot^7(c+dx)}{7d} + \frac{3a^3\csc(c+dx)}{d} - 10$$
$$= \frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{3d} + \frac{a^3\cot^5(c+dx)}{5d} - \frac{4a^3\cot^7(c+dx)}{7d} + 3a^3$$
$$= a^3x + \frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{3d} + \frac{a^3\cot^5(c+dx)}{5d} - \frac{4a^3\cot^7(c+dx)}{7d} +$$

Mathematica [A] time = 0.986898, size = 252, normalized size = 1.79

$$\frac{a^3\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\csc^7\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)(-23282\sin(c+dx)+23282\sin(2(c+dx))-9978\sin(3(c+dx))+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*Csc[c/2]*Csc[(c + d*x)/2]^7*Sec[c/2]*Sec[(c + d*x)/2]*(5880*d*x*Cos[d*
x] - 5880*d*x*Cos[2*c + d*x] - 5880*d*x*Cos[c + 2*d*x] + 5880*d*x*Cos[3*c +
 2*d*x] + 2520*d*x*Cos[2*c + 3*d*x] - 2520*d*x*Cos[4*c + 3*d*x] - 420*d*x*C
os[3*c + 4*d*x] + 420*d*x*Cos[5*c + 4*d*x] + 4200*Sin[c] - 11032*Sin[d*x] -
 23282*Sin[c + d*x] + 23282*Sin[2*(c + d*x)] - 9978*Sin[3*(c + d*x)] + 1663
*Sin[4*(c + d*x)] - 13720*Sin[2*c + d*x] + 15512*Sin[c + 2*d*x] + 9240*Sin[
3*c + 2*d*x] - 8088*Sin[2*c + 3*d*x] - 2520*Sin[4*c + 3*d*x] + 1768*Sin[3*c
 + 4*d*x]))/(215040*d)
```

Maple [B] time = 0.08, size = 293, normalized size = 2.1

$$\frac{1}{d}\left(a^3\left(-\frac{(\cot(dx+c))^7}{7}+\frac{(\cot(dx+c))^5}{5}-\frac{(\cot(dx+c))^3}{3}+\cot(dx+c)+dx+c\right)+3a^3\left(-1/7\frac{(\cos(dx+c))^8}{(\sin(dx+c))^7}+1/35\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^8*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/7*cot(d*x+c)^7+1/5*cot(d*x+c)^5-1/3*cot(d*x+c)^3+cot(d*x+c)+d*
x+c)+3*a^3*(-1/7/sin(d*x+c)^7*cos(d*x+c)^8+1/35/sin(d*x+c)^5*cos(d*x+c)^8-1
/35/sin(d*x+c)^3*cos(d*x+c)^8+1/7/sin(d*x+c)*cos(d*x+c)^8+1/7*(16/5+cos(d*x
+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-3/7*a^3/sin(d*x+c)^7*c
os(d*x+c)^7+a^3*(-1/7/sin(d*x+c)^7*cos(d*x+c)^6-1/35/sin(d*x+c)^5*cos(d*x+c
)^6+1/105/sin(d*x+c)^3*cos(d*x+c)^6-1/35/sin(d*x+c)*cos(d*x+c)^6-1/35*(8/3+
cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 1.67757, size = 205, normalized size = 1.45

$$\frac{\left(105\,dx+105\,c+\frac{105\,\tan(dx+c)^6-35\,\tan(dx+c)^4+21\,\tan(dx+c)^2-15}{\tan(dx+c)^7}\right)a^3+\frac{9\left(35\,\sin(dx+c)^6-35\,\sin(dx+c)^4+21\,\sin(dx+c)^2-5\right)a^3}{\sin(dx+c)^7}-\frac{\left(35\,\sin(dx+c)^4-\right.}{\sin}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/105*((105*d*x + 105*c + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan(
d*x + c)^2 - 15)/tan(d*x + c)^7)*a^3 + 9*(35*sin(d*x + c)^6 - 35*sin(d*x +
c)^4 + 21*sin(d*x + c)^2 - 5)*a^3/sin(d*x + c)^7 - (35*sin(d*x + c)^4 - 42*
sin(d*x + c)^2 + 15)*a^3/sin(d*x + c)^7 - 45*a^3/tan(d*x + c)^7)/d
```

Fricas [A] time = 1.13793, size = 402, normalized size = 2.85

$$\frac{221\,a^3\cos\left(dx+c\right)^4-348\,a^3\cos\left(dx+c\right)^3-25\,a^3\cos\left(dx+c\right)^2+303\,a^3\cos\left(dx+c\right)-136\,a^3+105\left(a^3dx\cos\left(dx+c\right)^3-\right.}{105\left(d\cos\left(dx+c\right)^3-3\,d\cos\left(dx+c\right)^2+3\,d\cos\left(dx+c\right)-d\right)\sin\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/105*(221*a^3*cos(d*x + c)^4 - 348*a^3*cos(d*x + c)^3 - 25*a^3*cos(d*x + c
)^2 + 303*a^3*cos(d*x + c) - 136*a^3 + 105*(a^3*d*x*cos(d*x + c)^3 - 3*a^3*
d*x*cos(d*x + c)^2 + 3*a^3*d*x*cos(d*x + c) - a^3*d*x)*sin(d*x + c))/((d*co
```

```
s(d*x + c)^3 - 3*d*cos(d*x + c)^2 + 3*d*cos(d*x + c) - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**8*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.58607, size = 130, normalized size = 0.92

$$\frac{1680\,(dx+c)a^3 - 105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + \frac{2730\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 - 560\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 126\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 15\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1680*(1680*(d*x + c)*a^3 - 105*a^3*tan(1/2*d*x + 1/2*c) + (2730*a^3*tan(1
/2*d*x + 1/2*c)^6 - 560*a^3*tan(1/2*d*x + 1/2*c)^4 + 126*a^3*tan(1/2*d*x +
1/2*c)^2 - 15*a^3)/tan(1/2*d*x + 1/2*c)^7)/d
```

3.54 $\int \cot^{10}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=179

$$-\frac{4a^3\cot^9(c+dx)}{9d}+\frac{a^3\cot^7(c+dx)}{7d}-\frac{a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^9(c+dx)}{9d}+\frac{15a^3\csc}{7}$$

```
[Out] -(a^3*x) - (a^3*Cot[c + d*x])/d + (a^3*Cot[c + d*x]^3)/(3*d) - (a^3*Cot[c +
 d*x]^5)/(5*d) + (a^3*Cot[c + d*x]^7)/(7*d) - (4*a^3*Cot[c + d*x]^9)/(9*d)
- (3*a^3*Csc[c + d*x])/d + (13*a^3*Csc[c + d*x]^3)/(3*d) - (21*a^3*Csc[c +
d*x]^5)/(5*d) + (15*a^3*Csc[c + d*x]^7)/(7*d) - (4*a^3*Csc[c + d*x]^9)/(9*d
)
```

Rubi [A] time = 0.199362, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3886, 3473, 8, 2606, 194, 2607, 30, 270}

$$-\frac{4a^3\cot^9(c+dx)}{9d}+\frac{a^3\cot^7(c+dx)}{7d}-\frac{a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^9(c+dx)}{9d}+\frac{15a^3\csc}{7}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^10*(a + a*Sec[c + d*x])^3,x]
```


```
[Out] -(a^3*x) - (a^3*Cot[c + d*x])/d + (a^3*Cot[c + d*x]^3)/(3*d) - (a^3*Cot[c +
 d*x]^5)/(5*d) + (a^3*Cot[c + d*x]^7)/(7*d) - (4*a^3*Cot[c + d*x]^9)/(9*d)
- (3*a^3*Csc[c + d*x])/d + (13*a^3*Csc[c + d*x]^3)/(3*d) - (21*a^3*Csc[c +
d*x]^5)/(5*d) + (15*a^3*Csc[c + d*x]^7)/(7*d) - (4*a^3*Csc[c + d*x]^9)/(9*d
)
```


Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\cot^{10}(c+dx)+3a^3\cot^9(c+dx)\csc(c+dx)+3a^3\cot^8(c+dx)\csc^2(c+\right.$$
$$= a^3\int \cot^{10}(c+dx)\,dx + a^3\int \cot^7(c+dx)\csc^3(c+dx)\,dx + \left(3a^3\right)\int \cot^9(c+$$
$$= -\frac{a^3\cot^9(c+dx)}{9d} - a^3\int \cot^8(c+dx)\,dx - \frac{a^3\,\text{Subst}\left(\int x^2\left(-1+x^2\right)^3\,dx,x,\csc\right.}{d}$$
$$= \frac{a^3\cot^7(c+dx)}{7d} - \frac{4a^3\cot^9(c+dx)}{9d} + a^3\int \cot^6(c+dx)\,dx - \frac{a^3\,\text{Subst}\left(\int\left(-x^2\right.\right.}{}$$
$$= -\frac{a^3\cot^5(c+dx)}{5d} + \frac{a^3\cot^7(c+dx)}{7d} - \frac{4a^3\cot^9(c+dx)}{9d} - \frac{3a^3\csc(c+dx)}{d} + \frac{13a}{}$$
$$= \frac{a^3\cot^3(c+dx)}{3d} - \frac{a^3\cot^5(c+dx)}{5d} + \frac{a^3\cot^7(c+dx)}{7d} - \frac{4a^3\cot^9(c+dx)}{9d} - \frac{3a^3}{}$$
$$= -\frac{a^3\cot(c+dx)}{d} + \frac{a^3\cot^3(c+dx)}{3d} - \frac{a^3\cot^5(c+dx)}{5d} + \frac{a^3\cot^7(c+dx)}{7d} - \frac{4a^3}{}$$
$$= -a^3x - \frac{a^3\cot(c+dx)}{d} + \frac{a^3\cot^3(c+dx)}{3d} - \frac{a^3\cot^5(c+dx)}{5d} + \frac{a^3\cot^7(c+dx)}{7d} -$$

Mathematica [B] time = 1.36962, size = 370, normalized size = 2.07

$$\frac{a^3\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\csc^9\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)(675036\sin(c+dx)-506277\sin(2(c+dx))-37502\sin(3(c+dx))+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^10*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*Csc[c/2]*Csc[(c + d*x)/2]^9*Sec[c/2]*Sec[(c + d*x)/2]^3*(-181440*d*x*C
os[d*x] + 181440*d*x*Cos[2*c + d*x] + 136080*d*x*Cos[c + 2*d*x] - 136080*d*
x*Cos[3*c + 2*d*x] + 10080*d*x*Cos[2*c + 3*d*x] - 10080*d*x*Cos[4*c + 3*d*x
] - 60480*d*x*Cos[3*c + 4*d*x] + 60480*d*x*Cos[5*c + 4*d*x] + 30240*d*x*Cos
[4*c + 5*d*x] - 30240*d*x*Cos[6*c + 5*d*x] - 5040*d*x*Cos[5*c + 6*d*x] + 50
40*d*x*Cos[7*c + 6*d*x] - 169344*Sin[c] + 338112*Sin[d*x] + 675036*Sin[c +
d*x] - 506277*Sin[2*(c + d*x)] - 37502*Sin[3*(c + d*x)] + 225012*Sin[4*(c +
 d*x)] - 112506*Sin[5*(c + d*x)] + 18751*Sin[6*(c + d*x)] + 431424*Sin[2*c
+ d*x] - 375552*Sin[c + 2*d*x] - 201600*Sin[3*c + 2*d*x] + 41248*Sin[2*c +
3*d*x] - 84000*Sin[4*c + 3*d*x] + 155712*Sin[3*c + 4*d*x] + 100800*Sin[5*c
+ 4*d*x] - 98016*Sin[4*c + 5*d*x] - 30240*Sin[6*c + 5*d*x] + 21376*Sin[5*c
+ 6*d*x]))/(41287680*d)
```

Maple [B] time = 0.086, size = 364, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^10*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/9*cot(d*x+c)^9+1/7*cot(d*x+c)^7-1/5*cot(d*x+c)^5+1/3*cot(d*x+c
)^3-cot(d*x+c)-d*x-c)+3*a^3*(-1/9/sin(d*x+c)^9*cos(d*x+c)^10+1/63/sin(d*x+c
)^7*cos(d*x+c)^10-1/105/sin(d*x+c)^5*cos(d*x+c)^10+1/63/sin(d*x+c)^3*cos(d*
x+c)^10-1/9/sin(d*x+c)*cos(d*x+c)^10-1/9*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c
)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))-1/3*a^3/sin(d*x+c)^9
*cos(d*x+c)^9+a^3*(-1/9/sin(d*x+c)^9*cos(d*x+c)^8-1/63/sin(d*x+c)^7*cos(d*x
+c)^8+1/315/sin(d*x+c)^5*cos(d*x+c)^8-1/315/sin(d*x+c)^3*cos(d*x+c)^8+1/63/
sin(d*x+c)*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*
x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 1.69417, size = 246, normalized size = 1.37

$$-\frac{\left(315\,dx+315\,c+\frac{315\,\tan(dx+c)^8-105\,\tan(dx+c)^6+63\,\tan(dx+c)^4-45\,\tan(dx+c)^2+35}{\tan(dx+c)^9}\right)a^3+\frac{3\left(315\,\sin(dx+c)^8-420\,\sin(dx+c)^6+378\,\sin(dx+c)^4-\right.}{\sin(dx+c)^9}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/315*((315*d*x + 315*c + (315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 + 63*ta
n(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)/tan(d*x + c)^9)*a^3 + 3*(315*sin(d*x
 + c)^8 - 420*sin(d*x + c)^6 + 378*sin(d*x + c)^4 - 180*sin(d*x + c)^2 + 35
)*a^3/sin(d*x + c)^9 - (105*sin(d*x + c)^6 - 189*sin(d*x + c)^4 + 135*sin(d
*x + c)^2 - 35)*a^3/sin(d*x + c)^9 + 105*a^3/tan(d*x + c)^9)/d
```

Fricas [A] time = 1.17243, size = 605, normalized size = 3.38

$$-\frac{668\,a^3\cos(dx+c)^6-1059\,a^3\cos(dx+c)^5-573\,a^3\cos(dx+c)^4+1813\,a^3\cos(dx+c)^3-393\,a^3\cos(dx+c)^2-789}{315\left(d\cos(dx+c)^5-3\,d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/315*(668*a^3*cos(d*x + c)^6 - 1059*a^3*cos(d*x + c)^5 - 573*a^3*cos(d*x
+ c)^4 + 1813*a^3*cos(d*x + c)^3 - 393*a^3*cos(d*x + c)^2 - 789*a^3*cos(d*x
 + c) + 368*a^3 + 315*(a^3*d*x*cos(d*x + c)^5 - 3*a^3*d*x*cos(d*x + c)^4 +
2*a^3*d*x*cos(d*x + c)^3 + 2*a^3*d*x*cos(d*x + c)^2 - 3*a^3*d*x*cos(d*x + c
) + a^3*d*x)*sin(d*x + c))/((d*cos(d*x + c)^5 - 3*d*cos(d*x + c)^4 + 2*d*co
s(d*x + c)^3 + 2*d*cos(d*x + c)^2 - 3*d*cos(d*x + c) + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**10*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.55588, size = 173, normalized size = 0.97

$$-\frac{105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20160\,(dx+c)a^3-2520\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{31185\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-6720\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+1827\,a^3\cdots}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\cdots}}{20160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/20160*(105*a^3*tan(1/2*d*x + 1/2*c)^3 + 20160*(d*x + c)*a^3 - 2520*a^3*t
an(1/2*d*x + 1/2*c) + (31185*a^3*tan(1/2*d*x + 1/2*c)^8 - 6720*a^3*tan(1/2*
d*x + 1/2*c)^6 + 1827*a^3*tan(1/2*d*x + 1/2*c)^4 - 360*a^3*tan(1/2*d*x + 1/
2*c)^2 + 35*a^3)/tan(1/2*d*x + 1/2*c)^9)/d
```

3.55 $\int \cot^{12}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=213

$$-\frac{4a^3\cot^{11}(c+dx)}{11d}+\frac{a^3\cot^9(c+dx)}{9d}-\frac{a^3\cot^7(c+dx)}{7d}+\frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}-4a^3\csc$$

```
[Out] a^3*x + (a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d) + (a^3*Cot[c + d*
x]^5)/(5*d) - (a^3*Cot[c + d*x]^7)/(7*d) + (a^3*Cot[c + d*x]^9)/(9*d) - (4*
a^3*Cot[c + d*x]^11)/(11*d) + (3*a^3*Csc[c + d*x])/d - (16*a^3*Csc[c + d*x]
^3)/(3*d) + (34*a^3*Csc[c + d*x]^5)/(5*d) - (36*a^3*Csc[c + d*x]^7)/(7*d) +
 (19*a^3*Csc[c + d*x]^9)/(9*d) - (4*a^3*Csc[c + d*x]^11)/(11*d)
```

Rubi [A] time = 0.221298, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3886, 3473, 8, 2606, 194, 2607, 30, 270}

$$-\frac{4a^3\cot^{11}(c+dx)}{11d}+\frac{a^3\cot^9(c+dx)}{9d}-\frac{a^3\cot^7(c+dx)}{7d}+\frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}-4a^3\csc$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^12*(a + a*Sec[c + d*x])^3,x]
```


```
[Out] a^3*x + (a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d) + (a^3*Cot[c + d*
x]^5)/(5*d) - (a^3*Cot[c + d*x]^7)/(7*d) + (a^3*Cot[c + d*x]^9)/(9*d) - (4*
a^3*Cot[c + d*x]^11)/(11*d) + (3*a^3*Csc[c + d*x])/d - (16*a^3*Csc[c + d*x]
^3)/(3*d) + (34*a^3*Csc[c + d*x]^5)/(5*d) - (36*a^3*Csc[c + d*x]^7)/(7*d) +
 (19*a^3*Csc[c + d*x]^9)/(9*d) - (4*a^3*Csc[c + d*x]^11)/(11*d)
```


Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cot^{12}(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\cot^{12}(c+dx)+3a^3\cot^{11}(c+dx)\csc(c+dx)+3a^3\cot^{10}(c+dx)\csc^2(\right.$$

$$= a^3\int \cot^{12}(c+dx)\,dx + a^3\int \cot^9(c+dx)\csc^3(c+dx)\,dx + \left(3a^3\right)\int \cot^{11}(c$$

$$= -\frac{a^3\cot^{11}(c+dx)}{11d} - a^3\int \cot^{10}(c+dx)\,dx - \frac{a^3\,\text{Subst}\left(\int x^2\left(-1+x^2\right)^4\,dx, x,\right.}{d}$$

$$= \frac{a^3\cot^9(c+dx)}{9d} - \frac{4a^3\cot^{11}(c+dx)}{11d} + a^3\int \cot^8(c+dx)\,dx - \frac{a^3\,\text{Subst}\left(\int (x\right.}{}$$

$$= -\frac{a^3\cot^7(c+dx)}{7d} + \frac{a^3\cot^9(c+dx)}{9d} - \frac{4a^3\cot^{11}(c+dx)}{11d} + \frac{3a^3\csc(c+dx)}{d} -$$

$$= \frac{a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+dx)}{7d} + \frac{a^3\cot^9(c+dx)}{9d} - \frac{4a^3\cot^{11}(c+dx)}{11d} + \frac{3a}{}$$

$$= -\frac{a^3\cot^3(c+dx)}{3d} + \frac{a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+dx)}{7d} + \frac{a^3\cot^9(c+dx)}{9d} - \frac{4a}{}$$

$$= \frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{3d} + \frac{a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+dx)}{7d} + \frac{a^3\text{co}}{}$$

$$= a^3x + \frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{3d} + \frac{a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+dx)}{7d} +$$

Mathematica [A] time = 6.01919, size = 268, normalized size = 1.26

$$-\frac{a^3\tan\left(\frac{c}{2}\right)(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(20\cot^2\left(\frac{c}{2}\right)(-4528480\cos(c+dx)+2388316\cos(2(c+dx))-750112\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^12*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(20*(2786111 - 4528480*Cos[c
+ d*x] + 2388316*Cos[2*(c + d*x)] - 750112*Cos[3*(c + d*x)] + 112229*Cos[4*
(c + d*x)])*Cot[c/2]^2*Csc[(c + d*x)/2]^10 - 5*Cot[c/2]*(90832896*d*x + (-3
2611198 + 54812150*Cos[c + d*x] - 32118776*Cos[2*(c + d*x)] + 12626567*Cos[
3*(c + d*x)] - 3023754*Cos[4*(c + d*x)] + 347267*Cos[5*(c + d*x)])*Csc[c/2]
*Csc[(c + d*x)/2]^11*Sin[(d*x)/2]) + 7392*Csc[c/2]*Sec[(c + d*x)/2]^5*(4370
*Sin[(d*x)/2] - 3060*Sin[c + (d*x)/2] + 2860*Sin[c + (3*d*x)/2] - 855*Sin[2
*c + (3*d*x)/2] + 743*Sin[2*c + (5*d*x)/2]))*Tan[c/2])/(3633315840*d)
```

Maple [B] time = 0.145, size = 425, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^12*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/11*cot(d*x+c)^11+1/9*cot(d*x+c)^9-1/7*cot(d*x+c)^7+1/5*cot(d*x
+c)^5-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+3*a^3*(-1/11/sin(d*x+c)^11*cos(d*x
+c)^12+1/99/sin(d*x+c)^9*cos(d*x+c)^12-1/231/sin(d*x+c)^7*cos(d*x+c)^12+1/2
31/sin(d*x+c)^5*cos(d*x+c)^12-1/99/sin(d*x+c)^3*cos(d*x+c)^12+1/11/sin(d*x+
c)*cos(d*x+c)^12+1/11*(256/63+cos(d*x+c)^10+10/9*cos(d*x+c)^8+80/63*cos(d*x
+c)^6+32/21*cos(d*x+c)^4+128/63*cos(d*x+c)^2)*sin(d*x+c))-3/11*a^3/sin(d*x+
c)^11*cos(d*x+c)^11+a^3*(-1/11/sin(d*x+c)^11*cos(d*x+c)^10-1/99/sin(d*x+c)^
9*cos(d*x+c)^10+1/693/sin(d*x+c)^7*cos(d*x+c)^10-1/1155/sin(d*x+c)^5*cos(d*
x+c)^10+1/693/sin(d*x+c)^3*cos(d*x+c)^10-1/99/sin(d*x+c)*cos(d*x+c)^10-1/99
*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^
2)*sin(d*x+c)))
```

Maxima [A] time = 1.65026, size = 286, normalized size = 1.34

$$\left(3465\,dx + 3465\,c + \frac{3465\,\tan(dx+c)^{10} - 1155\,\tan(dx+c)^{8} + 693\,\tan(dx+c)^{6} - 495\,\tan(dx+c)^{4} + 385\,\tan(dx+c)^{2} - 315}{\tan(dx+c)^{11}}\right)a^3 + \frac{15\left(693\,\sin(dx+c)^{10} - 1155\,\mathrm{si}\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^12*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3465*((3465*d*x + 3465*c + (3465*tan(d*x + c)^10 - 1155*tan(d*x + c)^8 +
693*tan(d*x + c)^6 - 495*tan(d*x + c)^4 + 385*tan(d*x + c)^2 - 315)/tan(d*x
 + c)^11)*a^3 + 15*(693*sin(d*x + c)^10 - 1155*sin(d*x + c)^8 + 1386*sin(d*
x + c)^6 - 990*sin(d*x + c)^4 + 385*sin(d*x + c)^2 - 63)*a^3/sin(d*x + c)^1
1 - (1155*sin(d*x + c)^8 - 2772*sin(d*x + c)^6 + 2970*sin(d*x + c)^4 - 1540
*sin(d*x + c)^2 + 315)*a^3/sin(d*x + c)^11 - 945*a^3/tan(d*x + c)^11)/d
```

Fricas [A] time = 1.22568, size = 805, normalized size = 3.78

$$\frac{7453\,a^3\cos(dx+c)^8 - 11964\,a^3\cos(dx+c)^7 - 11866\,a^3\cos(dx+c)^6 + 30542\,a^3\cos(dx+c)^5 + 90\,a^3\cos(dx+c)^4 - 26\cdots}{3465\left(a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^12*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3465*(7453*a^3*cos(d*x + c)^8 - 11964*a^3*cos(d*x + c)^7 - 11866*a^3*cos(
d*x + c)^6 + 30542*a^3*cos(d*x + c)^5 + 90*a^3*cos(d*x + c)^4 - 26438*a^3*c
os(d*x + c)^3 + 8539*a^3*cos(d*x + c)^2 + 7671*a^3*cos(d*x + c) - 3712*a^3
+ 3465*(a^3*d*x*cos(d*x + c)^7 - 3*a^3*d*x*cos(d*x + c)^6 + a^3*d*x*cos(d*x
 + c)^5 + 5*a^3*d*x*cos(d*x + c)^4 - 5*a^3*d*x*cos(d*x + c)^3 - a^3*d*x*cos
(d*x + c)^2 + 3*a^3*d*x*cos(d*x + c) - a^3*d*x)*sin(d*x + c))/((d*cos(d*x +
 c)^7 - 3*d*cos(d*x + c)^6 + d*cos(d*x + c)^5 + 5*d*cos(d*x + c)^4 - 5*d*co
s(d*x + c)^3 - d*cos(d*x + c)^2 + 3*d*cos(d*x + c) - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**12*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.58861, size = 217, normalized size = 1.02

$$-\frac{693\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-11550\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-887040\,(dx+c)a^3+159390\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{5\left(264726\,a^3\tan\right.}{}}{887040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^12*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/887040*(693*a^3*tan(1/2*d*x + 1/2*c)^5 - 11550*a^3*tan(1/2*d*x + 1/2*c)^
3 - 887040*(d*x + c)*a^3 + 159390*a^3*tan(1/2*d*x + 1/2*c) - 5*(264726*a^3*
tan(1/2*d*x + 1/2*c)^10 - 59136*a^3*tan(1/2*d*x + 1/2*c)^8 + 18018*a^3*tan(
1/2*d*x + 1/2*c)^6 - 4554*a^3*tan(1/2*d*x + 1/2*c)^4 + 770*a^3*tan(1/2*d*x
+ 1/2*c)^2 - 63*a^3)/tan(1/2*d*x + 1/2*c)^11)/d
```

3.56 $\int \frac{\tan^9(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=135

$$\frac{\sec^7(c+dx)}{7ad}-\frac{\sec^6(c+dx)}{6ad}-\frac{3\sec^5(c+dx)}{5ad}+\frac{3\sec^4(c+dx)}{4ad}+\frac{\sec^3(c+dx)}{ad}-\frac{3\sec^2(c+dx)}{2ad}-\frac{\sec(c+dx)}{ad}-\frac{\log(\cos(c+dx))}{ad}$$

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - Sec[c + d*x]/(a*d) - (3*Sec[c + d*x]^2)/(2*a*d
) + Sec[c + d*x]^3/(a*d) + (3*Sec[c + d*x]^4)/(4*a*d) - (3*Sec[c + d*x]^5)/
(5*a*d) - Sec[c + d*x]^6/(6*a*d) + Sec[c + d*x]^7/(7*a*d)
```

Rubi [A] time = 0.0784222, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{\sec^7(c+dx)}{7ad}-\frac{\sec^6(c+dx)}{6ad}-\frac{3\sec^5(c+dx)}{5ad}+\frac{3\sec^4(c+dx)}{4ad}+\frac{\sec^3(c+dx)}{ad}-\frac{3\sec^2(c+dx)}{2ad}-\frac{\sec(c+dx)}{ad}-\frac{\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^9/(a + a*Sec[c + d*x]),x]
```


```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - Sec[c + d*x]/(a*d) - (3*Sec[c + d*x]^2)/(2*a*d
) + Sec[c + d*x]^3/(a*d) + (3*Sec[c + d*x]^4)/(4*a*d) - (3*Sec[c + d*x]^5)/
(5*a*d) - Sec[c + d*x]^6/(6*a*d) + Sec[c + d*x]^7/(7*a*d)
```


Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\tan^9(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^4(a+ax)^3}{x^8}\,dx, x, \cos(c+dx)\right)}{a^8 d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^7}{x^8}-\frac{a^7}{x^7}-\frac{3a^7}{x^6}+\frac{3a^7}{x^5}+\frac{3a^7}{x^4}-\frac{3a^7}{x^3}-\frac{a^7}{x^2}+\frac{a^7}{x}\right)dx, x, \cos(c+dx)\right)}{a^8 d}$$
$$= -\frac{\log(\cos(c+dx))}{ad}-\frac{\sec(c+dx)}{ad}-\frac{3\sec^2(c+dx)}{2ad}+\frac{\sec^3(c+dx)}{ad}+\frac{3\sec^4(c+dx)}{4ad}-\frac{3\sec^5(c}{5a}$$

Mathematica [A] time = 0.560865, size = 137, normalized size = 1.01

$$-\frac{\sec^7(c+dx)(35\cos(c+dx)(105\log(\cos(c+dx))+104)+3(602\cos(2(c+dx))+140\cos(4(c+dx))+210\cos(5(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^9/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((35*Cos[c + d*x]*(104 + 105*Log[Cos[c + d*x]]) + 3*(212 + 602*Cos[2*(c +
d*x)] + 140*Cos[4*(c + d*x)] + 210*Cos[5*(c + d*x)] + 70*Cos[6*(c + d*x)] +
 245*Cos[5*(c + d*x)]*Log[Cos[c + d*x]] + 35*Cos[7*(c + d*x)]*Log[Cos[c + d
*x]] + 105*Cos[3*(c + d*x)]*(6 + 7*Log[Cos[c + d*x]])))*Sec[c + d*x]^7)/(67
20*a*d)
```

Maple [A] time = 0.089, size = 125, normalized size = 0.9

$$\frac{(\sec(dx+c))^7}{7da}-\frac{(\sec(dx+c))^6}{6da}-\frac{3(\sec(dx+c))^5}{5da}+\frac{3(\sec(dx+c))^4}{4da}+\frac{(\sec(dx+c))^3}{da}-\frac{3(\sec(dx+c))^2}{2da}-\frac{\sec(dx}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^9/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/7*sec(d*x+c)^7/d/a-1/6*sec(d*x+c)^6/d/a-3/5*sec(d*x+c)^5/d/a+3/4*sec(d*x+
c)^4/d/a+sec(d*x+c)^3/d/a-3/2*sec(d*x+c)^2/d/a-sec(d*x+c)/d/a+1/a/d*ln(sec(
d*x+c))
```

Maxima [A] time = 1.12235, size = 122, normalized size = 0.9

$$-\frac{\frac{420\log(\cos(dx+c))}{a}+\frac{420\cos(dx+c)^6+630\cos(dx+c)^5-420\cos(dx+c)^4-315\cos(dx+c)^3+252\cos(dx+c)^2+70\cos(dx+c)-60}{a\cos(dx+c)^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/420*(420*log(cos(d*x + c))/a + (420*cos(d*x + c)^6 + 630*cos(d*x + c)^5
- 420*cos(d*x + c)^4 - 315*cos(d*x + c)^3 + 252*cos(d*x + c)^2 + 70*cos(d*x
 + c) - 60)/(a*cos(d*x + c)^7))/d
```

Fricas [A] time = 1.23928, size = 266, normalized size = 1.97

$$-\frac{420\cos(dx+c)^7\log(-\cos(dx+c))+420\cos(dx+c)^6+630\cos(dx+c)^5-420\cos(dx+c)^4-315\cos(dx+c)^3+}{420\,ad\cos(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/420*(420*cos(d*x + c)^7*log(-cos(d*x + c)) + 420*cos(d*x + c)^6 + 630*co
s(d*x + c)^5 - 420*cos(d*x + c)^4 - 315*cos(d*x + c)^3 + 252*cos(d*x + c)^2
 + 70*cos(d*x + c) - 60)/(a*d*cos(d*x + c)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**9/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 17.6367, size = 331, normalized size = 2.45

$$\frac{\frac{420\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a}-\frac{420\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a}+\frac{\frac{5775\,(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{20685\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{42595\,(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{56035\,(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{28749\,(\cos(dx+}{(\cos(dx+c)+}}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/420*(420*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - 420*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a + (5775*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + 20685*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
42595*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*(cos(d*x + c) - 1)^
4/(cos(d*x + c) + 1)^4 + 28749*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 +
8463*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 1089*(cos(d*x + c) - 1)^7/
(cos(d*x + c) + 1)^7 + 705)/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^
7))/d
```

3.57 $\int \frac{\tan^7(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=97

$$\frac{\sec^5(c+dx)}{5ad}-\frac{\sec^4(c+dx)}{4ad}-\frac{2\sec^3(c+dx)}{3ad}+\frac{\sec^2(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}+\frac{\log(\cos(c+dx))}{ad}$$

```
[Out] Log[Cos[c + d*x]]/(a*d) + Sec[c + d*x]/(a*d) + Sec[c + d*x]^2/(a*d) - (2*Se
c[c + d*x]^3)/(3*a*d) - Sec[c + d*x]^4/(4*a*d) + Sec[c + d*x]^5/(5*a*d)
```

Rubi [A] time = 0.0684093, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{\sec^5(c+dx)}{5ad}-\frac{\sec^4(c+dx)}{4ad}-\frac{2\sec^3(c+dx)}{3ad}+\frac{\sec^2(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}+\frac{\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + a*Sec[c + d*x]),x]
```

```
[Out] Log[Cos[c + d*x]]/(a*d) + Sec[c + d*x]/(a*d) + Sec[c + d*x]^2/(a*d) - (2*Se
c[c + d*x]^3)/(3*a*d) - Sec[c + d*x]^4/(4*a*d) + Sec[c + d*x]^5/(5*a*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\tan^7(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^3(a+ax)^2}{x^6}\,dx, x, \cos(c+dx)\right)}{a^6d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^5}{x^6} - \frac{a^5}{x^5} - \frac{2a^5}{x^4} + \frac{2a^5}{x^3} + \frac{a^5}{x^2} - \frac{a^5}{x}\right) dx, x, \cos(c+dx)\right)}{a^6d}$$
$$= \frac{\log(\cos(c+dx))}{ad} + \frac{\sec(c+dx)}{ad} + \frac{\sec^2(c+dx)}{ad} - \frac{2\sec^3(c+dx)}{3ad} - \frac{\sec^4(c+dx)}{4ad} + \frac{\sec^5(c+d}{5ad}$$

Mathematica [A] time = 0.273942, size = 103, normalized size = 1.06

$$\frac{\sec^5(c+dx)(40\cos(2(c+dx)) + 60\cos(3(c+dx)) + 30\cos(4(c+dx)) + 75\cos(3(c+dx))\log(\cos(c+dx)) + 15\cos(5(}{240ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((58 + 40*Cos[2*(c + d*x)] + 60*Cos[3*(c + d*x)] + 30*Cos[4*(c + d*x)] + 75
*Cos[3*(c + d*x)]*Log[Cos[c + d*x]] + 15*Cos[5*(c + d*x)]*Log[Cos[c + d*x]]
 + 30*Cos[c + d*x]*(4 + 5*Log[Cos[c + d*x]]))*Sec[c + d*x]^5)/(240*a*d)
```

Maple [A] time = 0.074, size = 93, normalized size = 1.

$$\frac{(\sec(dx+c))^5}{5\,da} - \frac{(\sec(dx+c))^4}{4\,da} - \frac{2\,(\sec(dx+c))^3}{3\,da} + \frac{(\sec(dx+c))^2}{da} + \frac{\sec(dx+c)}{da} - \frac{\ln(\sec(dx+c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/5*sec(d*x+c)^5/d/a-1/4*sec(d*x+c)^4/d/a-2/3*sec(d*x+c)^3/d/a+sec(d*x+c)^2
/d/a+sec(d*x+c)/d/a-1/a/d*ln(sec(d*x+c))
```

Maxima [A] time = 1.10157, size = 95, normalized size = 0.98

$$\frac{\frac{60\,\log(\cos(dx+c))}{a} + \frac{60\,\cos(dx+c)^4+60\,\cos(dx+c)^3-40\,\cos(dx+c)^2-15\,\cos(dx+c)+12}{a\cos(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(60*log(cos(d*x + c))/a + (60*cos(d*x + c)^4 + 60*cos(d*x + c)^3 - 40*
cos(d*x + c)^2 - 15*cos(d*x + c) + 12)/(a*cos(d*x + c)^5))/d
```

Fricas [A] time = 1.16251, size = 201, normalized size = 2.07

$$\frac{60\cos(dx+c)^5\log(-\cos(dx+c)) + 60\cos(dx+c)^4 + 60\cos(dx+c)^3 - 40\cos(dx+c)^2 - 15\cos(dx+c) + 12}{60\,ad\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(60*cos(d*x + c)^5*log(-cos(d*x + c)) + 60*cos(d*x + c)^4 + 60*cos(d*x
 + c)^3 - 40*cos(d*x + c)^2 - 15*cos(d*x + c) + 12)/(a*d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^7(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**7/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**7/(sec(c + d*x) + 1), x)/a
```

Giac [B] time = 8.7259, size = 271, normalized size = 2.79

$$-\frac{\frac{60\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a} - \frac{60\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a} + \frac{\frac{485(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{1330(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{1970(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} + \frac{805(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} + \frac{137(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5} + 73}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - 60*log(a
bs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a + (485*(cos(d*x + c) - 1)
/(cos(d*x + c) + 1) + 1330*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1970
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 805*(cos(d*x + c) - 1)^4/(cos(
d*x + c) + 1)^4 + 137*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 73)/(a*((
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^5))/d
```

3.58 $\int \frac{\tan^5(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=66

$$\frac{\sec^3(c+dx)}{3ad}-\frac{\sec^2(c+dx)}{2ad}-\frac{\sec(c+dx)}{ad}-\frac{\log(\cos(c+dx))}{ad}$$

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - Sec[c + d*x]/(a*d) - Sec[c + d*x]^2/(2*a*d) +
Sec[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.0572111, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 75}

$$\frac{\sec^3(c+dx)}{3ad}-\frac{\sec^2(c+dx)}{2ad}-\frac{\sec(c+dx)}{ad}-\frac{\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - Sec[c + d*x]/(a*d) - Sec[c + d*x]^2/(2*a*d) +
Sec[c + d*x]^3/(3*a*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^2(a+ax)}{x^4}\,dx, x, \cos(c+dx)\right)}{a^4 d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^3}{x^4}-\frac{a^3}{x^3}-\frac{a^3}{x^2}+\frac{a^3}{x}\right)dx, x, \cos(c+dx)\right)}{a^4 d}$$
$$= -\frac{\log(\cos(c+dx))}{ad} - \frac{\sec(c+dx)}{ad} - \frac{\sec^2(c+dx)}{2ad} + \frac{\sec^3(c+dx)}{3ad}$$

Mathematica [A] time = 0.198023, size = 65, normalized size = 0.98

$$-\frac{\sec^3(c+dx)(6\cos(2(c+dx))+3\cos(3(c+dx))\log(\cos(c+dx))+\cos(c+dx)(9\log(\cos(c+dx))+6)+2)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((2 + 6*Cos[2*(c + d*x)] + 3*Cos[3*(c + d*x)]*Log[Cos[c + d*x]] + Cos[c +
d*x]*(6 + 9*Log[Cos[c + d*x]]))*Sec[c + d*x]^3)/(12*a*d)
```

Maple [A] time = 0.065, size = 62, normalized size = 0.9

$$\frac{(\sec(dx+c))^3}{3\,da} - \frac{(\sec(dx+c))^2}{2\,da} - \frac{\sec(dx+c)}{da} + \frac{\ln(\sec(dx+c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/3*sec(d*x+c)^3/d/a-1/2*sec(d*x+c)^2/d/a-sec(d*x+c)/d/a+1/a/d*ln(sec(d*x+c
))
```

Maxima [A] time = 1.18203, size = 68, normalized size = 1.03

$$-\frac{\frac{6\log(\cos(dx+c))}{a} + \frac{6\cos(dx+c)^2+3\cos(dx+c)-2}{a\cos(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(6*log(cos(d*x + c))/a + (6*cos(d*x + c)^2 + 3*cos(d*x + c) - 2)/(a*co
s(d*x + c)^3))/d
```

Fricas [A] time = 1.175, size = 142, normalized size = 2.15

$$-\frac{6\cos(dx+c)^3\log(-\cos(dx+c)) + 6\cos(dx+c)^2 + 3\cos(dx+c) - 2}{6\,ad\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(6*cos(d*x + c)^3*log(-cos(d*x + c)) + 6*cos(d*x + c)^2 + 3*cos(d*x +
c) - 2)/(a*d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\tan^5(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**5/(sec(c + d*x) + 1), x)/a
```

Giac [B] time = 3.77887, size = 212, normalized size = 3.21

$$\frac{\frac{6\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a} - \frac{6\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a} + \frac{\frac{21(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{45(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{11(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} + 3}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - 6*log(abs(-
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a + (21*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) + 45*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 11*(cos(d*x
 + c) - 1)^3/(cos(d*x + c) + 1)^3 + 3)/(a*((cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) + 1)^3))/d
```

3.59 $\int \frac{\tan^3(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=28

$$\frac{\sec(c+dx)}{ad}+\frac{\log(\cos(c+dx))}{ad}$$

```
[Out] Log[Cos[c + d*x]]/(a*d) + Sec[c + d*x]/(a*d)
```

Rubi [A] time = 0.0481519, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 43}

$$\frac{\sec(c+dx)}{ad}+\frac{\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] Log[Cos[c + d*x]]/(a*d) + Sec[c + d*x]/(a*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{a-ax}{x^2}\,dx,x,\cos(c+dx)\right)}{a^2d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a}{x^2}-\frac{a}{x}\right)dx,x,\cos(c+dx)\right)}{a^2d}$$
$$= \frac{\log(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{ad}$$

Mathematica [A] time = 0.065714, size = 21, normalized size = 0.75

$$\frac{\sec(c+dx)+\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Log[Cos[c + d*x]] + Sec[c + d*x])/(a*d)
```

Maple [A] time = 0.046, size = 30, normalized size = 1.1

$$\frac{\sec(dx+c)}{da}-\frac{\ln(\sec(dx+c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sec(d*x+c)),x)
```

```
[Out] sec(d*x+c)/d/a-1/a/d*ln(sec(d*x+c))
```

Maxima [A] time = 1.14619, size = 38, normalized size = 1.36

$$\frac{\frac{\log(\cos(dx+c))}{a}+\frac{1}{a\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(cos(d*x + c))/a + 1/(a*cos(d*x + c)))/d
```

Fricas [A] time = 1.15497, size = 78, normalized size = 2.79

$$\frac{\cos(dx+c)\log(-\cos(dx+c))+1}{ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] (cos(d*x + c)*log(-cos(d*x + c)) + 1)/(a*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\tan^3(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**3/(sec(c + d*x) + 1), x)/a
```

Giac [B] time = 1.97942, size = 150, normalized size = 5.36

$$-\frac{\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a}-\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a}+\frac{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -(log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - log(abs(-(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) - 1))/a + ((cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) - 1)/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

3.60 $\int \frac{\tan(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=17

$$-\frac{\log(\cos(c+dx)+1)}{ad}$$

```
[Out] -(Log[1 + Cos[c + d*x]]/(a*d))
```

Rubi [A] time = 0.0257337, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 31}

$$-\frac{\log(\cos(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(Log[1 + Cos[c + d*x]]/(a*d))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{a+ax}\,dx, x, \cos(c+dx)\right)}{d} = -\frac{\log(1+\cos(c+dx))}{ad}$$

Mathematica [A] time = 0.0177193, size = 19, normalized size = 1.12

$$-\frac{2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*Log[Cos[(c + d*x)/2]])/(a*d)
```

Maple [A] time = 0.02, size = 33, normalized size = 1.9

$$-\frac{\ln(1+\sec(dx+c))}{da} + \frac{\ln(\sec(dx+c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/d/a*ln(1+sec(d*x+c))+1/a/d*ln(sec(d*x+c))
```

Maxima [A] time = 1.15333, size = 23, normalized size = 1.35

$$-\frac{\log(\cos(dx+c)+1)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -log(cos(d*x + c) + 1)/(a*d)
```

Fricas [A] time = 1.11244, size = 49, normalized size = 2.88

$$-\frac{\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -log(1/2*cos(d*x + c) + 1/2)/(a*d)
```

Sympy [A] time = 4.0738, size = 41, normalized size = 2.41

$$\begin{cases} \frac{\log\left(\tan^2\left(c+dx\right)+1\right)}{2ad} - \frac{\log\left(\sec\left(c+dx\right)+1\right)}{ad} & \text{for } d \neq 0 \\ \frac{x\tan\left(c\right)}{a\sec\left(c\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c)),x)
```

```
[Out] Piecewise((log(tan(c + d*x)**2 + 1)/(2*a*d) - log(sec(c + d*x) + 1)/(a*d),
Ne(d, 0)), (x*tan(c)/(a*sec(c) + a), True))
```

Giac [A] time = 1.32052, size = 42, normalized size = 2.47

$$\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/(a*d)
```

3.61 $\int \frac{\cot(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=61

$$\frac{1}{2ad(\cos(c+dx)+1)}+\frac{\log(1-\cos(c+dx))}{4ad}+\frac{3\log(\cos(c+dx)+1)}{4ad}$$

```
[Out] 1/(2*a*d*(1 + Cos[c + d*x])) + Log[1 - Cos[c + d*x]]/(4*a*d) + (3*Log[1 + C
os[c + d*x]])/(4*a*d)
```

Rubi [A] time = 0.0556855, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 88}

$$\frac{1}{2ad(\cos(c+dx)+1)}+\frac{\log(1-\cos(c+dx))}{4ad}+\frac{3\log(\cos(c+dx)+1)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] 1/(2*a*d*(1 + Cos[c + d*x])) + Log[1 - Cos[c + d*x]]/(4*a*d) + (3*Log[1 + C
os[c + d*x]])/(4*a*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{a^2\,\mathrm{Subst}\left(\int \frac{x^2}{(a-ax)(a+ax)^2}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^2\,\mathrm{Subst}\left(\int \left(-\frac{1}{4a^3(-1+x)} + \frac{1}{2a^3(1+x)^2} - \frac{3}{4a^3(1+x)}\right) dx, x, \cos(c+dx)\right)}{d}$$
$$= \frac{1}{2ad(1+\cos(c+dx))} + \frac{\log(1-\cos(c+dx))}{4ad} + \frac{3\log(1+\cos(c+dx))}{4ad}$$

Mathematica [A] time = 0.120026, size = 67, normalized size = 1.1

$$\frac{\sec(c+dx)\left(2\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+1\right)}{2ad(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((1 + 2*Cos[(c + d*x)/2]^2*(3*Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]]
))*Sec[c + d*x])/(2*a*d*(1 + Sec[c + d*x]))
```

Maple [A] time = 0.067, size = 54, normalized size = 0.9

$$\frac{1}{2\,da\,(\cos(dx+c)+1)} + \frac{3\,\ln(\cos(dx+c)+1)}{4\,da} + \frac{\ln(-1+\cos(dx+c))}{4\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/2/a/d/(cos(d*x+c)+1)+3/4*ln(cos(d*x+c)+1)/a/d+1/4/d/a*ln(-1+cos(d*x+c))
```

Maxima [A] time = 1.25335, size = 63, normalized size = 1.03

$$\frac{\frac{3\,\log(\cos(dx+c)+1)}{a} + \frac{\log(\cos(dx+c)-1)}{a} + \frac{2}{a\cos(dx+c)+a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(3*log(cos(d*x + c) + 1)/a + log(cos(d*x + c) - 1)/a + 2/(a*cos(d*x + c
) + a))/d
```

Fricas [A] time = 1.16061, size = 182, normalized size = 2.98

$$\frac{3\left(\cos\left(dx+c\right)+1\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(\cos\left(dx+c\right)+1\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+2}{4\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(3*(cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + (cos(d*x + c) + 1)*
log(-1/2*cos(d*x + c) + 1/2) + 2)/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)/(sec(c + d*x) + 1), x)/a
```

Giac [A] time = 1.41428, size = 116, normalized size = 1.9

$$\frac{\frac{\log\left(\frac{\left|-\cos(dx+c)+1\right|}{\left|\cos(dx+c)+1\right|}\right)}{a}-\frac{4\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a}-\frac{\cos(dx+c)-1}{a(\cos(dx+c)+1)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a - 4*log(abs(-(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - (cos(d*x + c) - 1)/(a*(cos(d*x +
 c) + 1)))/d
```

3.62 $\int \frac{\cot^3(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=103

$$-\frac{1}{8ad(1-\cos(c+dx))}-\frac{3}{4ad(\cos(c+dx)+1)}+\frac{1}{8ad(\cos(c+dx)+1)^2}-\frac{5\log(1-\cos(c+dx))}{16ad}-\frac{11\log(\cos(c+dx)+$$

```
[Out] -1/(8*a*d*(1 - Cos[c + d*x])) + 1/(8*a*d*(1 + Cos[c + d*x])^2) - 3/(4*a*d*(
1 + Cos[c + d*x])) - (5*Log[1 - Cos[c + d*x]])/(16*a*d) - (11*Log[1 + Cos[c
 + d*x]])/(16*a*d)
```

Rubi [A] time = 0.0762475, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$-\frac{1}{8ad(1-\cos(c+dx))}-\frac{3}{4ad(\cos(c+dx)+1)}+\frac{1}{8ad(\cos(c+dx)+1)^2}-\frac{5\log(1-\cos(c+dx))}{16ad}-\frac{11\log(\cos(c+dx)+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/(8*a*d*(1 - Cos[c + d*x])) + 1/(8*a*d*(1 + Cos[c + d*x])^2) - 3/(4*a*d*(
1 + Cos[c + d*x])) - (5*Log[1 - Cos[c + d*x]])/(16*a*d) - (11*Log[1 + Cos[c
 + d*x]])/(16*a*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{a^4 \operatorname{Subst}\left(\int \frac{x^4}{(a-ax)^2(a+ax)^3}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^4 \operatorname{Subst}\left(\int \left(\frac{1}{8a^5(-1+x)^2} + \frac{5}{16a^5(-1+x)} + \frac{1}{4a^5(1+x)^3} - \frac{3}{4a^5(1+x)^2} + \frac{11}{16a^5(1+x)}\right) dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{1}{8ad(1-\cos(c+dx))} + \frac{1}{8ad(1+\cos(c+dx))^2} - \frac{3}{4ad(1+\cos(c+dx))} - \frac{5\log(1-\cos(c+dx}{16ad}$$

Mathematica [A] time = 0.634093, size = 107, normalized size = 1.04

$$-\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(2\csc^2\left(\frac{1}{2}(c+dx)\right) - \sec^4\left(\frac{1}{2}(c+dx)\right) + 12\sec^2\left(\frac{1}{2}(c+dx)\right) + 20\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 4}{16ad(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(Cos[(c + d*x)/2]^2*(2*Csc[(c + d*x)/2]^2 + 44*Log[Cos[(c + d*x)/2]] + 20*
Log[Sin[(c + d*x)/2]] + 12*Sec[(c + d*x)/2]^2 - Sec[(c + d*x)/2]^4)*Sec[c +
 d*x])/(16*a*d*(1 + Sec[c + d*x]))
```

Maple [A] time = 0.067, size = 90, normalized size = 0.9

$$\frac{1}{8\,da\,(\cos(dx+c)+1)^2} - \frac{3}{4\,da\,(\cos(dx+c)+1)} - \frac{11\,\ln(\cos(dx+c)+1)}{16\,da} + \frac{1}{8\,da\,(-1+\cos(dx+c))} - \frac{5\,\ln(-1+\cos(d}{16\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/8/a/d/(cos(d*x+c)+1)^2-3/4/a/d/(cos(d*x+c)+1)-11/16*ln(cos(d*x+c)+1)/a/d+
1/8/a/d/(-1+cos(d*x+c))-5/16/d/a*ln(-1+cos(d*x+c))
```

Maxima [A] time = 1.12747, size = 123, normalized size = 1.19

$$-\frac{\frac{2\left(5\cos(dx+c)^2-3\cos(dx+c)-6\right)}{a\cos(dx+c)^3+a\cos(dx+c)^2-a\cos(dx+c)-a}+\frac{11\log(\cos(dx+c)+1)}{a}+\frac{5\log(\cos(dx+c)-1)}{a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(2*(5*cos(d*x + c)^2 - 3*cos(d*x + c) - 6)/(a*cos(d*x + c)^3 + a*cos(
d*x + c)^2 - a*cos(d*x + c) - a) + 11*log(cos(d*x + c) + 1)/a + 5*log(cos(d
*x + c) - 1)/a)/d
```

Fricas [A] time = 1.22542, size = 389, normalized size = 3.78

$$-\frac{10\cos(dx+c)^2+11\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+5\left(\cos(dx+c)^3+\text{c}\right.}{16\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2-ad\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(10*cos(d*x + c)^2 + 11*(cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x +
c) - 1)*log(1/2*cos(d*x + c) + 1/2) + 5*(cos(d*x + c)^3 + cos(d*x + c)^2 -
cos(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) - 6*cos(d*x + c) - 12)/(a*d*
cos(d*x + c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\cot^3(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**3/(sec(c + d*x) + 1), x)/a
```

Giac [A] time = 1.53013, size = 212, normalized size = 2.06

$$\frac{\frac{2\left(\frac{5\,(\cos(dx+c)-1)}{\cos(dx+c)+1}+1\right)(\cos(dx+c)+1)}{a(\cos(dx+c)-1)}-\frac{10\,\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}+\frac{32\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a}+\frac{\frac{10\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(2*(5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)*(cos(d*x + c) + 1)/(a
*(cos(d*x + c) - 1)) - 10*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))
/a + 32*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a + (10*a*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2)/a^2)/d
```

3.63 $\int \frac{\cot^5(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=145

$$\frac{1}{4ad(1-\cos(c+dx))}+\frac{15}{16ad(\cos(c+dx)+1)}-\frac{1}{32ad(1-\cos(c+dx))^2}-\frac{9}{32ad(\cos(c+dx)+1)^2}+\frac{1}{24ad(\cos(c+dx)}$$

```
[Out] -1/(32*a*d*(1 - Cos[c + d*x])^2) + 1/(4*a*d*(1 - Cos[c + d*x])) + 1/(24*a*d
*(1 + Cos[c + d*x])^3) - 9/(32*a*d*(1 + Cos[c + d*x])^2) + 15/(16*a*d*(1 +
Cos[c + d*x])) + (11*Log[1 - Cos[c + d*x]])/(32*a*d) + (21*Log[1 + Cos[c +
d*x]])/(32*a*d)
```

Rubi [A] time = 0.0980931, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{1}{4ad(1-\cos(c+dx))}+\frac{15}{16ad(\cos(c+dx)+1)}-\frac{1}{32ad(1-\cos(c+dx))^2}-\frac{9}{32ad(\cos(c+dx)+1)^2}+\frac{1}{24ad(\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/(32*a*d*(1 - Cos[c + d*x])^2) + 1/(4*a*d*(1 - Cos[c + d*x])) + 1/(24*a*d
*(1 + Cos[c + d*x])^3) - 9/(32*a*d*(1 + Cos[c + d*x])^2) + 15/(16*a*d*(1 +
Cos[c + d*x])) + (11*Log[1 - Cos[c + d*x]])/(32*a*d) + (21*Log[1 + Cos[c +
d*x]])/(32*a*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{a^6\,\text{Subst}\left(\int \frac{x^6}{(a-ax)^3(a+ax)^4}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^6\,\text{Subst}\left(\int \left(-\frac{1}{16a^7(-1+x)^3} - \frac{1}{4a^7(-1+x)^2} - \frac{11}{32a^7(-1+x)} + \frac{1}{8a^7(1+x)^4} - \frac{9}{16a^7(1+x)^3} + \frac{15}{16a^7(1+x)^2} - \frac{21}{32a^7(1+\cdots}\right.\right.}{d}$$
$$= -\frac{1}{32ad(1-\cos(c+dx))^2} + \frac{1}{4ad(1-\cos(c+dx))} + \frac{1}{24ad(1+\cos(c+dx))^3} - \frac{9}{32ad(1+\cos(c+\cdots}$$

Mathematica [A] time = 0.538169, size = 135, normalized size = 0.93

$$-\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(3\csc^4\left(\frac{1}{2}(c+dx)\right) - 48\csc^2\left(\frac{1}{2}(c+dx)\right) - 2\sec^6\left(\frac{1}{2}(c+dx)\right) + 27\sec^4\left(\frac{1}{2}(c+dx)\right) - 180\,\text{s}\cdots\right.}{192ad(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(Cos[(c + d*x)/2]^2*(-48*Csc[(c + d*x)/2]^2 + 3*Csc[(c + d*x)/2]^4 - 504*L
og[Cos[(c + d*x)/2]] - 264*Log[Sin[(c + d*x)/2]] - 180*Sec[(c + d*x)/2]^2 +
 27*Sec[(c + d*x)/2]^4 - 2*Sec[(c + d*x)/2]^6)*Sec[c + d*x])/(192*a*d*(1 +
Sec[c + d*x]))
```

Maple [A] time = 0.068, size = 126, normalized size = 0.9

$$\frac{1}{24\,da\,(\cos(dx+c)+1)^3} - \frac{9}{32\,da\,(\cos(dx+c)+1)^2} + \frac{15}{16\,da\,(\cos(dx+c)+1)} + \frac{21\,\ln(\cos(dx+c)+1)}{32\,da} - \frac{1}{32\,da\,(-1+\text{c}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/24/a/d/(cos(d*x+c)+1)^3-9/32/a/d/(cos(d*x+c)+1)^2+15/16/a/d/(cos(d*x+c)+1
)+21/32*ln(cos(d*x+c)+1)/a/d-1/32/a/d/(-1+cos(d*x+c))^2-1/4/a/d/(-1+cos(d*x
```

```
+c))+11/32/d/a*ln(-1+cos(d*x+c))
```

Maxima [A] time = 1.17699, size = 176, normalized size = 1.21

$$\frac{\frac{2\left(33\cos(dx+c)^4-39\cos(dx+c)^3-79\cos(dx+c)^2+29\cos(dx+c)+44\right)}{a\cos(dx+c)^5+a\cos(dx+c)^4-2a\cos(dx+c)^3-2a\cos(dx+c)^2+a\cos(dx+c)+a}+\frac{63\log(\cos(dx+c)+1)}{a}+\frac{33\log(\cos(dx+c)-1)}{a}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(2*(33*cos(d*x + c)^4 - 39*cos(d*x + c)^3 - 79*cos(d*x + c)^2 + 29*cos
(d*x + c) + 44)/(a*cos(d*x + c)^5 + a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 -
 2*a*cos(d*x + c)^2 + a*cos(d*x + c) + a) + 63*log(cos(d*x + c) + 1)/a + 33
*log(cos(d*x + c) - 1)/a)/d
```

Fricas [A] time = 1.2057, size = 610, normalized size = 4.21

$$\frac{66\cos(dx+c)^4-78\cos(dx+c)^3-158\cos(dx+c)^2+63\left(\cos(dx+c)^5+\cos(dx+c)^4-2\cos(dx+c)^3-2\cos(dx\right.}{96\left(ad\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(66*cos(d*x + c)^4 - 78*cos(d*x + c)^3 - 158*cos(d*x + c)^2 + 63*(cos(
d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x
 + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 33*(cos(d*x + c)^5 + cos(d*x + c)^
4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*log(-1/2*cos(d*
x + c) + 1/2) + 58*cos(d*x + c) + 88)/(a*d*cos(d*x + c)^5 + a*d*cos(d*x + c
)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot^5(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**5/(sec(c + d*x) + 1), x)/a
```

Giac [A] time = 1.4128, size = 285, normalized size = 1.97

$$-\frac{\frac{3\left(\frac{14(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{66(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)^2}{a(\cos(dx+c)-1)^2}-\frac{132\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}+\frac{384\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a}+\frac{\frac{132a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{21a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^3}}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/384*(3*(14*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 66*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 + 1)*(cos(d*x + c) + 1)^2/(a*(cos(d*x + c) - 1)^2)
- 132*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a + 384*log(abs(-(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a + (132*a^2*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) + 21*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 2*a^
2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^3)/d
```

3.64 $\int \frac{\tan^8(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=105

$$-\frac{5\tanh^{-1}(\sin(c+dx))}{16ad}-\frac{\tan^5(c+dx)(6-5\sec(c+dx))}{30ad}+\frac{\tan^3(c+dx)(8-5\sec(c+dx))}{24ad}-\frac{\tan(c+dx)(16-5\sec(}{16ad}$$

```
[Out] x/a - (5*ArcTanh[Sin[c + d*x]])/(16*a*d) - ((16 - 5*Sec[c + d*x])*Tan[c + d
*x])/(16*a*d) + ((8 - 5*Sec[c + d*x])*Tan[c + d*x]^3)/(24*a*d) - ((6 - 5*Se
c[c + d*x])*Tan[c + d*x]^5)/(30*a*d)
```

Rubi [A] time = 0.144001, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3888, 3881, 3770}

$$-\frac{5\tanh^{-1}(\sin(c+dx))}{16ad}-\frac{\tan^5(c+dx)(6-5\sec(c+dx))}{30ad}+\frac{\tan^3(c+dx)(8-5\sec(c+dx))}{24ad}-\frac{\tan(c+dx)(16-5\sec(}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

```
[Out] x/a - (5*ArcTanh[Sin[c + d*x]])/(16*a*d) - ((16 - 5*Sec[c + d*x])*Tan[c + d
*x])/(16*a*d) + ((8 - 5*Sec[c + d*x])*Tan[c + d*x]^3)/(24*a*d) - ((6 - 5*Se
c[c + d*x])*Tan[c + d*x]^5)/(30*a*d)
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan^8(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int(-a+a\sec(c+dx))\tan^6(c+dx)\,dx}{a^2}$$
$$= -\frac{(6-5\sec(c+dx))\tan^5(c+dx)}{30ad} - \frac{\int(-6a+5a\sec(c+dx))\tan^4(c+dx)\,dx}{6a^2}$$
$$= \frac{(8-5\sec(c+dx))\tan^3(c+dx)}{24ad} - \frac{(6-5\sec(c+dx))\tan^5(c+dx)}{30ad} + \frac{\int(-24a+15a\sec(c+dx}{24a^2}$$
$$= -\frac{(16-5\sec(c+dx))\tan(c+dx)}{16ad} + \frac{(8-5\sec(c+dx))\tan^3(c+dx)}{24ad} - \frac{(6-5\sec(c+dx))\tan}{30ad}$$
$$= \frac{x}{a} - \frac{(16-5\sec(c+dx))\tan(c+dx)}{16ad} + \frac{(8-5\sec(c+dx))\tan^3(c+dx)}{24ad} - \frac{(6-5\sec(c+dx))\,t}{30ad}$$
$$= \frac{x}{a} - \frac{5\tanh^{-1}(\sin(c+dx))}{16ad} - \frac{(16-5\sec(c+dx))\tan(c+dx)}{16ad} + \frac{(8-5\sec(c+dx))\tan^3(c+d}{24ad}$$

Mathematica [B] time = 0.901728, size = 301, normalized size = 2.87

$$\cos^2\left(\tfrac{1}{2}(c+dx)\right)\sec(c+dx)\left(2400\left(\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)+\cos\left(\tfrac{1}{2}(c+dx)\right)\right)\right)+$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(2400*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c]*Sec[c + d*x]^6*(
2400*d*x*Cos[c] + 1800*d*x*Cos[c + 2*d*x] + 1800*d*x*Cos[3*c + 2*d*x] + 720
*d*x*Cos[3*c + 4*d*x] + 720*d*x*Cos[5*c + 4*d*x] + 120*d*x*Cos[5*c + 6*d*x]
 + 120*d*x*Cos[7*c + 6*d*x] + 3680*Sin[c] + 450*Sin[d*x] + 450*Sin[2*c + d*
x] - 3360*Sin[c + 2*d*x] + 2160*Sin[3*c + 2*d*x] - 25*Sin[2*c + 3*d*x] - 25
*Sin[4*c + 3*d*x] - 1488*Sin[3*c + 4*d*x] + 720*Sin[5*c + 4*d*x] + 165*Sin[
4*c + 5*d*x] + 165*Sin[6*c + 5*d*x] - 368*Sin[5*c + 6*d*x])))/(3840*a*d*(1
+ Sec[c + d*x]))
```

Maple [B] time = 0.093, size = 312, normalized size = 3.

$$2\,{\frac {\arctan \left(\tan \left(1/2\,dx+c/2 \right) \right) }{da}}-{\frac {1}{6\,da} \left(\tan \left({\frac {dx}{2}}+{\frac {c}{2}} \right) +1 \right) ^{-6}}+{\frac {7}{10\,da} \left(\tan \left({\frac {dx}{2}}+{\frac {c}{2}} \right) +1 \right) ^{-5}}-{\frac {3}{4\,da} \left(\tan \left({\frac {dx}{2}}+{\frac {c}{2}} \right) +1 \right) ^{-4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^8/(a+a*sec(d*x+c)),x)
```

```
[Out] 2/d/a*arctan(tan(1/2*d*x+1/2*c))-1/6/a/d/(tan(1/2*d*x+1/2*c)+1)^6+7/10/a/d/
(tan(1/2*d*x+1/2*c)+1)^5-3/4/a/d/(tan(1/2*d*x+1/2*c)+1)^4-5/12/a/d/(tan(1/2
*d*x+1/2*c)+1)^3+9/16/a/d/(tan(1/2*d*x+1/2*c)+1)^2+21/16/a/d/(tan(1/2*d*x+1
/2*c)+1)-5/16/a/d*ln(tan(1/2*d*x+1/2*c)+1)+1/6/a/d/(tan(1/2*d*x+1/2*c)-1)^6
+7/10/a/d/(tan(1/2*d*x+1/2*c)-1)^5+3/4/a/d/(tan(1/2*d*x+1/2*c)-1)^4-5/12/a/
d/(tan(1/2*d*x+1/2*c)-1)^3-9/16/a/d/(tan(1/2*d*x+1/2*c)-1)^2+21/16/a/d/(tan
(1/2*d*x+1/2*c)-1)+5/16/a/d*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.74257, size = 444, normalized size = 4.23

$$-\frac{\frac{2\left(\frac{165\sin(dx+c)}{\cos(dx+c)+1}-\frac{1095\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3138\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5118\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{1945\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{315\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)}{a-\frac{6a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{20a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{6a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{480\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{75\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*(2*(165*sin(d*x + c)/(cos(d*x + c) + 1) - 1095*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 3138*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5118*sin(d*x +
c)^7/(cos(d*x + c) + 1)^7 + 1945*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 315*
sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/(a - 6*a*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 15*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*a*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 15*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 6*a*sin(d*
x + c)^10/(cos(d*x + c) + 1)^10 + a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)
- 480*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 75*log(sin(d*x + c)/(cos(
d*x + c) + 1) + 1)/a - 75*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a)/d
```

Fricas [A] time = 1.22981, size = 354, normalized size = 3.37

$$\frac{480\,dx\cos\left(dx+c\right)^6-75\,\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)+75\,\cos\left(dx+c\right)^6\log\left(-\sin\left(dx+c\right)+1\right)-2\left(368\,\cos\left(dx-\right.\right.}{480\,ad\cos\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/480*(480*d*x*cos(d*x + c)^6 - 75*cos(d*x + c)^6*log(sin(d*x + c) + 1) + 7
5*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 2*(368*cos(d*x + c)^5 - 165*cos(d
*x + c)^4 - 176*cos(d*x + c)^3 + 130*cos(d*x + c)^2 + 48*cos(d*x + c) - 40)
*sin(d*x + c))/(a*d*cos(d*x + c)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^8\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**8/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**8/(sec(c + d*x) + 1), x)/a
```

Giac [A] time = 12.2761, size = 201, normalized size = 1.91

$$\frac{\frac{240\,(dx+c)}{a}-\frac{75\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}+\frac{75\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}+\frac{2\left(315\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-1945\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+5118\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-3138\,\mathrm{ta}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{6}}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/240*(240*(d*x + c)/a - 75*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + 75*log(a
bs(tan(1/2*d*x + 1/2*c) - 1))/a + 2*(315*tan(1/2*d*x + 1/2*c)^11 - 1945*tan
```

```
(1/2*d*x + 1/2*c)^9 + 5118*tan(1/2*d*x + 1/2*c)^7 - 3138*tan(1/2*d*x + 1/2*
c)^5 + 1095*tan(1/2*d*x + 1/2*c)^3 - 165*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*
x + 1/2*c)^2 - 1)^6*a))/d
```

3.65 $\int \frac{\tan^6(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=78

$$\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}-\frac{\tan^3(c+dx)(4-3\sec(c+dx))}{12ad}+\frac{\tan(c+dx)(8-3\sec(c+dx))}{8ad}-\frac{x}{a}$$

```
[Out] -(x/a) + (3*ArcTanh[Sin[c + d*x]])/(8*a*d) + ((8 - 3*Sec[c + d*x])*Tan[c +
d*x])/(8*a*d) - ((4 - 3*Sec[c + d*x])*Tan[c + d*x]^3)/(12*a*d)
```

Rubi [A] time = 0.108635, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3888, 3881, 3770}

$$\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}-\frac{\tan^3(c+dx)(4-3\sec(c+dx))}{12ad}+\frac{\tan(c+dx)(8-3\sec(c+dx))}{8ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(x/a) + (3*ArcTanh[Sin[c + d*x]])/(8*a*d) + ((8 - 3*Sec[c + d*x])*Tan[c +
d*x])/(8*a*d) - ((4 - 3*Sec[c + d*x])*Tan[c + d*x]^3)/(12*a*d)
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int(-a+a\sec(c+dx))\tan^4(c+dx)\,dx}{a^2}$$
$$= -\frac{(4-3\sec(c+dx))\tan^3(c+dx)}{12ad} - \frac{\int(-4a+3a\sec(c+dx))\tan^2(c+dx)\,dx}{4a^2}$$
$$= \frac{(8-3\sec(c+dx))\tan(c+dx)}{8ad} - \frac{(4-3\sec(c+dx))\tan^3(c+dx)}{12ad} + \frac{\int(-8a+3a\sec(c+dx))}{8a^2}$$
$$= -\frac{x}{a} + \frac{(8-3\sec(c+dx))\tan(c+dx)}{8ad} - \frac{(4-3\sec(c+dx))\tan^3(c+dx)}{12ad} + \frac{3\int\sec(c+dx)\,dx}{8a}$$
$$= -\frac{x}{a} + \frac{3\tanh^{-1}(\sin(c+dx))}{8ad} + \frac{(8-3\sec(c+dx))\tan(c+dx)}{8ad} - \frac{(4-3\sec(c+dx))\tan^3(c+}{12ad}$$

Mathematica [B] time = 6.42618, size = 893, normalized size = 11.45

$$-\frac{2x\sec(c+dx)\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{\sec(c+dx)a+a} - \frac{3\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec(c+dx)\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{4d(\sec(c+dx)a+a)} + \frac{3\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\right.}{4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*x*Cos[c/2 + (d*x)/2]^2*Sec[c + d*x])/(a + a*Sec[c + d*x]) - (3*Cos[c/2
+ (d*x)/2]^2*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x])/(4*
d*(a + a*Sec[c + d*x])) + (3*Cos[c/2 + (d*x)/2]^2*Log[Cos[c/2 + (d*x)/2] +
Sin[c/2 + (d*x)/2]]*Sec[c + d*x])/(4*d*(a + a*Sec[c + d*x])) + (Cos[c/2 + (
d*x)/2]^2*Sec[c + d*x])/(8*d*(a + a*Sec[c + d*x])*(Cos[c/2 + (d*x)/2] - Sin
[c/2 + (d*x)/2])^4) - (Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]*Sin[(d*x)/2])/(3*d
*(a + a*Sec[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 +
 (d*x)/2])^3) + (Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]*(-19*Cos[c/2] + 11*Sin[c
/2]))/(24*d*(a + a*Sec[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2]
- Sin[c/2 + (d*x)/2])^2) + (8*Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]*Sin[(d*x)/2
])/(3*d*(a + a*Sec[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Si
n[c/2 + (d*x)/2])) - (Cos[c/2 + (d*x)/2]^2*Sec[c + d*x])/(8*d*(a + a*Sec[c
+ d*x])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^4) - (Cos[c/2 + (d*x)/2]^
2*Sec[c + d*x]*Sin[(d*x)/2])/(3*d*(a + a*Sec[c + d*x])*(Cos[c/2] + Sin[c/2]
```

```
)*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + (Cos[c/2 + (d*x)/2]^2*Sec[
c + d*x]*(19*Cos[c/2] + 11*Sin[c/2]))/(24*d*(a + a*Sec[c + d*x])*(Cos[c/2]
+ Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (8*Cos[c/2 + (d*
x)/2]^2*Sec[c + d*x]*Sin[(d*x)/2])/(3*d*(a + a*Sec[c + d*x])*(Cos[c/2] + Si
n[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [B] time = 0.076, size = 228, normalized size = 2.9

$$-2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da}-\frac{1}{4\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-4}+\frac{5}{6\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}-\frac{3}{8\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+a*sec(d*x+c)),x)
```

```
[Out] -2/d/a*arctan(tan(1/2*d*x+1/2*c))-1/4/a/d/(tan(1/2*d*x+1/2*c)+1)^4+5/6/a/d/
(tan(1/2*d*x+1/2*c)+1)^3-3/8/a/d/(tan(1/2*d*x+1/2*c)+1)^2-11/8/a/d/(tan(1/2
*d*x+1/2*c)+1)+3/8/a/d*ln(tan(1/2*d*x+1/2*c)+1)+1/4/a/d/(tan(1/2*d*x+1/2*c)
-1)^4+5/6/a/d/(tan(1/2*d*x+1/2*c)-1)^3+3/8/a/d/(tan(1/2*d*x+1/2*c)-1)^2-11/
8/a/d/(tan(1/2*d*x+1/2*c)-1)-3/8/a/d*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.7429, size = 333, normalized size = 4.27

$$\frac{\frac{2\left(\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{71\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{137\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{33\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a-\frac{4\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\,a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4\,a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{48\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{9\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{9\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/24*(2*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 71*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 137*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 33*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7)/(a - 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 48*arctan(sin(d*x + c)/(cos(d*x +
c) + 1))/a + 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - 9*log(sin(d*x +
 c)/(cos(d*x + c) + 1) - 1)/a)/d
```

Fricas [A] time = 1.20081, size = 288, normalized size = 3.69

$$-\frac{48\,dx\cos\left(dx+c\right)^{4}-9\,\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)+9\,\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)-2\left(32\,\cos\left(dx+c\right)\right.}{48\,ad\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(48*d*x*cos(d*x + c)^4 - 9*cos(d*x + c)^4*log(sin(d*x + c) + 1) + 9*c
os(d*x + c)^4*log(-sin(d*x + c) + 1) - 2*(32*cos(d*x + c)^3 - 15*cos(d*x +
c)^2 - 8*cos(d*x + c) + 6)*sin(d*x + c))/(a*d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^{6}\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**6/(sec(c + d*x) + 1), x)/a
```

Giac [A] time = 5.1992, size = 166, normalized size = 2.13

$$-\frac{\frac{24\,(dx+c)}{a}-\frac{9\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}+\frac{9\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}+\frac{2\left(33\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-137\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+71\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,\tan\left(\frac{1}{2}\,dx\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{4}a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(24*(d*x + c)/a - 9*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + 9*log(abs(
tan(1/2*d*x + 1/2*c) - 1))/a + 2*(33*tan(1/2*d*x + 1/2*c)^7 - 137*tan(1/2*d
```

```
*x + 1/2*c)^5 + 71*tan(1/2*d*x + 1/2*c)^3 - 15*tan(1/2*d*x + 1/2*c))/((tan(
1/2*d*x + 1/2*c)^2 - 1)^4*a))/d
```

3.66 $\int \frac{\tan^4(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=49

$$-\frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{\tan(c+dx)(2-\sec(c+dx))}{2ad}+\frac{x}{a}$$

```
[Out] x/a - ArcTanh[Sin[c + d*x]]/(2*a*d) - ((2 - Sec[c + d*x])*Tan[c + d*x])/(2*
a*d)
```

Rubi [A] time = 0.0771262, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3888, 3881, 3770}

$$-\frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{\tan(c+dx)(2-\sec(c+dx))}{2ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] x/a - ArcTanh[Sin[c + d*x]]/(2*a*d) - ((2 - Sec[c + d*x])*Tan[c + d*x])/(2*
a*d)
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^4(c+dx)}{a+a\sec(c+dx)}\,dx &= \frac{\int (-a+a\sec(c+dx))\tan^2(c+dx)\,dx}{a^2} \\
&= -\frac{(2-\sec(c+dx))\tan(c+dx)}{2ad} - \frac{\int (-2a+a\sec(c+dx))\,dx}{2a^2} \\
&= \frac{x}{a} - \frac{(2-\sec(c+dx))\tan(c+dx)}{2ad} - \frac{\int \sec(c+dx)\,dx}{2a} \\
&= \frac{x}{a} - \frac{\tanh^{-1}(\sin(c+dx))}{2ad} - \frac{(2-\sec(c+dx))\tan(c+dx)}{2ad}
\end{aligned}
$$

Mathematica [B] time = 0.855145, size = 241, normalized size = 4.92

$$
\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-\frac{4\sin(dx)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{}{d\left(\cos\left(\frac{1}{2}(c+dx)\right.\right.}\right.}{2a(\sec(c+dx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(4*x + (2*Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]])/d - (2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + 1/(d*(Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2])^2) - 1/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^2) - (4*Sin[d*x])/(d*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(
c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(2
*a*(1 + Sec[c + d*x]))
```

Maple [B] time = 0.063, size = 144, normalized size = 2.9

$$
2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da}-\frac{1}{2\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}+\frac{3}{2\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-\frac{1}{2\,da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+a*sec(d*x+c)),x)
```

```
[Out] 2/d/a*arctan(tan(1/2*d*x+1/2*c))-1/2/a/d/(tan(1/2*d*x+1/2*c)+1)^2+3/2/a/d/(
tan(1/2*d*x+1/2*c)+1)-1/2/a/d*ln(tan(1/2*d*x+1/2*c)+1)+1/2/a/d/(tan(1/2*d*x
+1/2*c)-1)^2+3/2/a/d/(tan(1/2*d*x+1/2*c)-1)+1/2/a/d*ln(tan(1/2*d*x+1/2*c)-1
)
```

Maxima [B] time = 1.69882, size = 220, normalized size = 4.49

$$-\frac{\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{4\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4) - 4*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + log(sin(d
*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1) - 1
)/a)/d
```

Fricas [A] time = 1.19055, size = 224, normalized size = 4.57

$$\frac{4\,dx\cos\left(dx+c\right)^2-\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)+\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)-2\left(2\,\cos\left(dx+c\right)-1\right)\mathrm{si}}{4\,ad\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*d*x*cos(d*x + c)^2 - cos(d*x + c)^2*log(sin(d*x + c) + 1) + cos(d*x
+ c)^2*log(-sin(d*x + c) + 1) - 2*(2*cos(d*x + c) - 1)*sin(d*x + c))/(a*d*c
os(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

Giac [B] time = 2.51276, size = 130, normalized size = 2.65

$$\frac{\frac{2\,(dx+c)}{a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}+\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2 a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)/a - log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a + 2*(3*tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c))
/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a))/d
```

3.67 $\int \frac{\tan^2(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=21

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{x}{a}$$

```
[Out] -(x/a) + ArcTanh[Sin[c + d*x]]/(a*d)
```

Rubi [A] time = 0.0503932, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3888, 3770}

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(x/a) + ArcTanh[Sin[c + d*x]]/(a*d)
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int(-a+a\sec(c+dx))\,dx}{a^2}$$
$$= -\frac{x}{a} + \frac{\int \sec(c+dx)\,dx}{a}$$
$$= -\frac{x}{a} + \frac{\tanh^{-1}(\sin(c+dx))}{ad}$$

Mathematica [B] time = 0.0898872, size = 60, normalized size = 2.86

$$-\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)+dx}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((d*x + Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]])/(a*d))
```

Maple [B] time = 0.056, size = 59, normalized size = 2.8

$$-2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da}+\frac{1}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{1}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+a*sec(d*x+c)),x)
```

```
[Out] -2/d/a*arctan(tan(1/2*d*x+1/2*c))+1/a/d*ln(tan(1/2*d*x+1/2*c)+1)-1/a/d*ln(t
an(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.70849, size = 105, normalized size = 5.

$$-\frac{\frac{2\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - log(sin(d*x + c)/(cos(d*x +
 c) + 1) + 1)/a + log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a)/d
```

Fricas [A] time = 1.17183, size = 93, normalized size = 4.43

$$-\frac{2\,dx-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(-\sin\left(dx+c\right)+1\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*x - log(sin(d*x + c) + 1) + log(-sin(d*x + c) + 1))/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^2\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

Giac [B] time = 1.55859, size = 68, normalized size = 3.24

$$-\frac{\frac{dx+c}{a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)/a - log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/a)/d
```

3.68 $\int \frac{\cot^2(c+dx)}{a+a\sec(c+dx)} dx$

Optimal. Leaf size=61

$$\frac{\cot^3(c+dx)(1-\sec(c+dx))}{3ad}-\frac{\cot(c+dx)(3-2\sec(c+dx))}{3ad}-\frac{x}{a}$$

```
[Out] -(x/a) - (Cot[c + d*x]*(3 - 2*Sec[c + d*x]))/(3*a*d) + (Cot[c + d*x]^3*(1 -
 Sec[c + d*x]))/(3*a*d)
```

Rubi [A] time = 0.0973967, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3888, 3882, 8}

$$\frac{\cot^3(c+dx)(1-\sec(c+dx))}{3ad}-\frac{\cot(c+dx)(3-2\sec(c+dx))}{3ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(x/a) - (Cot[c + d*x]*(3 - 2*Sec[c + d*x]))/(3*a*d) + (Cot[c + d*x]^3*(1 -
 Sec[c + d*x]))/(3*a*d)
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(c+dx)}{a+a\sec(c+dx)}\,dx &= \frac{\int \cot^4(c+dx)(-a+a\sec(c+dx))\,dx}{a^2} \\
&= \frac{\cot^3(c+dx)(1-\sec(c+dx))}{3ad} + \frac{\int \cot^2(c+dx)(3a-2a\sec(c+dx))\,dx}{3a^2} \\
&= -\frac{\cot(c+dx)(3-2\sec(c+dx))}{3ad} + \frac{\cot^3(c+dx)(1-\sec(c+dx))}{3ad} + \frac{\int -3a\,dx}{3a^2} \\
&= -\frac{x}{a} - \frac{\cot(c+dx)(3-2\sec(c+dx))}{3ad} + \frac{\cot^3(c+dx)(1-\sec(c+dx))}{3ad}
\end{aligned}
$$

Mathematica [A] time = 0.768036, size = 100, normalized size = 1.64

$$
\frac{\sec(c+dx)\left(-12dx\cos^2\left(\frac{1}{2}(c+dx)\right)-\tan\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{dx}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(3\csc\left(\frac{c}{2}\right)\cot\left(\frac{1}{2}(c+dx)\right)+13\sec\left(\frac{c}{2}\right)\right)\right)}{6ad(\sec(c+dx)+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]*(-12*d*x*Cos[(c + d*x)/2]^2 + Cos[(c + d*x)/2]*(3*Cot[(c + d*
x)/2]*Csc[c/2] + 13*Sec[c/2])*Sin[(d*x)/2] - Tan[(c + d*x)/2]))/(6*a*d*(1 +
 Sec[c + d*x]))
```

Maple [A] time = 0.056, size = 74, normalized size = 1.2

$$
-\frac{1}{12\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da}-\frac{1}{4\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/12/a/d*tan(1/2*d*x+1/2*c)^3+1/a/d*tan(1/2*d*x+1/2*c)-2/d/a*arctan(tan(1/
2*d*x+1/2*c))-1/4/a/d/tan(1/2*d*x+1/2*c)
```

Maxima [A] time = 1.73576, size = 126, normalized size = 2.07

$$\frac{\frac{\frac{12 \sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a} - \frac{24 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{3(\cos(dx+c)+1)}{a\sin(dx+c)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*((12*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) +
1)^3)/a - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 3*(cos(d*x + c) +
1)/(a*sin(d*x + c)))/d
```

Fricas [A] time = 1.08635, size = 170, normalized size = 2.79

$$-\frac{4\,\cos\left(dx+c\right)^2 + 3\left(dx\cos\left(dx+c\right) + dx\right)\sin\left(dx+c\right) + \cos\left(dx+c\right) - 2}{3\left(ad\cos\left(dx+c\right) + ad\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(4*cos(d*x + c)^2 + 3*(d*x*cos(d*x + c) + d*x)*sin(d*x + c) + cos(d*x
+ c) - 2)/((a*d*cos(d*x + c) + a*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

Giac [A] time = 1.33486, size = 89, normalized size = 1.46

$$-\frac{\frac{12\,(dx+c)}{a}+\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}+\frac{3}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(12*(d*x + c)/a + (a^2*tan(1/2*d*x + 1/2*c)^3 - 12*a^2*tan(1/2*d*x +
1/2*c))/a^3 + 3/(a*tan(1/2*d*x + 1/2*c)))/d
```

3.69 $\int \frac{\cot^4(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=88

$$\frac{\cot^5(c+dx)(1-\sec(c+dx))}{5ad}-\frac{\cot^3(c+dx)(5-4\sec(c+dx))}{15ad}+\frac{\cot(c+dx)(15-8\sec(c+dx))}{15ad}+\frac{x}{a}$$

```
[Out] x/a + (Cot[c + d*x]*(15 - 8*Sec[c + d*x]))/(15*a*d) - (Cot[c + d*x]^3*(5 -
4*Sec[c + d*x]))/(15*a*d) + (Cot[c + d*x]^5*(1 - Sec[c + d*x]))/(5*a*d)
```

Rubi [A] time = 0.127382, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3888, 3882, 8}

$$\frac{\cot^5(c+dx)(1-\sec(c+dx))}{5ad}-\frac{\cot^3(c+dx)(5-4\sec(c+dx))}{15ad}+\frac{\cot(c+dx)(15-8\sec(c+dx))}{15ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```


```
[Out] x/a + (Cot[c + d*x]*(15 - 8*Sec[c + d*x]))/(15*a*d) - (Cot[c + d*x]^3*(5 -
4*Sec[c + d*x]))/(15*a*d) + (Cot[c + d*x]^5*(1 - Sec[c + d*x]))/(5*a*d)
```


Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \cot^6(c+dx)(-a+a\sec(c+dx))\,dx}{a^2}$$
$$= \frac{\cot^5(c+dx)(1-\sec(c+dx))}{5ad} + \frac{\int \cot^4(c+dx)(5a-4a\sec(c+dx))\,dx}{5a^2}$$
$$= -\frac{\cot^3(c+dx)(5-4\sec(c+dx))}{15ad} + \frac{\cot^5(c+dx)(1-\sec(c+dx))}{5ad} + \frac{\int \cot^2(c+dx)(-15a+8a$$
$$= \frac{\cot(c+dx)(15-8\sec(c+dx))}{15ad} - \frac{\cot^3(c+dx)(5-4\sec(c+dx))}{15ad} + \frac{\cot^5(c+dx)(1-\sec(c+a$$
$$= \frac{x}{a} + \frac{\cot(c+dx)(15-8\sec(c+dx))}{15ad} - \frac{\cot^3(c+dx)(5-4\sec(c+dx))}{15ad} + \frac{\cot^5(c+dx)(1-\sec($$

Mathematica [B] time = 0.810169, size = 254, normalized size = 2.89

$$\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\csc^3(c+dx)\sec(c+dx)(534\sin(c+dx)+178\sin(2(c+dx))-178\sin(3(c+dx))-89\sin(4(c+dx))-5$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Csc[c/2]*Csc[c + d*x]^3*Sec[c/2]*Sec[c + d*x]*(360*d*x*Cos[d*x] - 360*d*x*
Cos[2*c + d*x] + 120*d*x*Cos[c + 2*d*x] - 120*d*x*Cos[3*c + 2*d*x] - 120*d*
x*Cos[2*c + 3*d*x] + 120*d*x*Cos[4*c + 3*d*x] - 60*d*x*Cos[3*c + 4*d*x] + 6
0*d*x*Cos[5*c + 4*d*x] - 200*Sin[c] - 584*Sin[d*x] + 534*Sin[c + d*x] + 178
*Sin[2*(c + d*x)] - 178*Sin[3*(c + d*x)] - 89*Sin[4*(c + d*x)] - 520*Sin[2*
c + d*x] - 248*Sin[c + 2*d*x] - 120*Sin[3*c + 2*d*x] + 248*Sin[2*c + 3*d*x]
 + 120*Sin[4*c + 3*d*x] + 184*Sin[3*c + 4*d*x]))/(1920*a*d*(1 + Sec[c + d*x
]))
```

Maple [A] time = 0.068, size = 113, normalized size = 1.3

$$-\frac{1}{80\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 + \frac{1}{8\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + 2\,\frac{\arctan(\tan(1/2\,dx+c/2))}{da} - \frac{1}{48\,da}\left(\tan\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/80/a/d*tan(1/2*d*x+1/2*c)^5+1/8/a/d*tan(1/2*d*x+1/2*c)^3-1/a/d*tan(1/2*d
*x+1/2*c)+2/d/a*arctan(tan(1/2*d*x+1/2*c))-1/48/a/d/tan(1/2*d*x+1/2*c)^3+3/
8/a/d/tan(1/2*d*x+1/2*c)
```

Maxima [A] time = 1.71367, size = 185, normalized size = 2.1

$$-\frac{\frac{3\left(\frac{80\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a}-\frac{480\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{5\left(\frac{18\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^3}{a\sin(dx+c)^3}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*(3*(80*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a - 480*arctan(sin(d*x +
 c)/(cos(d*x + c) + 1))/a - 5*(18*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)*
(cos(d*x + c) + 1)^3/(a*sin(d*x + c)^3))/d
```

Fricas [A] time = 1.13097, size = 342, normalized size = 3.89

$$\frac{23\cos(dx+c)^4+8\cos(dx+c)^3-27\cos(dx+c)^2+15\left(dx\cos(dx+c)^3+dx\cos(dx+c)^2-dx\cos(dx+c)-dx\right)\mathrm{si}}{15\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2-ad\cos(dx+c)-ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(23*cos(d*x + c)^4 + 8*cos(d*x + c)^3 - 27*cos(d*x + c)^2 + 15*(d*x*co
s(d*x + c)^3 + d*x*cos(d*x + c)^2 - d*x*cos(d*x + c) - d*x)*sin(d*x + c) -
7*cos(d*x + c) + 8)/((a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x
 + c) - a*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot^4(c+dx)}{\sec(c+dx)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

Giac [A] time = 1.36358, size = 132, normalized size = 1.5

$$\frac{\frac{240\,(dx+c)}{a} + \frac{5\left(18\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3} - \frac{3\left(a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-10\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+80\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^5}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/240*(240*(d*x + c)/a + 5*(18*tan(1/2*d*x + 1/2*c)^2 - 1)/(a*tan(1/2*d*x +
 1/2*c)^3) - 3*(a^4*tan(1/2*d*x + 1/2*c)^5 - 10*a^4*tan(1/2*d*x + 1/2*c)^3
+ 80*a^4*tan(1/2*d*x + 1/2*c))/a^5)/d
```

3.70 $\int \frac{\cot^6(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=117

$$\frac{\cot^7(c+dx)(1-\sec(c+dx))}{7ad}-\frac{\cot^5(c+dx)(7-6\sec(c+dx))}{35ad}+\frac{\cot^3(c+dx)(35-24\sec(c+dx))}{105ad}-\frac{\cot(c+dx)(35-}{35}$$

```
[Out] -(x/a) + (Cot[c + d*x]^3*(35 - 24*Sec[c + d*x]))/(105*a*d) - (Cot[c + d*x]*
(35 - 16*Sec[c + d*x]))/(35*a*d) - (Cot[c + d*x]^5*(7 - 6*Sec[c + d*x]))/(3
5*a*d) + (Cot[c + d*x]^7*(1 - Sec[c + d*x]))/(7*a*d)
```

Rubi [A] time = 0.161961, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3888, 3882, 8}

$$\frac{\cot^7(c+dx)(1-\sec(c+dx))}{7ad}-\frac{\cot^5(c+dx)(7-6\sec(c+dx))}{35ad}+\frac{\cot^3(c+dx)(35-24\sec(c+dx))}{105ad}-\frac{\cot(c+dx)(35-}{35}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(x/a) + (Cot[c + d*x]^3*(35 - 24*Sec[c + d*x]))/(105*a*d) - (Cot[c + d*x]*
(35 - 16*Sec[c + d*x]))/(35*a*d) - (Cot[c + d*x]^5*(7 - 6*Sec[c + d*x]))/(3
5*a*d) + (Cot[c + d*x]^7*(1 - Sec[c + d*x]))/(7*a*d)
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \cot^8(c+dx)(-a+a\sec(c+dx))\,dx}{a^2}$$
$$= \frac{\cot^7(c+dx)(1-\sec(c+dx))}{7ad} + \frac{\int \cot^6(c+dx)(7a-6a\sec(c+dx))\,dx}{7a^2}$$
$$= -\frac{\cot^5(c+dx)(7-6\sec(c+dx))}{35ad} + \frac{\cot^7(c+dx)(1-\sec(c+dx))}{7ad} + \frac{\int \cot^4(c+dx)(-35a+24a}{35a^2}$$
$$= \frac{\cot^3(c+dx)(35-24\sec(c+dx))}{105ad} - \frac{\cot^5(c+dx)(7-6\sec(c+dx))}{35ad} + \frac{\cot^7(c+dx)(1-\sec(c-}{7ad}$$
$$= \frac{\cot^3(c+dx)(35-24\sec(c+dx))}{105ad} - \frac{\cot(c+dx)(35-16\sec(c+dx))}{35ad} - \frac{\cot^5(c+dx)(7-6\sec}{35ad}$$
$$= -\frac{x}{a} + \frac{\cot^3(c+dx)(35-24\sec(c+dx))}{105ad} - \frac{\cot(c+dx)(35-16\sec(c+dx))}{35ad} - \frac{\cot^5(c+dx)(7-}{35a}$$

Mathematica [B] time = 1.07246, size = 359, normalized size = 3.07

$$\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\csc^5(c+dx)\sec(c+dx)(-22860\sin(c+dx)-5715\sin(2(c+dx))+11430\sin(3(c+dx))+4572\sin(4(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Csc[c/2]*Csc[c + d*x]^5*Sec[c/2]*Sec[c + d*x]*(-16800*d*x*Cos[d*x] + 16800
*d*x*Cos[2*c + d*x] - 4200*d*x*Cos[c + 2*d*x] + 4200*d*x*Cos[3*c + 2*d*x] +
 8400*d*x*Cos[2*c + 3*d*x] - 8400*d*x*Cos[4*c + 3*d*x] + 3360*d*x*Cos[3*c +
 4*d*x] - 3360*d*x*Cos[5*c + 4*d*x] - 1680*d*x*Cos[4*c + 5*d*x] + 1680*d*x*
Cos[6*c + 5*d*x] - 840*d*x*Cos[5*c + 6*d*x] + 840*d*x*Cos[7*c + 6*d*x] + 31
36*Sin[c] + 30112*Sin[d*x] - 22860*Sin[c + d*x] - 5715*Sin[2*(c + d*x)] + 1
1430*Sin[3*(c + d*x)] + 4572*Sin[4*(c + d*x)] - 2286*Sin[5*(c + d*x)] - 114
3*Sin[6*(c + d*x)] + 26208*Sin[2*c + d*x] + 14080*Sin[c + 2*d*x] - 16400*Si
n[2*c + 3*d*x] - 11760*Sin[4*c + 3*d*x] - 7904*Sin[3*c + 4*d*x] - 3360*Sin[
5*c + 4*d*x] + 3952*Sin[4*c + 5*d*x] + 1680*Sin[6*c + 5*d*x] + 2816*Sin[5*c
 + 6*d*x]))/(107520*a*d*(1 + Sec[c + d*x]))
```

Maple [A] time = 0.067, size = 150, normalized size = 1.3

$$-\frac{1}{448\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}+\frac{1}{40\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}-\frac{29}{192\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}+\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\,\frac{\arctan\left(\tan\left(1/\ldots\right.\right.}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/448/a/d*tan(1/2*d*x+1/2*c)^7+1/40/a/d*tan(1/2*d*x+1/2*c)^5-29/192/a/d*ta
n(1/2*d*x+1/2*c)^3+1/a/d*tan(1/2*d*x+1/2*c)-2/d/a*arctan(tan(1/2*d*x+1/2*c)
)-1/320/a/d/tan(1/2*d*x+1/2*c)^5+1/24/a/d/tan(1/2*d*x+1/2*c)^3-29/64/a/d/ta
n(1/2*d*x+1/2*c)
```

Maxima [A] time = 1.69761, size = 239, normalized size = 2.04

$$\frac{\frac{\frac{6720\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{1015\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{168\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{15\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}{a}-\frac{13440\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{7\left(\frac{40\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{435\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-3\right)(\cos(dx+c)+1)^{5}}{a\sin(dx+c)^{5}}}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6720*((6720*sin(d*x + c)/(cos(d*x + c) + 1) - 1015*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 168*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/a - 13440*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
+ 7*(40*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 435*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4 - 3)*(cos(d*x + c) + 1)^5/(a*sin(d*x + c)^5))/d
```

Fricas [A] time = 1.16881, size = 533, normalized size = 4.56

$$-\frac{176\,\cos(dx+c)^{6}+71\,\cos(dx+c)^{5}-335\,\cos(dx+c)^{4}-125\,\cos(dx+c)^{3}+225\,\cos(dx+c)^{2}+105\left(dx\cos(dx+\ldots\right.}{105\left(ad\cos(dx+c)^{5}+ad\cos(dx+c)^{4}-2\,ad\cos(dx+\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/105*(176*cos(d*x + c)^6 + 71*cos(d*x + c)^5 - 335*cos(d*x + c)^4 - 125*c
os(d*x + c)^3 + 225*cos(d*x + c)^2 + 105*(d*x*cos(d*x + c)^5 + d*x*cos(d*x
+ c)^4 - 2*d*x*cos(d*x + c)^3 - 2*d*x*cos(d*x + c)^2 + d*x*cos(d*x + c) + d
*x)*sin(d*x + c) + 57*cos(d*x + c) - 48)/((a*d*cos(d*x + c)^5 + a*d*cos(d*x
 + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) +
a*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot^6(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**6/(sec(c + d*x) + 1), x)/a
```

Giac [A] time = 1.32248, size = 171, normalized size = 1.46

$$-\frac{\frac{6720\,(dx+c)}{a}+\frac{7\left(435\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-40\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}+\frac{15\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-168\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1015\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6720\,a^6\tan\left(\frac{1}{2}\,a\right.}{a^7}}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6720*(6720*(d*x + c)/a + 7*(435*tan(1/2*d*x + 1/2*c)^4 - 40*tan(1/2*d*x
+ 1/2*c)^2 + 3)/(a*tan(1/2*d*x + 1/2*c)^5) + (15*a^6*tan(1/2*d*x + 1/2*c)^7
 - 168*a^6*tan(1/2*d*x + 1/2*c)^5 + 1015*a^6*tan(1/2*d*x + 1/2*c)^3 - 6720*
a^6*tan(1/2*d*x + 1/2*c))/a^7)/d
```

3.71 $\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=120

$$\frac{\sec^6(c+dx)}{6a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}-\frac{\sec^4(c+dx)}{4a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{\sec^2(c+dx)}{2a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{\log(\cos(c+dx))}{a^2d}$$

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) - (2*Sec[c + d*x])/(a^2*d) - Sec[c + d*x]^2/(2
*a^2*d) + (4*Sec[c + d*x]^3)/(3*a^2*d) - Sec[c + d*x]^4/(4*a^2*d) - (2*Sec[
c + d*x]^5)/(5*a^2*d) + Sec[c + d*x]^6/(6*a^2*d)
```

Rubi [A] time = 0.0736004, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{\sec^6(c+dx)}{6a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}-\frac{\sec^4(c+dx)}{4a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{\sec^2(c+dx)}{2a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{\log(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^9/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) - (2*Sec[c + d*x])/(a^2*d) - Sec[c + d*x]^2/(2
*a^2*d) + (4*Sec[c + d*x]^3)/(3*a^2*d) - Sec[c + d*x]^4/(4*a^2*d) - (2*Sec[
c + d*x]^5)/(5*a^2*d) + Sec[c + d*x]^6/(6*a^2*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^4(a+ax)^2}{x^7}\,dx, x, \cos(c+dx)\right)}{a^8 d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^6}{x^7}-\frac{2a^6}{x^6}-\frac{a^6}{x^5}+\frac{4a^6}{x^4}-\frac{a^6}{x^3}-\frac{2a^6}{x^2}+\frac{a^6}{x}\right)dx, x, \cos(c+dx)\right)}{a^8 d}$$
$$= -\frac{\log(\cos(c+dx))}{a^2 d}-\frac{2\sec(c+dx)}{a^2 d}-\frac{\sec^2(c+dx)}{2a^2 d}+\frac{4\sec^3(c+dx)}{3a^2 d}-\frac{\sec^4(c+dx)}{4a^2 d}-\frac{2\sec^5}{5a}$$

Mathematica [A] time = 0.517759, size = 125, normalized size = 1.04

$$-\frac{\sec^6(c+dx)(312\cos(c+dx)+5(28\cos(3(c+dx))+6\cos(4(c+dx))+12\cos(5(c+dx))+18\cos(4(c+dx))\log(\cos(c}{480a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^9/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -((312*Cos[c + d*x] + 5*(14 + 28*Cos[3*(c + d*x)] + 6*Cos[4*(c + d*x)] + 12
*Cos[5*(c + d*x)] + 30*Log[Cos[c + d*x]] + 18*Cos[4*(c + d*x)]*Log[Cos[c +
d*x]] + 3*Cos[6*(c + d*x)]*Log[Cos[c + d*x]] + 9*Cos[2*(c + d*x)]*(4 + 5*Lo
g[Cos[c + d*x]])))*Sec[c + d*x]^6)/(480*a^2*d)
```

Maple [A] time = 0.084, size = 110, normalized size = 0.9

$$\frac{(\sec(dx+c))^6}{6da^2}-\frac{2(\sec(dx+c))^5}{5da^2}-\frac{(\sec(dx+c))^4}{4da^2}+\frac{4(\sec(dx+c))^3}{3da^2}-\frac{(\sec(dx+c))^2}{2da^2}-2\frac{\sec(dx+c)}{da^2}+\frac{\ln(\sec(dx+}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^9/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6*sec(d*x+c)^6/d/a^2-2/5*sec(d*x+c)^5/d/a^2-1/4*sec(d*x+c)^4/d/a^2+4/3*se
c(d*x+c)^3/d/a^2-1/2*sec(d*x+c)^2/d/a^2-2*sec(d*x+c)/d/a^2+1/d/a^2*ln(sec(d
*x+c))
```

Maxima [A] time = 1.14996, size = 108, normalized size = 0.9

$$-\frac{\frac{60 \log(\cos(dx+c))}{a^2}+\frac{120 \cos(dx+c)^5+30 \cos(dx+c)^4-80 \cos(dx+c)^3+15 \cos(dx+c)^2+24 \cos(dx+c)-10}{a^2 \cos(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/60*(60*log(cos(d*x + c))/a^2 + (120*cos(d*x + c)^5 + 30*cos(d*x + c)^4 -
 80*cos(d*x + c)^3 + 15*cos(d*x + c)^2 + 24*cos(d*x + c) - 10)/(a^2*cos(d*x
 + c)^6))/d
```

Fricas [A] time = 1.22501, size = 234, normalized size = 1.95

$$-\frac{60 \cos(dx+c)^6 \log(-\cos(dx+c)) + 120 \cos(dx+c)^5 + 30 \cos(dx+c)^4 - 80 \cos(dx+c)^3 + 15 \cos(dx+c)^2 + 24}{60\,a^2 d \cos(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(60*cos(d*x + c)^6*log(-cos(d*x + c)) + 120*cos(d*x + c)^5 + 30*cos(d
*x + c)^4 - 80*cos(d*x + c)^3 + 15*cos(d*x + c)^2 + 24*cos(d*x + c) - 10)/(
a^2*d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**9/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 11.4248, size = 301, normalized size = 2.51

$$\frac{\frac{60\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}-\frac{60\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^2}+\frac{\frac{234(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{1005(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{2220(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{2925(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{1002(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+14}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - 60*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^2 + (234*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + 1005*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 2
220*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 2925*(cos(d*x + c) - 1)^4/(
cos(d*x + c) + 1)^4 + 1002*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 147*
(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 19)/(a^2*((cos(d*x + c) - 1)/(c
os(d*x + c) + 1) + 1)^6))/d
```

3.72 $\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=65

$$\frac{\sec^4(c+dx)}{4a^2d} - \frac{2\sec^3(c+dx)}{3a^2d} + \frac{2\sec(c+dx)}{a^2d} + \frac{\log(\cos(c+dx))}{a^2d}$$

```
[Out] Log[Cos[c + d*x]]/(a^2*d) + (2*Sec[c + d*x])/(a^2*d) - (2*Sec[c + d*x]^3)/(
3*a^2*d) + Sec[c + d*x]^4/(4*a^2*d)
```

Rubi [A] time = 0.0578504, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 75}

$$\frac{\sec^4(c+dx)}{4a^2d} - \frac{2\sec^3(c+dx)}{3a^2d} + \frac{2\sec(c+dx)}{a^2d} + \frac{\log(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] Log[Cos[c + d*x]]/(a^2*d) + (2*Sec[c + d*x])/(a^2*d) - (2*Sec[c + d*x]^3)/(
3*a^2*d) + Sec[c + d*x]^4/(4*a^2*d)
```


Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^3(a+ax)}{x^5}\,dx, x, \cos(c+dx)\right)}{a^6 d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^4}{x^5}-\frac{2a^4}{x^4}+\frac{2a^4}{x^2}-\frac{a^4}{x}\right)dx, x, \cos(c+dx)\right)}{a^6 d}$$
$$= \frac{\log(\cos(c+dx))}{a^2 d}+\frac{2\sec(c+dx)}{a^2 d}-\frac{2\sec^3(c+dx)}{3a^2 d}+\frac{\sec^4(c+dx)}{4a^2 d}$$

Mathematica [A] time = 0.20201, size = 83, normalized size = 1.28

$$\frac{\sec^4(c+dx)(20\cos(c+dx)+3(4\cos(3(c+dx))+4\cos(2(c+dx))\log(\cos(c+dx))+\cos(4(c+dx))\log(\cos(c+dx))+}{24a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((20*Cos[c + d*x] + 3*(2 + 4*Cos[3*(c + d*x)] + 3*Log[Cos[c + d*x]] + 4*Cos
[2*(c + d*x)]*Log[Cos[c + d*x]] + Cos[4*(c + d*x)]*Log[Cos[c + d*x]]))*Sec[
c + d*x]^4)/(24*a^2*d)
```

Maple [A] time = 0.075, size = 63, normalized size = 1.

$$\frac{(\sec(dx+c))^4}{4\,da^2}-\frac{2\,(\sec(dx+c))^3}{3\,da^2}+2\,\frac{\sec(dx+c)}{da^2}-\frac{\ln(\sec(dx+c))}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/4*sec(d*x+c)^4/d/a^2-2/3*sec(d*x+c)^3/d/a^2+2*sec(d*x+c)/d/a^2-1/d/a^2*ln
(sec(d*x+c))
```

Maxima [A] time = 1.13033, size = 68, normalized size = 1.05

$$\frac{\frac{12\,\log(\cos(dx+c))}{a^2}+\frac{24\,\cos(dx+c)^3-8\,\cos(dx+c)+3}{a^2\cos(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(12*log(cos(d*x + c))/a^2 + (24*cos(d*x + c)^3 - 8*cos(d*x + c) + 3)/(
a^2*cos(d*x + c)^4))/d
```

Fricas [A] time = 1.18856, size = 147, normalized size = 2.26

$$\frac{12\cos\left(dx+c\right)^{4}\log\left(-\cos\left(dx+c\right)\right)+24\cos\left(dx+c\right)^{3}-8\cos\left(dx+c\right)+3}{12\,a^{2}d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(12*cos(d*x + c)^4*log(-cos(d*x + c)) + 24*cos(d*x + c)^3 - 8*cos(d*x
+ c) + 3)/(a^2*d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\tan^{7}\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**7/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**7/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [B] time = 7.42592, size = 243, normalized size = 3.74

$$-\frac{\frac{12\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^{2}}-\frac{12\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^{2}}-\frac{\frac{4\,(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{54\,(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{124\,(\cos(dx+c)-1)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{25\,(\cos(dx+c)-1)^{4}}{(\cos(dx+c)+1)^{4}}+7}{a^{2}\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^{4}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/12*(12*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - 12*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^2 - (4*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) - 54*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 124*
(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 25*(cos(d*x + c) - 1)^4/(cos(d*
x + c) + 1)^4 + 7)/(a^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^4))/d
```

3.73 $\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=48

$$\frac{\sec^2(c+dx)}{2a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{\log(\cos(c+dx))}{a^2d}$$

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) - (2*Sec[c + d*x])/(a^2*d) + Sec[c + d*x]^2/(2
*a^2*d)
```

Rubi [A] time = 0.0497635, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 43}

$$\frac{\sec^2(c+dx)}{2a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{\log(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) - (2*Sec[c + d*x])/(a^2*d) + Sec[c + d*x]^2/(2
*a^2*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^2}{x^3}\,dx, x, \cos(c+dx)\right)}{a^4d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^2}{x^3}-\frac{2a^2}{x^2}+\frac{a^2}{x}\right)dx, x, \cos(c+dx)\right)}{a^4d}$$
$$= -\frac{\log(\cos(c+dx))}{a^2d} - \frac{2\sec(c+dx)}{a^2d} + \frac{\sec^2(c+dx)}{2a^2d}$$

Mathematica [A] time = 0.114018, size = 51, normalized size = 1.06

$$-\frac{\sec^2(c+dx)(4\cos(c+dx)+\cos(2(c+dx))\log(\cos(c+dx))+\log(\cos(c+dx))-1)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -((-1 + 4*Cos[c + d*x] + Log[Cos[c + d*x]] + Cos[2*(c + d*x)]*Log[Cos[c + d
*x]])*Sec[c + d*x]^2)/(2*a^2*d)
```

Maple [A] time = 0.058, size = 46, normalized size = 1.

$$\frac{(\sec(dx+c))^2}{2\,da^2} - 2\,\frac{\sec(dx+c)}{da^2} + \frac{\ln(\sec(dx+c))}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/2*sec(d*x+c)^2/d/a^2-2*sec(d*x+c)/d/a^2+1/d/a^2*ln(sec(d*x+c))
```

Maxima [A] time = 1.17518, size = 54, normalized size = 1.12

$$-\frac{\frac{2\log(\cos(dx+c))}{a^2} + \frac{4\cos(dx+c)-1}{a^2\cos(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*log(cos(d*x + c))/a^2 + (4*cos(d*x + c) - 1)/(a^2*cos(d*x + c)^2))/
d
```

Fricas [A] time = 1.20419, size = 119, normalized size = 2.48

$$-\frac{2\cos(dx+c)^2\log(-\cos(dx+c))+4\cos(dx+c)-1}{2a^2d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*cos(d*x + c)^2*log(-cos(d*x + c)) + 4*cos(d*x + c) - 1)/(a^2*d*cos(
d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\tan^5(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [B] time = 3.51986, size = 184, normalized size = 3.83

$$\frac{\frac{2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}-\frac{2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^2}-\frac{\frac{6(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+5}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - 2*log(abs
(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^2 - (6*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) - 3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 5)/(a^2*(
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2))/d
```

3.74 $\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=33

$$\frac{2\log(\cos(c+dx)+1)}{a^2d}-\frac{\log(\cos(c+dx))}{a^2d}$$

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) + (2*Log[1 + Cos[c + d*x]])/(a^2*d)
```

Rubi [A] time = 0.0448387, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 72}

$$\frac{2\log(\cos(c+dx)+1)}{a^2d}-\frac{\log(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) + (2*Log[1 + Cos[c + d*x]])/(a^2*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{a-ax}{x(a+ax)}\,dx, x, \cos(c+dx)\right)}{a^2d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{x}-\frac{2}{1+x}\right)dx, x, \cos(c+dx)\right)}{a^2d}$$
$$= -\frac{\log(\cos(c+dx))}{a^2d} + \frac{2\log(1+\cos(c+dx))}{a^2d}$$

Mathematica [A] time = 0.060438, size = 30, normalized size = 0.91

$$\frac{4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*Log[Cos[(c + d*x)/2]] - Log[Cos[c + d*x]])/(a^2*d)
```

Maple [A] time = 0.072, size = 34, normalized size = 1.

$$2\,\frac{\ln(1+\sec(dx+c))}{da^2} - \frac{\ln(\sec(dx+c))}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*ln(1+sec(d*x+c))-1/d/a^2*ln(sec(d*x+c))
```

Maxima [A] time = 1.12223, size = 42, normalized size = 1.27

$$\frac{\frac{2\,\log(\cos(dx+c)+1)}{a^2}-\frac{\log(\cos(dx+c))}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (2*log(cos(d*x + c) + 1)/a^2 - log(cos(d*x + c))/a^2)/d
```

Fricas [A] time = 1.19847, size = 85, normalized size = 2.58

$$-\frac{\log\left(-\cos\left(dx+c\right)\right)-2\,\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(log(-cos(d*x + c)) - 2*log(1/2*cos(d*x + c) + 1/2))/(a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\tan^3\left(c+dx\right)}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.91302, size = 45, normalized size = 1.36

$$-\frac{\log\left(\left|\frac{\left(\cos(dx+c)-1\right)^2}{\left(\cos(dx+c)+1\right)^2}-1\right|\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -log(abs((cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1))/(a^2*d)
```

3.75 $\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=36

$$-\frac{1}{a^2d(\cos(c+dx)+1)}-\frac{\log(\cos(c+dx)+1)}{a^2d}$$

```
[Out] -(1/(a^2*d*(1 + Cos[c + d*x]))) - Log[1 + Cos[c + d*x]]/(a^2*d)
```

Rubi [A] time = 0.0347561, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 43}

$$-\frac{1}{a^2d(\cos(c+dx)+1)}-\frac{\log(\cos(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(1/(a^2*d*(1 + Cos[c + d*x]))) - Log[1 + Cos[c + d*x]]/(a^2*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x}{(a+ax)^2}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{a^2(1+x)^2}+\frac{1}{a^2(1+x)}\right)dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{1}{a^2d(1+\cos(c+dx))} - \frac{\log(1+\cos(c+dx))}{a^2d}$$

Mathematica [A] time = 0.132192, size = 56, normalized size = 1.56

$$-\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\left(2\cos(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+1\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -((1 + 2*Log[Cos[(c + d*x)/2]] + 2*Cos[c + d*x]*Log[Cos[(c + d*x)/2]])*Sec[
(c + d*x)/2]^2)/(2*a^2*d)
```

Maple [A] time = 0.024, size = 50, normalized size = 1.4

$$\frac{1}{da^2\left(1+\sec(dx+c)\right)} - \frac{\ln\left(1+\sec(dx+c)\right)}{da^2} + \frac{\ln\left(\sec(dx+c)\right)}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/d/a^2/(1+sec(d*x+c))-1/d/a^2*ln(1+sec(d*x+c))+1/d/a^2*ln(sec(d*x+c))
```

Maxima [A] time = 1.13091, size = 47, normalized size = 1.31

$$-\frac{\frac{1}{a^2\cos(dx+c)+a^2}+\frac{\log(\cos(dx+c)+1)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(1/(a^2*cos(d*x + c) + a^2) + log(cos(d*x + c) + 1)/a^2)/d
```

Fricas [A] time = 1.12163, size = 113, normalized size = 3.14

$$-\frac{(\cos(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+1}{a^2d\cos(dx+c)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -((cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 1)/(a^2*d*cos(d*x + c) +
 a^2*d)
```

Sympy [A] time = 29.8006, size = 177, normalized size = 4.92

$$\begin{cases} \frac{\log\left(\tan^2(c+dx)+1\right)\sec(c+dx)}{2a^2d\sec(c+dx)+2a^2d}+\frac{\log\left(\tan^2(c+dx)+1\right)}{2a^2d\sec(c+dx)+2a^2d}-\frac{2\log(\sec(c+dx)+1)\sec(c+dx)}{2a^2d\sec(c+dx)+2a^2d}-\frac{2\log(\sec(c+dx)+1)}{2a^2d\sec(c+dx)+2a^2d}+\frac{2}{2a^2d\sec(c+dx)+2a^2d} & \text{for } d \neq \\ \frac{x\tan(c)}{(a\sec(c)+a)^2} & \text{otherw} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Piecewise((log(tan(c + d*x)**2 + 1)*sec(c + d*x)/(2*a**2*d*sec(c + d*x) + 2
*a**2*d) + log(tan(c + d*x)**2 + 1)/(2*a**2*d*sec(c + d*x) + 2*a**2*d) - 2*
log(sec(c + d*x) + 1)*sec(c + d*x)/(2*a**2*d*sec(c + d*x) + 2*a**2*d) - 2*l
og(sec(c + d*x) + 1)/(2*a**2*d*sec(c + d*x) + 2*a**2*d) + 2/(2*a**2*d*sec(c
 + d*x) + 2*a**2*d), Ne(d, 0)), (x*tan(c)/(a*sec(c) + a)**2, True))
```

Giac [A] time = 1.26897, size = 77, normalized size = 2.14

$$\frac{\frac{2 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}+\frac{\cos(dx+c)-1}{a^2(\cos(dx+c)+1)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 + (cos(d*x
+ c) - 1)/(a^2*(cos(d*x + c) + 1)))/d
```

3.76 $\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=81

$$\frac{5}{4a^2d(\cos(c+dx)+1)}-\frac{1}{4a^2d(\cos(c+dx)+1)^2}+\frac{\log(1-\cos(c+dx))}{8a^2d}+\frac{7\log(\cos(c+dx)+1)}{8a^2d}$$

```
[Out] -1/(4*a^2*d*(1 + Cos[c + d*x])^2) + 5/(4*a^2*d*(1 + Cos[c + d*x])) + Log[1
- Cos[c + d*x]]/(8*a^2*d) + (7*Log[1 + Cos[c + d*x]])/(8*a^2*d)
```

Rubi [A] time = 0.060841, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 88}

$$\frac{5}{4a^2d(\cos(c+dx)+1)}-\frac{1}{4a^2d(\cos(c+dx)+1)^2}+\frac{\log(1-\cos(c+dx))}{8a^2d}+\frac{7\log(\cos(c+dx)+1)}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/(4*a^2*d*(1 + Cos[c + d*x])^2) + 5/(4*a^2*d*(1 + Cos[c + d*x])) + Log[1
- Cos[c + d*x]]/(8*a^2*d) + (7*Log[1 + Cos[c + d*x]])/(8*a^2*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{a^2\,\text{Subst}\left(\int \frac{x^3}{(a-ax)(a+ax)^3}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \left(-\frac{1}{8a^4(-1+x)} - \frac{1}{2a^4(1+x)^3} + \frac{5}{4a^4(1+x)^2} - \frac{7}{8a^4(1+x)}\right) dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{1}{4a^2d(1+\cos(c+dx))^2} + \frac{5}{4a^2d(1+\cos(c+dx))} + \frac{\log(1-\cos(c+dx))}{8a^2d} + \frac{7\log(1+\cos(}{8a^2d}$$

Mathematica [A] time = 0.186381, size = 83, normalized size = 1.02

$$\frac{\sec^2(c+dx)\left(10\cos^2\left(\frac{1}{2}(c+dx)\right)+4\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+7\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)-1\right)}{4a^2d(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((-1 + 10*Cos[(c + d*x)/2]^2 + 4*Cos[(c + d*x)/2]^4*(7*Log[Cos[(c + d*x)/2]
] + Log[Sin[(c + d*x)/2]]))*Sec[c + d*x]^2)/(4*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 0.076, size = 72, normalized size = 0.9

$$-\frac{1}{4\,da^2\,(\cos(dx+c)+1)^2}+\frac{5}{4\,da^2\,(\cos(dx+c)+1)}+\frac{7\,\ln(\cos(dx+c)+1)}{8\,da^2}+\frac{\ln(-1+\cos(dx+c))}{8\,da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/4/d/a^2/(cos(d*x+c)+1)^2+5/4/d/a^2/(cos(d*x+c)+1)+7/8*ln(cos(d*x+c)+1)/a
^2/d+1/8/d/a^2*ln(-1+cos(d*x+c))
```

Maxima [A] time = 1.19077, size = 100, normalized size = 1.23

$$\frac{\frac{2\,(5\,\cos(dx+c)+4)}{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2}+\frac{7\,\log(\cos(dx+c)+1)}{a^2}+\frac{\log(\cos(dx+c)-1)}{a^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2*(5*cos(d*x + c) + 4)/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)
 + 7*log(cos(d*x + c) + 1)/a^2 + log(cos(d*x + c) - 1)/a^2)/d
```

Fricas [A] time = 1.19867, size = 297, normalized size = 3.67

$$\frac{7\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)\cdots\right.}{8\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(7*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) +
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 10*cos
(d*x + c) + 8)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.32623, size = 158, normalized size = 1.95

$$\frac{\frac{2\,\log\left(\frac{\left|-\cos\left(dx+c\right)+1\right|}{\left|\cos\left(dx+c\right)+1\right|}\right)}{a^{2}}-\frac{16\,\log\left(\left|-\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}+1\right|\right)}{a^{2}}-\frac{\frac{8\,a^{2}\left(\cos\left(dx+c\right)-1\right)}{\cos\left(dx+c\right)+1}+\frac{a^{2}\left(\cos\left(dx+c\right)-1\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}}{a^{4}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2 - 16*log(abs(
-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - (8*a^2*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/a^4)/
d
```

3.77 $\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=123

$$-\frac{1}{16a^2d(1-\cos(c+dx))}-\frac{23}{16a^2d(\cos(c+dx)+1)}+\frac{1}{2a^2d(\cos(c+dx)+1)^2}-\frac{1}{12a^2d(\cos(c+dx)+1)^3}-\frac{3\log(1-\cos(c}{16a^2d}$$

```
[Out] -1/(16*a^2*d*(1 - Cos[c + d*x])) - 1/(12*a^2*d*(1 + Cos[c + d*x])^3) + 1/(2
*a^2*d*(1 + Cos[c + d*x])^2) - 23/(16*a^2*d*(1 + Cos[c + d*x])) - (3*Log[1
- Cos[c + d*x]])/(16*a^2*d) - (13*Log[1 + Cos[c + d*x]])/(16*a^2*d)
```

Rubi [A] time = 0.086958, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$-\frac{1}{16a^2d(1-\cos(c+dx))}-\frac{23}{16a^2d(\cos(c+dx)+1)}+\frac{1}{2a^2d(\cos(c+dx)+1)^2}-\frac{1}{12a^2d(\cos(c+dx)+1)^3}-\frac{3\log(1-\cos(c}{16a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```



```
[Out] -1/(16*a^2*d*(1 - Cos[c + d*x])) - 1/(12*a^2*d*(1 + Cos[c + d*x])^3) + 1/(2
*a^2*d*(1 + Cos[c + d*x])^2) - 23/(16*a^2*d*(1 + Cos[c + d*x])) - (3*Log[1
- Cos[c + d*x]])/(16*a^2*d) - (13*Log[1 + Cos[c + d*x]])/(16*a^2*d)
```


Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{a^4\,\text{Subst}\left(\int \frac{x^5}{(a-ax)^2(a+ax)^4}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a^4\,\text{Subst}\left(\int \left(\frac{1}{16a^6(-1+x)^2} + \frac{3}{16a^6(-1+x)} - \frac{1}{4a^6(1+x)^4} + \frac{1}{a^6(1+x)^3} - \frac{23}{16a^6(1+x)^2} + \frac{13}{16a^6(1+x)}\right) dx, x, c\right.}{d}$$

$$= -\frac{1}{16a^2d(1-\cos(c+dx))} - \frac{1}{12a^2d(1+\cos(c+dx))^3} + \frac{1}{2a^2d(1+\cos(c+dx))^2} - \frac{}{16a^2d(1+}$$

Mathematica [A] time = 0.369981, size = 121, normalized size = 0.98

$$-\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(3\csc^2\left(\frac{1}{2}(c+dx)\right)+\sec^6\left(\frac{1}{2}(c+dx)\right)-12\sec^4\left(\frac{1}{2}(c+dx)\right)+69\sec^2\left(\frac{1}{2}(c+dx)\right)+36\,\mathrm{l}\right.}{24a^2d(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Cos[(c + d*x)/2]^4*(3*Csc[(c + d*x)/2]^2 + 156*Log[Cos[(c + d*x)/2]] + 36
*Log[Sin[(c + d*x)/2]] + 69*Sec[(c + d*x)/2]^2 - 12*Sec[(c + d*x)/2]^4 + Se
c[(c + d*x)/2]^6)*Sec[c + d*x]^2)/(24*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 0.077, size = 108, normalized size = 0.9

$$-\frac{1}{12\,da^2\left(\cos\left(dx+c\right)+1\right)^3}+\frac{1}{2\,da^2\left(\cos\left(dx+c\right)+1\right)^2}-\frac{23}{16\,da^2\left(\cos\left(dx+c\right)+1\right)}-\frac{13\,\ln\left(\cos\left(dx+c\right)+1\right)}{16\,da^2}+\frac{}{16\,da^2\,(-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/12/d/a^2/(cos(d*x+c)+1)^3+1/2/d/a^2/(cos(d*x+c)+1)^2-23/16/d/a^2/(cos(d*
x+c)+1)-13/16*ln(cos(d*x+c)+1)/a^2/d+1/16/d/a^2/(-1+cos(d*x+c))-3/16/d/a^2*
ln(-1+cos(d*x+c))
```

Maxima [A] time = 1.14885, size = 149, normalized size = 1.21

$$-\frac{\frac{2\left(33\cos(dx+c)^3+18\cos(dx+c)^2-37\cos(dx+c)-26\right)}{a^2\cos(dx+c)^4+2a^2\cos(dx+c)^3-2a^2\cos(dx+c)-a^2}+\frac{39\log(\cos(dx+c)+1)}{a^2}+\frac{9\log(\cos(dx+c)-1)}{a^2}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/48*(2*(33*cos(d*x + c)^3 + 18*cos(d*x + c)^2 - 37*cos(d*x + c) - 26)/(a^
2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c) - a^2) + 39*lo
g(cos(d*x + c) + 1)/a^2 + 9*log(cos(d*x + c) - 1)/a^2)/d
```

Fricas [A] time = 1.21099, size = 444, normalized size = 3.61

$$-\frac{66\cos(dx+c)^3+36\cos(dx+c)^2+39\left(\cos(dx+c)^4+2\cos(dx+c)^3-2\cos(dx+c)-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{48\left(a^2d\cos(dx+c)^4+2a^2d\cos(dx+c)^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/48*(66*cos(d*x + c)^3 + 36*cos(d*x + c)^2 + 39*(cos(d*x + c)^4 + 2*cos(d
*x + c)^3 - 2*cos(d*x + c) - 1)*log(1/2*cos(d*x + c) + 1/2) + 9*(cos(d*x +
c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2)
- 74*cos(d*x + c) - 52)/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 - 2*
a^2*d*cos(d*x + c) - a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\cot^3(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.39026, size = 251, normalized size = 2.04

$$\frac{\frac{3\left(\frac{6\,(\cos(dx+c)-1)}{\cos(dx+c)+1}+1\right)(\cos(dx+c)+1)}{a^2(\cos(dx+c)-1)}-\frac{18\,\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2}+\frac{96\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}+\frac{\frac{48\,a^4(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9\,a^4(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{a^4(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{a^6}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/96*(3*(6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)*(cos(d*x + c) + 1)/(a
^2*(cos(d*x + c) - 1)) - 18*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1
))/a^2 + 96*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 + (48*
a^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^4*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2 + a^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^6)/d
```

3.78 $\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=165

$$\frac{9}{64a^2d(1-\cos(c+dx))}+\frac{51}{32a^2d(\cos(c+dx)+1)}-\frac{1}{64a^2d(1-\cos(c+dx))^2}-\frac{3}{4a^2d(\cos(c+dx)+1)^2}+\frac{11}{48a^2d(\cos(c+dx)+1)^3}$$

```
[Out] -1/(64*a^2*d*(1 - Cos[c + d*x])^2) + 9/(64*a^2*d*(1 - Cos[c + d*x])) - 1/(3
2*a^2*d*(1 + Cos[c + d*x])^4) + 11/(48*a^2*d*(1 + Cos[c + d*x])^3) - 3/(4*a
^2*d*(1 + Cos[c + d*x])^2) + 51/(32*a^2*d*(1 + Cos[c + d*x])) + (29*Log[1 -
 Cos[c + d*x]])/(128*a^2*d) + (99*Log[1 + Cos[c + d*x]])/(128*a^2*d)
```

Rubi [A] time = 0.110681, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{9}{64a^2d(1-\cos(c+dx))}+\frac{51}{32a^2d(\cos(c+dx)+1)}-\frac{1}{64a^2d(1-\cos(c+dx))^2}-\frac{3}{4a^2d(\cos(c+dx)+1)^2}+\frac{11}{48a^2d(\cos(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] -1/(64*a^2*d*(1 - Cos[c + d*x])^2) + 9/(64*a^2*d*(1 - Cos[c + d*x])) - 1/(3
2*a^2*d*(1 + Cos[c + d*x])^4) + 11/(48*a^2*d*(1 + Cos[c + d*x])^3) - 3/(4*a
^2*d*(1 + Cos[c + d*x])^2) + 51/(32*a^2*d*(1 + Cos[c + d*x])) + (29*Log[1 -
 Cos[c + d*x]])/(128*a^2*d) + (99*Log[1 + Cos[c + d*x]])/(128*a^2*d)
```


Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{a^6\,\text{Subst}\left(\int \frac{x^7}{(a-ax)^3(a+ax)^5}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^6\,\text{Subst}\left(\int \left(-\frac{1}{32a^8(-1+x)^3} - \frac{9}{64a^8(-1+x)^2} - \frac{29}{128a^8(-1+x)} - \frac{1}{8a^8(1+x)^5} + \frac{11}{16a^8(1+x)^4} - \frac{3}{2a^8(1+x)^3} + \frac{3}{}\right.\right.}{d}$$
$$= -\frac{1}{64a^2d(1-\cos(c+dx))^2} + \frac{9}{64a^2d(1-\cos(c+dx))} - \frac{1}{32a^2d(1+\cos(c+dx))^4} + \frac{}{48a^2d(1}$$

Mathematica [A] time = 0.805033, size = 154, normalized size = 0.93

$$-\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(6\csc^4\left(\frac{1}{2}(c+dx)\right) - 108\csc^2\left(\frac{1}{2}(c+dx)\right) + 3\sec^8\left(\frac{1}{2}(c+dx)\right) - 44\sec^6\left(\frac{1}{2}(c+dx)\right) + 2\right.}{384a^2d(\sec(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Cos[(c + d*x)/2]^4*(-108*Csc[(c + d*x)/2]^2 + 6*Csc[(c + d*x)/2]^4 - 24*(
99*Log[Cos[(c + d*x)/2]] + 29*Log[Sin[(c + d*x)/2]]) - 1224*Sec[(c + d*x)/2
]^2 + 288*Sec[(c + d*x)/2]^4 - 44*Sec[(c + d*x)/2]^6 + 3*Sec[(c + d*x)/2]^8
)*Sec[c + d*x]^2)/(384*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 0.075, size = 144, normalized size = 0.9

$$-\frac{1}{32\,da^2\left(\cos(dx+c)+1\right)^4} + \frac{11}{48\,da^2\left(\cos(dx+c)+1\right)^3} - \frac{3}{4\,da^2\left(\cos(dx+c)+1\right)^2} + \frac{51}{32\,da^2\left(\cos(dx+c)+1\right)} + \frac{99\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/32/d/a^2/(cos(d*x+c)+1)^4+11/48/d/a^2/(cos(d*x+c)+1)^3-3/4/d/a^2/(cos(d*
x+c)+1)^2+51/32/d/a^2/(cos(d*x+c)+1)+99/128*ln(cos(d*x+c)+1)/a^2/d-1/64/d/a
```

```
^2/(-1+cos(d*x+c))^2-9/64/d/a^2/(-1+cos(d*x+c))+29/128/d/a^2*ln(-1+cos(d*x+
c))
```

Maxima [A] time = 1.15676, size = 225, normalized size = 1.36

$$\frac{\frac{2\left(279\cos(dx+c)^5+78\cos(dx+c)^4-634\cos(dx+c)^3-338\cos(dx+c)^2+343\cos(dx+c)+224\right)}{a^2\cos(dx+c)^6+2a^2\cos(dx+c)^5-a^2\cos(dx+c)^4-4a^2\cos(dx+c)^3-a^2\cos(dx+c)^2+2a^2\cos(dx+c)+a^2}+\frac{297\log(\cos(dx+c)+1)}{a^2}+\frac{87\log(\cos(dx+c)-1)}{a^2}}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/384*(2*(279*cos(d*x + c)^5 + 78*cos(d*x + c)^4 - 634*cos(d*x + c)^3 - 338
*cos(d*x + c)^2 + 343*cos(d*x + c) + 224)/(a^2*cos(d*x + c)^6 + 2*a^2*cos(d
*x + c)^5 - a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^3 - a^2*cos(d*x + c)^2
+ 2*a^2*cos(d*x + c) + a^2) + 297*log(cos(d*x + c) + 1)/a^2 + 87*log(cos(d*
x + c) - 1)/a^2)/d
```

Fricas [A] time = 1.22873, size = 749, normalized size = 4.54

$$\frac{558\cos(dx+c)^5+156\cos(dx+c)^4-1268\cos(dx+c)^3-676\cos(dx+c)^2+297\left(\cos(dx+c)^6+2\cos(dx+c)^5-\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/384*(558*cos(d*x + c)^5 + 156*cos(d*x + c)^4 - 1268*cos(d*x + c)^3 - 676*
cos(d*x + c)^2 + 297*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 -
4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(1/2*cos(d*x + c
) + 1/2) + 87*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 - 4*cos(d
*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/
2) + 686*cos(d*x + c) + 448)/(a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5
 - a^2*d*cos(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2
*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot^5(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.41409, size = 319, normalized size = 1.93

$$-\frac{\frac{6\left(\frac{16(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{87(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)^2}{a^2(\cos(dx+c)-1)^2}-\frac{348\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2}+\frac{1536\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}+\frac{\frac{768a^6(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{174a^6(\cos(dx+c)}{(\cos(dx+c)+1)}}{}}{1536d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/1536*(6*(16*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 87*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2 + 1)*(cos(d*x + c) + 1)^2/(a^2*(cos(d*x + c) - 1)^
2) - 348*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2 + 1536*log(a
bs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 + (768*a^6*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 174*a^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2 + 32*a^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 3*a^6*(cos(d*x + c)
 - 1)^4/(cos(d*x + c) + 1)^4)/a^8)/d
```

3.79 $\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=119

$$\frac{\tan^5(c+dx)}{5a^2d}+\frac{\tan^3(c+dx)}{3a^2d}-\frac{\tan(c+dx)}{a^2d}-\frac{3\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{\tan^3(c+dx)\sec(c+dx)}{2a^2d}+\frac{3\tan(c+dx)\sec(c+dx)}{4a^2d}$$

```
[Out] x/a^2 - (3*ArcTanh[Sin[c + d*x]])/(4*a^2*d) - Tan[c + d*x]/(a^2*d) + (3*Sec
[c + d*x]*Tan[c + d*x])/(4*a^2*d) + Tan[c + d*x]^3/(3*a^2*d) - (Sec[c + d*x
]*Tan[c + d*x]^3)/(2*a^2*d) + Tan[c + d*x]^5/(5*a^2*d)
```

Rubi [A] time = 0.190259, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3888, 3886, 3473, 8, 2611, 3770, 2607, 30}

$$\frac{\tan^5(c+dx)}{5a^2d}+\frac{\tan^3(c+dx)}{3a^2d}-\frac{\tan(c+dx)}{a^2d}-\frac{3\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{\tan^3(c+dx)\sec(c+dx)}{2a^2d}+\frac{3\tan(c+dx)\sec(c+dx)}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] x/a^2 - (3*ArcTanh[Sin[c + d*x]])/(4*a^2*d) - Tan[c + d*x]/(a^2*d) + (3*Sec
[c + d*x]*Tan[c + d*x])/(4*a^2*d) + Tan[c + d*x]^3/(3*a^2*d) - (Sec[c + d*x
]*Tan[c + d*x]^3)/(2*a^2*d) + Tan[c + d*x]^5/(5*a^2*d)
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int(-a+a\sec(c+dx))^2\tan^4(c+dx)\,dx}{a^4}$$
$$= \frac{\int\left(a^2\tan^4(c+dx)-2a^2\sec(c+dx)\tan^4(c+dx)+a^2\sec^2(c+dx)\tan^4(c+dx)\right)\,dx}{a^4}$$
$$= \frac{\int\tan^4(c+dx)\,dx}{a^2}+\frac{\int\sec^2(c+dx)\tan^4(c+dx)\,dx}{a^2}-\frac{2\int\sec(c+dx)\tan^4(c+dx)\,dx}{a^2}$$
$$= \frac{\tan^3(c+dx)}{3a^2d}-\frac{\sec(c+dx)\tan^3(c+dx)}{2a^2d}-\frac{\int\tan^2(c+dx)\,dx}{a^2}+\frac{3\int\sec(c+dx)\tan^2(c+dx}{2a^2}$$
$$= -\frac{\tan(c+dx)}{a^2d}+\frac{3\sec(c+dx)\tan(c+dx)}{4a^2d}+\frac{\tan^3(c+dx)}{3a^2d}-\frac{\sec(c+dx)\tan^3(c+dx)}{2a^2d}+\ \text{tan}$$
$$= \frac{x}{a^2}-\frac{3\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{\tan(c+dx)}{a^2d}+\frac{3\sec(c+dx)\tan(c+dx)}{4a^2d}+\frac{\tan^3(c+dx)}{3a^2d}-\ \text{se}$$

Mathematica [B] time = 5.58872, size = 495, normalized size = 4.16

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(-\frac{151\sin\left(\frac{c}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{151\sin\left(\frac{c}{2}\right)}{d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{}{d(\cos(}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(240*x + (180*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]])/d - (180*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d - ((29
3*Cos[(d*x)/2] + 333*Cos[2*c + (3*d*x)/2] + 287*Cos[2*c + (5*d*x)/2] + 67*C
os[4*c + (7*d*x)/2] + 68*Cos[4*c + (9*d*x)/2])*Sec[c]*Sec[c + d*x]^5*Sin[(d
*x)/2])/(2*d) + (36*Sin[c/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] -
Sin[(c + d*x)/2])^4) - (151*Sin[c/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2])^2) + (36*Sin[c/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2])^4) - (151*Sin[c/2])/(d*(Cos[c/2] + Sin[c/2
])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (Cos[c/2]*Sec[c]*Sec[c + d*x]
^4*(308*Sin[c/2] - 43*Sin[c/2 + d*x] - 43*Sin[(3*c)/2 + d*x] - 346*Sin[(3*c
)/2 + 2*d*x] + 346*Sin[(5*c)/2 + 2*d*x] + 149*Sin[(5*c)/2 + 3*d*x] + 149*Si
n[(7*c)/2 + 3*d*x]))/(4*d)))/(60*a^2*(1 + Sec[c + d*x])^2)
```

Maple [B] time = 0.089, size = 269, normalized size = 2.3

$$2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^{2}}-\frac{1}{5\,da^{2}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-5}+\frac{1}{da^{2}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-4}-\frac{19}{12\,da^{2}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^8/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*arctan(tan(1/2*d*x+1/2*c))-1/5/d/a^2/(tan(1/2*d*x+1/2*c)+1)^5+1/d/a
^2/(tan(1/2*d*x+1/2*c)+1)^4-19/12/d/a^2/(tan(1/2*d*x+1/2*c)+1)^3+1/8/d/a^2/
(tan(1/2*d*x+1/2*c)+1)^2+7/4/d/a^2/(tan(1/2*d*x+1/2*c)+1)-3/4/d/a^2*ln(tan(
1/2*d*x+1/2*c)+1)-1/5/d/a^2/(tan(1/2*d*x+1/2*c)-1)^5-1/d/a^2/(tan(1/2*d*x+1
/2*c)-1)^4-19/12/d/a^2/(tan(1/2*d*x+1/2*c)-1)^3-1/8/d/a^2/(tan(1/2*d*x+1/2*
c)-1)^2+7/4/d/a^2/(tan(1/2*d*x+1/2*c)-1)+3/4/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.75619, size = 406, normalized size = 3.41

$$-\frac{\frac{2\left(\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{110\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{328\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{530\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{105\,\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}\right)}{a^{2}-\frac{5\,a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{10\,a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{10\,a^{2}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{5\,a^{2}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{a^{2}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{120\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{2}}+\frac{45\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^{2}}-\frac{45\,\log\left(\frac{\sin}{\cos}\right.}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/60*(2*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 110*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 328*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 530*sin(d*x + c)^7/
(cos(d*x + c) + 1)^7 + 105*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^2 - 5*a^
2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^2*sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 - 10*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*a^2*sin(d*x + c)^8
/(cos(d*x + c) + 1)^8 - a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - 120*ar
ctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 45*log(sin(d*x + c)/(cos(d*x +
c) + 1) + 1)/a^2 - 45*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

Fricas [A] time = 1.19154, size = 323, normalized size = 2.71

$$\frac{120\,dx\cos\left(dx+c\right)^{5}-45\,\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)+45\,\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+1\right)-2\left(68\,\cos\left(dx\right.\right.}{120\,a^{2}d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/120*(120*d*x*cos(d*x + c)^5 - 45*cos(d*x + c)^5*log(sin(d*x + c) + 1) + 4
5*cos(d*x + c)^5*log(-sin(d*x + c) + 1) - 2*(68*cos(d*x + c)^4 - 75*cos(d*x
 + c)^3 + 4*cos(d*x + c)^2 + 30*cos(d*x + c) - 12)*sin(d*x + c))/(a^2*d*cos
(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^8(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**8/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**8/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A] time = 10.3405, size = 184, normalized size = 1.55

$$\frac{\frac{60\,(dx+c)}{a^2} - \frac{45\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2} + \frac{45\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2} + \frac{2\left(105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-530\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+328\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-110\tan\left(\frac{1}{2}d\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^5a^2}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(60*(d*x + c)/a^2 - 45*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 + 45*log
(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 2*(105*tan(1/2*d*x + 1/2*c)^9 - 530*t
an(1/2*d*x + 1/2*c)^7 + 328*tan(1/2*d*x + 1/2*c)^5 - 110*tan(1/2*d*x + 1/2*
c)^3 + 15*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*a^2))/d
```

3.80 $\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=72

$$\frac{\tan^3(c+dx)}{3a^2d}+\frac{\tan(c+dx)}{a^2d}+\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{\tan(c+dx)\sec(c+dx)}{a^2d}-\frac{x}{a^2}$$

```
[Out] -(x/a^2) + ArcTanh[Sin[c + d*x]]/(a^2*d) + Tan[c + d*x]/(a^2*d) - (Sec[c +
d*x]*Tan[c + d*x])/(a^2*d) + Tan[c + d*x]^3/(3*a^2*d)
```

Rubi [A] time = 0.14916, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3888, 3886, 3473, 8, 2611, 3770, 2607, 30}

$$\frac{\tan^3(c+dx)}{3a^2d}+\frac{\tan(c+dx)}{a^2d}+\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{\tan(c+dx)\sec(c+dx)}{a^2d}-\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(x/a^2) + ArcTanh[Sin[c + d*x]]/(a^2*d) + Tan[c + d*x]/(a^2*d) - (Sec[c +
d*x]*Tan[c + d*x])/(a^2*d) + Tan[c + d*x]^3/(3*a^2*d)
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int(-a+a\sec(c+dx))^2\tan^2(c+dx)\,dx}{a^4}$$
$$= \frac{\int\left(a^2\tan^2(c+dx)-2a^2\sec(c+dx)\tan^2(c+dx)+a^2\sec^2(c+dx)\tan^2(c+dx)\right)dx}{a^4}$$
$$= \frac{\int\tan^2(c+dx)\,dx}{a^2}+\frac{\int\sec^2(c+dx)\tan^2(c+dx)\,dx}{a^2}-\frac{2\int\sec(c+dx)\tan^2(c+dx)\,dx}{a^2}$$
$$= \frac{\tan(c+dx)}{a^2d}-\frac{\sec(c+dx)\tan(c+dx)}{a^2d}-\frac{\int 1\,dx}{a^2}+\frac{\int\sec(c+dx)\,dx}{a^2}+\frac{\mathrm{Subst}\left(\int x^2\,dx,x,\mathrm{t}\right.}{a^2d}$$
$$= -\frac{x}{a^2}+\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{\tan(c+dx)}{a^2d}-\frac{\sec(c+dx)\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{3a^2d}$$

Mathematica [B] time = 6.2941, size = 767, normalized size = 10.65

$$-\frac{4x\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)}{(a\sec(c+dx)+a)^2}+\frac{8\sin\left(\frac{dx}{2}\right)\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)}{3d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)(a\sec(c+dx)+a)^2\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{\quad}{3d\left(\sin\left(\frac{c}{2}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*x*Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^2 - (4*Cos[
c/2 + (d*x)/2]^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^
2)/(d*(a + a*Sec[c + d*x])^2) + (4*Cos[c/2 + (d*x)/2]^4*Log[Cos[c/2 + (d*x)
/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^2)/(d*(a + a*Sec[c + d*x])^2) + (2*C
os[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*Sin[(d*x)/2])/(3*d*(a + a*Sec[c + d*x])^
2*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + (Cos
[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*(-5*Cos[c/2] + 7*Sin[c/2]))/(3*d*(a + a*Se
c[c + d*x])^2*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2
])^2) + (8*Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*Sin[(d*x)/2])/(3*d*(a + a*Se
c[c + d*x])^2*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2
])) + (2*Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*Sin[(d*x)/2])/(3*d*(a + a*Sec[
c + d*x])^2*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])
^3) + (Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*(5*Cos[c/2] + 7*Sin[c/2]))/(3*d*
(a + a*Sec[c + d*x])^2*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2
+ (d*x)/2])^2) + (8*Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*Sin[(d*x)/2])/(3*d*
(a + a*Sec[c + d*x])^2*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2
+ (d*x)/2]))
```

Maple [B] time = 0.076, size = 185, normalized size = 2.6

$$-2\frac{\arctan(\tan(1/2\,dx+c/2))}{da^2}-\frac{1}{3\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}+\frac{3}{2\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}-2\,\frac{1}{da^2\,(\tan(1/2\,dx+c/2)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -2/d/a^2*arctan(tan(1/2*d*x+1/2*c))-1/3/d/a^2/(tan(1/2*d*x+1/2*c)+1)^3+3/2/
d/a^2/(tan(1/2*d*x+1/2*c)+1)^2-2/d/a^2/(tan(1/2*d*x+1/2*c)+1)+1/d/a^2*ln(ta
n(1/2*d*x+1/2*c)+1)-1/3/d/a^2/(tan(1/2*d*x+1/2*c)-1)^3-3/2/d/a^2/(tan(1/2*d
*x+1/2*c)-1)^2-2/d/a^2/(tan(1/2*d*x+1/2*c)-1)-1/d/a^2*ln(tan(1/2*d*x+1/2*c)
-1)
```

Maxima [B] time = 1.7966, size = 265, normalized size = 3.68

$$-\frac{\frac{4\left(\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2-\frac{3\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\,a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{6\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{3\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{3\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(4*(sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3*sin(d*x + c)^5/(cos(d*x +
c) + 1)^5)/(a^2 - 3*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 - a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + 6
*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - 3*log(sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)/a^2 + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

Fricas [A] time = 1.3059, size = 259, normalized size = 3.6

$$-\frac{6\,dx\cos(dx+c)^3-3\,\cos(dx+c)^3\log(\sin(dx+c)+1)+3\,\cos(dx+c)^3\log(-\sin(dx+c)+1)-2\left(2\,\cos(dx+c)^2-\right.}{6\,a^2d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*d*x*cos(d*x + c)^3 - 3*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 3*cos
(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(2*cos(d*x + c)^2 - 3*cos(d*x + c) +
 1)*sin(d*x + c))/(a^2*d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^6(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**6/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A] time = 4.87682, size = 134, normalized size = 1.86

$$-\frac{\frac{3\,(dx+c)}{a^2}-\frac{3\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}+\frac{3\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}+\frac{4\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3a^2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)/a^2 - 3*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 + 3*log(ab
s(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 4*(3*tan(1/2*d*x + 1/2*c)^5 - tan(1/2*d*
x + 1/2*c)^3)/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^2))/d
```

3.81 $\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=34

$$\frac{\tan(c+dx)}{a^2d}-\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{x}{a^2}$$

```
[Out] x/a^2 - (2*ArcTanh[Sin[c + d*x]])/(a^2*d) + Tan[c + d*x]/(a^2*d)
```

Rubi [A] time = 0.0653077, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3888, 3773, 3770, 3767, 8}

$$\frac{\tan(c+dx)}{a^2d}-\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] x/a^2 - (2*ArcTanh[Sin[c + d*x]])/(a^2*d) + Tan[c + d*x]/(a^2*d)
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3773

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \frac{\int (-a+a\sec(c+dx))^2\,dx}{a^4}\\
&= \frac{x}{a^2}+\frac{\int \sec^2(c+dx)\,dx}{a^2}-\frac{2\int \sec(c+dx)\,dx}{a^2}\\
&= \frac{x}{a^2}-\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{\mathrm{Subst}(\int 1\,dx, x, -\tan(c+dx))}{a^2d}\\
&= \frac{x}{a^2}-\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{\tan(c+dx)}{a^2d}
\end{aligned}
$$

Mathematica [B] time = 0.48694, size = 177, normalized size = 5.21

$$
\frac{4\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(\frac{\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}+2\log\left(\cos\right.\right.}{a^2d(\sec(c+dx)+1)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(d*x + 2*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] - 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sin[d*x]/((Cos[c
/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [B] time = 0.062, size = 102, normalized size = 3.

$$
2\,\frac{\arctan(\tan(1/2\,dx+c/2))}{da^2}-\frac{1}{da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-2\,\frac{\ln(\tan(1/2\,dx+c/2)+1)}{da^2}-\frac{1}{da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*arctan(tan(1/2*d*x+1/2*c))-1/d/a^2/(tan(1/2*d*x+1/2*c)+1)-2/d/a^2*l
n(tan(1/2*d*x+1/2*c)+1)-1/d/a^2/(tan(1/2*d*x+1/2*c)-1)+2/d/a^2*ln(tan(1/2*d
*x+1/2*c)-1)
```

Maxima [B] time = 1.78972, size = 166, normalized size = 4.88

$$\frac{2\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}+\frac{\sin(dx+c)}{\left(a^2-\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2*(arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - log(sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)/a^2 + log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2 + sin(d*x
 + c)/((a^2 - a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)))
/d
```

Fricas [A] time = 1.18004, size = 177, normalized size = 5.21

$$\frac{dx\cos(dx+c)-\cos(dx+c)\log(\sin(dx+c)+1)+\cos(dx+c)\log(-\sin(dx+c)+1)+\sin(dx+c)}{a^2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (d*x*cos(d*x + c) - cos(d*x + c)*log(sin(d*x + c) + 1) + cos(d*x + c)*log(-
sin(d*x + c) + 1) + sin(d*x + c))/(a^2*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [B] time = 2.48726, size = 107, normalized size = 3.15

$$\frac{\frac{dx+c}{a^2} - \frac{2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2} + \frac{2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2} - \frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)/a^2 - 2*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 + 2*log(abs(tan(1
/2*d*x + 1/2*c) - 1))/a^2 - 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
 - 1)*a^2))/d
```

3.82 $\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=33

$$\frac{2\tan(c+dx)}{ad(a\sec(c+dx)+a)}-\frac{x}{a^2}$$

```
[Out] -(x/a^2) + (2*Tan[c + d*x])/(a*d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.111743, antiderivative size = 35, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3888, 3886, 3473, 8, 2606, 3767}

$$-\frac{2\cot(c+dx)}{a^2d}+\frac{2\csc(c+dx)}{a^2d}-\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - (2*Cot[c + d*x])/(a^2*d) + (2*Csc[c + d*x])/(a^2*d)
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \frac{\int \cot^2(c+dx)(-a+a\sec(c+dx))^2\,dx}{a^4}\\ &= \frac{\int \left(a^2\cot^2(c+dx)-2a^2\cot(c+dx)\csc(c+dx)+a^2\csc^2(c+dx)\right)dx}{a^4}\\ &= \frac{\int \cot^2(c+dx)\,dx}{a^2}+\frac{\int \csc^2(c+dx)\,dx}{a^2}-\frac{2\int \cot(c+dx)\csc(c+dx)\,dx}{a^2}\\ &= -\frac{\cot(c+dx)}{a^2d}-\frac{\int 1\,dx}{a^2}-\frac{\mathrm{Subst}(\int 1\,dx,x,\cot(c+dx))}{a^2d}+\frac{2\,\mathrm{Subst}(\int 1\,dx,x,\csc(c+dx))}{a^2d}\\ &= -\frac{x}{a^2}-\frac{2\cot(c+dx)}{a^2d}+\frac{2\csc(c+dx)}{a^2d}\end{aligned}$$

Mathematica [A] time = 0.0218923, size = 42, normalized size = 1.27

$$\frac{\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{2\tan^{-1}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((-2*ArcTan[Tan[c/2 + (d*x)/2]])/d + (2*Tan[c/2 + (d*x)/2])/d)/a^2
```

Maple [A] time = 0.069, size = 37, normalized size = 1.1

$$2\,\frac{\tan\left(1/2\,dx+c/2\right)}{da^2}-2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*tan(1/2*d*x+1/2*c)-2/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.76568, size = 66, normalized size = 2.

$$-\frac{2\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\sin(dx+c)}{a^2(\cos(dx+c)+1)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -2*(arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - sin(d*x + c)/(a^2*(cos(d*
x + c) + 1)))/d
```

Fricas [A] time = 1.12642, size = 99, normalized size = 3.

$$-\frac{dx\cos\left(dx+c\right)+dx-2\,\sin\left(dx+c\right)}{a^2d\cos\left(dx+c\right)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(d*x*cos(d*x + c) + d*x - 2*sin(d*x + c))/(a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.51605, size = 39, normalized size = 1.18

$$-\frac{\frac{dx+c}{a^2} - \frac{2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -((d*x + c)/a^2 - 2*tan(1/2*d*x + 1/2*c)/a^2)/d
```

3.83 $\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=107

$$-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot(c+dx)}{a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{4\csc^3(c+dx)}{3a^2d}+\frac{2\csc(c+dx)}{a^2d}-\frac{x}{a^2}$$

```
[Out] -(x/a^2) - Cot[c + d*x]/(a^2*d) + Cot[c + d*x]^3/(3*a^2*d) - (2*Cot[c + d*x
]^5)/(5*a^2*d) + (2*Csc[c + d*x])/(a^2*d) - (4*Csc[c + d*x]^3)/(3*a^2*d) +
(2*Csc[c + d*x]^5)/(5*a^2*d)
```

Rubi [A] time = 0.173748, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3888, 3886, 3473, 8, 2606, 194, 2607, 30}

$$-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot(c+dx)}{a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{4\csc^3(c+dx)}{3a^2d}+\frac{2\csc(c+dx)}{a^2d}-\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] -(x/a^2) - Cot[c + d*x]/(a^2*d) + Cot[c + d*x]^3/(3*a^2*d) - (2*Cot[c + d*x
]^5)/(5*a^2*d) + (2*Csc[c + d*x])/(a^2*d) - (4*Csc[c + d*x]^3)/(3*a^2*d) +
(2*Csc[c + d*x]^5)/(5*a^2*d)
```


Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \cot^6(c+dx)(-a+a\sec(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cot^6(c+dx) - 2a^2\cot^5(c+dx)\csc(c+dx) + a^2\cot^4(c+dx)\csc^2(c+dx)\right)dx}{a^4}$$
$$= \frac{\int \cot^6(c+dx)\,dx}{a^2} + \frac{\int \cot^4(c+dx)\csc^2(c+dx)\,dx}{a^2} - \frac{2\int \cot^5(c+dx)\csc(c+dx)\,dx}{a^2}$$
$$= -\frac{\cot^5(c+dx)}{5a^2d} - \frac{\int \cot^4(c+dx)\,dx}{a^2} + \frac{\text{Subst}\left(\int x^4\,dx, x, -\cot(c+dx)\right)}{a^2d} + \frac{2\,\text{Subst}\left(\int \left(-1+\right.\right.}{}$$
$$= \frac{\cot^3(c+dx)}{3a^2d} - \frac{2\cot^5(c+dx)}{5a^2d} + \frac{\int \cot^2(c+dx)\,dx}{a^2} + \frac{2\,\text{Subst}\left(\int \left(1-2x^2+x^4\right)dx, x, \csc(c\right.}{a^2d}$$
$$= -\frac{\cot(c+dx)}{a^2d} + \frac{\cot^3(c+dx)}{3a^2d} - \frac{2\cot^5(c+dx)}{5a^2d} + \frac{2\csc(c+dx)}{a^2d} - \frac{4\csc^3(c+dx)}{3a^2d} + \frac{2\csc^5(c}{5a^2d}$$
$$= -\frac{x}{a^2} - \frac{\cot(c+dx)}{a^2d} + \frac{\cot^3(c+dx)}{3a^2d} - \frac{2\cot^5(c+dx)}{5a^2d} + \frac{2\csc(c+dx)}{a^2d} - \frac{4\csc^3(c+dx)}{3a^2d} + \frac{2\,\text{cs}}{}$$

Mathematica [A] time = 1.34006, size = 149, normalized size = 1.39

$$\frac{\sec^2(c+dx)\left(-120dx\cos^4\left(\frac{1}{2}(c+dx)\right) + 3\tan\left(\frac{1}{2}(c+dx)\right) - 31\tan\left(\frac{c}{2}\right)\cos^2\left(\frac{1}{2}(c+dx)\right) - 31\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\cos\left(\frac{1}{2}(c+\right.\right.}{30a^2d(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^2*(-120*d*x*Cos[(c + d*x)/2]^4 - 31*Cos[(c + d*x)/2]*Sec[c/2]
*Sin[(d*x)/2] + Cos[(c + d*x)/2]^3*(15*Cot[(c + d*x)/2]*Csc[c/2] + 193*Sec[
c/2])*Sin[(d*x)/2] - 31*Cos[(c + d*x)/2]^2*Tan[c/2] + 3*Tan[(c + d*x)/2]))/
(30*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 0.062, size = 94, normalized size = 0.9

$$\frac{1}{40\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 - \frac{5}{24\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{11}{8\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - 2\frac{\arctan(\tan(1/2\,dx+c/2))}{da^2} - \frac{1}{8\,da^2}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/40/d/a^2*tan(1/2*d*x+1/2*c)^5-5/24/d/a^2*tan(1/2*d*x+1/2*c)^3+11/8/d/a^2*
tan(1/2*d*x+1/2*c)-2/d/a^2*arctan(tan(1/2*d*x+1/2*c))-1/8/d/a^2/tan(1/2*d*x
+1/2*c)
```

Maxima [A] time = 1.73956, size = 153, normalized size = 1.43

$$\frac{\frac{\frac{165 \sin(dx+c)}{\cos(dx+c)+1} - \frac{25 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^2} - \frac{240 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2} - \frac{15 (\cos(dx+c)+1)}{a^2 \sin(dx+c)}}{120\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/120*((165*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^2 - 240*arctan(sin(d*x
 + c)/(cos(d*x + c) + 1))/a^2 - 15*(cos(d*x + c) + 1)/(a^2*sin(d*x + c)))/d
```

Fricas [A] time = 1.10047, size = 277, normalized size = 2.59

$$-\frac{26 \cos(dx+c)^3 + 22 \cos(dx+c)^2 + 15 \left(dx \cos(dx+c)^2 + 2\, dx \cos(dx+c) + dx\right) \sin(dx+c) - 17 \cos(dx+c) - 16}{15 \left(a^2 d \cos(dx+c)^2 + 2\, a^2 d \cos(dx+c) + a^2 d\right) \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(26*cos(d*x + c)^3 + 22*cos(d*x + c)^2 + 15*(d*x*cos(d*x + c)^2 + 2*d
*x*cos(d*x + c) + d*x)*sin(d*x + c) - 17*cos(d*x + c) - 16)/((a^2*d*cos(d*x
 + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\, dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.39127, size = 113, normalized size = 1.06

$$-\frac{\frac{120\,(dx+c)}{a^2}+\frac{15}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{3\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-25\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+165\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{10}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/120*(120*(d*x + c)/a^2 + 15/(a^2*tan(1/2*d*x + 1/2*c)) - (3*a^8*tan(1/2*
d*x + 1/2*c)^5 - 25*a^8*tan(1/2*d*x + 1/2*c)^3 + 165*a^8*tan(1/2*d*x + 1/2*
c))/a^10)/d
```

3.84 $\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=139

$$-\frac{2\cot^7(c+dx)}{7a^2d}+\frac{\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)}{a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}-\frac{6\csc^5(c+dx)}{5a^2d}+\frac{2\csc^3(c+dx)}{a^2d}-\frac{2\csc(c+dx)}{a^2d}+\frac{x}{a^2}$$

```
[Out] x/a^2 + Cot[c + d*x]/(a^2*d) - Cot[c + d*x]^3/(3*a^2*d) + Cot[c + d*x]^5/(5
*a^2*d) - (2*Cot[c + d*x]^7)/(7*a^2*d) - (2*Csc[c + d*x])/(a^2*d) + (2*Csc[
c + d*x]^3)/(a^2*d) - (6*Csc[c + d*x]^5)/(5*a^2*d) + (2*Csc[c + d*x]^7)/(7*
a^2*d)
```

Rubi [A] time = 0.190764, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3888, 3886, 3473, 8, 2606, 194, 2607, 30}

$$-\frac{2\cot^7(c+dx)}{7a^2d}+\frac{\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)}{a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}-\frac{6\csc^5(c+dx)}{5a^2d}+\frac{2\csc^3(c+dx)}{a^2d}-\frac{2\csc(c+dx)}{a^2d}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```



```
[Out] x/a^2 + Cot[c + d*x]/(a^2*d) - Cot[c + d*x]^3/(3*a^2*d) + Cot[c + d*x]^5/(5
*a^2*d) - (2*Cot[c + d*x]^7)/(7*a^2*d) - (2*Csc[c + d*x])/(a^2*d) + (2*Csc[
c + d*x]^3)/(a^2*d) - (6*Csc[c + d*x]^5)/(5*a^2*d) + (2*Csc[c + d*x]^7)/(7*
a^2*d)
```


Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \cot^8(c+dx)(-a+a\sec(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cot^8(c+dx)-2a^2\cot^7(c+dx)\csc(c+dx)+a^2\cot^6(c+dx)\csc^2(c+dx)\right)dx}{a^4}$$
$$= \frac{\int \cot^8(c+dx)\,dx}{a^2}+\frac{\int \cot^6(c+dx)\csc^2(c+dx)\,dx}{a^2}-\frac{2\int \cot^7(c+dx)\csc(c+dx)\,dx}{a^2}$$
$$= -\frac{\cot^7(c+dx)}{7a^2d}-\frac{\int \cot^6(c+dx)\,dx}{a^2}+\frac{\text{Subst}\left(\int x^6\,dx,x,-\cot(c+dx)\right)}{a^2d}+\frac{2\,\text{Subst}\left(\int \left(-1\right.\right.}{}$$
$$= \frac{\cot^5(c+dx)}{5a^2d}-\frac{2\cot^7(c+dx)}{7a^2d}+\frac{\int \cot^4(c+dx)\,dx}{a^2}+\frac{2\,\text{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)dx\right.}{a^2d}$$
$$= -\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot^5(c+dx)}{5a^2d}-\frac{2\cot^7(c+dx)}{7a^2d}-\frac{2\csc(c+dx)}{a^2d}+\frac{2\csc^3(c+dx)}{a^2d}-\frac{6\csc^5(}{5a}$$
$$= \frac{\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot^5(c+dx)}{5a^2d}-\frac{2\cot^7(c+dx)}{7a^2d}-\frac{2\csc(c+dx)}{a^2d}+\frac{2\csc^3(c+}{a^2d}$$
$$= \frac{x}{a^2}+\frac{\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot^5(c+dx)}{5a^2d}-\frac{2\cot^7(c+dx)}{7a^2d}-\frac{2\csc(c+dx)}{a^2d}+\frac{2\csc}{}$$

Mathematica [B] time = 0.986489, size = 314, normalized size = 2.26

$$\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\csc^3(c+dx)\sec^2(c+dx)(16002\sin(c+dx)+9144\sin(2(c+dx))-3429\sin(3(c+dx))-4572\sin(4(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Csc[c/2]*Csc[c + d*x]^3*Sec[c/2]*Sec[c + d*x]^2*(5880*d*x*Cos[d*x] - 5880*
d*x*Cos[2*c + d*x] + 3360*d*x*Cos[c + 2*d*x] - 3360*d*x*Cos[3*c + 2*d*x] -
1260*d*x*Cos[2*c + 3*d*x] + 1260*d*x*Cos[4*c + 3*d*x] - 1680*d*x*Cos[3*c +
4*d*x] + 1680*d*x*Cos[5*c + 4*d*x] - 420*d*x*Cos[4*c + 5*d*x] + 420*d*x*Cos
[6*c + 5*d*x] - 4032*Sin[c] - 9632*Sin[d*x] + 16002*Sin[c + d*x] + 9144*Sin
[2*(c + d*x)] - 3429*Sin[3*(c + d*x)] - 4572*Sin[4*(c + d*x)] - 1143*Sin[5*
(c + d*x)] - 11760*Sin[2*c + d*x] - 8864*Sin[c + 2*d*x] - 3360*Sin[3*c + 2*
d*x] + 2064*Sin[2*c + 3*d*x] + 2520*Sin[4*c + 3*d*x] + 4432*Sin[3*c + 4*d*x
] + 1680*Sin[5*c + 4*d*x] + 1528*Sin[4*c + 5*d*x]))/(26880*a^2*d*(1 + Sec[c
 + d*x])^2)
```

Maple [A] time = 0.074, size = 132, normalized size = 1.

$$\frac{1}{224\,d a^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{7}{160\,d a^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{11}{48\,d a^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{21}{16\,d a^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{\arctan(\mathrm{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/224/d/a^2*tan(1/2*d*x+1/2*c)^7-7/160/d/a^2*tan(1/2*d*x+1/2*c)^5+11/48/d/a
^2*tan(1/2*d*x+1/2*c)^3-21/16/d/a^2*tan(1/2*d*x+1/2*c)+2/d/a^2*arctan(tan(1
/2*d*x+1/2*c))-1/96/d/a^2/tan(1/2*d*x+1/2*c)^3+7/32/d/a^2/tan(1/2*d*x+1/2*c
)
```

Maxima [A] time = 1.97822, size = 212, normalized size = 1.53

$$-\frac{\frac{\frac{4410\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{770\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^2}-\frac{6720\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{35\left(\frac{21\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^3}{a^2\sin(dx+c)^3}}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3360*((4410*sin(d*x + c)/(cos(d*x + c) + 1) - 770*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/a^2 - 6720*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
^2 - 35*(21*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^3/(
a^2*sin(d*x + c)^3))/d
```

Fricas [A] time = 1.262, size = 402, normalized size = 2.89

$$\frac{191\,\cos(dx+c)^5+172\,\cos(dx+c)^4-253\,\cos(dx+c)^3-258\,\cos(dx+c)^2+105\left(dx\cos(dx+c)^4+2\,dx\cos(dx+c)^3\right.}{105\left(a^2d\cos(dx+c)^4+2\,a^2d\cos(dx+c)^3-2\,a^2d\cos(dx+c)-a^2d\right)\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(191*cos(d*x + c)^5 + 172*cos(d*x + c)^4 - 253*cos(d*x + c)^3 - 258*c
os(d*x + c)^2 + 105*(d*x*cos(d*x + c)^4 + 2*d*x*cos(d*x + c)^3 - 2*d*x*cos(
d*x + c) - d*x)*sin(d*x + c) + 87*cos(d*x + c) + 96)/((a^2*d*cos(d*x + c)^4
 + 2*a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.30733, size = 154, normalized size = 1.11

$$\frac{\frac{3360\,(dx+c)}{a^2} + \frac{35\left(21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3} + \frac{15\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-147\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+770\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4410\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{14}}}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3360*(3360*(d*x + c)/a^2 + 35*(21*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^2*tan(1/
2*d*x + 1/2*c)^3) + (15*a^12*tan(1/2*d*x + 1/2*c)^7 - 147*a^12*tan(1/2*d*x
+ 1/2*c)^5 + 770*a^12*tan(1/2*d*x + 1/2*c)^3 - 4410*a^12*tan(1/2*d*x + 1/2*
c))/a^14)/d
```

3.85 $\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=179

$$-\frac{2\cot^9(c+dx)}{9a^2d}+\frac{\cot^7(c+dx)}{7a^2d}-\frac{\cot^5(c+dx)}{5a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot(c+dx)}{a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}-\frac{8\csc^7(c+dx)}{7a^2d}+\frac{12\csc}{5}$$

```
[Out] -(x/a^2) - Cot[c + d*x]/(a^2*d) + Cot[c + d*x]^3/(3*a^2*d) - Cot[c + d*x]^5
/(5*a^2*d) + Cot[c + d*x]^7/(7*a^2*d) - (2*Cot[c + d*x]^9)/(9*a^2*d) + (2*C
sc[c + d*x])/(a^2*d) - (8*Csc[c + d*x]^3)/(3*a^2*d) + (12*Csc[c + d*x]^5)/(
5*a^2*d) - (8*Csc[c + d*x]^7)/(7*a^2*d) + (2*Csc[c + d*x]^9)/(9*a^2*d)
```

Rubi [A] time = 0.207881, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3888, 3886, 3473, 8, 2606, 194, 2607, 30}

$$-\frac{2\cot^9(c+dx)}{9a^2d}+\frac{\cot^7(c+dx)}{7a^2d}-\frac{\cot^5(c+dx)}{5a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot(c+dx)}{a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}-\frac{8\csc^7(c+dx)}{7a^2d}+\frac{12\csc}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] -(x/a^2) - Cot[c + d*x]/(a^2*d) + Cot[c + d*x]^3/(3*a^2*d) - Cot[c + d*x]^5
/(5*a^2*d) + Cot[c + d*x]^7/(7*a^2*d) - (2*Cot[c + d*x]^9)/(9*a^2*d) + (2*C
sc[c + d*x])/(a^2*d) - (8*Csc[c + d*x]^3)/(3*a^2*d) + (12*Csc[c + d*x]^5)/(
5*a^2*d) - (8*Csc[c + d*x]^7)/(7*a^2*d) + (2*Csc[c + d*x]^9)/(9*a^2*d)
```


Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \cot^{10}(c+dx)(-a+a\sec(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cot^{10}(c+dx)-2a^2\cot^9(c+dx)\csc(c+dx)+a^2\cot^8(c+dx)\csc^2(c+dx)\right)dx}{a^4}$$
$$= \frac{\int \cot^{10}(c+dx)\,dx}{a^2}+\frac{\int \cot^8(c+dx)\csc^2(c+dx)\,dx}{a^2}-\frac{2\int \cot^9(c+dx)\csc(c+dx)\,dx}{a^2}$$
$$= -\frac{\cot^9(c+dx)}{9a^2d}-\frac{\int \cot^8(c+dx)\,dx}{a^2}+\frac{\text{Subst}\left(\int x^8\,dx,x,-\cot(c+dx)\right)}{a^2d}+\frac{2\,\text{Subst}\left(\int \left(-1+\right.\right.}{}$$
$$= \frac{\cot^7(c+dx)}{7a^2d}-\frac{2\cot^9(c+dx)}{9a^2d}+\frac{\int \cot^6(c+dx)\,dx}{a^2}+\frac{2\,\text{Subst}\left(\int \left(1-4x^2+6x^4-4x^6+x^8\right)\right.}{a^2d}$$
$$= -\frac{\cot^5(c+dx)}{5a^2d}+\frac{\cot^7(c+dx)}{7a^2d}-\frac{2\cot^9(c+dx)}{9a^2d}+\frac{2\csc(c+dx)}{a^2d}-\frac{8\csc^3(c+dx)}{3a^2d}+\frac{12\csc^5(}{5a^2}$$
$$= \frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5a^2d}+\frac{\cot^7(c+dx)}{7a^2d}-\frac{2\cot^9(c+dx)}{9a^2d}+\frac{2\csc(c+dx)}{a^2d}-\frac{8\csc^3(c+}{3a^2d}$$
$$= -\frac{\cot(c+dx)}{a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5a^2d}+\frac{\cot^7(c+dx)}{7a^2d}-\frac{2\cot^9(c+dx)}{9a^2d}+\frac{2\csc(c+d}{a^2d}$$
$$= -\frac{x}{a^2}-\frac{\cot(c+dx)}{a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5a^2d}+\frac{\cot^7(c+dx)}{7a^2d}-\frac{2\cot^9(c+dx)}{9a^2d}+\frac{2\csc(}{a}$$

Mathematica [B] time = 6.57088, size = 802, normalized size = 4.48

$$\frac{\sec\left(\frac{c}{2}\right)\sec^2(c+dx)\sin\left(\frac{dx}{2}\right)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{288d(\sec(c+dx)a+a)^2}+\frac{\sec^2(c+dx)\tan\left(\frac{c}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{288d(\sec(c+dx)a+a)^2}-\frac{109\sec\left(\frac{c}{2}\right)\sec^2(c+dx)\sin\left(\frac{dx}{2}\right)\sec^3}{2016d(\sec(c+dx)a+a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*x*Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^2 + (17*Cos
[c/2 + (d*x)/2]^2*Cot[c/2]*Cot[c/2 + (d*x)/2]^2*Sec[c + d*x]^2)/(160*d*(a +
 a*Sec[c + d*x])^2) - (Cot[c/2]*Cot[c/2 + (d*x)/2]^4*Sec[c + d*x]^2)/(160*d
*(a + a*Sec[c + d*x])^2) + (201*Cos[c/2 + (d*x)/2]^3*Cot[c/2 + (d*x)/2]*Csc
[c/2]*Sec[c + d*x]^2*Sin[(d*x)/2])/(160*d*(a + a*Sec[c + d*x])^2) - (17*Cos
[c/2 + (d*x)/2]*Cot[c/2 + (d*x)/2]^3*Csc[c/2]*Sec[c + d*x]^2*Sin[(d*x)/2])/
(160*d*(a + a*Sec[c + d*x])^2) + (Cot[c/2 + (d*x)/2]^4*Csc[c/2]*Csc[c/2 + (
d*x)/2]*Sec[c + d*x]^2*Sin[(d*x)/2])/(160*d*(a + a*Sec[c + d*x])^2) - (7891
*Cos[c/2 + (d*x)/2]*Sec[c/2]*Sec[c + d*x]^2*Sin[(d*x)/2])/(5040*d*(a + a*Se
c[c + d*x])^2) + (63881*Cos[c/2 + (d*x)/2]^3*Sec[c/2]*Sec[c + d*x]^2*Sin[(d
```

```
*x)/2])/(10080*d*(a + a*Sec[c + d*x])^2) + (313*Sec[c/2]*Sec[c/2 + (d*x)/2]
*Sec[c + d*x]^2*Sin[(d*x)/2])/(840*d*(a + a*Sec[c + d*x])^2) - (109*Sec[c/2
]*Sec[c/2 + (d*x)/2]^3*Sec[c + d*x]^2*Sin[(d*x)/2])/(2016*d*(a + a*Sec[c +
d*x])^2) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^5*Sec[c + d*x]^2*Sin[(d*x)/2])/(288
*d*(a + a*Sec[c + d*x])^2) + (313*Sec[c + d*x]^2*Tan[c/2])/(840*d*(a + a*Se
c[c + d*x])^2) - (7891*Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]^2*Tan[c/2])/(5040*
d*(a + a*Sec[c + d*x])^2) - (109*Sec[c/2 + (d*x)/2]^2*Sec[c + d*x]^2*Tan[c/
2])/(2016*d*(a + a*Sec[c + d*x])^2) + (Sec[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*
Tan[c/2])/(288*d*(a + a*Sec[c + d*x])^2)
```

Maple [A] time = 0.074, size = 170, normalized size = 1.

$$\frac{1}{1152\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9-\frac{9}{896\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{37}{640\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{31}{128\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{16}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/1152/d/a^2*tan(1/2*d*x+1/2*c)^9-9/896/d/a^2*tan(1/2*d*x+1/2*c)^7+37/640/d
/a^2*tan(1/2*d*x+1/2*c)^5-31/128/d/a^2*tan(1/2*d*x+1/2*c)^3+163/128/d/a^2*t
an(1/2*d*x+1/2*c)-2/d/a^2*arctan(tan(1/2*d*x+1/2*c))-1/640/d/a^2/tan(1/2*d*
x+1/2*c)^5+3/128/d/a^2/tan(1/2*d*x+1/2*c)^3-37/128/d/a^2/tan(1/2*d*x+1/2*c)
```

Maxima [A] time = 1.7423, size = 266, normalized size = 1.49

$$\frac{\frac{\frac{51345\sin(dx+c)}{\cos(dx+c)+1}-\frac{9765\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2331\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{405\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^2}-\frac{80640\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{63\left(\frac{15\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{185\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-1\right)(\cos}{a^2\sin(dx+c)^5}}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/40320*((51345*sin(d*x + c)/(cos(d*x + c) + 1) - 9765*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 2331*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 405*sin(d*x +
c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^2 - 8
0640*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 63*(15*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 - 185*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 1)*(cos(d*x
```

```
+ c) + 1)^5/(a^2*sin(d*x + c)^5))/d
```

Fricas [A] time = 1.26167, size = 651, normalized size = 3.64

$$-\frac{598 \cos(dx+c)^7 + 566 \cos(dx+c)^6 - 1212 \cos(dx+c)^5 - 1310 \cos(dx+c)^4 + 860 \cos(dx+c)^3 + 1014 \cos(dx+c}{315\left(a^2 d \cos(dx+c)^6 + 2\,a^2 d \cos(dx+c)^5 - a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/315*(598*cos(d*x + c)^7 + 566*cos(d*x + c)^6 - 1212*cos(d*x + c)^5 - 131
0*cos(d*x + c)^4 + 860*cos(d*x + c)^3 + 1014*cos(d*x + c)^2 + 315*(d*x*cos(
d*x + c)^6 + 2*d*x*cos(d*x + c)^5 - d*x*cos(d*x + c)^4 - 4*d*x*cos(d*x + c)
^3 - d*x*cos(d*x + c)^2 + 2*d*x*cos(d*x + c) + d*x)*sin(d*x + c) - 197*cos(
d*x + c) - 256)/((a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a^2*d*cos
(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d
*x + c) + a^2*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32933, size = 194, normalized size = 1.08

$$-\frac{\frac{40320\,(dx+c)}{a^2} + \frac{63\left(185 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 15 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 1\right)}{a^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5} - \frac{35\,a^{16} \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 405\,a^{16} \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 2331\,a^{16} \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 9765\,a^{16} \mathrm{ta}}{a^{18}}}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/40320*(40320*(d*x + c)/a^2 + 63*(185*tan(1/2*d*x + 1/2*c)^4 - 15*tan(1/2
*d*x + 1/2*c)^2 + 1)/(a^2*tan(1/2*d*x + 1/2*c)^5) - (35*a^16*tan(1/2*d*x +
1/2*c)^9 - 405*a^16*tan(1/2*d*x + 1/2*c)^7 + 2331*a^16*tan(1/2*d*x + 1/2*c)
^5 - 9765*a^16*tan(1/2*d*x + 1/2*c)^3 + 51345*a^16*tan(1/2*d*x + 1/2*c))/a^
18)/d
```

3.86 $\int \frac{\tan^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=137

$$\frac{\sec^7(c+dx)}{7a^3d}-\frac{\sec^6(c+dx)}{2a^3d}+\frac{\sec^5(c+dx)}{5a^3d}+\frac{5\sec^4(c+dx)}{4a^3d}-\frac{5\sec^3(c+dx)}{3a^3d}-\frac{\sec^2(c+dx)}{2a^3d}+\frac{3\sec(c+dx)}{a^3d}+\frac{\log(\cos}{a}$$

```
[Out] Log[Cos[c + d*x]]/(a^3*d) + (3*Sec[c + d*x])/(a^3*d) - Sec[c + d*x]^2/(2*a^
3*d) - (5*Sec[c + d*x]^3)/(3*a^3*d) + (5*Sec[c + d*x]^4)/(4*a^3*d) + Sec[c
+ d*x]^5/(5*a^3*d) - Sec[c + d*x]^6/(2*a^3*d) + Sec[c + d*x]^7/(7*a^3*d)
```

Rubi [A] time = 0.0768255, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{\sec^7(c+dx)}{7a^3d}-\frac{\sec^6(c+dx)}{2a^3d}+\frac{\sec^5(c+dx)}{5a^3d}+\frac{5\sec^4(c+dx)}{4a^3d}-\frac{5\sec^3(c+dx)}{3a^3d}-\frac{\sec^2(c+dx)}{2a^3d}+\frac{3\sec(c+dx)}{a^3d}+\frac{\log(\cos}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^11/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Log[Cos[c + d*x]]/(a^3*d) + (3*Sec[c + d*x])/(a^3*d) - Sec[c + d*x]^2/(2*a^
3*d) - (5*Sec[c + d*x]^3)/(3*a^3*d) + (5*Sec[c + d*x]^4)/(4*a^3*d) + Sec[c
+ d*x]^5/(5*a^3*d) - Sec[c + d*x]^6/(2*a^3*d) + Sec[c + d*x]^7/(7*a^3*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\tan^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^5(a+ax)^2}{x^8}\,dx, x, \cos(c+dx)\right)}{a^{10}d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^7}{x^8}-\frac{3a^7}{x^7}+\frac{a^7}{x^6}+\frac{5a^7}{x^5}-\frac{5a^7}{x^4}-\frac{a^7}{x^3}+\frac{3a^7}{x^2}-\frac{a^7}{x}\right) dx, x, \cos(c+dx)\right)}{a^{10}d}$$
$$= \frac{\log(\cos(c+dx))}{a^3d}+\frac{3\sec(c+dx)}{a^3d}-\frac{\sec^2(c+dx)}{2a^3d}-\frac{5\sec^3(c+dx)}{3a^3d}+\frac{5\sec^4(c+dx)}{4a^3d}+\frac{\sec^5}{5}$$

Mathematica [A] time = 0.316213, size = 140, normalized size = 1.02

$$\frac{\sec^7(c+dx)(4522\cos(2(c+dx))+1050\cos(3(c+dx))+2380\cos(4(c+dx))-210\cos(5(c+dx))+630\cos(6(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^11/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((3732 + 4522*Cos[2*(c + d*x)] + 1050*Cos[3*(c + d*x)] + 2380*Cos[4*(c + d*
x)] - 210*Cos[5*(c + d*x)] + 630*Cos[6*(c + d*x)] + 2205*Cos[3*(c + d*x)]*L
og[Cos[c + d*x]] + 735*Cos[5*(c + d*x)]*Log[Cos[c + d*x]] + 105*Cos[7*(c +
d*x)]*Log[Cos[c + d*x]] + 105*Cos[c + d*x]*(8 + 35*Log[Cos[c + d*x]]))*Sec[
c + d*x]^7)/(6720*a^3*d)
```

Maple [A] time = 0.097, size = 127, normalized size = 0.9

$$\frac{(\sec(dx+c))^7}{7\,da^3}-\frac{(\sec(dx+c))^6}{2\,da^3}+\frac{(\sec(dx+c))^5}{5\,da^3}+\frac{5\,(\sec(dx+c))^4}{4\,da^3}-\frac{5\,(\sec(dx+c))^3}{3\,da^3}-\frac{(\sec(dx+c))^2}{2\,da^3}+3\,\frac{\sec(dx}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^11/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/7*sec(d*x+c)^7/a^3/d-1/2*sec(d*x+c)^6/a^3/d+1/5*sec(d*x+c)^5/a^3/d+5/4*se
c(d*x+c)^4/a^3/d-5/3*sec(d*x+c)^3/a^3/d-1/2*sec(d*x+c)^2/a^3/d+3*sec(d*x+c)
/a^3/d-1/d/a^3*ln(sec(d*x+c))
```

Maxima [A] time = 1.08609, size = 122, normalized size = 0.89

$$\frac{\frac{420\,\log(\cos(dx+c))}{a^3}+\frac{1260\,\cos(dx+c)^6-210\,\cos(dx+c)^5-700\,\cos(dx+c)^4+525\,\cos(dx+c)^3+84\,\cos(dx+c)^2-210\,\cos(dx+c)+60}{a^3\cos(dx+c)^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/420*(420*log(cos(d*x + c))/a^3 + (1260*cos(d*x + c)^6 - 210*cos(d*x + c)^
5 - 700*cos(d*x + c)^4 + 525*cos(d*x + c)^3 + 84*cos(d*x + c)^2 - 210*cos(d
*x + c) + 60)/(a^3*cos(d*x + c)^7))/d
```

Fricas [A] time = 1.24943, size = 269, normalized size = 1.96

$$\frac{420\,\cos\left(dx+c\right)^7\log\left(-\cos\left(dx+c\right)\right)+1260\,\cos\left(dx+c\right)^6-210\,\cos\left(dx+c\right)^5-700\,\cos\left(dx+c\right)^4+525\,\cos\left(dx+c\right)^3+}{420\,a^3d\cos\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/420*(420*cos(d*x + c)^7*log(-cos(d*x + c)) + 1260*cos(d*x + c)^6 - 210*co
s(d*x + c)^5 - 700*cos(d*x + c)^4 + 525*cos(d*x + c)^3 + 84*cos(d*x + c)^2
- 210*cos(d*x + c) + 60)/(a^3*d*cos(d*x + c)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**11/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 36.512, size = 332, normalized size = 2.42

$$-\frac{\frac{420\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}-\frac{420\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^3}-\frac{\frac{1393(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{819(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{6755(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{20195(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{28749(\cos(dx+c)}{(\cos(dx+c)+1}}{a^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/420*(420*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 - 420*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^3 - (1393*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) + 819*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
 - 6755*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 20195*(cos(d*x + c) - 1
)^4/(cos(d*x + c) + 1)^4 - 28749*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5
- 8463*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 1089*(cos(d*x + c) - 1)^
7/(cos(d*x + c) + 1)^7 + 319)/(a^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 1)^7))/d
```

3.87 $\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^3} dx$

Optimal. Leaf size=99

$$\frac{\sec^5(c+dx)}{5a^3d} - \frac{3\sec^4(c+dx)}{4a^3d} + \frac{2\sec^3(c+dx)}{3a^3d} + \frac{\sec^2(c+dx)}{a^3d} - \frac{3\sec(c+dx)}{a^3d} - \frac{\log(\cos(c+dx))}{a^3d}$$

```
[Out] -(Log[Cos[c + d*x]]/(a^3*d)) - (3*Sec[c + d*x])/(a^3*d) + Sec[c + d*x]^2/(a
^3*d) + (2*Sec[c + d*x]^3)/(3*a^3*d) - (3*Sec[c + d*x]^4)/(4*a^3*d) + Sec[c
 + d*x]^5/(5*a^3*d)
```

Rubi [A] time = 0.0670373, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 75}

$$\frac{\sec^5(c+dx)}{5a^3d} - \frac{3\sec^4(c+dx)}{4a^3d} + \frac{2\sec^3(c+dx)}{3a^3d} + \frac{\sec^2(c+dx)}{a^3d} - \frac{3\sec(c+dx)}{a^3d} - \frac{\log(\cos(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^9/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^3*d)) - (3*Sec[c + d*x])/(a^3*d) + Sec[c + d*x]^2/(a
^3*d) + (2*Sec[c + d*x]^3)/(3*a^3*d) - (3*Sec[c + d*x]^4)/(4*a^3*d) + Sec[c
 + d*x]^5/(5*a^3*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{(a-ax)^4(a+ax)}{x^6}\,dx, x, \cos(c+dx)\right)}{a^8 d}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(\frac{a^5}{x^6} - \frac{3a^5}{x^5} + \frac{2a^5}{x^4} + \frac{2a^5}{x^3} - \frac{3a^5}{x^2} + \frac{a^5}{x}\right) dx, x, \cos(c+dx)\right)}{a^8 d}$$
$$= -\frac{\log(\cos(c+dx))}{a^3 d} - \frac{3\sec(c+dx)}{a^3 d} + \frac{\sec^2(c+dx)}{a^3 d} + \frac{2\sec^3(c+dx)}{3a^3 d} - \frac{3\sec^4(c+dx)}{4a^3 d} + \frac{\text{sec}}{}$$

Mathematica [A] time = 0.38488, size = 93, normalized size = 0.94

$$-\frac{\sec^5(c+dx)(280\cos(2(c+dx)) + 90\cos(4(c+dx)) + 150\cos(c+dx)\log(\cos(c+dx)) + 15\cos(5(c+dx))\log(\cos(c+}{240a^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^9/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -((142 + 280*Cos[2*(c + d*x)] + 90*Cos[4*(c + d*x)] + 150*Cos[c + d*x]*Log[
Cos[c + d*x]] + 15*Cos[5*(c + d*x)]*Log[Cos[c + d*x]] + 15*Cos[3*(c + d*x)]
*(-4 + 5*Log[Cos[c + d*x]]))*Sec[c + d*x]^5)/(240*a^3*d)
```

Maple [A] time = 0.084, size = 93, normalized size = 0.9

$$\frac{(\sec(dx+c))^5}{5\,da^3} - \frac{3\,(\sec(dx+c))^4}{4\,da^3} + \frac{2\,(\sec(dx+c))^3}{3\,da^3} + \frac{(\sec(dx+c))^2}{da^3} - 3\,\frac{\sec(dx+c)}{da^3} + \frac{\ln(\sec(dx+c))}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^9/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/5*sec(d*x+c)^5/a^3/d-3/4*sec(d*x+c)^4/a^3/d+2/3*sec(d*x+c)^3/a^3/d+sec(d*
x+c)^2/a^3/d-3*sec(d*x+c)/a^3/d+1/d/a^3*ln(sec(d*x+c))
```

Maxima [A] time = 1.15816, size = 95, normalized size = 0.96

$$-\frac{\frac{60\,\log\left(\cos\left(dx+c\right)\right)}{a^3}+\frac{180\,\cos\left(dx+c\right)^4-60\,\cos\left(dx+c\right)^3-40\,\cos\left(dx+c\right)^2+45\,\cos\left(dx+c\right)-12}{a^3\cos\left(dx+c\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(60*log(cos(d*x + c))/a^3 + (180*cos(d*x + c)^4 - 60*cos(d*x + c)^3 -
 40*cos(d*x + c)^2 + 45*cos(d*x + c) - 12)/(a^3*cos(d*x + c)^5))/d
```

Fricas [A] time = 1.26944, size = 207, normalized size = 2.09

$$-\frac{60\,\cos\left(dx+c\right)^5\log\left(-\cos\left(dx+c\right)\right)+180\,\cos\left(dx+c\right)^4-60\,\cos\left(dx+c\right)^3-40\,\cos\left(dx+c\right)^2+45\,\cos\left(dx+c\right)-12}{60\,a^3d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(60*cos(d*x + c)^5*log(-cos(d*x + c)) + 180*cos(d*x + c)^4 - 60*cos(d
*x + c)^3 - 40*cos(d*x + c)^2 + 45*cos(d*x + c) - 12)/(a^3*d*cos(d*x + c)^5
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**9/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 13.9043, size = 273, normalized size = 2.76

$$\frac{\frac{60 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3} - \frac{60 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^3} - \frac{\frac{475 (\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{590 (\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{50 (\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{805 (\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} - \frac{137 (\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5} + 119}{a^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 - 60*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^3 - (475*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + 590*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 50
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 805*(cos(d*x + c) - 1)^4/(cos(
d*x + c) + 1)^4 - 137*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 119)/(a^3
*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^5))/d
```

3.88 $\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=65

$$\frac{\sec^3(c+dx)}{3a^3d}-\frac{3\sec^2(c+dx)}{2a^3d}+\frac{3\sec(c+dx)}{a^3d}+\frac{\log(\cos(c+dx))}{a^3d}$$

```
[Out] Log[Cos[c + d*x]]/(a^3*d) + (3*Sec[c + d*x])/(a^3*d) - (3*Sec[c + d*x]^2)/(
2*a^3*d) + Sec[c + d*x]^3/(3*a^3*d)
```

Rubi [A] time = 0.0568776, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 43}

$$\frac{\sec^3(c+dx)}{3a^3d}-\frac{3\sec^2(c+dx)}{2a^3d}+\frac{3\sec(c+dx)}{a^3d}+\frac{\log(\cos(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Log[Cos[c + d*x]]/(a^3*d) + (3*Sec[c + d*x])/(a^3*d) - (3*Sec[c + d*x]^2)/(
2*a^3*d) + Sec[c + d*x]^3/(3*a^3*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^3}{x^4}\,dx, x, \cos(c+dx)\right)}{a^6 d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^3}{x^4}-\frac{3a^3}{x^3}+\frac{3a^3}{x^2}-\frac{a^3}{x}\right)dx, x, \cos(c+dx)\right)}{a^6 d}$$
$$= \frac{\log(\cos(c+dx))}{a^3 d}+\frac{3\sec(c+dx)}{a^3 d}-\frac{3\sec^2(c+dx)}{2a^3 d}+\frac{\sec^3(c+dx)}{3a^3 d}$$

Mathematica [A] time = 0.183869, size = 64, normalized size = 0.98

$$\frac{\sec^3(c+dx)(18\cos(2(c+dx))+9\cos(c+dx)(\log(\cos(c+dx))-2)+3\cos(3(c+dx))\log(\cos(c+dx))+22)}{12a^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((22 + 18*Cos[2*(c + d*x)] + 9*Cos[c + d*x]*(-2 + Log[Cos[c + d*x]]) + 3*Co
s[3*(c + d*x)]*Log[Cos[c + d*x]])*Sec[c + d*x]^3)/(12*a^3*d)
```

Maple [A] time = 0.075, size = 63, normalized size = 1.

$$\frac{(\sec(dx+c))^3}{3\,da^3}-\frac{3\,(\sec(dx+c))^2}{2\,da^3}+3\,\frac{\sec(dx+c)}{da^3}-\frac{\ln(\sec(dx+c))}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/3*sec(d*x+c)^3/a^3/d-3/2*sec(d*x+c)^2/a^3/d+3*sec(d*x+c)/a^3/d-1/d/a^3*ln
(sec(d*x+c))
```

Maxima [A] time = 1.07671, size = 68, normalized size = 1.05

$$\frac{\frac{6\log(\cos(dx+c))}{a^3}+\frac{18\cos(dx+c)^2-9\cos(dx+c)+2}{a^3\cos(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(6*log(cos(d*x + c))/a^3 + (18*cos(d*x + c)^2 - 9*cos(d*x + c) + 2)/(a^
3*cos(d*x + c)^3))/d
```

Fricas [A] time = 1.18473, size = 144, normalized size = 2.22

$$\frac{6\cos(dx+c)^3\log(-\cos(dx+c))+18\cos(dx+c)^2-9\cos(dx+c)+2}{6\,a^3d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(6*cos(d*x + c)^3*log(-cos(d*x + c)) + 18*cos(d*x + c)^2 - 9*cos(d*x +
c) + 2)/(a^3*d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\tan^7(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**7/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**7/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [B] time = 7.96975, size = 213, normalized size = 3.28

$$-\frac{\frac{6\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}-\frac{6\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^3}-\frac{\frac{75(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{51(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{11(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+29}{a^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(6*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 - 6*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^3 - (75*(cos(d*x + c) - 1)
/(cos(d*x + c) + 1) + 51*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 11*(co
s(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 29)/(a^3*((cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + 1)^3))/d
```

3.89 $\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=46

$$\frac{\sec(c+dx)}{a^3d}+\frac{3\log(\cos(c+dx))}{a^3d}-\frac{4\log(\cos(c+dx)+1)}{a^3d}$$

```
[Out] (3*Log[Cos[c + d*x]])/(a^3*d) - (4*Log[1 + Cos[c + d*x]])/(a^3*d) + Sec[c +
 d*x]/(a^3*d)
```

Rubi [A] time = 0.0541435, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{\sec(c+dx)}{a^3d}+\frac{3\log(\cos(c+dx))}{a^3d}-\frac{4\log(\cos(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3*Log[Cos[c + d*x]])/(a^3*d) - (4*Log[1 + Cos[c + d*x]])/(a^3*d) + Sec[c +
 d*x]/(a^3*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^2}{x^2(a+ax)}\,dx, x, \cos(c+dx)\right)}{a^4d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a}{x^2}-\frac{3a}{x}+\frac{4a}{1+x}\right)dx, x, \cos(c+dx)\right)}{a^4d}$$
$$= \frac{3\log(\cos(c+dx))}{a^3d} - \frac{4\log(1+\cos(c+dx))}{a^3d} + \frac{\sec(c+dx)}{a^3d}$$

Mathematica [A] time = 0.113691, size = 36, normalized size = 0.78

$$\frac{\sec(c+dx) - 8\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 3\log(\cos(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-8*Log[Cos[(c + d*x)/2]] + 3*Log[Cos[c + d*x]] + Sec[c + d*x])/(a^3*d)
```

Maple [A] time = 0.071, size = 46, normalized size = 1.

$$\frac{\sec(dx+c)}{a^3d} - 4\,\frac{\ln(1+\sec(dx+c))}{a^3d} + \frac{\ln(\sec(dx+c))}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sec(d*x+c))^3,x)
```

```
[Out] sec(d*x+c)/a^3/d-4/d/a^3*ln(1+sec(d*x+c))+1/d/a^3*ln(sec(d*x+c))
```

Maxima [A] time = 1.16601, size = 61, normalized size = 1.33

$$-\frac{\frac{4\log(\cos(dx+c)+1)}{a^3} - \frac{3\log(\cos(dx+c))}{a^3} - \frac{1}{a^3\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(4*log(cos(d*x + c) + 1)/a^3 - 3*log(cos(d*x + c))/a^3 - 1/(a^3*cos(d*x +
c)))/d
```

Fricas [A] time = 1.22972, size = 144, normalized size = 3.13

$$\frac{3\cos(dx+c)\log(-\cos(dx+c)) - 4\cos(dx+c)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) + 1}{a^3 d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (3*cos(d*x + c)*log(-cos(d*x + c)) - 4*cos(d*x + c)*log(1/2*cos(d*x + c) +
1/2) + 1)/(a^3*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [B] time = 3.67744, size = 151, normalized size = 3.28

$$\frac{\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3} + \frac{3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^3} - \frac{\frac{3(\cos(dx+c)-1)}{\cos(dx+c)+1}+1}{a^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + 3*log(abs(-(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^3 - (3*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) + 1)/(a^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

3.90 $\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=35

$$\frac{2}{a^3d(\cos(c+dx)+1)}+\frac{\log(\cos(c+dx)+1)}{a^3d}$$

```
[Out] 2/(a^3*d*(1 + Cos[c + d*x])) + Log[1 + Cos[c + d*x]]/(a^3*d)
```

Rubi [A] time = 0.0514823, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 43}

$$\frac{2}{a^3d(\cos(c+dx)+1)}+\frac{\log(\cos(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] 2/(a^3*d*(1 + Cos[c + d*x])) + Log[1 + Cos[c + d*x]]/(a^3*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{a-ax}{(a+ax)^2}\,dx, x, \cos(c+dx)\right)}{a^2d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{2}{a(1+x)^2}-\frac{1}{a(1+x)}\right)dx, x, \cos(c+dx)\right)}{a^2d}$$
$$= \frac{2}{a^3d(1+\cos(c+dx))}+\frac{\log(1+\cos(c+dx))}{a^3d}$$

Mathematica [A] time = 0.062097, size = 33, normalized size = 0.94

$$\frac{\tan^2\left(\frac{1}{2}(c+dx)\right)+2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Log[Cos[(c + d*x)/2]] + Tan[(c + d*x)/2]^2)/(a^3*d)
```

Maple [A] time = 0.092, size = 51, normalized size = 1.5

$$-2\,\frac{1}{da^3\,(1+\sec(dx+c))}+\frac{\ln(1+\sec(dx+c))}{da^3}-\frac{\ln(\sec(dx+c))}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -2/d/a^3/(1+sec(d*x+c))+1/d/a^3*ln(1+sec(d*x+c))-1/d/a^3*ln(sec(d*x+c))
```

Maxima [A] time = 1.06252, size = 49, normalized size = 1.4

$$\frac{\frac{2}{a^3\cos(dx+c)+a^3}+\frac{\log(\cos(dx+c)+1)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (2/(a^3*cos(d*x + c) + a^3) + log(cos(d*x + c) + 1)/a^3)/d
```

Fricas [A] time = 1.11642, size = 112, normalized size = 3.2

$$\frac{(\cos(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+2}{a^3d\cos(dx+c)+a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] ((cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 2)/(a^3*d*cos(d*x + c) +
a^3*d)
```

Sympy [A] time = 27.4519, size = 457, normalized size = 13.06

$$\begin{cases} -\frac{\log\left(\tan^2(c+dx)+1\right)\sec^2(c+dx)}{2a^3d\sec^2(c+dx)+4a^3d\sec(c+dx)+2a^3d} - \frac{2\log\left(\tan^2(c+dx)+1\right)\sec(c+dx)}{2a^3d\sec^2(c+dx)+4a^3d\sec(c+dx)+2a^3d} - \frac{\log\left(\tan^2(c+dx)+1\right)}{2a^3d\sec^2(c+dx)+4a^3d\sec(c+dx)+2a^3d} + \frac{2\log(\sec(c+dx)+1}{2a^3d\sec^2(c+dx)+4a^3d} \\ \frac{x\tan^3(c)}{(a\sec(c)+a)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Piecewise((-log(tan(c + d*x)**2 + 1)*sec(c + d*x)**2/(2*a**3*d*sec(c + d*x)
**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) - 2*log(tan(c + d*x)**2 + 1)*sec(c
+ d*x)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) - log(
tan(c + d*x)**2 + 1)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*
a**3*d) + 2*log(sec(c + d*x) + 1)*sec(c + d*x)**2/(2*a**3*d*sec(c + d*x)**2
 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) + 4*log(sec(c + d*x) + 1)*sec(c + d*x)
/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) + 2*log(sec(
c + d*x) + 1)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d)
 + tan(c + d*x)**2/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a*
*3*d) - 2*sec(c + d*x)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) +
2*a**3*d) - 2/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d)
, Ne(d, 0)), (x*tan(c)**3/(a*sec(c) + a)**3, True))
```

Giac [A] time = 1.84627, size = 76, normalized size = 2.17

$$-\frac{\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}+\frac{\cos(dx+c)-1}{a^3(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + (cos(d*x + c)
- 1)/(a^3*(cos(d*x + c) + 1)))/d
```

3.91 $\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=56

$$-\frac{2}{a^3d(\cos(c+dx)+1)}+\frac{1}{2a^3d(\cos(c+dx)+1)^2}-\frac{\log(\cos(c+dx)+1)}{a^3d}$$

```
[Out] 1/(2*a^3*d*(1 + Cos[c + d*x])^2) - 2/(a^3*d*(1 + Cos[c + d*x])) - Log[1 + C
os[c + d*x]]/(a^3*d)
```

Rubi [A] time = 0.0413221, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 43}

$$-\frac{2}{a^3d(\cos(c+dx)+1)}+\frac{1}{2a^3d(\cos(c+dx)+1)^2}-\frac{\log(\cos(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] 1/(2*a^3*d*(1 + Cos[c + d*x])^2) - 2/(a^3*d*(1 + Cos[c + d*x])) - Log[1 + C
os[c + d*x]]/(a^3*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^2}{(a+ax)^3}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{a^3(1+x)^3} - \frac{2}{a^3(1+x)^2} + \frac{1}{a^3(1+x)}\right)dx, x, \cos(c+dx)\right)}{d}$$
$$= \frac{1}{2a^3d(1+\cos(c+dx))^2} - \frac{2}{a^3d(1+\cos(c+dx))} - \frac{\log(1+\cos(c+dx))}{a^3d}$$

Mathematica [A] time = 0.129941, size = 79, normalized size = 1.41

$$-\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(8\cos^2\left(\frac{1}{2}(c+dx)\right)+16\cos^4\left(\frac{1}{2}(c+dx)\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-1\right)}{a^3d(\sec(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -((Cos[(c + d*x)/2]^2*(-1 + 8*Cos[(c + d*x)/2]^2 + 16*Cos[(c + d*x)/2]^4*Lo
g[Cos[(c + d*x)/2]])*Sec[c + d*x]^3)/(a^3*d*(1 + Sec[c + d*x])^3))
```

Maple [A] time = 0.027, size = 68, normalized size = 1.2

$$\frac{1}{2\,da^3\,(1+\sec(dx+c))^2} + \frac{1}{da^3\,(1+\sec(dx+c))} - \frac{\ln(1+\sec(dx+c))}{da^3} + \frac{\ln(\sec(dx+c))}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3/(1+sec(d*x+c))^2+1/d/a^3/(1+sec(d*x+c))-1/d/a^3*ln(1+sec(d*x+c))+
1/d/a^3*ln(sec(d*x+c))
```

Maxima [A] time = 1.11954, size = 81, normalized size = 1.45

$$-\frac{\frac{4\,\cos(dx+c)+3}{a^3\cos(dx+c)^2+2\,a^3\cos(dx+c)+a^3} + \frac{2\,\log(\cos(dx+c)+1)}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((4*cos(d*x + c) + 3)/(a^3*cos(d*x + c)^2 + 2*a^3*cos(d*x + c) + a^3)
+ 2*log(cos(d*x + c) + 1)/a^3)/d
```

Fricas [A] time = 1.10605, size = 204, normalized size = 3.64

$$-\frac{2\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+4\cos\left(dx+c\right)+3}{2\left(a^{3}d\cos\left(dx+c\right)^{2}+2\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) +
 4*cos(d*x + c) + 3)/(a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [A] time = 27.8445, size = 411, normalized size = 7.34

$$\begin{cases}\frac{\log\left(\tan^{2}\left(c+dx\right)+1\right)\sec^{2}\left(c+dx\right)}{2a^{3}d\sec^{2}\left(c+dx\right)+4a^{3}d\sec\left(c+dx\right)+2a^{3}d}+\frac{2\log\left(\tan^{2}\left(c+dx\right)+1\right)\sec\left(c+dx\right)}{2a^{3}d\sec^{2}\left(c+dx\right)+4a^{3}d\sec\left(c+dx\right)+2a^{3}d}+\frac{\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2a^{3}d\sec^{2}\left(c+dx\right)+4a^{3}d\sec\left(c+dx\right)+2a^{3}d}-\frac{2\log\left(\sec\left(c+dx\right)+1\right)s}{2a^{3}d\sec^{2}\left(c+dx\right)+4a^{3}d\,se} \\ \frac{x\tan\left(c\right)}{\left(a\sec\left(c\right)+a\right)^{3}}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Piecewise((log(tan(c + d*x)**2 + 1)*sec(c + d*x)**2/(2*a**3*d*sec(c + d*x)*
*2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) + 2*log(tan(c + d*x)**2 + 1)*sec(c +
 d*x)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) + log(t
an(c + d*x)**2 + 1)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a
**3*d) - 2*log(sec(c + d*x) + 1)*sec(c + d*x)**2/(2*a**3*d*sec(c + d*x)**2
+ 4*a**3*d*sec(c + d*x) + 2*a**3*d) - 4*log(sec(c + d*x) + 1)*sec(c + d*x)/
(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) - 2*log(sec(c
 + d*x) + 1)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d)
+ 2*sec(c + d*x)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3
```

```
*d) + 3/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d), Ne(d
, 0)), (x*tan(c)/(a*sec(c) + a)**3, True))
```

Giac [A] time = 1.3264, size = 117, normalized size = 2.09

$$\frac{\frac{8 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}+\frac{\frac{6 a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^6}}{8 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + (6*a^3*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2)/a^6)/d
```

3.92 $\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=101

$$\frac{17}{8a^3d(\cos(c+dx)+1)}-\frac{7}{8a^3d(\cos(c+dx)+1)^2}+\frac{1}{6a^3d(\cos(c+dx)+1)^3}+\frac{\log(1-\cos(c+dx))}{16a^3d}+\frac{15\log(\cos(c+dx)+}{16a^3d}$$

```
[Out] 1/(6*a^3*d*(1 + Cos[c + d*x])^3) - 7/(8*a^3*d*(1 + Cos[c + d*x])^2) + 17/(8
*a^3*d*(1 + Cos[c + d*x])) + Log[1 - Cos[c + d*x]]/(16*a^3*d) + (15*Log[1 +
 Cos[c + d*x]])/(16*a^3*d)
```

Rubi [A] time = 0.0686666, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 88}

$$\frac{17}{8a^3d(\cos(c+dx)+1)}-\frac{7}{8a^3d(\cos(c+dx)+1)^2}+\frac{1}{6a^3d(\cos(c+dx)+1)^3}+\frac{\log(1-\cos(c+dx))}{16a^3d}+\frac{15\log(\cos(c+dx)+}{16a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] 1/(6*a^3*d*(1 + Cos[c + d*x])^3) - 7/(8*a^3*d*(1 + Cos[c + d*x])^2) + 17/(8
*a^3*d*(1 + Cos[c + d*x])) + Log[1 - Cos[c + d*x]]/(16*a^3*d) + (15*Log[1 +
 Cos[c + d*x]])/(16*a^3*d)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{a^2\operatorname{Subst}\left(\int \frac{x^4}{(a-ax)(a+ax)^4}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^2\operatorname{Subst}\left(\int \left(-\frac{1}{16a^5(-1+x)} + \frac{1}{2a^5(1+x)^4} - \frac{7}{4a^5(1+x)^3} + \frac{17}{8a^5(1+x)^2} - \frac{15}{16a^5(1+x)}\right) dx, x, \cos(c+dx)\right)}{d}$$
$$= \frac{1}{6a^3d(1+\cos(c+dx))^3} - \frac{7}{8a^3d(1+\cos(c+dx))^2} + \frac{17}{8a^3d(1+\cos(c+dx))} + \frac{\log(1-\cos(c}{16a^3d}$$

Mathematica [A] time = 0.350372, size = 97, normalized size = 0.96

$$\frac{\sec^3(c+dx)\left(102\cos^4\left(\frac{1}{2}(c+dx)\right) - 21\cos^2\left(\frac{1}{2}(c+dx)\right) + 12\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 15\log\left(\cos\left(\frac{1}{2}(c\right.\right.\right.\right.}{12a^3d(\sec(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((2 - 21*Cos[(c + d*x)/2]^2 + 102*Cos[(c + d*x)/2]^4 + 12*Cos[(c + d*x)/2]^
6*(15*Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]]))*Sec[c + d*x]^3)/(12*a
^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 0.112, size = 90, normalized size = 0.9

$$\frac{1}{6\,da^3\,(\cos(dx+c)+1)^3} - \frac{7}{8\,da^3\,(\cos(dx+c)+1)^2} + \frac{17}{8\,da^3\,(\cos(dx+c)+1)} + \frac{15\,\ln(\cos(dx+c)+1)}{16\,da^3} + \frac{\ln(-1+\cos}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/6/d/a^3/(cos(d*x+c)+1)^3-7/8/d/a^3/(cos(d*x+c)+1)^2+17/8/d/a^3/(cos(d*x+c
)+1)+15/16*ln(cos(d*x+c)+1)/a^3/d+1/16/d/a^3*ln(-1+cos(d*x+c))
```

Maxima [A] time = 1.14988, size = 132, normalized size = 1.31

$$\frac{\frac{2\left(51\,\cos(dx+c)^2+81\,\cos(dx+c)+34\right)}{a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3}+\frac{45\,\log(\cos(dx+c)+1)}{a^3}+\frac{3\,\log(\cos(dx+c)-1)}{a^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/48*(2*(51*cos(d*x + c)^2 + 81*cos(d*x + c) + 34)/(a^3*cos(d*x + c)^3 + 3*
a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3) + 45*log(cos(d*x + c) + 1)/a
^3 + 3*log(cos(d*x + c) - 1)/a^3)/d
```

Fricas [A] time = 1.18833, size = 419, normalized size = 4.15

$$\frac{102\,\cos\left(dx+c\right)^2+45\left(\cos\left(dx+c\right)^3+3\,\cos\left(dx+c\right)^2+3\,\cos\left(dx+c\right)+1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+3\left(\cos\left(dx+c\right)^3+\right.}{48\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(102*cos(d*x + c)^2 + 45*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*
x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 3*(cos(d*x + c)^3 + 3*cos(d*x + c
)^2 + 3*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 162*cos(d*x + c) +
 68)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c)
+ a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot\left(c+dx\right)}{\sec^3\left(c+dx\right)+3\sec^2\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x)
 + 1), x)/a**3
```

Giac [A] time = 1.33082, size = 193, normalized size = 1.91

$$\frac{\frac{6 \log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3} - \frac{96 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3} - \frac{\frac{66 a^6(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{15 a^6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{2 a^6(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{a^9}}{96 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/96*(6*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 - 96*log(abs(
-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 - (66*a^6*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) + 15*a^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
2*a^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^9)/d
```

3.93 $\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=143

$$-\frac{1}{32a^3d(1-\cos(c+dx))}-\frac{9}{4a^3d(\cos(c+dx)+1)}+\frac{39}{32a^3d(\cos(c+dx)+1)^2}-\frac{5}{12a^3d(\cos(c+dx)+1)^3}+\frac{1}{16a^3d(\cos(c+$$

```
[Out] -1/(32*a^3*d*(1 - Cos[c + d*x])) + 1/(16*a^3*d*(1 + Cos[c + d*x])^4) - 5/(1
2*a^3*d*(1 + Cos[c + d*x])^3) + 39/(32*a^3*d*(1 + Cos[c + d*x])^2) - 9/(4*a
^3*d*(1 + Cos[c + d*x])) - (7*Log[1 - Cos[c + d*x]])/(64*a^3*d) - (57*Log[1
 + Cos[c + d*x]])/(64*a^3*d)
```

Rubi [A] time = 0.0966191, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$-\frac{1}{32a^3d(1-\cos(c+dx))}-\frac{9}{4a^3d(\cos(c+dx)+1)}+\frac{39}{32a^3d(\cos(c+dx)+1)^2}-\frac{5}{12a^3d(\cos(c+dx)+1)^3}+\frac{1}{16a^3d(\cos(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] -1/(32*a^3*d*(1 - Cos[c + d*x])) + 1/(16*a^3*d*(1 + Cos[c + d*x])^4) - 5/(1
2*a^3*d*(1 + Cos[c + d*x])^3) + 39/(32*a^3*d*(1 + Cos[c + d*x])^2) - 9/(4*a
^3*d*(1 + Cos[c + d*x])) - (7*Log[1 - Cos[c + d*x]])/(64*a^3*d) - (57*Log[1
 + Cos[c + d*x]])/(64*a^3*d)
```


Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{a^4\,\text{Subst}\left(\int \frac{x^6}{(a-ax)^2(a+ax)^5}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^4\,\text{Subst}\left(\int \left(\frac{1}{32a^7(-1+x)^2}+\frac{7}{64a^7(-1+x)}+\frac{1}{4a^7(1+x)^5}-\frac{5}{4a^7(1+x)^4}+\frac{39}{16a^7(1+x)^3}-\frac{9}{4a^7(1+x)^2}+\frac{57}{64a^7(1}\right.\right.}{d}$$
$$= -\frac{1}{32a^3d(1-\cos(c+dx))}+\frac{1}{16a^3d(1+\cos(c+dx))^4}-\frac{5}{12a^3d(1+\cos(c+dx))^3}+\frac{}{32a^3d(1}$$

Mathematica [A] time = 0.63356, size = 140, normalized size = 0.98

$$-\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(12\csc^2\left(\frac{1}{2}(c+dx)\right)-3\sec^8\left(\frac{1}{2}(c+dx)\right)+40\sec^6\left(\frac{1}{2}(c+dx)\right)-234\sec^4\left(\frac{1}{2}(c+dx)\right)+\right.}{96a^3d(\sec(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(Cos[(c + d*x)/2]^6*(12*Csc[(c + d*x)/2]^2 + 24*(57*Log[Cos[(c + d*x)/2]]
+ 7*Log[Sin[(c + d*x)/2]]) + 864*Sec[(c + d*x)/2]^2 - 234*Sec[(c + d*x)/2]^
4 + 40*Sec[(c + d*x)/2]^6 - 3*Sec[(c + d*x)/2]^8)*Sec[c + d*x]^3)/(96*a^3*d
*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 0.088, size = 126, normalized size = 0.9

$$\frac{1}{16\,da^3\left(\cos(dx+c)+1\right)^4}-\frac{5}{12\,da^3\left(\cos(dx+c)+1\right)^3}+\frac{39}{32\,da^3\left(\cos(dx+c)+1\right)^2}-\frac{9}{4\,da^3\left(\cos(dx+c)+1\right)}-\frac{57\ln(c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/16/d/a^3/(cos(d*x+c)+1)^4-5/12/d/a^3/(cos(d*x+c)+1)^3+39/32/d/a^3/(cos(d*
x+c)+1)^2-9/4/d/a^3/(cos(d*x+c)+1)-57/64*ln(cos(d*x+c)+1)/a^3/d+1/32/d/a^3/
```

```
(-1+cos(d*x+c))-7/64/d/a^3*ln(-1+cos(d*x+c))
```

Maxima [A] time = 1.18638, size = 197, normalized size = 1.38

$$-\frac{\frac{2\left(213\cos(dx+c)^4+303\cos(dx+c)^3-95\cos(dx+c)^2-333\cos(dx+c)-136\right)}{a^3\cos(dx+c)^5+3a^3\cos(dx+c)^4+2a^3\cos(dx+c)^3-2a^3\cos(dx+c)^2-3a^3\cos(dx+c)-a^3}+\frac{171\log(\cos(dx+c)+1)}{a^3}+\frac{21\log(\cos(dx+c)-1)}{a^3}}{192d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/192*(2*(213*cos(d*x + c)^4 + 303*cos(d*x + c)^3 - 95*cos(d*x + c)^2 - 33
3*cos(d*x + c) - 136)/(a^3*cos(d*x + c)^5 + 3*a^3*cos(d*x + c)^4 + 2*a^3*co
s(d*x + c)^3 - 2*a^3*cos(d*x + c)^2 - 3*a^3*cos(d*x + c) - a^3) + 171*log(c
os(d*x + c) + 1)/a^3 + 21*log(cos(d*x + c) - 1)/a^3)/d
```

Fricas [A] time = 1.20036, size = 652, normalized size = 4.56

$$-\frac{426\cos(dx+c)^4+606\cos(dx+c)^3-190\cos(dx+c)^2+171\left(\cos(dx+c)^5+3\cos(dx+c)^4+2\cos(dx+c)^3-2\cos\right.}{192\left(a^3d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/192*(426*cos(d*x + c)^4 + 606*cos(d*x + c)^3 - 190*cos(d*x + c)^2 + 171*
(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 -
3*cos(d*x + c) - 1)*log(1/2*cos(d*x + c) + 1/2) + 21*(cos(d*x + c)^5 + 3*co
s(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*lo
g(-1/2*cos(d*x + c) + 1/2) - 666*cos(d*x + c) - 272)/(a^3*d*cos(d*x + c)^5
+ 3*a^3*d*cos(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2
- 3*a^3*d*cos(d*x + c) - a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\cot^3(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A] time = 1.42353, size = 286, normalized size = 2.

$$\frac{\frac{12\left(\frac{7(\cos(dx+c)-1)}{\cos(dx+c)+1}+1\right)(\cos(dx+c)+1)}{a^3(\cos(dx+c)-1)} - \frac{84\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3} + \frac{768\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3} + \frac{\frac{504a^9(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{132a^9(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{28a^9(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{a^{12}}}{768d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/768*(12*(7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)*(cos(d*x + c) + 1)/
(a^3*(cos(d*x + c) - 1)) - 84*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) +
 1))/a^3 + 768*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + (
504*a^9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 132*a^9*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2 + 28*a^9*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 +
 3*a^9*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/a^12)/d
```

3.94 $\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=185

$$\frac{5}{64a^3d(1-\cos(c+dx))}+\frac{303}{128a^3d(\cos(c+dx)+1)}-\frac{1}{128a^3d(1-\cos(c+dx))^2}-\frac{99}{64a^3d(\cos(c+dx)+1)^2}+\frac{3}{48a^3d(\cos(c$$

```
[Out] -1/(128*a^3*d*(1 - Cos[c + d*x])^2) + 5/(64*a^3*d*(1 - Cos[c + d*x])) + 1/(
40*a^3*d*(1 + Cos[c + d*x])^5) - 13/(64*a^3*d*(1 + Cos[c + d*x])^4) + 35/(4
8*a^3*d*(1 + Cos[c + d*x])^3) - 99/(64*a^3*d*(1 + Cos[c + d*x])^2) + 303/(1
28*a^3*d*(1 + Cos[c + d*x])) + (37*Log[1 - Cos[c + d*x]])/(256*a^3*d) + (21
9*Log[1 + Cos[c + d*x]])/(256*a^3*d)
```

Rubi [A] time = 0.123572, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{5}{64a^3d(1-\cos(c+dx))}+\frac{303}{128a^3d(\cos(c+dx)+1)}-\frac{1}{128a^3d(1-\cos(c+dx))^2}-\frac{99}{64a^3d(\cos(c+dx)+1)^2}+\frac{3}{48a^3d(\cos(c$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```



```
[Out] -1/(128*a^3*d*(1 - Cos[c + d*x])^2) + 5/(64*a^3*d*(1 - Cos[c + d*x])) + 1/(
40*a^3*d*(1 + Cos[c + d*x])^5) - 13/(64*a^3*d*(1 + Cos[c + d*x])^4) + 35/(4
8*a^3*d*(1 + Cos[c + d*x])^3) - 99/(64*a^3*d*(1 + Cos[c + d*x])^2) + 303/(1
28*a^3*d*(1 + Cos[c + d*x])) + (37*Log[1 - Cos[c + d*x]])/(256*a^3*d) + (21
9*Log[1 + Cos[c + d*x]])/(256*a^3*d)
```


Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{a^6\,\text{Subst}\left(\int \frac{x^8}{(a-ax)^3(a+ax)^6}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a^6\,\text{Subst}\left(\int \left(-\frac{1}{64a^9(-1+x)^3} - \frac{5}{64a^9(-1+x)^2} - \frac{37}{256a^9(-1+x)} + \frac{1}{8a^9(1+x)^6} - \frac{13}{16a^9(1+x)^5} + \frac{35}{16a^9(1+x)^4} - \right.\right.}{d}$$

$$= -\frac{1}{128a^3d(1-\cos(c+dx))^2} + \frac{5}{64a^3d(1-\cos(c+dx))} + \frac{1}{40a^3d(1+\cos(c+dx))^5} - \frac{}{64a^3d(}$$

Mathematica [A] time = 1.18225, size = 169, normalized size = 0.91

$$\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(1400\cos^4\left(\frac{1}{2}(c+dx)\right) - 195\cos^2\left(\frac{1}{2}(c+dx)\right) + 60\cos^8\left(\frac{1}{2}(c+dx)\right)\left(10\cot^2\left(\frac{1}{2}(c+dx)\right)\right.\right.}{192}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((12 - 195*Cos[(c + d*x)/2]^2 + 1400*Cos[(c + d*x)/2]^4 + 60*Cos[(c + d*x)/
2]^8*(303 + 10*Cot[(c + d*x)/2]^2) - 30*Cos[(c + d*x)/2]^6*(198 + Cot[(c +
d*x)/2]^4) + 120*Cos[(c + d*x)/2]^10*(219*Log[Cos[(c + d*x)/2]] + 37*Log[Si
n[(c + d*x)/2]]))*Sec[(c + d*x)/2]^4*Sec[c + d*x]^3)/(1920*a^3*d*(1 + Sec[c
 + d*x])^3)
```

Maple [A] time = 0.082, size = 162, normalized size = 0.9

$$\frac{1}{40\,da^3\left(\cos(dx+c)+1\right)^5} - \frac{13}{64\,da^3\left(\cos(dx+c)+1\right)^4} + \frac{35}{48\,da^3\left(\cos(dx+c)+1\right)^3} - \frac{99}{64\,da^3\left(\cos(dx+c)+1\right)^2} + \frac{}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sec(d*x+c))^3,x)
```

[Out] 1/40/a^3/d/(cos(d*x+c)+1)^5-13/64/d/a^3/(cos(d*x+c)+1)^4+35/48/d/a^3/(cos(d*x+c)+1)^3-99/64/d/a^3/(cos(d*x+c)+1)^2+303/128/d/a^3/(cos(d*x+c)+1)+219/256*ln(cos(d*x+c)+1)/a^3/d-1/128/d/a^3/(-1+cos(d*x+c))^2-5/64/d/a^3/(-1+cos(d*x+c))+37/256/d/a^3*ln(-1+cos(d*x+c))

Maxima [A] time = 1.14742, size = 254, normalized size = 1.37

$$\frac{\frac{2\left(4395\cos(dx+c)^6+5505\cos(dx+c)^5-6940\cos(dx+c)^4-12780\cos(dx+c)^3-367\cos(dx+c)^2+6939\cos(dx+c)+2768\right)}{a^3\cos(dx+c)^7+3a^3\cos(dx+c)^6+a^3\cos(dx+c)^5-5a^3\cos(dx+c)^4-5a^3\cos(dx+c)^3+a^3\cos(dx+c)^2+3a^3\cos(dx+c)+a^3}+\frac{3285\log(\cos(dx+c)+1)}{a^3}+\frac{555\log(c}{}}{3840d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] 1/3840*(2*(4395*cos(d*x + c)^6 + 5505*cos(d*x + c)^5 - 6940*cos(d*x + c)^4 - 12780*cos(d*x + c)^3 - 367*cos(d*x + c)^2 + 6939*cos(d*x + c) + 2768)/(a^3*cos(d*x + c)^7 + 3*a^3*cos(d*x + c)^6 + a^3*cos(d*x + c)^5 - 5*a^3*cos(d*x + c)^4 - 5*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3) + 3285*log(cos(d*x + c) + 1)/a^3 + 555*log(cos(d*x + c) - 1)/a^3)/d

---

Fricas [A] time = 1.29477, size = 879, normalized size = 4.75

$$\frac{8790\cos(dx+c)^6+11010\cos(dx+c)^5-13880\cos(dx+c)^4-25560\cos(dx+c)^3-734\cos(dx+c)^2+3285\left(\cos(c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] 1/3840*(8790*cos(d*x + c)^6 + 11010*cos(d*x + c)^5 - 13880*cos(d*x + c)^4 - 25560*cos(d*x + c)^3 - 734*cos(d*x + c)^2 + 3285*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 555*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 13878*cos(d*x + c) + 5536)/(a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a^3*d*cos(d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A] time = 1.36609, size = 352, normalized size = 1.9

$$-\frac{\frac{30\left(\frac{18(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{111(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)^2}{a^3(\cos(dx+c)-1)^2}-\frac{2220\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3}+\frac{15360\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}+\frac{\frac{9780a^{12}(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2790a^{12}(\cdots}{(\cos\cdots}}{\cdots}}{15360d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/15360*(30*(18*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 111*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2 + 1)*(cos(d*x + c) + 1)^2/(a^3*(cos(d*x + c) -
1)^2) - 2220*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 + 15360*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + (9780*a^12*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + 2790*a^12*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2 + 740*a^12*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 135*a^12
*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 12*a^12*(cos(d*x + c) - 1)^5/(
cos(d*x + c) + 1)^5)/a^15)/d
```

3.95 $\int \frac{\tan^{12}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=237

$$-\frac{3\tan^7(c+dx)}{7a^3d}-\frac{\tan^5(c+dx)}{5a^3d}+\frac{\tan^3(c+dx)}{3a^3d}-\frac{\tan(c+dx)}{a^3d}-\frac{125\tanh^{-1}(\sin(c+dx))}{128a^3d}+\frac{\tan^5(c+dx)\sec^3(c+dx)}{8a^3d}-$$

```
[Out] x/a^3 - (125*ArcTanh[Sin[c + d*x]])/(128*a^3*d) - Tan[c + d*x]/(a^3*d) + (1
15*Sec[c + d*x]*Tan[c + d*x])/(128*a^3*d) + (5*Sec[c + d*x]^3*Tan[c + d*x])
/(64*a^3*d) + Tan[c + d*x]^3/(3*a^3*d) - (5*Sec[c + d*x]*Tan[c + d*x]^3)/(8
*a^3*d) - (5*Sec[c + d*x]^3*Tan[c + d*x]^3)/(48*a^3*d) - Tan[c + d*x]^5/(5*
a^3*d) + (Sec[c + d*x]*Tan[c + d*x]^5)/(2*a^3*d) + (Sec[c + d*x]^3*Tan[c +
d*x]^5)/(8*a^3*d) - (3*Tan[c + d*x]^7)/(7*a^3*d)
```

Rubi [A] time = 0.362998, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3888, 3886, 3473, 8, 2611, 3770, 2607, 30, 3768}

$$-\frac{3\tan^7(c+dx)}{7a^3d}-\frac{\tan^5(c+dx)}{5a^3d}+\frac{\tan^3(c+dx)}{3a^3d}-\frac{\tan(c+dx)}{a^3d}-\frac{125\tanh^{-1}(\sin(c+dx))}{128a^3d}+\frac{\tan^5(c+dx)\sec^3(c+dx)}{8a^3d}-$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^12/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] x/a^3 - (125*ArcTanh[Sin[c + d*x]])/(128*a^3*d) - Tan[c + d*x]/(a^3*d) + (1
15*Sec[c + d*x]*Tan[c + d*x])/(128*a^3*d) + (5*Sec[c + d*x]^3*Tan[c + d*x])
/(64*a^3*d) + Tan[c + d*x]^3/(3*a^3*d) - (5*Sec[c + d*x]*Tan[c + d*x]^3)/(8
*a^3*d) - (5*Sec[c + d*x]^3*Tan[c + d*x]^3)/(48*a^3*d) - Tan[c + d*x]^5/(5*
a^3*d) + (Sec[c + d*x]*Tan[c + d*x]^5)/(2*a^3*d) + (Sec[c + d*x]^3*Tan[c +
d*x]^5)/(8*a^3*d) - (3*Tan[c + d*x]^7)/(7*a^3*d)
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

IntegerQ[2*n]

Rubi steps

$$\int \frac{\tan^{12}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int (-a+a\sec(c+dx))^3\tan^6(c+dx)\,dx}{a^6}$$

$$= \frac{\int \left(-a^3\tan^6(c+dx)+3a^3\sec(c+dx)\tan^6(c+dx)-3a^3\sec^2(c+dx)\tan^6(c+dx)+a^3\sec\right.}{a^6}$$

$$= -\frac{\int \tan^6(c+dx)\,dx}{a^3}+\frac{\int \sec^3(c+dx)\tan^6(c+dx)\,dx}{a^3}+\frac{3\int \sec(c+dx)\tan^6(c+dx)\,dx}{a^3}-\frac{3}{}$$

$$= -\frac{\tan^5(c+dx)}{5a^3d}+\frac{\sec(c+dx)\tan^5(c+dx)}{2a^3d}+\frac{\sec^3(c+dx)\tan^5(c+dx)}{8a^3d}-\frac{5\int \sec^3(c+dx)\,\mathrm{t}}{8a^3}$$

$$= \frac{\tan^3(c+dx)}{3a^3d}-\frac{5\sec(c+dx)\tan^3(c+dx)}{8a^3d}-\frac{5\sec^3(c+dx)\tan^3(c+dx)}{48a^3d}-\frac{\tan^5(c+dx)}{5a^3d}+$$

$$= -\frac{\tan(c+dx)}{a^3d}+\frac{15\sec(c+dx)\tan(c+dx)}{16a^3d}+\frac{5\sec^3(c+dx)\tan(c+dx)}{64a^3d}+\frac{\tan^3(c+dx)}{3a^3d}-$$

$$= \frac{x}{a^3}-\frac{15\tanh^{-1}(\sin(c+dx))}{16a^3d}-\frac{\tan(c+dx)}{a^3d}+\frac{115\sec(c+dx)\tan(c+dx)}{128a^3d}+\frac{5\sec^3(c+dx)}{64a^3}$$

$$= \frac{x}{a^3}-\frac{125\tanh^{-1}(\sin(c+dx))}{128a^3d}-\frac{\tan(c+dx)}{a^3d}+\frac{115\sec(c+dx)\tan(c+dx)}{128a^3d}+\frac{5\sec^3(c+dx)}{64a}$$

Mathematica [A] time = 1.3231, size = 362, normalized size = 1.53

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(1680000\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^12/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(1680000*(Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c]*Sec[c + d*x
]^8*(470400*d*x*Cos[c] + 376320*d*x*Cos[c + 2*d*x] + 376320*d*x*Cos[3*c + 2
*d*x] + 188160*d*x*Cos[3*c + 4*d*x] + 188160*d*x*Cos[5*c + 4*d*x] + 53760*d
*x*Cos[5*c + 6*d*x] + 53760*d*x*Cos[7*c + 6*d*x] + 6720*d*x*Cos[7*c + 8*d*x
] + 6720*d*x*Cos[9*c + 8*d*x] + 519680*Sin[c] + 133175*Sin[d*x] + 133175*Si
n[2*c + d*x] - 544768*Sin[c + 2*d*x] + 286720*Sin[3*c + 2*d*x] + 63595*Sin[
2*c + 3*d*x] + 63595*Sin[4*c + 3*d*x] - 254464*Sin[3*c + 4*d*x] + 161280*Si
n[5*c + 4*d*x] + 65135*Sin[4*c + 5*d*x] + 65135*Sin[6*c + 5*d*x] - 118784*S
```

```
in[5*c + 6*d*x] + 27195*Sin[6*c + 7*d*x] + 27195*Sin[8*c + 7*d*x] - 14848*S
in[7*c + 8*d*x])))/(215040*a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 0.115, size = 396, normalized size = 1.7

$$2\,{\frac {\arctan \left(\tan \left(1/2\,dx+c/2 \right) \right) }{d{a}^{3}}}-{\frac {1}{8\,d{a}^{3}} \left(\tan \left({\frac {dx}{2}}+{\frac {c}{2}} \right) +1 \right) ^{-8}}+{\frac {13}{14\,d{a}^{3}} \left(\tan \left({\frac {dx}{2}}+{\frac {c}{2}} \right) +1 \right) ^{-7}}-{\frac {65}{24\,d{a}^{3}} \left(\tan \left({\frac {dx}{2}}+{\frac {c}{2}} \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^12/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 2/d/a^3*arctan(tan(1/2*d*x+1/2*c))-1/8/d/a^3/(tan(1/2*d*x+1/2*c)+1)^8+13/14
/d/a^3/(tan(1/2*d*x+1/2*c)+1)^7-65/24/d/a^3/(tan(1/2*d*x+1/2*c)+1)^6+143/40
/d/a^3/(tan(1/2*d*x+1/2*c)+1)^5-79/64/d/a^3/(tan(1/2*d*x+1/2*c)+1)^4-49/32/
d/a^3/(tan(1/2*d*x+1/2*c)+1)^3+29/128/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2+253/12
8/d/a^3/(tan(1/2*d*x+1/2*c)+1)-125/128/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)+1/8/d
/a^3/(tan(1/2*d*x+1/2*c)-1)^8+13/14/d/a^3/(tan(1/2*d*x+1/2*c)-1)^7+65/24/d/
a^3/(tan(1/2*d*x+1/2*c)-1)^6+143/40/d/a^3/(tan(1/2*d*x+1/2*c)-1)^5+79/64/d/
a^3/(tan(1/2*d*x+1/2*c)-1)^4-49/32/d/a^3/(tan(1/2*d*x+1/2*c)-1)^3-29/128/d/
a^3/(tan(1/2*d*x+1/2*c)-1)^2+253/128/d/a^3/(tan(1/2*d*x+1/2*c)-1)+125/128/d
/a^3*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [A] time = 1.59826, size = 579, normalized size = 2.44

$$-\frac{\frac{2\left(\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{11375\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{79723\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{269879\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{550089\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{749973\,\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{212625\,\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{26565\,\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}\right)}{a^3-\frac{8a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{56a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{70a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{56a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{28a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}-\frac{8a^3\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{a^3\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}}-\frac{26880}{}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^12/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/13440*(2*(315*sin(d*x + c)/(cos(d*x + c) + 1) - 11375*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3 + 79723*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 269879*sin(
d*x + c)^7/(cos(d*x + c) + 1)^7 + 550089*sin(d*x + c)^9/(cos(d*x + c) + 1)^
9 - 749973*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 212625*sin(d*x + c)^13/(
cos(d*x + c) + 1)^13 - 26565*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)/(a^3 -
```

```
8*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^3*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 - 56*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^3*sin(d*x +
 c)^8/(cos(d*x + c) + 1)^8 - 56*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 +
 28*a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a^3*sin(d*x + c)^14/(cos(
d*x + c) + 1)^14 + a^3*sin(d*x + c)^16/(cos(d*x + c) + 1)^16) - 26880*arcta
n(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 + 13125*log(sin(d*x + c)/(cos(d*x +
c) + 1) + 1)/a^3 - 13125*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

Fricas [A] time = 1.29227, size = 446, normalized size = 1.88

$$\frac{26880\,dx\cos\left(dx+c\right)^{8}-13125\,\cos\left(dx+c\right)^{8}\log\left(\sin\left(dx+c\right)+1\right)+13125\,\cos\left(dx+c\right)^{8}\log\left(-\sin\left(dx+c\right)+1\right)-2\left(148\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^12/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/26880*(26880*d*x*cos(d*x + c)^8 - 13125*cos(d*x + c)^8*log(sin(d*x + c) +
 1) + 13125*cos(d*x + c)^8*log(-sin(d*x + c) + 1) - 2*(14848*cos(d*x + c)^7
 - 27195*cos(d*x + c)^6 + 7424*cos(d*x + c)^5 + 17710*cos(d*x + c)^4 - 1459
2*cos(d*x + c)^3 - 1960*cos(d*x + c)^2 + 5760*cos(d*x + c) - 1680)*sin(d*x
+ c))/(a^3*d*cos(d*x + c)^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**12/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 69.7811, size = 236, normalized size = 1.

$$\frac{\frac{13440\,(dx+c)}{a^{3}}-\frac{13125\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{3}}+\frac{13125\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{3}}+\frac{2\left(26565\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}-212625\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+749973\,\tan\left(\frac{1}{2}\ldots\right.\right.}{}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^12/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/13440*(13440*(d*x + c)/a^3 - 13125*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3
 + 13125*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 2*(26565*tan(1/2*d*x + 1/
2*c)^15 - 212625*tan(1/2*d*x + 1/2*c)^13 + 749973*tan(1/2*d*x + 1/2*c)^11 -
 550089*tan(1/2*d*x + 1/2*c)^9 + 269879*tan(1/2*d*x + 1/2*c)^7 - 79723*tan(
1/2*d*x + 1/2*c)^5 + 11375*tan(1/2*d*x + 1/2*c)^3 - 315*tan(1/2*d*x + 1/2*c
))/((tan(1/2*d*x + 1/2*c)^2 - 1)^8*a^3))/d
```

3.96 $\int \frac{\tan^{10}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=169

$$-\frac{3\tan^5(c+dx)}{5a^3d}-\frac{\tan^3(c+dx)}{3a^3d}+\frac{\tan(c+dx)}{a^3d}+\frac{19\tanh^{-1}(\sin(c+dx))}{16a^3d}+\frac{\tan^3(c+dx)\sec^3(c+dx)}{6a^3d}-\frac{\tan(c+dx)\sec}{8a^3d}$$

```
[Out] -(x/a^3) + (19*ArcTanh[Sin[c + d*x]])/(16*a^3*d) + Tan[c + d*x]/(a^3*d) - (
17*Sec[c + d*x]*Tan[c + d*x])/(16*a^3*d) - (Sec[c + d*x]^3*Tan[c + d*x])/(8
*a^3*d) - Tan[c + d*x]^3/(3*a^3*d) + (3*Sec[c + d*x]*Tan[c + d*x]^3)/(4*a^3
*d) + (Sec[c + d*x]^3*Tan[c + d*x]^3)/(6*a^3*d) - (3*Tan[c + d*x]^5)/(5*a^3
*d)
```

Rubi [A] time = 0.268065, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3888, 3886, 3473, 8, 2611, 3770, 2607, 30, 3768}

$$-\frac{3\tan^5(c+dx)}{5a^3d}-\frac{\tan^3(c+dx)}{3a^3d}+\frac{\tan(c+dx)}{a^3d}+\frac{19\tanh^{-1}(\sin(c+dx))}{16a^3d}+\frac{\tan^3(c+dx)\sec^3(c+dx)}{6a^3d}-\frac{\tan(c+dx)\sec}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^10/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] -(x/a^3) + (19*ArcTanh[Sin[c + d*x]])/(16*a^3*d) + Tan[c + d*x]/(a^3*d) - (
17*Sec[c + d*x]*Tan[c + d*x])/(16*a^3*d) - (Sec[c + d*x]^3*Tan[c + d*x])/(8
*a^3*d) - Tan[c + d*x]^3/(3*a^3*d) + (3*Sec[c + d*x]*Tan[c + d*x]^3)/(4*a^3
*d) + (Sec[c + d*x]^3*Tan[c + d*x]^3)/(6*a^3*d) - (3*Tan[c + d*x]^5)/(5*a^3
*d)
```


Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
```

```
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\tan^{10}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int(-a+a\sec(c+dx))^3\tan^4(c+dx)\,dx}{a^6}$$
$$= \frac{\int\left(-a^3\tan^4(c+dx)+3a^3\sec(c+dx)\tan^4(c+dx)-3a^3\sec^2(c+dx)\tan^4(c+dx)+a^3\sec\right.}{a^6}$$
$$= -\frac{\int\tan^4(c+dx)\,dx}{a^3}+\frac{\int\sec^3(c+dx)\tan^4(c+dx)\,dx}{a^3}+\frac{3\int\sec(c+dx)\tan^4(c+dx)\,dx}{a^3}-\frac{3}{}$$
$$= -\frac{\tan^3(c+dx)}{3a^3d}+\frac{3\sec(c+dx)\tan^3(c+dx)}{4a^3d}+\frac{\sec^3(c+dx)\tan^3(c+dx)}{6a^3d}-\frac{\int\sec^3(c+dx)\,\mathrm{t}}{2a}$$
$$= \frac{\tan(c+dx)}{a^3d}-\frac{9\sec(c+dx)\tan(c+dx)}{8a^3d}-\frac{\sec^3(c+dx)\tan(c+dx)}{8a^3d}-\frac{\tan^3(c+dx)}{3a^3d}+\frac{3\sec}{}$$
$$= -\frac{x}{a^3}+\frac{9\tanh^{-1}(\sin(c+dx))}{8a^3d}+\frac{\tan(c+dx)}{a^3d}-\frac{17\sec(c+dx)\tan(c+dx)}{16a^3d}-\frac{\sec^3(c+dx)\tan}{8a^3d}$$
$$= -\frac{x}{a^3}+\frac{19\tanh^{-1}(\sin(c+dx))}{16a^3d}+\frac{\tan(c+dx)}{a^3d}-\frac{17\sec(c+dx)\tan(c+dx)}{16a^3d}-\frac{\sec^3(c+dx)\,\mathrm{t}}{8a^3d}$$

Mathematica [A] time = 0.852181, size = 303, normalized size = 1.79

$$-\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(9120\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^10/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(9120*(Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c]*Sec[c + d*x]^
6*(2400*d*x*Cos[c] + 1800*d*x*Cos[c + 2*d*x] + 1800*d*x*Cos[3*c + 2*d*x] +
720*d*x*Cos[3*c + 4*d*x] + 720*d*x*Cos[5*c + 4*d*x] + 120*d*x*Cos[5*c + 6*d
*x] + 120*d*x*Cos[7*c + 6*d*x] + 1760*Sin[c] - 210*Sin[d*x] - 210*Sin[2*c +
 d*x] - 1440*Sin[c + 2*d*x] + 1200*Sin[3*c + 2*d*x] + 865*Sin[2*c + 3*d*x]
+ 865*Sin[4*c + 3*d*x] - 1296*Sin[3*c + 4*d*x] - 240*Sin[5*c + 4*d*x] + 435
*Sin[4*c + 5*d*x] + 435*Sin[6*c + 5*d*x] - 176*Sin[5*c + 6*d*x])))/(960*a^3
*d*(1 + Sec[c + d*x])^3)
```

Maple [B] time = 0.102, size = 312, normalized size = 1.9

$$-2\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^{3}}-\frac{1}{6\,da^{3}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-6}+\frac{11}{10\,da^{3}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-5}-\frac{11}{4\,da^{3}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^10/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -2/d/a^3*arctan(tan(1/2*d*x+1/2*c))-1/6/d/a^3/(tan(1/2*d*x+1/2*c)+1)^6+11/1
0/d/a^3/(tan(1/2*d*x+1/2*c)+1)^5-11/4/d/a^3/(tan(1/2*d*x+1/2*c)+1)^4+11/4/d
/a^3/(tan(1/2*d*x+1/2*c)+1)^3+5/16/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2-35/16/d/a
^3/(tan(1/2*d*x+1/2*c)+1)+19/16/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)+1/6/d/a^3/(t
an(1/2*d*x+1/2*c)-1)^6+11/10/d/a^3/(tan(1/2*d*x+1/2*c)-1)^5+11/4/d/a^3/(tan
(1/2*d*x+1/2*c)-1)^4+11/4/d/a^3/(tan(1/2*d*x+1/2*c)-1)^3-5/16/d/a^3/(tan(1/
2*d*x+1/2*c)-1)^2-35/16/d/a^3/(tan(1/2*d*x+1/2*c)-1)-19/16/d/a^3*ln(tan(1/2
*d*x+1/2*c)-1)
```

Maxima [B] time = 1.71528, size = 463, normalized size = 2.74

$$-\frac{\frac{2\left(\frac{45\sin(dx+c)}{\cos(dx+c)+1}-\frac{95\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{366\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{1746\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{3135\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}+\frac{525\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)}{a^{3}-\frac{6a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{15a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{20a^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{15a^{3}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{6a^{3}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^{3}\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}+\frac{480\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{3}}-\frac{285\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right.}{a^{3}}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^10/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/240*(2*(45*sin(d*x + c)/(cos(d*x + c) + 1) - 95*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 - 366*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1746*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 - 3135*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 525*sin(
d*x + c)^11/(cos(d*x + c) + 1)^11)/(a^3 - 6*a^3*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*a^3*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 + 15*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 6*a^
3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^3*sin(d*x + c)^12/(cos(d*x + c)
 + 1)^12) + 480*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 285*log(sin(d
*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 285*log(sin(d*x + c)/(cos(d*x + c) +
1) - 1)/a^3)/d
```

Fricas [A] time = 1.25241, size = 362, normalized size = 2.14

$$-\frac{480\,dx\cos\left(dx+c\right)^{6}-285\,\cos\left(dx+c\right)^{6}\log\left(\sin\left(dx+c\right)+1\right)+285\,\cos\left(dx+c\right)^{6}\log\left(-\sin\left(dx+c\right)+1\right)-2\left(176\,\cos\left(\right.\right.}{480\,a^{3}d\cos\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^10/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/480*(480*d*x*cos(d*x + c)^6 - 285*cos(d*x + c)^6*log(sin(d*x + c) + 1) +
 285*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 2*(176*cos(d*x + c)^5 - 435*co
s(d*x + c)^4 + 208*cos(d*x + c)^3 + 110*cos(d*x + c)^2 - 144*cos(d*x + c) +
 40)*sin(d*x + c))/(a^3*d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**10/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 42.3985, size = 201, normalized size = 1.19

$$-\frac{\frac{240\,(dx+c)}{a^{3}}-\frac{285\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{3}}+\frac{285\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{3}}+\frac{2\left(525\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-3135\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+1746\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-366\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right.}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^10/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/240*(240*(d*x + c)/a^3 - 285*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 + 28
5*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 2*(525*tan(1/2*d*x + 1/2*c)^11 -
 3135*tan(1/2*d*x + 1/2*c)^9 + 1746*tan(1/2*d*x + 1/2*c)^7 - 366*tan(1/2*d*
x + 1/2*c)^5 - 95*tan(1/2*d*x + 1/2*c)^3 + 45*tan(1/2*d*x + 1/2*c))/((tan(1
/2*d*x + 1/2*c)^2 - 1)^6*a^3))/d
```

3.97 $\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=99

$$-\frac{\tan^3(c+dx)}{a^3d}-\frac{\tan(c+dx)}{a^3d}-\frac{13\tanh^{-1}(\sin(c+dx))}{8a^3d}+\frac{\tan(c+dx)\sec^3(c+dx)}{4a^3d}+\frac{11\tan(c+dx)\sec(c+dx)}{8a^3d}+\frac{x}{a^3}$$

```
[Out] x/a^3 - (13*ArcTanh[Sin[c + d*x]])/(8*a^3*d) - Tan[c + d*x]/(a^3*d) + (11*S
ec[c + d*x]*Tan[c + d*x])/(8*a^3*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(4*a^3*
d) - Tan[c + d*x]^3/(a^3*d)
```

Rubi [A] time = 0.204299, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3888, 3886, 3473, 8, 2611, 3770, 2607, 30, 3768}

$$-\frac{\tan^3(c+dx)}{a^3d}-\frac{\tan(c+dx)}{a^3d}-\frac{13\tanh^{-1}(\sin(c+dx))}{8a^3d}+\frac{\tan(c+dx)\sec^3(c+dx)}{4a^3d}+\frac{11\tan(c+dx)\sec(c+dx)}{8a^3d}+\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] x/a^3 - (13*ArcTanh[Sin[c + d*x]])/(8*a^3*d) - Tan[c + d*x]/(a^3*d) + (11*S
ec[c + d*x]*Tan[c + d*x])/(8*a^3*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(4*a^3*
d) - Tan[c + d*x]^3/(a^3*d)
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int(-a+a\sec(c+dx))^3\tan^2(c+dx)\,dx}{a^6}$$
$$= \frac{\int\left(-a^3\tan^2(c+dx)+3a^3\sec(c+dx)\tan^2(c+dx)-3a^3\sec^2(c+dx)\tan^2(c+dx)+a^3\operatorname{se}\right.}{a^6}$$
$$= -\frac{\int\tan^2(c+dx)\,dx}{a^3}+\frac{\int\sec^3(c+dx)\tan^2(c+dx)\,dx}{a^3}+\frac{3\int\sec(c+dx)\tan^2(c+dx)\,dx}{a^3}-$$
$$= -\frac{\tan(c+dx)}{a^3d}+\frac{3\sec(c+dx)\tan(c+dx)}{2a^3d}+\frac{\sec^3(c+dx)\tan(c+dx)}{4a^3d}-\frac{\int\sec^3(c+dx)\,dx}{4a^3}$$
$$= \frac{x}{a^3}-\frac{3\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{\tan(c+dx)}{a^3d}+\frac{11\sec(c+dx)\tan(c+dx)}{8a^3d}+\frac{\sec^3(c+dx)\tan}{4a^3d}$$
$$= \frac{x}{a^3}-\frac{13\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{\tan(c+dx)}{a^3d}+\frac{11\sec(c+dx)\tan(c+dx)}{8a^3d}+\frac{\sec^3(c+dx)\tan}{4a^3d}$$

Mathematica [B] time = 0.729249, size = 230, normalized size = 2.32

$$\frac{\sec^4(c+dx)\left(38\sin(c+dx)-32\sin(2(c+dx))+22\sin(3(c+dx))+39\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\operatorname{co}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^4*(24*d*x + 39*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*C
os[2*(c + d*x)]*(8*d*x + 13*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 13*L
og[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[4*(c + d*x)]*(8*d*x + 13*Log
[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 13*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]]) - 39*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 38*Sin[c + d*x] -
 32*Sin[2*(c + d*x)] + 22*Sin[3*(c + d*x)]))/(64*a^3*d)
```

Maple [B] time = 0.088, size = 228, normalized size = 2.3

$$2\,\frac{\arctan(\tan(1/2\,dx+c/2))}{da^3}-\frac{1}{4\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-4}+\frac{3}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}-\frac{27}{8\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^8/(a+a*sec(d*x+c))^3,x)
```

[Out] 2/d/a^3*arctan(tan(1/2*d*x+1/2*c))-1/4/d/a^3/(tan(1/2*d*x+1/2*c)+1)^4+3/2/d
/a^3/(tan(1/2*d*x+1/2*c)+1)^3-27/8/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2+21/8/d/a^
3/(tan(1/2*d*x+1/2*c)+1)-13/8/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)+1/4/d/a^3/(tan
(1/2*d*x+1/2*c)-1)^4+3/2/d/a^3/(tan(1/2*d*x+1/2*c)-1)^3+27/8/d/a^3/(tan(1/2
*d*x+1/2*c)-1)^2+21/8/d/a^3/(tan(1/2*d*x+1/2*c)-1)+13/8/d/a^3*ln(tan(1/2*d*
x+1/2*c)-1)

Maxima [B] time = 1.64603, size = 347, normalized size = 3.51

$$\frac{\frac{2\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{13\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{21\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^3-\frac{4a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{16\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{13\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{13\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] 1/8*(2*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 13*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 21*sin(d*x + c)^7/(cos(d*
x + c) + 1)^7)/(a^3 - 4*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^3*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) + 16*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a^3 - 13*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 13*l
og(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d

Fricas [A] time = 1.19605, size = 265, normalized size = 2.68

$$\frac{16\,dx\cos(dx+c)^4-13\,\cos(dx+c)^4\log(\sin(dx+c)+1)+13\,\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left(11\,\cos(dx+c\right.}{16\,a^3d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] 1/16*(16*d*x*cos(d*x + c)^4 - 13*cos(d*x + c)^4*log(sin(d*x + c) + 1) + 13*
cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(11*cos(d*x + c)^2 - 8*cos(d*x +
c) + 2)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**8/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 13.369, size = 166, normalized size = 1.68

$$\frac{\frac{8\,(dx+c)}{a^3} - \frac{13\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3} + \frac{13\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3} + \frac{2\left(21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-13\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4 a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*(d*x + c)/a^3 - 13*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 + 13*log(a
bs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 2*(21*tan(1/2*d*x + 1/2*c)^7 + 3*tan(1/
2*d*x + 1/2*c)^5 - 13*tan(1/2*d*x + 1/2*c)^3 + 5*tan(1/2*d*x + 1/2*c))/((ta
n(1/2*d*x + 1/2*c)^2 - 1)^4*a^3))/d
```

3.98 $\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=66

$$-\frac{5\tan(c+dx)}{2a^3d}+\frac{7\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{\tan(c+dx)(1-\sec(c+dx))}{2a^3d}-\frac{x}{a^3}$$

```
[Out] -(x/a^3) + (7*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (5*Tan[c + d*x])/(2*a^3*d)
 - ((1 - Sec[c + d*x])*Tan[c + d*x])/(2*a^3*d)
```

Rubi [A] time = 0.0908163, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3888, 3775, 3914, 3767, 8, 3770}

$$-\frac{5\tan(c+dx)}{2a^3d}+\frac{7\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{\tan(c+dx)(1-\sec(c+dx))}{2a^3d}-\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(x/a^3) + (7*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (5*Tan[c + d*x])/(2*a^3*d)
 - ((1 - Sec[c + d*x])*Tan[c + d*x])/(2*a^3*d)
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3775

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[a/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integer
Q[2*n]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= \frac{\int(-a+a\sec(c+dx))^3\,dx}{a^6} \\
&= -\frac{(1-\sec(c+dx))\tan(c+dx)}{2a^3d} - \frac{\int(-a+a\sec(c+dx))(-2a+5a\sec(c+dx))\,dx}{2a^5} \\
&= -\frac{x}{a^3} - \frac{(1-\sec(c+dx))\tan(c+dx)}{2a^3d} - \frac{5\int\sec^2(c+dx)\,dx}{2a^3} + \frac{7\int\sec(c+dx)\,dx}{2a^3} \\
&= -\frac{x}{a^3} + \frac{7\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{(1-\sec(c+dx))\tan(c+dx)}{2a^3d} + \frac{5\,\mathrm{Subst}(\int 1\,dx, x, -\tan(c}{2a^3d} \\
&= -\frac{x}{a^3} + \frac{7\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{5\tan(c+dx)}{2a^3d} - \frac{(1-\sec(c+dx))\tan(c+dx)}{2a^3d}
\end{aligned}
$$

Mathematica [B] time = 0.906319, size = 241, normalized size = 3.65

$$
\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-\frac{12\sin(dx)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{}{d\left(\cos\left(\frac{1}{2}(\right.\right.}\right.}{a^3(\sec(c+a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(-4*x - (14*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]])/d + (14*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + 1/(d*
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - 1/(d*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^2) - (12*Sin[d*x])/(d*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2]
)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]))))/(a^3*(1 + Sec[c + d*x])^3)
```

Maple [B] time = 0.08, size = 144, normalized size = 2.2

$$-2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^{3}}-\frac{1}{2\,da^{3}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}+\frac{7}{2\,da^{3}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}+\frac{7}{2\,da^{3}}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -2/d/a^3*arctan(tan(1/2*d*x+1/2*c))-1/2/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2+7/2/
d/a^3/(tan(1/2*d*x+1/2*c)+1)+7/2/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)+1/2/d/a^3/(
tan(1/2*d*x+1/2*c)-1)^2+7/2/d/a^3/(tan(1/2*d*x+1/2*c)-1)-7/2/d/a^3*ln(tan(1
/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.67691, size = 231, normalized size = 3.5

$$-\frac{\frac{2\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{7\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{a^{3}-\frac{2a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}+\frac{4\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{3}}-\frac{7\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^{3}}+\frac{7\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^{3}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3)/(a^3 - 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4) + 4*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 -
 7*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 7*log(sin(d*x + c)/(cos(d
```

```
*x + c) + 1) - 1)/a^3)/d
```

Fricas [A] time = 1.18403, size = 234, normalized size = 3.55

$$-\frac{4\,dx\cos\left(dx+c\right)^{2}-7\,\cos\left(dx+c\right)^{2}\log\left(\sin\left(dx+c\right)+1\right)+7\,\cos\left(dx+c\right)^{2}\log\left(-\sin\left(dx+c\right)+1\right)+2\,(6\,\cos\left(dx+c\right)-}{4\,a^{3}d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(4*d*x*cos(d*x + c)^2 - 7*cos(d*x + c)^2*log(sin(d*x + c) + 1) + 7*cos
(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(6*cos(d*x + c) - 1)*sin(d*x + c))/(
a^3*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^{6}\left(c+dx\right)}{\sec^{3}\left(c+dx\right)+3\sec^{2}\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**6/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A] time = 5.85243, size = 131, normalized size = 1.98

$$-\frac{\frac{2\,(dx+c)}{a^{3}}-\frac{7\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{3}}+\frac{7\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{3}}-\frac{2\left(7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}a^{3}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)/a^3 - 7*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 + 7*log(ab
s(tan(1/2*d*x + 1/2*c) - 1))/a^3 - 2*(7*tan(1/2*d*x + 1/2*c)^3 - 5*tan(1/2*
d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3))/d
```

3.99 $\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=46

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{4\tan(c+dx)}{a^2d(a\sec(c+dx)+a)}+\frac{x}{a^3}$$

```
[Out] x/a^3 + ArcTanh[Sin[c + d*x]]/(a^3*d) - (4*Tan[c + d*x])/(a^2*d*(a + a*Sec[
c + d*x]))
```

Rubi [A] time = 0.140903, antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3888, 3886, 3473, 8, 2606, 3767, 2621, 321, 207}

$$\frac{4\cot(c+dx)}{a^3d}-\frac{4\csc(c+dx)}{a^3d}+\frac{\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] x/a^3 + ArcTanh[Sin[c + d*x]]/(a^3*d) + (4*Cot[c + d*x])/(a^3*d) - (4*Csc[c
 + d*x])/(a^3*d)
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \cot^2(c+dx)(-a+a\sec(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-a^3\cot^2(c+dx)+3a^3\cot(c+dx)\csc(c+dx)-3a^3\csc^2(c+dx)+a^3\csc^2(c+dx)\sec\right.}{a^6}$$
$$= -\frac{\int \cot^2(c+dx)\,dx}{a^3}+\frac{\int \csc^2(c+dx)\sec(c+dx)\,dx}{a^3}+\frac{3\int \cot(c+dx)\csc(c+dx)\,dx}{a^3}-\frac{3}{}$$
$$= \frac{\cot(c+dx)}{a^3d}+\frac{\int 1\,dx}{a^3}-\frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{a^3d}+\frac{3\,\text{Subst}(\int 1\,dx,x,\cot(c+dx}{a^3d}$$
$$= \frac{x}{a^3}+\frac{4\cot(c+dx)}{a^3d}-\frac{4\csc(c+dx)}{a^3d}-\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{a^3d}$$
$$= \frac{x}{a^3}+\frac{\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{4\cot(c+dx)}{a^3d}-\frac{4\csc(c+dx)}{a^3d}$$

Mathematica [B] time = 0.259056, size = 117, normalized size = 2.54

$$\frac{8\cos^5\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\right.\right.\right.}{a^3d(\sec(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (8*Cos[(c + d*x)/2]^5*Sec[c + d*x]^3*(Cos[(c + d*x)/2]*(d*x - Log[Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 4
*Sec[c/2]*Sin[(d*x)/2]))/(a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 0.082, size = 76, normalized size = 1.7

$$-4\,\frac{\tan(1/2\,dx+c/2)}{da^3}+2\,\frac{\arctan(\tan(1/2\,dx+c/2))}{da^3}+\frac{1}{da^3}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{1}{da^3}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -4/d/a^3*tan(1/2*d*x+1/2*c)+2/d/a^3*arctan(tan(1/2*d*x+1/2*c))+1/d/a^3*ln(t
an(1/2*d*x+1/2*c)+1)-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.70578, size = 132, normalized size = 2.87

$$\frac{\frac{2\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}-\frac{4\,\sin(dx+c)}{a^3(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 + log(sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)/a^3 - log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3 - 4*sin(d
*x + c)/(a^3*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.15144, size = 225, normalized size = 4.89

$$\frac{2\,dx\cos\left(dx+c\right)+2\,dx+\left(\cos\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(\cos\left(dx+c\right)+1\right)\log\left(-\sin\left(dx+c\right)+1\right)-8\,\sin\left(dx\right.}{2\left(a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*d*x*cos(d*x + c) + 2*d*x + (cos(d*x + c) + 1)*log(sin(d*x + c) + 1)
- (cos(d*x + c) + 1)*log(-sin(d*x + c) + 1) - 8*sin(d*x + c))/(a^3*d*cos(d*
x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^4\left(c+dx\right)}{\sec^3\left(c+dx\right)+3\sec^2\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A] time = 2.67768, size = 85, normalized size = 1.85

$$\frac{\frac{dx+c}{a^3} + \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3} - \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3} - \frac{4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((d*x + c)/a^3 + log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - log(abs(tan(1/2*d
*x + 1/2*c) - 1))/a^3 - 4*tan(1/2*d*x + 1/2*c)/a^3)/d
```

3.100 $\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=60

$$\frac{2\tan(c+dx)}{a^2d(a\sec(c+dx)+a)}-\frac{x}{a^3}-\frac{\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^3}$$

```
[Out] -(x/a^3) + (2*Tan[c + d*x])/(a^2*d*(a + a*Sec[c + d*x])) - Tan[c + d*x]^3/(
3*d*(a + a*Sec[c + d*x])^3)
```

Rubi [A] time = 0.173323, antiderivative size = 71, normalized size of antiderivative = 1.18, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3888, 3886, 3473, 8, 2606, 2607, 30}

$$\frac{4\cot^3(c+dx)}{3a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\csc^3(c+dx)}{3a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(x/a^3) - Cot[c + d*x]/(a^3*d) + (4*Cot[c + d*x]^3)/(3*a^3*d) + (3*Csc[c +
 d*x])/(a^3*d) - (4*Csc[c + d*x]^3)/(3*a^3*d)
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= \frac{\int \cot^4(c+dx)(-a+a\sec(c+dx))^3\,dx}{a^6}\\
&= \frac{\int \left(-a^3\cot^4(c+dx)+3a^3\cot^3(c+dx)\csc(c+dx)-3a^3\cot^2(c+dx)\csc^2(c+dx)+a^3\operatorname{co}\right.}{a^6}\\
&= -\frac{\int \cot^4(c+dx)\,dx}{a^3}+\frac{\int \cot(c+dx)\csc^3(c+dx)\,dx}{a^3}+\frac{3\int \cot^3(c+dx)\csc(c+dx)\,dx}{a^3}-\frac{3}{}\\
&= \frac{\cot^3(c+dx)}{3a^3d}+\frac{\int \cot^2(c+dx)\,dx}{a^3}-\frac{\operatorname{Subst}\left(\int x^2\,dx,x,\csc(c+dx)\right)}{a^3d}-\frac{3\operatorname{Subst}\left(\int x^2\,dx,x,\right.}{a^3d}\\
&= -\frac{\cot(c+dx)}{a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{4\csc^3(c+dx)}{3a^3d}-\frac{\int 1\,dx}{a^3}\\
&= -\frac{x}{a^3}-\frac{\cot(c+dx)}{a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{4\csc^3(c+dx)}{3a^3d}
\end{aligned}$$

Mathematica [B] time = 0.380248, size = 125, normalized size = 2.08

$$-\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(351\sin\left(c+\frac{dx}{2}\right)-277\sin\left(c+\frac{3dx}{2}\right)-3\sin\left(2c+\frac{3dx}{2}\right)+180dx\cos\left(c+\frac{dx}{2}\right)+60dx\cos\left(c+\frac{3dx}{2}\right)\right.}{480a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(Sec[c/2]*Sec[(c + d*x)/2]^3*(180*d*x*Cos[(d*x)/2] + 180*d*x*Cos[c + (d*x)
/2] + 60*d*x*Cos[c + (3*d*x)/2] + 60*d*x*Cos[2*c + (3*d*x)/2] - 471*Sin[(d*
x)/2] + 351*Sin[c + (d*x)/2] - 277*Sin[c + (3*d*x)/2] - 3*Sin[2*c + (3*d*x)
/2]))/(480*a^3*d)
```

Maple [A] time = 0.082, size = 56, normalized size = 0.9

$$-\frac{1}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+2\,\frac{\tan(1/2\,dx+c/2)}{da^3}-2\,\frac{\arctan(\tan(1/2\,dx+c/2))}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/3/d/a^3*tan(1/2*d*x+1/2*c)^3+2/d/a^3*tan(1/2*d*x+1/2*c)-2/d/a^3*arctan(t
an(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.63608, size = 97, normalized size = 1.62

$$\frac{\frac{\frac{6\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^3}-\frac{6\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*((6*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/a^3 - 6*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 1.09627, size = 200, normalized size = 3.33

$$-\frac{3\,dx\cos\left(dx+c\right)^{2}+6\,dx\cos\left(dx+c\right)+3\,dx-\left(7\,\cos\left(dx+c\right)+5\right)\sin\left(dx+c\right)}{3\,\left(a^{3}d\cos\left(dx+c\right)^{2}+2\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*d*x*cos(d*x + c)^2 + 6*d*x*cos(d*x + c) + 3*d*x - (7*cos(d*x + c) +
 5)*sin(d*x + c))/(a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^{2}\left(c+dx\right)}{\sec^{3}\left(c+dx\right)+3\sec^{2}\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A] time = 1.65397, size = 68, normalized size = 1.13

$$-\frac{\frac{3\,(dx+c)}{a^{3}}+\frac{a^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-6\,a^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{9}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)/a^3 + (a^6*tan(1/2*d*x + 1/2*c)^3 - 6*a^6*tan(1/2*d*x + 1
/2*c))/a^9)/d
```

3.101 $\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=143

$$\frac{4\cot^7(c+dx)}{7a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\csc^7(c+dx)}{7a^3d}+\frac{11\csc^5(c+dx)}{5a^3d}-\frac{10\csc^3(c+dx)}{3a^3d}+\frac{3\,\mathrm{cs}}{}$$

```
[Out] -(x/a^3) - Cot[c + d*x]/(a^3*d) + Cot[c + d*x]^3/(3*a^3*d) - Cot[c + d*x]^5
/(5*a^3*d) + (4*Cot[c + d*x]^7)/(7*a^3*d) + (3*Csc[c + d*x])/(a^3*d) - (10*
Csc[c + d*x]^3)/(3*a^3*d) + (11*Csc[c + d*x]^5)/(5*a^3*d) - (4*Csc[c + d*x]
^7)/(7*a^3*d)
```

Rubi [A] time = 0.236258, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3888, 3886, 3473, 8, 2606, 194, 2607, 30, 270}

$$\frac{4\cot^7(c+dx)}{7a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\csc^7(c+dx)}{7a^3d}+\frac{11\csc^5(c+dx)}{5a^3d}-\frac{10\csc^3(c+dx)}{3a^3d}+\frac{3\,\mathrm{cs}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(x/a^3) - Cot[c + d*x]/(a^3*d) + Cot[c + d*x]^3/(3*a^3*d) - Cot[c + d*x]^5
/(5*a^3*d) + (4*Cot[c + d*x]^7)/(7*a^3*d) + (3*Csc[c + d*x])/(a^3*d) - (10*
Csc[c + d*x]^3)/(3*a^3*d) + (11*Csc[c + d*x]^5)/(5*a^3*d) - (4*Csc[c + d*x]
^7)/(7*a^3*d)
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \cot^8(c+dx)(-a+a\sec(c+dx))^3\,dx}{a^6}$$

$$= \frac{\int \left(-a^3\cot^8(c+dx)+3a^3\cot^7(c+dx)\csc(c+dx)-3a^3\cot^6(c+dx)\csc^2(c+dx)+a^3\cot^5\right.}{a^6}$$

$$= -\frac{\int \cot^8(c+dx)\,dx}{a^3}+\frac{\int \cot^5(c+dx)\csc^3(c+dx)\,dx}{a^3}+\frac{3\int \cot^7(c+dx)\csc(c+dx)\,dx}{a^3}-3$$

$$= \frac{\cot^7(c+dx)}{7a^3d}+\frac{\int \cot^6(c+dx)\,dx}{a^3}-\frac{\mathrm{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx,x,\csc(c+dx)\right)}{a^3d}-\frac{3\,\mathrm{Subst}\left(\right.}{}$$

$$= -\frac{\cot^5(c+dx)}{5a^3d}+\frac{4\cot^7(c+dx)}{7a^3d}-\frac{\int \cot^4(c+dx)\,dx}{a^3}-\frac{\mathrm{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx,x,\csc(\right.}{a^3d}$$

$$= \frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{4\cot^7(c+dx)}{7a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{10\csc^3(c+dx)}{3a^3d}+\frac{11\csc^5(}{5a^3}$$

$$= -\frac{\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{4\cot^7(c+dx)}{7a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{10\csc^3(c+}{3a^3d}$$

$$= -\frac{x}{a^3}-\frac{\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{4\cot^7(c+dx)}{7a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{10\,\mathrm{cs}}{}$$

Mathematica [A] time = 1.27293, size = 252, normalized size = 1.76

$$\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\csc\left(\frac{1}{2}(c+dx)\right)\sec^7\left(\frac{1}{2}(c+dx)\right)(-23282\sin(c+dx)-23282\sin(2(c+dx))-9978\sin(3(c+dx))-1663}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Csc[c/2]*Csc[(c + d*x)/2]*Sec[c/2]*Sec[(c + d*x)/2]^7*(-5880*d*x*Cos[d*x]
+ 5880*d*x*Cos[2*c + d*x] - 5880*d*x*Cos[c + 2*d*x] + 5880*d*x*Cos[3*c + 2*
d*x] - 2520*d*x*Cos[2*c + 3*d*x] + 2520*d*x*Cos[4*c + 3*d*x] - 420*d*x*Cos[
3*c + 4*d*x] + 420*d*x*Cos[5*c + 4*d*x] + 4200*Sin[c] + 11032*Sin[d*x] - 23
282*Sin[c + d*x] - 23282*Sin[2*(c + d*x)] - 9978*Sin[3*(c + d*x)] - 1663*Si
n[4*(c + d*x)] + 13720*Sin[2*c + d*x] + 15512*Sin[c + 2*d*x] + 9240*Sin[3*c
 + 2*d*x] + 8088*Sin[2*c + 3*d*x] + 2520*Sin[4*c + 3*d*x] + 1768*Sin[3*c +
4*d*x]))/(215040*a^3*d)
```

Maple [A] time = 0.066, size = 113, normalized size = 0.8

$$-\frac{1}{112\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{3}{40\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{1}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{13}{8\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\,\frac{\arctan(\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/112/d/a^3*tan(1/2*d*x+1/2*c)^7+3/40/d/a^3*tan(1/2*d*x+1/2*c)^5-1/3/d/a^3
*tan(1/2*d*x+1/2*c)^3+13/8/d/a^3*tan(1/2*d*x+1/2*c)-2/d/a^3*arctan(tan(1/2*
d*x+1/2*c))-1/16/d/a^3/tan(1/2*d*x+1/2*c)
```

Maxima [A] time = 1.64185, size = 180, normalized size = 1.26

$$\frac{\frac{\frac{2730\sin(dx+c)}{\cos(dx+c)+1}-\frac{560\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{126\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^3}-\frac{3360\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{105\,(\cos(dx+c)+1)}{a^3\sin(dx+c)}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/1680*((2730*sin(d*x + c)/(cos(d*x + c) + 1) - 560*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 126*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/
(cos(d*x + c) + 1)^7)/a^3 - 3360*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^
3 - 105*(cos(d*x + c) + 1)/(a^3*sin(d*x + c)))/d
```

Fricas [A] time = 1.2438, size = 377, normalized size = 2.64

$$-\frac{221\,\cos(dx+c)^4+348\,\cos(dx+c)^3-25\,\cos(dx+c)^2+105\left(dx\cos(dx+c)^3+3\,dx\cos(dx+c)^2+3\,dx\cos(dx+\right.}{105\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+c)+a^3d\right)\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/105*(221*cos(d*x + c)^4 + 348*cos(d*x + c)^3 - 25*cos(d*x + c)^2 + 105*(
d*x*cos(d*x + c)^3 + 3*d*x*cos(d*x + c)^2 + 3*d*x*cos(d*x + c) + d*x)*sin(d
```

```
*x + c) - 303*cos(d*x + c) - 136)/((a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x
+ c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A] time = 1.48011, size = 134, normalized size = 0.94

$$-\frac{\frac{1680\,(dx+c)}{a^3}+\frac{105}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{15\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-126\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+560\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2730\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{21}}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/1680*(1680*(d*x + c)/a^3 + 105/(a^3*tan(1/2*d*x + 1/2*c)) + (15*a^18*tan
(1/2*d*x + 1/2*c)^7 - 126*a^18*tan(1/2*d*x + 1/2*c)^5 + 560*a^18*tan(1/2*d*
x + 1/2*c)^3 - 2730*a^18*tan(1/2*d*x + 1/2*c))/a^21)/d
```

3.102 $\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^3} dx$

Optimal. Leaf size=177

$$\frac{4\cot^9(c+dx)}{9a^3d}-\frac{\cot^7(c+dx)}{7a^3d}+\frac{\cot^5(c+dx)}{5a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot(c+dx)}{a^3d}-\frac{4\csc^9(c+dx)}{9a^3d}+\frac{15\csc^7(c+dx)}{7a^3d}-\frac{21\,\mathrm{cs}}{}$$

```
[Out] x/a^3 + Cot[c + d*x]/(a^3*d) - Cot[c + d*x]^3/(3*a^3*d) + Cot[c + d*x]^5/(5
*a^3*d) - Cot[c + d*x]^7/(7*a^3*d) + (4*Cot[c + d*x]^9)/(9*a^3*d) - (3*Csc[
c + d*x])/(a^3*d) + (13*Csc[c + d*x]^3)/(3*a^3*d) - (21*Csc[c + d*x]^5)/(5*
a^3*d) + (15*Csc[c + d*x]^7)/(7*a^3*d) - (4*Csc[c + d*x]^9)/(9*a^3*d)
```

Rubi [A] time = 0.253495, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3888, 3886, 3473, 8, 2606, 194, 2607, 30, 270}

$$\frac{4\cot^9(c+dx)}{9a^3d}-\frac{\cot^7(c+dx)}{7a^3d}+\frac{\cot^5(c+dx)}{5a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot(c+dx)}{a^3d}-\frac{4\csc^9(c+dx)}{9a^3d}+\frac{15\csc^7(c+dx)}{7a^3d}-\frac{21\,\mathrm{cs}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] x/a^3 + Cot[c + d*x]/(a^3*d) - Cot[c + d*x]^3/(3*a^3*d) + Cot[c + d*x]^5/(5
*a^3*d) - Cot[c + d*x]^7/(7*a^3*d) + (4*Cot[c + d*x]^9)/(9*a^3*d) - (3*Csc[
c + d*x])/(a^3*d) + (13*Csc[c + d*x]^3)/(3*a^3*d) - (21*Csc[c + d*x]^5)/(5*
a^3*d) + (15*Csc[c + d*x]^7)/(7*a^3*d) - (4*Csc[c + d*x]^9)/(9*a^3*d)
```


Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \cot^{10}(c+dx)(-a+a\sec(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-a^3\cot^{10}(c+dx)+3a^3\cot^9(c+dx)\csc(c+dx)-3a^3\cot^8(c+dx)\csc^2(c+dx)+a^3\cot\right.}{a^6}$$
$$= -\frac{\int \cot^{10}(c+dx)\,dx}{a^3}+\frac{\int \cot^7(c+dx)\csc^3(c+dx)\,dx}{a^3}+\frac{3\int \cot^9(c+dx)\csc(c+dx)\,dx}{a^3}-$$
$$= \frac{\cot^9(c+dx)}{9a^3d}+\frac{\int \cot^8(c+dx)\,dx}{a^3}-\frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^3\,dx,x,\csc(c+dx)\right)}{a^3d}-\frac{3\,\text{Subst}}{}$$
$$= -\frac{\cot^7(c+dx)}{7a^3d}+\frac{4\cot^9(c+dx)}{9a^3d}-\frac{\int \cot^6(c+dx)\,dx}{a^3}-\frac{\text{Subst}\left(\int \left(-x^2+3x^4-3x^6+x^8\right)\right.}{a^3d}$$
$$= \frac{\cot^5(c+dx)}{5a^3d}-\frac{\cot^7(c+dx)}{7a^3d}+\frac{4\cot^9(c+dx)}{9a^3d}-\frac{3\csc(c+dx)}{a^3d}+\frac{13\csc^3(c+dx)}{3a^3d}-\frac{21\csc^5}{5a}$$
$$= -\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot^5(c+dx)}{5a^3d}-\frac{\cot^7(c+dx)}{7a^3d}+\frac{4\cot^9(c+dx)}{9a^3d}-\frac{3\csc(c+dx)}{a^3d}+\frac{13\csc^3(}{3a}$$
$$= \frac{\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot^5(c+dx)}{5a^3d}-\frac{\cot^7(c+dx)}{7a^3d}+\frac{4\cot^9(c+dx)}{9a^3d}-\frac{3\csc(c+d}{a^3d}$$
$$= \frac{x}{a^3}+\frac{\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot^5(c+dx)}{5a^3d}-\frac{\cot^7(c+dx)}{7a^3d}+\frac{4\cot^9(c+dx)}{9a^3d}-\frac{3\csc(}{a}$$

Mathematica [B] time = 1.17305, size = 366, normalized size = 2.07

$$\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\csc^3(2(c+dx))(675036\sin(c+dx)+506277\sin(2(c+dx))-37502\sin(3(c+dx))-225012\sin(4(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Csc[c/2]*Csc[2*(c + d*x)]^3*Sec[c/2]*(181440*d*x*Cos[d*x] - 181440*d*x*Cos
[2*c + d*x] + 136080*d*x*Cos[c + 2*d*x] - 136080*d*x*Cos[3*c + 2*d*x] - 100
80*d*x*Cos[2*c + 3*d*x] + 10080*d*x*Cos[4*c + 3*d*x] - 60480*d*x*Cos[3*c +
4*d*x] + 60480*d*x*Cos[5*c + 4*d*x] - 30240*d*x*Cos[4*c + 5*d*x] + 30240*d*
x*Cos[6*c + 5*d*x] - 5040*d*x*Cos[5*c + 6*d*x] + 5040*d*x*Cos[7*c + 6*d*x]
- 169344*Sin[c] - 338112*Sin[d*x] + 675036*Sin[c + d*x] + 506277*Sin[2*(c +
 d*x)] - 37502*Sin[3*(c + d*x)] - 225012*Sin[4*(c + d*x)] - 112506*Sin[5*(c
 + d*x)] - 18751*Sin[6*(c + d*x)] - 431424*Sin[2*c + d*x] - 375552*Sin[c +
2*d*x] - 201600*Sin[3*c + 2*d*x] - 41248*Sin[2*c + 3*d*x] + 84000*Sin[4*c +
 3*d*x] + 155712*Sin[3*c + 4*d*x] + 100800*Sin[5*c + 4*d*x] + 98016*Sin[4*c
 + 5*d*x] + 30240*Sin[6*c + 5*d*x] + 21376*Sin[5*c + 6*d*x]))/(80640*a^3*d*
```

```
(1 + Sec[c + d*x])^3)
```

Maple [A] time = 0.079, size = 151, normalized size = 0.9

$$-\frac{1}{576\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9+\frac{1}{56\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{29}{320\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{1}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{99}{64\,da^3}\,\mathrm{ta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/576/d/a^3*tan(1/2*d*x+1/2*c)^9+1/56/d/a^3*tan(1/2*d*x+1/2*c)^7-29/320/d/
a^3*tan(1/2*d*x+1/2*c)^5+1/3/d/a^3*tan(1/2*d*x+1/2*c)^3-99/64/d/a^3*tan(1/2
*d*x+1/2*c)+2/d/a^3*arctan(tan(1/2*d*x+1/2*c))-1/192/d/a^3/tan(1/2*d*x+1/2*
c)^3+1/8/d/a^3/tan(1/2*d*x+1/2*c)
```

Maxima [A] time = 1.61626, size = 239, normalized size = 1.35

$$-\frac{\frac{\frac{31185\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{6720\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1827\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{360\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^3}-\frac{40320\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{105\left(\frac{24\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^3}{a^3\sin(dx+c)^3}}{20160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/20160*((31185*sin(d*x + c)/(cos(d*x + c) + 1) - 6720*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3 + 1827*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 360*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^3 -
40320*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 105*(24*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^3/(a^3*sin(d*x + c)^3))/d
```

Fricas [A] time = 1.48758, size = 566, normalized size = 3.2

$$\frac{668\,\cos(dx+c)^6+1059\,\cos(dx+c)^5-573\,\cos(dx+c)^4-1813\,\cos(dx+c)^3-393\,\cos(dx+c)^2+315\left(dx\cos(dx+\right.}{315\left(a^3d\cos(dx+c)^5+3\,a^3d\cos(dx+c)^4+2\,a^3d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/315*(668*cos(d*x + c)^6 + 1059*cos(d*x + c)^5 - 573*cos(d*x + c)^4 - 1813
*cos(d*x + c)^3 - 393*cos(d*x + c)^2 + 315*(d*x*cos(d*x + c)^5 + 3*d*x*cos(
d*x + c)^4 + 2*d*x*cos(d*x + c)^3 - 2*d*x*cos(d*x + c)^2 - 3*d*x*cos(d*x +
c) - d*x)*sin(d*x + c) + 789*cos(d*x + c) + 368)/((a^3*d*cos(d*x + c)^5 + 3
*a^3*d*cos(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3
*a^3*d*cos(d*x + c) - a^3*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A] time = 1.44733, size = 177, normalized size = 1.

$$\frac{\frac{20160\,(dx+c)}{a^3} + \frac{105\left(24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3} - \frac{35\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-360\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1827\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6720\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3118}{a^{27}}}{20160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/20160*(20160*(d*x + c)/a^3 + 105*(24*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^3*tan
(1/2*d*x + 1/2*c)^3) - (35*a^24*tan(1/2*d*x + 1/2*c)^9 - 360*a^24*tan(1/2*d
*x + 1/2*c)^7 + 1827*a^24*tan(1/2*d*x + 1/2*c)^5 - 6720*a^24*tan(1/2*d*x +
1/2*c)^3 + 31185*a^24*tan(1/2*d*x + 1/2*c))/a^27)/d
```

3.103 $\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=215

$$\frac{4\cot^{11}(c+dx)}{11a^3d}-\frac{\cot^9(c+dx)}{9a^3d}+\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\csc^{11}(c+dx)}{11a^3d}+\frac{19\csc^9}{9a}$$

```
[Out] -(x/a^3) - Cot[c + d*x]/(a^3*d) + Cot[c + d*x]^3/(3*a^3*d) - Cot[c + d*x]^5
/(5*a^3*d) + Cot[c + d*x]^7/(7*a^3*d) - Cot[c + d*x]^9/(9*a^3*d) + (4*Cot[c
 + d*x]^11)/(11*a^3*d) + (3*Csc[c + d*x])/(a^3*d) - (16*Csc[c + d*x]^3)/(3*
a^3*d) + (34*Csc[c + d*x]^5)/(5*a^3*d) - (36*Csc[c + d*x]^7)/(7*a^3*d) + (1
9*Csc[c + d*x]^9)/(9*a^3*d) - (4*Csc[c + d*x]^11)/(11*a^3*d)
```

Rubi [A] time = 0.278093, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3888, 3886, 3473, 8, 2606, 194, 2607, 30, 270}

$$\frac{4\cot^{11}(c+dx)}{11a^3d}-\frac{\cot^9(c+dx)}{9a^3d}+\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\csc^{11}(c+dx)}{11a^3d}+\frac{19\csc^9}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] -(x/a^3) - Cot[c + d*x]/(a^3*d) + Cot[c + d*x]^3/(3*a^3*d) - Cot[c + d*x]^5
/(5*a^3*d) + Cot[c + d*x]^7/(7*a^3*d) - Cot[c + d*x]^9/(9*a^3*d) + (4*Cot[c
 + d*x]^11)/(11*a^3*d) + (3*Csc[c + d*x])/(a^3*d) - (16*Csc[c + d*x]^3)/(3*
a^3*d) + (34*Csc[c + d*x]^5)/(5*a^3*d) - (36*Csc[c + d*x]^7)/(7*a^3*d) + (1
9*Csc[c + d*x]^9)/(9*a^3*d) - (4*Csc[c + d*x]^11)/(11*a^3*d)
```


Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
```

```
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \cot^{12}(c+dx)(-a+a\sec(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-a^3\cot^{12}(c+dx)+3a^3\cot^{11}(c+dx)\csc(c+dx)-3a^3\cot^{10}(c+dx)\csc^2(c+dx)+a^3 c\right.}{a^6}$$
$$= -\frac{\int \cot^{12}(c+dx)\,dx}{a^3}+\frac{\int \cot^9(c+dx)\csc^3(c+dx)\,dx}{a^3}+\frac{3\int \cot^{11}(c+dx)\csc(c+dx)\,dx}{a^3}-$$
$$= \frac{\cot^{11}(c+dx)}{11a^3d}+\frac{\int \cot^{10}(c+dx)\,dx}{a^3}-\frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^4\,dx,x,\csc(c+dx)\right)}{a^3d}-\frac{3\,\text{Subst}}{}$$
$$= -\frac{\cot^9(c+dx)}{9a^3d}+\frac{4\cot^{11}(c+dx)}{11a^3d}-\frac{\int \cot^8(c+dx)\,dx}{a^3}-\frac{\text{Subst}\left(\int \left(x^2-4x^4+6x^6-4x^8+x\right.\right.}{a^3d}$$
$$= \frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^9(c+dx)}{9a^3d}+\frac{4\cot^{11}(c+dx)}{11a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{16\csc^3(c+dx)}{3a^3d}+\frac{34\csc^5}{5a}$$
$$= -\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^9(c+dx)}{9a^3d}+\frac{4\cot^{11}(c+dx)}{11a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{16\csc^3(c}{3a^3}$$
$$= \frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^9(c+dx)}{9a^3d}+\frac{4\cot^{11}(c+dx)}{11a^3d}+\frac{3\csc(c+d}{a^3d}$$
$$= -\frac{\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^9(c+dx)}{9a^3d}+\frac{4\cot^{11}(c+d}{11a^3d}$$
$$= -\frac{x}{a^3}-\frac{\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^9(c+dx)}{9a^3d}+\frac{4\cot^{11}}{11a}$$

Mathematica [A] time = 3.59001, size = 394, normalized size = 1.83

$$-\frac{\tan\left(\frac{c}{2}\right)\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(315\sec^{10}\left(\frac{1}{2}(c+dx)\right)-5425\sec^8\left(\frac{1}{2}(c+dx)\right)+41320\sec^6\left(\frac{1}{2}(c+dx)\right)-184650\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(231*(-25 + 28*Cos[c + d*x])*Cot[c/2]^2
*Csc[(c + d*x)/2]^4 + 561145*Sec[(c + d*x)/2]^2 - 184650*Sec[(c + d*x)/2]^4
 + 41320*Sec[(c + d*x)/2]^6 - 5425*Sec[(c + d*x)/2]^8 + 315*Sec[(c + d*x)/2
]^10 - 1736335*Csc[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] + 561145*Csc[c/2]*Sec
[(c + d*x)/2]^3*Sin[(d*x)/2] - 184650*Csc[c/2]*Sec[(c + d*x)/2]^5*Sin[(d*x)
/2] + 41320*Csc[c/2]*Sec[(c + d*x)/2]^7*Sin[(d*x)/2] - 5425*Csc[c/2]*Sec[(c
 + d*x)/2]^9*Sin[(d*x)/2] + 315*Csc[c/2]*Sec[(c + d*x)/2]^11*Sin[(d*x)/2] +
 6468*Csc[c/2]^3*Csc[(c + d*x)/2]^3*Sin[c]*Sin[(d*x)/2] + 231*Cot[c/2]*(384
```

```
0*d*x - Csc[c/2]*Csc[(c + d*x)/2]*(743 + 3*Csc[(c + d*x)/2]^4)*Sin[(d*x)/2]
))*Tan[c/2])/(110880*a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 0.082, size = 189, normalized size = 0.9

$$-\frac{1}{2816\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{11}+\frac{5}{1152\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{9}-\frac{23}{896\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}+\frac{13}{128\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}-\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/2816/d/a^3*tan(1/2*d*x+1/2*c)^11+5/1152/d/a^3*tan(1/2*d*x+1/2*c)^9-23/89
6/d/a^3*tan(1/2*d*x+1/2*c)^7+13/128/d/a^3*tan(1/2*d*x+1/2*c)^5-1/3/d/a^3*ta
n(1/2*d*x+1/2*c)^3+191/128/d/a^3*tan(1/2*d*x+1/2*c)-2/d/a^3*arctan(tan(1/2*
d*x+1/2*c))-1/1280/d/a^3/tan(1/2*d*x+1/2*c)^5+5/384/d/a^3/tan(1/2*d*x+1/2*c
)^3-23/128/d/a^3/tan(1/2*d*x+1/2*c)
```

Maxima [A] time = 1.55563, size = 294, normalized size = 1.37

$$\frac{\frac{5\left(\frac{264726\sin(dx+c)}{\cos(dx+c)+1}-\frac{59136\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{18018\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{4554\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{770\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{63\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)}{a^3}-\frac{1774080\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{231\left(\frac{50\sin(dx+\ldots}{(\cos(dx+c)+\ldots}\right.}{\ldots}}{887040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/887040*(5*(264726*sin(d*x + c)/(cos(d*x + c) + 1) - 59136*sin(d*x + c)^3/
(cos(d*x + c) + 1)^3 + 18018*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 4554*sin
(d*x + c)^7/(cos(d*x + c) + 1)^7 + 770*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
- 63*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/a^3 - 1774080*arctan(sin(d*x +
c)/(cos(d*x + c) + 1))/a^3 + 231*(50*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 -
690*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 3)*(cos(d*x + c) + 1)^5/(a^3*sin(
d*x + c)^5))/d
```

Fricas [A] time = 1.59559, size = 757, normalized size = 3.52

$$-\frac{7453\cos(dx+c)^8+11964\cos(dx+c)^7-11866\cos(dx+c)^6-30542\cos(dx+c)^5+90\cos(dx+c)^4+26438\cos(}{3465\left(a^3d\cos(dx+c)^7+3a^3d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3465*(7453*cos(d*x + c)^8 + 11964*cos(d*x + c)^7 - 11866*cos(d*x + c)^6
- 30542*cos(d*x + c)^5 + 90*cos(d*x + c)^4 + 26438*cos(d*x + c)^3 + 8539*co
s(d*x + c)^2 + 3465*(d*x*cos(d*x + c)^7 + 3*d*x*cos(d*x + c)^6 + d*x*cos(d*
x + c)^5 - 5*d*x*cos(d*x + c)^4 - 5*d*x*cos(d*x + c)^3 + d*x*cos(d*x + c)^2
 + 3*d*x*cos(d*x + c) + d*x)*sin(d*x + c) - 7671*cos(d*x + c) - 3712)/((a^3
*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a^3*d*cos(d*x + c)^5 - 5*a^3*d
*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2 + 3*a^3*d*c
os(d*x + c) + a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.56771, size = 216, normalized size = 1.

$$-\frac{\frac{887040(dx+c)}{a^3}+\frac{231\left(690\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-50\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3\right)}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}+\frac{5\left(63a^{30}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-770a^{30}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+4554a^{30}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-1801\right.}{a^{33}}}{887040d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/887040*(887040*(d*x + c)/a^3 + 231*(690*tan(1/2*d*x + 1/2*c)^4 - 50*tan(
1/2*d*x + 1/2*c)^2 + 3)/(a^3*tan(1/2*d*x + 1/2*c)^5) + 5*(63*a^30*tan(1/2*d
*x + 1/2*c)^11 - 770*a^30*tan(1/2*d*x + 1/2*c)^9 + 4554*a^30*tan(1/2*d*x +
1/2*c)^7 - 18018*a^30*tan(1/2*d*x + 1/2*c)^5 + 59136*a^30*tan(1/2*d*x + 1/2
*c)^3 - 264726*a^30*tan(1/2*d*x + 1/2*c))/a^33)/d
```

3.104 $\int (a + a\sec(c + dx))(e\tan(c + dx))^{5/2}\,dx$

Optimal. Leaf size=310

$$\frac{ae^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}-\frac{ae^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}-\frac{ae^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d}$$

```
[Out] (a*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d)
- (a*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d
) - (a*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
d*x]]])/(2*Sqrt[2]*d) + (a*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqr
t[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) + (6*a*e^2*Cos[c + d*x]*EllipticE
[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*d*Sqrt[Sin[2*c + 2*d*x]]) - (6
*a*e*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(5*d) + (2*e*(5*a + 3*a*Sec[c + d
*x])*(e*Tan[c + d*x])^(3/2))/(15*d)
```

Rubi [A] time = 0.320431, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {3881, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$\frac{ae^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}-\frac{ae^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}-\frac{ae^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] (a*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d)
- (a*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d
) - (a*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
d*x]]])/(2*Sqrt[2]*d) + (a*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqr
t[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) + (6*a*e^2*Cos[c + d*x]*EllipticE
[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*d*Sqrt[Sin[2*c + 2*d*x]]) - (6
*a*e*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(5*d) + (2*e*(5*a + 3*a*Sec[c + d
*x])*(e*Tan[c + d*x])^(3/2))/(15*d)
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
```

```
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))(e\tan(c+dx))^{5/2}\,dx = \frac{2e(5a+3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d} - \frac{1}{5}\left(2e^2\right)\int\left(\frac{5a}{2}+\frac{3}{2}a\sec(c+dx\right.$$

$$= \frac{2e(5a+3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d} - \frac{1}{5}\left(3ae^2\right)\int \sec(c+dx)\sqrt{e\tan(}$$

$$= -\frac{6ae\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{2e(5a+3a\sec(c+dx))(e\tan(c+dx)}{15d}$$

$$= -\frac{6ae\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{2e(5a+3a\sec(c+dx))(e\tan(c+dx)}{15d}$$

$$= -\frac{6ae\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{2e(5a+3a\sec(c+dx))(e\tan(c+dx)}{15d}$$

$$= \frac{6ae^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5d\sqrt{\sin(2c+2dx)}} - \frac{6ae\cos(c+dx)(e\tan(}{5d}$$

$$= -\frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} + \frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$

$$= \frac{ae^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{ae^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{ae^{5/2}\log}{}$$

Mathematica [C] time = 2.23151, size = 186, normalized size = 0.6

$$\frac{a(\cos(c+dx)+1)\csc(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(e\tan(c+dx))^{5/2}\left(\frac{24\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(c+dx)\right)}{\sqrt{\sec^2(c+dx)}}-36\cos^2(c+dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Csc[c + d*x]*Sec[(c + d*x)/2]^2*(12 + 20*Cos[c + d*x]
 - 36*Cos[c + d*x]^2 + (24*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[c + d*x]^2
])/Sqrt[Sec[c + d*x]^2] + 15*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Cot[c + d*
x]^2*Sqrt[Sin[2*(c + d*x)]] + 15*Cot[c + d*x]^2*Log[Cos[c + d*x] + Sin[c +
d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]])*(e*Tan[c + d*x])^(5/
2))/(60*d)
```

Maple [C] time = 0.296, size = 1495, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*tan(d*x+c))^(5/2),x)
```

```
[Out] 1/30*a/d*2^(1/2)*(-1+cos(d*x+c))^2*(15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/
2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^3-15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-
1/2*I,1/2*2^(1/2))*cos(d*x+c)^3+15*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*
I,1/2*2^(1/2))*cos(d*x+c)^3+15*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/
2*2^(1/2))*cos(d*x+c)^3-36*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(
d*x+c)^3+18*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((
1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^3+15*I*(
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-15*I*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+15*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+15*((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-36*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2+18*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
```

```
,1/2*2^(1/2))*cos(d*x+c)^2+8*cos(d*x+c)^3*2^(1/2)-24*cos(d*x+c)^2*2^(1/2)+1
0*cos(d*x+c)*2^(1/2)+6*2^(1/2))*(cos(d*x+c)+1)^2*(e*sin(d*x+c)/cos(d*x+c))^
(5/2)/sin(d*x+c)^7
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)(e\tan(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(5/2), x)
```

3.105 $\int (a+a\sec (c+dx))(e\tan (c+dx))^{3/2}\,dx$

Optimal. Leaf size=282

$$-\frac{ae^2\sqrt{\sin(2c+2dx)}\sec(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3d\sqrt{e\tan(c+dx)}}+\frac{ae^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}-\frac{ae^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right.}{\sqrt{2}d}$$

```
[Out] (a*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d)
- (a*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d
) + (a*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
d*x]]])/(2*Sqrt[2]*d) - (a*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqr
t[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (a*e^2*EllipticF[c - Pi/4 + d*x
, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*d*Sqrt[e*Tan[c + d*x]]) + (2*e
*(3*a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]])/(3*d)
```

Rubi [A] time = 0.274087, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3881, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$\frac{ae^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}-\frac{ae^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}+\frac{ae^{3/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right.}{2\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d)
- (a*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d
) + (a*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
d*x]]])/(2*Sqrt[2]*d) - (a*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqr
t[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (a*e^2*EllipticF[c - Pi/4 + d*x
, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*d*Sqrt[e*Tan[c + d*x]]) + (2*e
*(3*a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]])/(3*d)
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
```

```
]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))(e\tan(c+dx))^{3/2}\,dx = \frac{2e(3a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d} - \frac{1}{3}\left(2e^2\right)\int \frac{\frac{3a}{2}+\frac{1}{2}a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx$$

$$= \frac{2e(3a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d} - \frac{1}{3}\left(ae^2\right)\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx - \left(ae^2\right)$$

$$= \frac{2e(3a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d} - \frac{\left(ae^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x, e\tan(\right.}{d}$$

$$= \frac{2e(3a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d} - \frac{\left(2ae^3\right)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(\right.}{d}$$

$$= -\frac{ae^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3d\sqrt{e\tan(c+dx)}} + \frac{2e(3a+a\sec(c+dx))\sqrt{}}{3d}$$

$$= -\frac{ae^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3d\sqrt{e\tan(c+dx)}} + \frac{2e(3a+a\sec(c+dx))\sqrt{}}{3d}$$

$$= \frac{ae^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{ae^{3/2}\log\left(\sqrt{e}+\sqrt{e}\,\text{ta}\right.}{}$$

$$= \frac{ae^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{ae^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{ae^{3/2}\log\left(\right.}{}$$

Mathematica [C] time = 2.12653, size = 214, normalized size = 0.76

$$-\frac{ae\cos(2(c+dx))\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec^2(c+dx)}\sqrt{e\tan(c+dx)}\left(\sqrt{\sec^2(c+dx)}\left(12\sin(c+dx)+4\tan(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] -(a*e*Cos[2*(c + d*x)]*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]^
2]*Sqrt[e*Tan[c + d*x]]*(4*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[T
an[c + d*x]]], -1]*Sqrt[Tan[c + d*x]] + Sqrt[Sec[c + d*x]^2]*(12*Sin[c + d*
x] + 3*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[Sin[2*(c + d*x)]] - 3*Log[C
os[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]]
 + 4*Tan[c + d*x])))/(12*d*(-1 + Tan[c + d*x]^2))
```

Maple [C] time = 0.239, size = 688, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*tan(d*x+c))^(3/2),x)
```

```
[Out] -1/6*a/d*2^(1/2)*(-1+cos(d*x+c))*(3*I*sin(d*x+c)*cos(d*x+c)*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*I*sin(d*x+c)*cos(d*x+c)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*sin(d*x+c)*cos(d*x+c)*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*sin(d*x+c)*cos(d*x+c)*(
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+4*sin(d*x+c)*cos(d*x+
c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-6*cos(d*x+c)^2*2^(1/2)+4*cos
(d*x+c)*2^(1/2)+2*2^(1/2))*(cos(d*x+c)+1)^2*(e*sin(d*x+c)/cos(d*x+c))^(3/2)
/sin(d*x+c)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)\,(e\tan(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(3/2), x)
```

3.106 $\int (a + a\sec(c + dx))\sqrt{e\tan(c + dx)}\,dx$

Optimal. Leaf size=272

$$-\frac{a\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}+\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}+\frac{a\sqrt{e}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d}$$

```
[Out] -((a*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d
)) + (a*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2
]*d) + (a*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c
 + d*x]]])/(2*Sqrt[2]*d) - (a*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (2*a*Cos[c + d*x]*EllipticE[
c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(d*Sqrt[Sin[2*c + 2*d*x]]) + (2*a*
Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(d*e)
```

Rubi [A] time = 0.243957, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$-\frac{a\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}+\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}+\frac{a\sqrt{e}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]],x]
```



```
[Out] -((a*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d
)) + (a*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2
]*d) + (a*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c
 + d*x]]])/(2*Sqrt[2]*d) - (a*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (2*a*Cos[c + d*x]*EllipticE[
c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(d*Sqrt[Sin[2*c + 2*d*x]]) + (2*a*
Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(d*e)
```


Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\sec(c+dx))\sqrt{e\tan(c+dx)}\,dx = a\int \sqrt{e\tan(c+dx)}\,dx + a\int \sec(c+dx)\sqrt{e\tan(c+dx)}\,dx$$

$$= \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - (2a)\int \cos(c+dx)\sqrt{e\tan(c+dx)}\,dx + \frac{(ae)}{\quad}$$

$$= \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} + \frac{(2ae)\,\mathrm{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d}$$

$$= \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - \frac{(ae)\,\mathrm{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d} +$$

$$= -\frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}} + \frac{2a\cos(c+dx)(e\tan(c+dx}{de}$$

$$= \frac{a\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{a\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c\right.}{2}$$

$$= -\frac{a\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{a\sqrt{e}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{a\sqrt{e}\log\left(\sqrt{\right.}}{}$$

Mathematica [C] time = 1.50688, size = 182, normalized size = 0.67

$$-\frac{a(\cos(c+dx)+1)\csc(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{e\tan(c+dx)}\left(8\tan^2(c+dx)\mathrm{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] -(a*(1 + Cos[c + d*x])*Csc[c + d*x]*Sec[(c + d*x)/2]^2*Sqrt[e*Tan[c + d*x]]
*(3*Sqrt[Sec[c + d*x]^2]*(-4*Sin[c + d*x]^2 + ArcSin[Cos[c + d*x] - Sin[c +
 d*x]]*Sqrt[Sin[2*(c + d*x)]] + Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[
2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]]) + 8*Hypergeometric2F1[3/4, 3/2, 7/4,
 -Tan[c + d*x]^2]*Tan[c + d*x]^2))/(12*d*Sqrt[Sec[c + d*x]^2])
```

Maple [C] time = 0.28, size = 1405, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*tan(d*x+c))^(1/2),x)
```

```
[Out] -1/2*a/d*2^(1/2)*(e*sin(d*x+c)/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d
*x+c))^2*(-I*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/
2))+I*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-I*(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+I*((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2),1/2-1/2*I,1/2*2^(1/2))+cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1
/2*I,1/2*2^(1/2))-4*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+
2*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/
2*I,1/2*2^(1/2))-4*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellip
ticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+2*((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2),1/2*2^(1/2))+2*cos(d*x+c)*2^(1/2)-2*2^(1/2))/sin(d*x+c
)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \sqrt{e\tan\left(c+dx\right)}\,dx+\int \sqrt{e\tan\left(c+dx\right)}\sec\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(sqrt(e*tan(c + d*x)), x) + Integral(sqrt(e*tan(c + d*x))*sec(c
+ d*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(dx+c\right)+a\right)\sqrt{e\tan\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sqrt(e*tan(d*x + c)), x)
```

3.107 $\int \frac{a+a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx$

Optimal. Leaf size=244

$$\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}}+\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\sqrt{e}}-\frac{a\log}{}$$

```
[Out] -((a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*Sqrt[e]
)) + (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*Sqrt
[e]) - (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]
])/(2*Sqrt[2]*d*Sqrt[e]) + (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*
Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) + (a*EllipticF[c - Pi/4 + d*x,
 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(d*Sqrt[e*Tan[c + d*x]])
```

Rubi [A] time = 0.212879, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}}+\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\sqrt{e}}-\frac{a\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\sqrt{e}}+\frac{a\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] -((a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*Sqrt[e]
)) + (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*Sqrt
[e]) - (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]
])/(2*Sqrt[2]*d*Sqrt[e]) + (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*
Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) + (a*EllipticF[c - Pi/4 + d*x,
 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(d*Sqrt[e*Tan[c + d*x]])
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx = a\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx + a\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx$$

$$= \frac{(ae)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x, e\tan(c+dx)\right)}{d} + \frac{\left(a\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$= \frac{(2ae)\,\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d} + \frac{\left(a\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{\sqrt{e\tan(c+dx)}}$$

$$= \frac{aF\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d\sqrt{e\tan(c+dx)}} + \frac{a\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d} + \frac{a}{}$$

$$= \frac{aF\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d\sqrt{e\tan(c+dx)}} + \frac{a\,\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2d}$$

$$= -\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} + \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan}\right.}{2\sqrt{2}d\sqrt{e}}$$

$$= -\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} + \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} - \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\right.}{2\sqrt{2}d\sqrt{e}}$$

Mathematica [C] time = 1.68266, size = 220, normalized size = 0.9

$$\frac{20a\sin(c+dx)\cos^2\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)F_1\left(\frac{1}{4};\frac{1}{2},1;\right.}{d\sqrt{e\tan(c+dx)}\left(2(\cos(c+dx)-1)\left(2F_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-F_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\tan^2\left(\frac{1}{2}(c+\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] (20*a*AppellF1[1/4, 1/2, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*C
os[(c + d*x)/2]^2*(1 + Sec[c + d*x])*Sin[c + d*x])/(d*(2*(2*AppellF1[5/4, 1
/2, 2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[5/4, 3/2, 1
, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + 5*Ap
pellF1[1/4, 1/2, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[
c + d*x]))*Sqrt[e*Tan[c + d*x]])
```

Maple [C] time = 0.243, size = 284, normalized size = 1.2

$$-\frac{a\sqrt{2}\,(-1+\cos(dx+c))\,(\cos(dx+c)+1)^2}{2\,d\,(\sin(dx+c))^2\cos(dx+c)}\left(i\text{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\text{EllipticPi}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x)
```

```
[Out] -1/2*a/d*2^(1/2)*(I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
,1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2),1/2+1/2*I,1/2*2^(1/2))+EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*(-1+cos(d*x+c))/sin(d*x+c)^2/cos(d*x+c)*(cos(d*x+c)+1)^2/(e*
sin(d*x+c)/cos(d*x+c))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{1}{\sqrt{e\tan{(c+dx)}}}\,dx+\int \frac{\sec{(c+dx)}}{\sqrt{e\tan{(c+dx)}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(1/sqrt(e*tan(c + d*x)), x) + Integral(sec(c + d*x)/sqrt(e*tan(c
 + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sec{(dx+c)}+a}{\sqrt{e\tan{(dx+c)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(e*tan(d*x + c)), x)
```

3.108 $\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=305

$$\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}-\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{3/2}}-\frac{a\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}}+\frac{a\log\left(\sqrt{\right.}}{}$$

```
[Out] (a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(3/2))
- (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(3/2)
) - (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/
(2*Sqrt[2]*d*e^(3/2)) + (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqr
t[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(3/2)) - (2*(a + a*Sec[c + d*x]))/(d*e*S
qrt[e*Tan[c + d*x]]) - (2*a*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[
e*Tan[c + d*x]])/(d*e^2*Sqrt[Sin[2*c + 2*d*x]]) + (2*a*Cos[c + d*x]*(e*Tan[
c + d*x])^(3/2))/(d*e^3)
```

Rubi [A] time = 0.303329, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {3882, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}-\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{3/2}}-\frac{a\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}}+\frac{a\log\left(\sqrt{\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(3/2),x]
```


```
[Out] (a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(3/2))
- (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(3/2)
) - (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/
(2*Sqrt[2]*d*e^(3/2)) + (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqr
t[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(3/2)) - (2*(a + a*Sec[c + d*x]))/(d*e*S
qrt[e*Tan[c + d*x]]) - (2*a*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[
e*Tan[c + d*x]])/(d*e^2*Sqrt[Sin[2*c + 2*d*x]]) + (2*a*Cos[c + d*x]*(e*Tan[
c + d*x])^(3/2))/(d*e^3)
```


Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx = -\frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}} + \frac{2\int\left(-\frac{a}{2}+\frac{1}{2}a\sec(c+dx)\right)\sqrt{e\tan(c+dx)}\,dx}{e^2}$$

$$= -\frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}} - \frac{a\int\sqrt{e\tan(c+dx)}\,dx}{e^2} + \frac{a\int\sec(c+dx)\sqrt{e\tan(c+dx)}\,dx}{e^2}$$

$$= -\frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}} + \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de^3} - \frac{(2a)\int\cos(c+dx)\sqrt{e\tan(c+dx)}}{e^2}$$

$$= -\frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}} + \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de^3} - \frac{(2a)\,\text{Subst}\left(\int\frac{x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan}\right.}{de}$$

$$= -\frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}} + \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de^3} + \frac{a\,\text{Subst}\left(\int\frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c}\right.}{de}$$

$$= -\frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}} - \frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{de^2\sqrt{\sin(2c+2dx)}} + \frac{2a\cos(c+dx)(e\,t}{de^3}$$

$$= -\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{3/2}} + \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\,ta}\right.}{2\sqrt{2}de^{3/2}}$$

$$= \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} - \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} - \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\right.}{2\sqrt{2}de^{3/2}}$$

Mathematica [C] time = 2.60705, size = 196, normalized size = 0.64

$$-\frac{a(\cos(c+dx)+1)\csc(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{e\tan(c+dx)}\left(8\tan^2(c+dx)\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] -(a*(1 + Cos[c + d*x])*Csc[c + d*x]*Sec[(c + d*x)/2]^2*Sqrt[e*Tan[c + d*x]]
*(3*Sqrt[Sec[c + d*x]^2]*(2 + 4*Cos[c + d*x] + 2*Cos[2*(c + d*x)] - ArcSin[
```

```
Cos[c + d*x] - Sin[c + d*x]]*Sqrt[Sin[2*(c + d*x)]] - Log[Cos[c + d*x] + Si
n[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]]) + 8*Hypergeome
tric2F1[3/4, 3/2, 7/4, -Tan[c + d*x]^2]*Tan[c + d*x]^2))/(12*d*e^2*Sqrt[Sec
[c + d*x]^2])
```

Maple [C] time = 0.247, size = 1414, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x)
```

```
[Out] -1/2*a/d*2^(1/2)*(I*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*El
lipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/
2))*cos(d*x+c)-I*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellip
ticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))
*cos(d*x+c)+I*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Elliptic
Pi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-(-
(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x
+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)-I*(-(-1
+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)
-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-(-(-1+cos(d*x+c)-sin(
d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin
(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)-4*(-(-1+cos(d*x+c)-sin(d*x
+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x
+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)+2*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c
))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2
*2^(1/2))*cos(d*x+c)-(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*E
llipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1
/2))-(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+
cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-4*(-(-1+cos
```

```
(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1+cos(d*x+c)-sin(
d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+2*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)
,1/2*2^(1/2))+4*cos(d*x+c)*2^(1/2))*sin(d*x+c)/cos(d*x+c)^2/(e*sin(d*x+c)/c
os(d*x+c))^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{1}{(e\tan(c+dx))^{\frac{3}{2}}}\,dx + \int \frac{\sec(c+dx)}{(e\tan(c+dx))^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))**(3/2),x)
```

```
[Out] a*(Integral((e*tan(c + d*x))**(-3/2), x) + Integral(sec(c + d*x)/(e*tan(c +
 d*x))**(3/2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sec(dx+c)+a}{(e\tan(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/(e*tan(d*x + c))^(3/2), x)
```

3.109 $\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=282

$$-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3de^2\sqrt{e\tan(c+dx)}}+\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}}-\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{5/2}}+\cdots$$

```
[Out] (a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(5/2))
- (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(5/2)
) + (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/
(2*Sqrt[2]*d*e^(5/2)) - (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqr
t[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(5/2)) - (2*(a + a*Sec[c + d*x]))/(3*d*e
*(e*Tan[c + d*x])^(3/2)) - (a*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqr
t[Sin[2*c + 2*d*x]])/(3*d*e^2*Sqrt[e*Tan[c + d*x]])
```

Rubi [A] time = 0.275203, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3882, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}}-\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{5/2}}+\frac{a\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{5/2}}-\frac{a\log(\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(5/2),x]
```


```
[Out] (a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(5/2))
- (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(5/2)
) + (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/
(2*Sqrt[2]*d*e^(5/2)) - (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqr
t[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(5/2)) - (2*(a + a*Sec[c + d*x]))/(3*d*e
*(e*Tan[c + d*x])^(3/2)) - (a*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqr
t[Sin[2*c + 2*d*x]])/(3*d*e^2*Sqrt[e*Tan[c + d*x]])
```


Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
```

```
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx = -\frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}} + \frac{2\int \frac{-\frac{3a}{2}-\frac{1}{2}a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3e^2}$$

$$= -\frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}} - \frac{a\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3e^2} - \frac{a\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{e^2}$$

$$= -\frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}} - \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x, e\tan(c+dx)\right)}{de} - \frac{\left(a\sqrt{\sin(c+dx)}\right)\int \frac{}{\sqrt{\cos(c}}}{3e^2\sqrt{\cos(c+dx)}\sqrt{e\,t}}$$

$$= -\frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{de} - \frac{\left(a\sec(c+dx)\sqrt{\sin(2c+}\right.}{3e^2\sqrt{e\tan}}$$

$$= -\frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}} - \frac{aF\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3de^2\sqrt{e\tan(c+dx)}} - \frac{a\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx,\right.}{de}$$

$$= -\frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}} - \frac{aF\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3de^2\sqrt{e\tan(c+dx)}} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2}{-e-\sqrt{2}\sqrt{e}x}\right.}{2}$$

$$= \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{5/2}} - \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c\right.}{2\sqrt{2}de^{5/2}}$$

$$= \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} - \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} + \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\right.}{2\sqrt{2}de^{5/2}}$$

Mathematica [C] time = 1.62, size = 200, normalized size = 0.71

$$-\frac{a\csc(c+dx)\sqrt{e\tan(c+dx)}\left(\sqrt{\sec^2(c+dx)}\left(2\cot\left(\frac{1}{2}(c+dx)\right)-3\sqrt{\sin(2(c+dx))}\sin^{-1}(\cos(c+dx)-\sin(c+dx))+2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] -(a*Csc[c + d*x]*(Sqrt[Sec[c + d*x]^2]*(2*Cot[(c + d*x)/2] + 2*Cos[(3*(c +
d*x))/2]*Csc[(c + d*x)/2] - 3*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[Sin[
2*(c + d*x)]] + 3*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]
*Sqrt[Sin[2*(c + d*x)]]) - 4*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt
[Tan[c + d*x]]], -1]*Sqrt[Tan[c + d*x]])*Sqrt[e*Tan[c + d*x]])/(6*d*e^3*Sqr
t[Sec[c + d*x]^2])
```

Maple [C] time = 0.216, size = 658, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x)
```

```
[Out] 1/6*a/d*2^(1/2)*(-1+cos(d*x+c))*(3*I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c
)-sin(d*x+c))/sin(d*x+c))^(1/2)-3*I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)
-sin(d*x+c))/sin(d*x+c))^(1/2)+3*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/si
n(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-si
n(d*x+c))/sin(d*x+c))^(1/2)+3*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d
*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d
*x+c))/sin(d*x+c))^(1/2)-4*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c
))^(1/2),1/2*2^(1/2))*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x
+c))^(1/2)+2*cos(d*x+c)*2^(1/2))*(cos(d*x+c)+1)^2/cos(d*x+c)^3/(e*sin(d*x+c
)/cos(d*x+c))^(5/2)/sin(d*x+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sec(dx+c)+a}{(e\tan(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/(e*tan(d*x + c))^(5/2), x)
```

3.110 $\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=346

$$-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}}+\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{7/2}}+\frac{a\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{7/2}}-\frac{a\log}{}$$

```
[Out] -((a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(7/2)
)) + (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(7
/2)) + (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]
])/(2*Sqrt[2]*d*e^(7/2)) - (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*
Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(7/2)) - (2*(a + a*Sec[c + d*x]))/(5*
d*e*(e*Tan[c + d*x])^(5/2)) + (2*(5*a + 3*a*Sec[c + d*x]))/(5*d*e^3*Sqrt[e*
Tan[c + d*x]]) + (6*a*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[
c + d*x]])/(5*d*e^4*Sqrt[Sin[2*c + 2*d*x]]) - (6*a*Cos[c + d*x]*(e*Tan[c +
d*x])^(3/2))/(5*d*e^5)
```

Rubi [A] time = 0.365497, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {3882, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}}+\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{7/2}}+\frac{a\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{7/2}}-\frac{a\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(7/2),x]
```


```
[Out] -((a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(7/2)
)) + (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(7
/2)) + (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]
])/(2*Sqrt[2]*d*e^(7/2)) - (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*
Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(7/2)) - (2*(a + a*Sec[c + d*x]))/(5*
d*e*(e*Tan[c + d*x])^(5/2)) + (2*(5*a + 3*a*Sec[c + d*x]))/(5*d*e^3*Sqrt[e*
Tan[c + d*x]]) + (6*a*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[
c + d*x]])/(5*d*e^4*Sqrt[Sin[2*c + 2*d*x]]) - (6*a*Cos[c + d*x]*(e*Tan[c +
d*x])^(3/2))/(5*d*e^5)
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
```

```
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx = -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}}+\frac{2\int\frac{-\frac{5a}{2}-\frac{3}{2}a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx}{5e^2}$$

$$= -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}}+\frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}}+\frac{4\int\left(\frac{5a}{4}-\frac{3}{4}a\sec(c+dx)\right)\sqrt{e\tan(c+dx)}\,dx}{5e^4}$$

$$= -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}}+\frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}}-\frac{(3a)\int\sec(c+dx)\sqrt{e\tan(c+dx)}\,dx}{5e^4}+\frac{a\int}{}$$

$$= -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}}+\frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}}-\frac{6a\cos(c+dx)(e\tan(c+dx))^{3/2}}{5de^5}+\frac{(6a)\int c}{}$$

$$= -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}}+\frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}}-\frac{6a\cos(c+dx)(e\tan(c+dx))^{3/2}}{5de^5}+\frac{(2a)\,\mathrm{Sub}}{}$$

$$= -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}}+\frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}}-\frac{6a\cos(c+dx)(e\tan(c+dx))^{3/2}}{5de^5}-\frac{a\,\mathrm{Subst}}{}$$

$$= -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}}+\frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}}+\frac{6a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+}}{5de^4\sqrt{\sin(2c+2dx)}}$$

$$= \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{7/2}}-\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c}\right.}{2\sqrt{2}de^{7/2}}$$

$$= -\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}}+\frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}}+\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{}\right.}{2\sqrt{2}de^{7/2}}$$

Mathematica [C] time = 2.47236, size = 254, normalized size = 0.73

$$-\frac{a\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)\left(-8\sin^2(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec^2(c+dx)}\mathrm{Hypergeometric2F}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(7/2),x]
```

```
[Out] -(a*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(1 + Sec[c + d*x])*(2*Cot[(c + d*x)/2
] - 19*Sin[c + d*x] + 12*Sin[c + d*x]^2*Tan[(c + d*x)/2] - 8*Hypergeometric
2F1[3/4, 3/2, 7/4, -Tan[c + d*x]^2]*Sqrt[Sec[c + d*x]^2]*Sin[c + d*x]^2*Tan
[(c + d*x)/2] + 5*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[Sin[2*(c + d*x)]
]*Tan[(c + d*x)/2] + 5*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*
x)]]]*Sqrt[Sin[2*(c + d*x)]]*Tan[(c + d*x)/2] + 5*Sin[c + d*x]*Tan[(c + d*x
)/2]^2))/(20*d*e^3*Sqrt[e*Tan[c + d*x]])
```

Maple [C] time = 0.269, size = 1427, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*tan(d*x+c))^(7/2),x)
```

```
[Out] -1/10*a/d*2^(1/2)*(5*I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*E
llipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2
))*cos(d*x+c)^2-5*I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Elli
pticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*
cos(d*x+c)^2+5*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticP
i(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d
*x+c)^2+5*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)
^2+12*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2-6*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2-5*I*((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))+5*I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*
I,1/2*2^(1/2))-5*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
```

```
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-5*(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-12*((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2),1/2*2^(1/2))+6*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-18*
cos(d*x+c)^2*2^(1/2)+16*cos(d*x+c)*2^(1/2))*sin(d*x+c)^3/(-1+cos(d*x+c))/co
s(d*x+c)^4/(e*sin(d*x+c)/cos(d*x+c))^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sec(dx+c)+a}{(e\tan(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/(e*tan(d*x + c))^(7/2), x)
```

3.111 $\int (a+a\sec(c+dx))^2(e\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=366

$$\frac{a^2e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}-\frac{a^2e^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}-\frac{a^2e^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d}$$

```
[Out] (a^2*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d
) - (a^2*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[
2]*d) - (a^2*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Ta
n[c + d*x]]])/(2*Sqrt[2]*d) + (a^2*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*
x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) + (12*a^2*e^2*Cos[c + d*x
]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*d*Sqrt[Sin[2*c + 2*
d*x]]) + (2*a^2*e*(e*Tan[c + d*x])^(3/2))/(3*d) - (12*a^2*e*Cos[c + d*x]*(e
*Tan[c + d*x])^(3/2))/(5*d) + (4*a^2*e*Sec[c + d*x]*(e*Tan[c + d*x])^(3/2))
/(5*d) + (2*a^2*(e*Tan[c + d*x])^(7/2))/(7*d*e)
```

Rubi [A] time = 0.432896, antiderivative size = 366, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.68, Rules used = {3886, 3473, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2611, 2613, 2615, 2572, 2639, 2607, 32}

$$\frac{a^2e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}-\frac{a^2e^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}-\frac{a^2e^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^(5/2),x]
```


```
[Out] (a^2*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d
) - (a^2*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[
2]*d) - (a^2*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Ta
n[c + d*x]]])/(2*Sqrt[2]*d) + (a^2*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*
x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) + (12*a^2*e^2*Cos[c + d*x
]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*d*Sqrt[Sin[2*c + 2*
d*x]]) + (2*a^2*e*(e*Tan[c + d*x])^(3/2))/(3*d) - (12*a^2*e*Cos[c + d*x]*(e
*Tan[c + d*x])^(3/2))/(5*d) + (4*a^2*e*Sec[c + d*x]*(e*Tan[c + d*x])^(3/2))
/(5*d) + (2*a^2*(e*Tan[c + d*x])^(7/2))/(7*d*e)
```


Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
```

```
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2(e\tan(c+dx))^{5/2}\,dx = \int \left(a^2(e\tan(c+dx))^{5/2}+2a^2\sec(c+dx)(e\tan(c+dx))^{5/2}+a^2\sec^2(c+dx)\right.$$

$$= a^2\int (e\tan(c+dx))^{5/2}\,dx + a^2\int \sec^2(c+dx)(e\tan(c+dx))^{5/2}\,dx + \left(2a^2\right)$$

$$= \frac{2a^2e(e\tan(c+dx))^{3/2}}{3d} + \frac{4a^2e\sec(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{a^2\,\text{Subst}\left(\int(e\right.}{}$$

$$= \frac{2a^2e(e\tan(c+dx))^{3/2}}{3d} - \frac{12a^2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{4a^2e\sec(c+}{}$$

$$= \frac{2a^2e(e\tan(c+dx))^{3/2}}{3d} - \frac{12a^2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{4a^2e\sec(c+}{}$$

$$= \frac{2a^2e(e\tan(c+dx))^{3/2}}{3d} - \frac{12a^2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{4a^2e\sec(c+}{}$$

$$= \frac{12a^2e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5d\sqrt{\sin(2c+2dx)}} + \frac{2a^2e(e\tan(c+dx))^{3/2}}{3d}$$

$$= -\frac{a^2e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} + \frac{a^2e^{5/2}\log\left(\sqrt{e}+\right.}{}$$

$$= \frac{a^2e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{a^2e^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{a^2e^{5/2}}{}$$

Mathematica [C] time = 6.11792, size = 117, normalized size = 0.32

$$\frac{2a^2e\cos^4\left(\frac{1}{2}(c+dx)\right)(e\tan(c+dx))^{3/2}\sec^4\left(\frac{1}{2}\tan^{-1}(\tan(c+dx))\right)\left(-42\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\tan^2(c+dx)\right)\right.}{105d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*e*Cos[(c + d*x)/2]^4*Sec[ArcTan[Tan[c + d*x]]/2]^4*(e*Tan[c + d*x])^
(3/2)*(35 - 42*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] - 35*Hyper
geometric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2] + 42*Sqrt[Sec[c + d*x]^2] + 15*T
an[c + d*x]^2))/(105*d)
```

Maple [C] time = 0.294, size = 1518, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(5/2),x)
```

```
[Out] 1/210*a^2/d*2^(1/2)*(-1+cos(d*x+c))^2*(-105*I*EllipticPi(((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3+105*I*EllipticPi(((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^4+105*EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^
(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*cos(d*x+c)^4+105*EllipticPi(((1-cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*cos(d*x+c)^4-504*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)*cos(d*x+c)^4+252*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c
)^4+105*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I
,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)
^3-105*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,
1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^
4+105*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^3+1
05*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^3-504*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^3+252*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^3+212*2^(1/2)*cos(d*x+c)^4-336*cos
```

```
(d*x+c)^3*2^(1/2)+10*cos(d*x+c)^2*2^(1/2)+84*cos(d*x+c)*2^(1/2)+30*2^(1/2))
*(cos(d*x+c)+1)^2*(e*sin(d*x+c)/cos(d*x+c))^(5/2)/cos(d*x+c)/sin(d*x+c)^7
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^2 (e\tan(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*(e*tan(d*x + c))^(5/2), x)
```

3.112 $\int (a + a\sec(c+dx))^2 (e\tan(c+dx))^{3/2}\, dx$

Optimal. Leaf size=335

$$-\frac{2a^2e^2\sqrt{\sin(2c+2dx)}\sec(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3d\sqrt{e\tan(c+dx)}}+\frac{a^2e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}-\frac{a^2e^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+}}{\sqrt{e}}\right.}{\sqrt{2}d}$$

```
[Out] (a^2*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d
) - (a^2*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[
2]*d) + (a^2*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Ta
n[c + d*x]]])/(2*Sqrt[2]*d) - (a^2*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*
x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (2*a^2*e^2*EllipticF[c
- Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*d*Sqrt[e*Tan[c + d
*x]]) + (2*a^2*e*Sqrt[e*Tan[c + d*x]])/d + (4*a^2*e*Sec[c + d*x]*Sqrt[e*Tan
[c + d*x]])/(3*d) + (2*a^2*(e*Tan[c + d*x])^(5/2))/(5*d*e)
```

Rubi [A] time = 0.385315, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {3886, 3473, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2611, 2614, 2573, 2641, 2607, 32}

$$\frac{a^2e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}-\frac{a^2e^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}+\frac{a^2e^{3/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{\right.}}{2\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^(3/2),x]
```


```
[Out] (a^2*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d
) - (a^2*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[
2]*d) + (a^2*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Ta
n[c + d*x]]])/(2*Sqrt[2]*d) - (a^2*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*
x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (2*a^2*e^2*EllipticF[c
- Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*d*Sqrt[e*Tan[c + d
*x]]) + (2*a^2*e*Sqrt[e*Tan[c + d*x]])/d + (4*a^2*e*Sec[c + d*x]*Sqrt[e*Tan
[c + d*x]])/(3*d) + (2*a^2*(e*Tan[c + d*x])^(5/2))/(5*d*e)
```


Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
```

```
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2(e\tan(c+dx))^{3/2}\,dx = \int \left(a^2(e\tan(c+dx))^{3/2}+2a^2\sec(c+dx)(e\tan(c+dx))^{3/2}+a^2\sec^2(c+d\right.$$

$$= a^2\int (e\tan(c+dx))^{3/2}\,dx + a^2\int \sec^2(c+dx)(e\tan(c+dx))^{3/2}\,dx + \left(2a^2\right)$$

$$= \frac{2a^2e\sqrt{e\tan(c+dx)}}{d} + \frac{4a^2e\sec(c+dx)\sqrt{e\tan(c+dx)}}{3d} + \frac{a^2\,\mathrm{Subst}\left(\int (ex)^3\right.}{}$$

$$= \frac{2a^2e\sqrt{e\tan(c+dx)}}{d} + \frac{4a^2e\sec(c+dx)\sqrt{e\tan(c+dx)}}{3d} + \frac{2a^2(e\tan(c+dx}{5de}$$

$$= \frac{2a^2e\sqrt{e\tan(c+dx)}}{d} + \frac{4a^2e\sec(c+dx)\sqrt{e\tan(c+dx)}}{3d} + \frac{2a^2(e\tan(c+dx}{5de}$$

$$= -\frac{2a^2e^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3d\sqrt{e\tan(c+dx)}} + \frac{2a^2e\sqrt{e\tan(c+dx)}}{d}$$

$$= -\frac{2a^2e^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3d\sqrt{e\tan(c+dx)}} + \frac{2a^2e\sqrt{e\tan(c+dx)}}{d}$$

$$= \frac{a^2e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{a^2e^{3/2}\log\left(\sqrt{e}+\sqrt{\right.}{}$$

$$= \frac{a^2e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{a^2e^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{a^2e^{3/}}{}$$

Mathematica [C] time = 11.823, size = 257, normalized size = 0.77

$$\frac{a^2\cos^4\left(\frac{1}{2}(c+dx)\right)(e\tan(c+dx))^{3/2}\sec^4\left(\frac{1}{2}\tan^{-1}(\tan(c+dx))\right)\left(-80\sqrt{\tan(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a^2*Cos[(c + d*x)/2]^4*Sec[ArcTan[Tan[c + d*x]]/2]^4*(e*Tan[c + d*x])^(3/2
)*(30*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 30*Sqrt[2]*ArcTan[1
+ Sqrt[2]*Sqrt[Tan[c + d*x]]] + 15*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x
]] + Tan[c + d*x]] - 15*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
+ d*x]] + 120*Sqrt[Tan[c + d*x]] - 80*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan
[c + d*x]^2]*Sqrt[Tan[c + d*x]] + 80*Sqrt[Sec[c + d*x]^2]*Sqrt[Tan[c + d*x]
] + 24*Tan[c + d*x]^(5/2)))/(60*d*Tan[c + d*x]^(3/2))
```

Maple [C] time = 0.26, size = 721, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(3/2),x)
```

```
[Out] 1/30*a^2/d*2^(1/2)*(-1+cos(d*x+c))*(-15*I*sin(d*x+c)*EllipticPi(((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+15*I*sin(d*x+c)*EllipticPi(((
1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+15*sin(d*x+c)*Ellipt
icPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+15*sin(d*x+c)
*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1
/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-10*sin
(d*x+c)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
```

```
^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+24*cos(d*x
+c)^3*2^(1/2)-4*cos(d*x+c)^2*2^(1/2)-14*cos(d*x+c)*2^(1/2)-6*2^(1/2))*(cos(
d*x+c)+1)^2*(e*sin(d*x+c)/cos(d*x+c))^(3/2)/cos(d*x+c)/sin(d*x+c)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(3/2),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*tan(d*x+c))**(3/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^2 (e\tan(dx+c))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*(e*tan(d*x + c))^(3/2), x)
```

3.113 $\int (a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}\,dx$

Optimal. Leaf size=309

$$-\frac{a^2\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}+\frac{a^2\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}+\frac{2a^2(e\tan(c+dx))^{3/2}}{3de}+\frac{a^2\sqrt{e}\log\left(\sqrt{e}\tan(c+dx)\cdots\right)}{2\cdots}$$

```
[Out] -((a^2*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]
*d)) + (a^2*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sq
rt[2]*d) + (a^2*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e
*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (a^2*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c +
 d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (4*a^2*Cos[c + d*x]*
EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(d*Sqrt[Sin[2*c + 2*d*x]
]) + (2*a^2*(e*Tan[c + d*x])^(3/2))/(3*d*e) + (4*a^2*Cos[c + d*x]*(e*Tan[c
+ d*x])^(3/2))/(d*e)
```

Rubi [A] time = 0.335517, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3886, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639, 2607, 32}

$$-\frac{a^2\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}+\frac{a^2\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}+\frac{2a^2(e\tan(c+dx))^{3/2}}{3de}+\frac{a^2\sqrt{e}\log\left(\sqrt{e}\tan(c+dx)\cdots\right)}{2\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] -((a^2*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]
*d)) + (a^2*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sq
rt[2]*d) + (a^2*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e
*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (a^2*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c +
 d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (4*a^2*Cos[c + d*x]*
EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(d*Sqrt[Sin[2*c + 2*d*x]
]) + (2*a^2*(e*Tan[c + d*x])^(3/2))/(3*d*e) + (4*a^2*Cos[c + d*x]*(e*Tan[c
+ d*x])^(3/2))/(d*e)
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
```

```
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}\,dx = \int \left(a^2\sqrt{e\tan(c+dx)}+2a^2\sec(c+dx)\sqrt{e\tan(c+dx)}+a^2\sec^2(c+dx)\sqrt{e\tan\right.$$

$$= a^2\int \sqrt{e\tan(c+dx)}\,dx + a^2\int \sec^2(c+dx)\sqrt{e\tan(c+dx)}\,dx + \left(2a^2\right)\int \sec($$

$$= \frac{4a^2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - \left(4a^2\right)\int \cos(c+dx)\sqrt{e\tan(c+dx)}\,dx + $$

$$= \frac{2a^2(e\tan(c+dx))^{3/2}}{3de} + \frac{4a^2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} + \frac{\left(2a^2e\right)\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{2a^2(e\tan(c+dx))^{3/2}}{3de} + \frac{4a^2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - \frac{\left(a^2e\right)\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}} + \frac{2a^2(e\tan(c+dx))^{3/2}}{3de} + $$

$$= \frac{a^2\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{a^2\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan\right.}{}$$

$$= -\frac{a^2\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{a^2\sqrt{e}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + $$

Mathematica [C] time = 1.27132, size = 106, normalized size = 0.34

$$\frac{4a^2\sin\left(\frac{1}{2}(c+dx)\right)\cos^5\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{e\tan(c+dx)}\sec^4\left(\frac{1}{2}\tan^{-1}(\tan(c+dx))\right)\left(2\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\right.\right.}{3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] (4*a^2*Cos[(c + d*x)/2]^5*(1 + 2*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c +
d*x]^2] + Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2])*Sec[c + d*x]*Sec
```

```
[ArcTan[Tan[c + d*x]]/2]^4*Sin[(c + d*x)/2]*Sqrt[e*Tan[c + d*x]])/(3*d)
```

Maple [C] time = 0.263, size = 1480, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(1/2),x)
```

```
[Out] -1/6*a^2/d*2^(1/2)*(cos(d*x+c)+1)^2*(e*sin(d*x+c)/cos(d*x+c))^(1/2)*(-1+cos
(d*x+c))^2*(-3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/
2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos
(d*x+c)^2+3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1
/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*
x+c)^2+3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^
2+3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-24*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2+12*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2-3*I*cos(d*x+c)*EllipticPi(((1-cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+3*I*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
),1/2-1/2*I,1/2*2^(1/2))+3*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1
/2*I,1/2*2^(1/2))-24*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c)
```

```
)^(1/2)*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))
+12*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF
(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+14*cos(d*x+c)^2*
2^(1/2)-12*cos(d*x+c)*2^(1/2)-2*2^(1/2))/sin(d*x+c)^5/cos(d*x+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int\sqrt{e\tan{(c+dx)}}\,dx+\int 2\sqrt{e\tan{(c+dx)}}\sec{(c+dx)}\,dx+\int\sqrt{e\tan{(c+dx)}}\sec^2{(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*tan(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(sqrt(e*tan(c + d*x)), x) + Integral(2*sqrt(e*tan(c + d*x))*s
ec(c + d*x), x) + Integral(sqrt(e*tan(c + d*x))*sec(c + d*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sec (dx + c) + a)^2 \sqrt{e \tan (dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sqrt(e*tan(d*x + c)), x)
```

3.114 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}\,dx$

Optimal. Leaf size=278

$$\frac{2a^2\sqrt{\sin(2c+2dx)}\sec(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}-\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}}+\frac{a^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\sqrt{e}}+$$

```
[Out] -((a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*Sqrt[
e])) + (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*
Sqrt[e]) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) + (2*a^2*EllipticF[c -
 Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(d*Sqrt[e*Tan[c + d*x]
]) + (2*a^2*Sqrt[e*Tan[c + d*x]])/(d*e)
```

Rubi [A] time = 0.303158, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.56, Rules used = {3886, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641, 2607, 32}

$$-\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}}+\frac{a^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\sqrt{e}}+\frac{2a^2\sqrt{e\tan(c+dx)}}{de}-\frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan}\right.}{2\sqrt{2}d\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sqrt[e*Tan[c + d*x]],x]
```


```
[Out] -((a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*Sqrt[
e])) + (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*
Sqrt[e]) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) + (2*a^2*EllipticF[c -
 Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(d*Sqrt[e*Tan[c + d*x]
]) + (2*a^2*Sqrt[e*Tan[c + d*x]])/(d*e)
```


Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
```

```
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}\,dx = \int \left(\frac{a^2}{\sqrt{e\tan(c+dx)}} + \frac{2a^2\sec(c+dx)}{\sqrt{e\tan(c+dx)}} + \frac{a^2\sec^2(c+dx)}{\sqrt{e\tan(c+dx)}}\right)dx$$

$$= a^2\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx + a^2\int \frac{\sec^2(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx + \left(2a^2\right)\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx$$

$$= \frac{a^2\,\text{Subst}\left(\int \frac{1}{\sqrt{ex}}\,dx, x, \tan(c+dx)\right)}{d} + \frac{\left(a^2e\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(e^2+x^2\right)}\,dx, x, e\tan(c+dx)\right)}{d} + \frac{(2a}{}$$

$$= \frac{2a^2\sqrt{e\tan(c+dx)}}{de} + \frac{\left(2a^2e\right)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d} + \frac{\left(2a^2\sec(c+dx)\sqrt{\sin}\right.}{\sqrt{e}}$$

$$= \frac{2a^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d\sqrt{e\tan(c+dx)}} + \frac{2a^2\sqrt{e\tan(c+dx)}}{de} + \frac{a^2\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\right.}{}$$

$$= \frac{2a^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d\sqrt{e\tan(c+dx)}} + \frac{2a^2\sqrt{e\tan(c+dx)}}{de} + \frac{a^2\,\text{Subst}\left(\int \frac{}{e-\sqrt{2}}\right.}{}$$

$$= -\frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} + \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{}\right.}{2\sqrt{2}d\sqrt{e}}$$

$$= -\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} + \frac{a^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+d\right.}{2\sqrt{2}d}$$

Mathematica [C] time = 2.50198, size = 220, normalized size = 0.79

$$\frac{a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\tan(c+dx)}\sec^4\left(\frac{1}{2}\tan^{-1}(\tan(c+dx))\right)\left(16\sqrt{\tan(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(c\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] (a^2*Cos[(c + d*x)/2]^4*Sec[ArcTan[Tan[c + d*x]]/2]^4*(-2*Sqrt[2]*ArcTan[1
- Sqrt[2]*Sqrt[Tan[c + d*x]]] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d
*x]]] - Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + Sqrt[2
]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 8*Sqrt[Tan[c + d*x]]
 + 16*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c + d*x]^2]*Sqrt[Tan[c + d*x]])
*Sqrt[Tan[c + d*x]])/(4*d*Sqrt[e*Tan[c + d*x]])
```

Maple [C] time = 0.463, size = 655, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x)
```

```
[Out] 1/2*a^2/d*2^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(-I*sin(d*x+c)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+I*sin(d*x+c)*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2),1/2-1/2*I,1/2*2^(1/2))-sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/
2+1/2*I,1/2*2^(1/2))-2*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2
))+2*cos(d*x+c)*2^(1/2)-2*2^(1/2))/sin(d*x+c)^3/cos(d*x+c)/(e*sin(d*x+c)/co
s(d*x+c))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int \frac{1}{\sqrt{e\tan{(c+dx)}}}\,dx + \int \frac{2\sec{(c+dx)}}{\sqrt{e\tan{(c+dx)}}}\,dx + \int \frac{\sec^2{(c+dx)}}{\sqrt{e\tan{(c+dx)}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*tan(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(1/sqrt(e*tan(c + d*x)), x) + Integral(2*sec(c + d*x)/sqrt(e*
tan(c + d*x)), x) + Integral(sec(c + d*x)**2/sqrt(e*tan(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec{(dx+c)}+a)^2}{\sqrt{e\tan{(dx+c)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(e*tan(d*x + c)), x)
```

3.115 $\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=310

$$\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}-\frac{a^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{3/2}}-\frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}}+\frac{a^2\log}{}$$

```
[Out] (a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(3/2)
) - (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(
3/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*
x]]])/(2*Sqrt[2]*d*e^(3/2)) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqr
t[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(3/2)) - (4*a^2)/(d*e*Sqrt[e*Tan
[c + d*x]]) - (4*a^2*Cos[c + d*x])/(d*e*Sqrt[e*Tan[c + d*x]]) - (4*a^2*Cos[
c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(d*e^2*Sqrt[Sin
[2*c + 2*d*x]])
```

Rubi [A] time = 0.387875, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {3886, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2608, 2615, 2572, 2639, 2607, 32}

$$\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}-\frac{a^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{3/2}}-\frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}}+\frac{a^2\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(3/2),x]
```


```
[Out] (a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(3/2)
) - (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(
3/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*
x]]])/(2*Sqrt[2]*d*e^(3/2)) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqr
t[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(3/2)) - (4*a^2)/(d*e*Sqrt[e*Tan
[c + d*x]]) - (4*a^2*Cos[c + d*x])/(d*e*Sqrt[e*Tan[c + d*x]]) - (4*a^2*Cos[
c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(d*e^2*Sqrt[Sin
[2*c + 2*d*x]])
```


Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2608

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(n + 1)), x] - Dist[(a^2*(m - 2))/(b^2*(n + 1)), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx = \int \left(\frac{a^2}{(e\tan(c+dx))^{3/2}}+\frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{3/2}}+\frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{3/2}}\right)dx$$

$$= a^2\int \frac{1}{(e\tan(c+dx))^{3/2}}\,dx + a^2\int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx + \left(2a^2\right)\int \frac{\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx$$

$$= -\frac{2a^2}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{(ex)^{3/2}}\,dx, x, \tan(c+dx)\right)}{d} - \frac{a^2\int \sqrt{e}\cdots}{\cdots}$$

$$= -\frac{4a^2}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}} - \frac{a^2\,\text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\tan(c+dx)\right)}{de} - \frac{\left(4a^2\sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{4a^2}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}} - \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{de} - \frac{(\cdots}{\cdots}$$

$$= -\frac{4a^2}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{de^2\sqrt{\sin(2c+2dx)}}\cdots$$

$$= -\frac{4a^2}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{de^2\sqrt{\sin(2c+2dx)}}\cdots$$

$$= -\frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{3/2}} + \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{\cdots}\right.}{2\sqrt{2}de^{3/2}}\cdots$$

$$= \frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} - \frac{a^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)\cdots\right.}{2\sqrt{2}de^{3\cdots}}$$

Mathematica [C] time = 7.22865, size = 238, normalized size = 0.77

$$\frac{a^2\left(8e^{3i(c+dx)}\sqrt{1-e^{4i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},e^{4i(c+dx)}\right)-24\left(e^{i(c+dx)}+e^{2i(c+dx)}+e^{3i(c+dx)}+1\right)-3\sqrt{-1+e^{4i(c+dx)}}\right.}{6de\left(1+e^{2i(c+dx)}\right)\sqrt{e\tan(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a^2*(-24*(1 + E^(I*(c + d*x)) + E^((2*I)*(c + d*x)) + E^((3*I)*(c + d*x)))
 - 3*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]]
+ 6*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sq
rt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]] + 8*E^((3*I)*(c +
 d*x))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4
*I)*(c + d*x))]))/(6*d*e*(1 + E^((2*I)*(c + d*x)))*Sqrt[e*Tan[c + d*x]])
```

Maple [C] time = 0.24, size = 1392, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(3/2),x)
```

```
[Out] -1/2*a^2/d*2^(1/2)*(I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*El
lipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)*cos(d*x+c)-I*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1
+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(
1/2))-cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+I*(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-cos(d*x+c)*((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-I*((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x
```

```
+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2),1/2+1/2*I,1/2*2^(1/2))-8*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*
2^(1/2))+4*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*El
lipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2),1/2+1/2*I,1/2*2^(1/2))-8*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+4*(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+8*cos(d*x+c)*2^(1/2))*sin(d*x+c)
/(e*sin(d*x+c)/cos(d*x+c))^(3/2)/cos(d*x+c)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{2}\left(\int \frac{1}{(e\tan{(c+dx)})^{\frac{3}{2}}}\,dx+\int \frac{2\sec{(c+dx)}}{(e\tan{(c+dx)})^{\frac{3}{2}}}\,dx+\int \frac{\sec^{2}{(c+dx)}}{(e\tan{(c+dx)})^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*tan(d*x+c))**(3/2),x)
```

```
[Out] a**2*(Integral((e*tan(c + d*x))**(-3/2), x) + Integral(2*sec(c + d*x)/(e*ta
n(c + d*x))**(3/2), x) + Integral(sec(c + d*x)**2/(e*tan(c + d*x))**(3/2),
x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec{(dx+c)}+a)^{2}}{(e\tan{(dx+c)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/(e*tan(d*x + c))^(3/2), x)
```

3.116 $\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=316

$$-\frac{2a^2\sqrt{\sin(2c+2dx)}\sec(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3de^2\sqrt{e\tan(c+dx)}}+\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}}-\frac{a^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{5/2}}$$

```
[Out] (a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(5/2)
) - (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(
5/2)) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*
x]]])/(2*Sqrt[2]*d*e^(5/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqr
t[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(5/2)) - (4*a^2)/(3*d*e*(e*Tan[c
 + d*x])^(3/2)) - (4*a^2*Sec[c + d*x])/(3*d*e*(e*Tan[c + d*x])^(3/2)) - (2*
a^2*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*d*
e^2*Sqrt[e*Tan[c + d*x]])
```

Rubi [A] time = 0.386216, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {3886, 3474, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2609, 2614, 2573, 2641, 2607, 32}

$$\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}}-\frac{a^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{5/2}}+\frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{5/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] (a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(5/2)
) - (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(
5/2)) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*
x]]])/(2*Sqrt[2]*d*e^(5/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqr
t[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(5/2)) - (4*a^2)/(3*d*e*(e*Tan[c
 + d*x])^(3/2)) - (4*a^2*Sec[c + d*x])/(3*d*e*(e*Tan[c + d*x])^(3/2)) - (2*
a^2*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*d*
e^2*Sqrt[e*Tan[c + d*x]])
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{5/2}}\,dx = \int \left(\frac{a^2}{(e\tan(c+dx))^{5/2}}+\frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{5/2}}+\frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{5/2}}\right)dx$$

$$= a^2\int \frac{1}{(e\tan(c+dx))^{5/2}}\,dx + a^2\int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx + \left(2a^2\right)\int \frac{\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx$$

$$= -\frac{2a^2}{3de(e\tan(c+dx))^{3/2}} - \frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}} + \frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{(ex)^{5/2}}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2$$

$$= -\frac{4a^2}{3de(e\tan(c+dx))^{3/2}} - \frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}} - \frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\left(e^2+x^2\right)}\,dx, x, e\tan(c+dx)\right)}{de}$$

$$= -\frac{4a^2}{3de(e\tan(c+dx))^{3/2}} - \frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}} - \frac{\left(2a^2\right)\mathrm{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{de}$$

$$= -\frac{4a^2}{3de(e\tan(c+dx))^{3/2}} - \frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}} - \frac{2a^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+}}{3de^2\sqrt{e\tan(c+dx)}}$$

$$= -\frac{4a^2}{3de(e\tan(c+dx))^{3/2}} - \frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}} - \frac{2a^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+}}{3de^2\sqrt{e\tan(c+dx)}}$$

$$= \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{5/2}} - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\,\mathrm{ta}}\right.}{2\sqrt{2}de^{5/2}}$$

$$= \frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} - \frac{a^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} + \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\right.}{2\sqrt{2}de^{5/2}}$$

Mathematica [C] time = 4.8991, size = 224, normalized size = 0.71

$$-\frac{a^2\cos^2\left(\frac{1}{2}(c+dx)\right)\cos(c+dx)\cot\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}\tan^{-1}(\tan(c+dx))\right)\left(16\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-\tan\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] -(a^2*Cos[(c + d*x)/2]^2*Cos[c + d*x]*Cot[(c + d*x)/2]*Sec[ArcTan[Tan[c + d
*x]]/2]^4*(16*Hypergeometric2F1[-3/4, 1/2, 1/4, -Tan[c + d*x]^2] + 16*Hyper
geometric2F1[-3/4, 1, 1/4, -Tan[c + d*x]^2] + 3*Sqrt[2]*(2*ArcTan[1 - Sqrt[
2]*Sqrt[Tan[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + Log[1 -
 Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c +
d*x]] + Tan[c + d*x]])*Tan[c + d*x]^(3/2)))/(24*d*e^2*Sqrt[e*Tan[c + d*x]])
```

Maple [C] time = 0.226, size = 650, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(5/2),x)
```

```
[Out] 1/6*a^2/d*2^(1/2)*(-1+cos(d*x+c))*(3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*sin(d*x+c)-3*I*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
),1/2+1/2*I,1/2*2^(1/2))+3*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1
/2*I,1/2*2^(1/2))+3*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/
2*2^(1/2))-2*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*
EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+4*cos(d
*x+c)*2^(1/2))*(cos(d*x+c)+1)^2/sin(d*x+c)/cos(d*x+c)^3/(e*sin(d*x+c)/cos(d
```

```
*x+c))^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^2}{(e\tan(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/(e*tan(d*x + c))^(5/2), x)
```

3.117 $\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=370

$$-\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}}+\frac{a^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{7/2}}+\frac{2a^2}{de^3\sqrt{e\tan(c+dx)}}+\frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan}\right.}{2\sqrt{2}de^{7/2}}$$

```
[Out] -((a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(7/
2))) + (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*
e^(7/2)) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*d*e^(7/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(7/2)) - (4*a^2)/(5*d*e*(e*Ta
n[c + d*x])^(5/2)) - (4*a^2*Sec[c + d*x])/(5*d*e*(e*Tan[c + d*x])^(5/2)) +
(2*a^2)/(d*e^3*Sqrt[e*Tan[c + d*x]]) + (12*a^2*Cos[c + d*x])/(5*d*e^3*Sqrt[
e*Tan[c + d*x]]) + (12*a^2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e
*Tan[c + d*x]])/(5*d*e^4*Sqrt[Sin[2*c + 2*d*x]])
```

Rubi [A] time = 0.455875, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.68, Rules used = {3886, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2609, 2608, 2615, 2572, 2639, 2607, 32}

$$-\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}}+\frac{a^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{7/2}}+\frac{2a^2}{de^3\sqrt{e\tan(c+dx)}}+\frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan}\right.}{2\sqrt{2}de^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(7/2),x]
```


```
[Out] -((a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(7/
2))) + (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*
e^(7/2)) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*d*e^(7/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(7/2)) - (4*a^2)/(5*d*e*(e*Ta
n[c + d*x])^(5/2)) - (4*a^2*Sec[c + d*x])/(5*d*e*(e*Tan[c + d*x])^(5/2)) +
(2*a^2)/(d*e^3*Sqrt[e*Tan[c + d*x]]) + (12*a^2*Cos[c + d*x])/(5*d*e^3*Sqrt[
e*Tan[c + d*x]]) + (12*a^2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e
*Tan[c + d*x]])/(5*d*e^4*Sqrt[Sin[2*c + 2*d*x]])
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2608

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(n + 1)), x] - Dist[(a^2*(m - 2))/(b^2*(n + 1)), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx = \int \left(\frac{a^2}{(e\tan(c+dx))^{7/2}}+\frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{7/2}}+\frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{7/2}}\right)dx$$

$$= a^2\int \frac{1}{(e\tan(c+dx))^{7/2}}\,dx + a^2\int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx + \left(2a^2\right)\int \frac{\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx$$

$$= -\frac{2a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{(ex)^{7/2}}\,dx, x, \tan(c+dx)\right)}{d} - \frac{a^2}{}$$

$$= -\frac{4a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} + \frac{2a^2}{de^3\sqrt{e\tan(c+dx)}} + \frac{12a^2\cos(c+dx)}{5de^3\sqrt{e\tan(c+dx)}}$$

$$= -\frac{4a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} + \frac{2a^2}{de^3\sqrt{e\tan(c+dx)}} + \frac{12a^2\cos(c+dx)}{5de^3\sqrt{e\tan(c+dx)}}$$

$$= -\frac{4a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} + \frac{2a^2}{de^3\sqrt{e\tan(c+dx)}} + \frac{12a^2\cos(c+dx)}{5de^3\sqrt{e\tan(c+dx)}}$$

$$= -\frac{4a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} + \frac{2a^2}{de^3\sqrt{e\tan(c+dx)}} + \frac{12a^2\cos(c+dx)}{5de^3\sqrt{e\tan(c+dx)}}$$

$$= -\frac{4a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} + \frac{2a^2}{de^3\sqrt{e\tan(c+dx)}} + \frac{12a^2\cos(c+dx)}{5de^3\sqrt{e\tan(c+dx)}}$$

$$= \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{7/2}} - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan}\right.}{2\sqrt{2}de^{7/2}}$$

$$= -\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} + \frac{a^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} + \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx\right.}{2\sqrt{2}de^{7}}$$

Mathematica [C] time = 13.9828, size = 2820, normalized size = 7.62

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(7/2),x]
```

```
[Out] (Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((7*Cot[c/2])/(10*d) - (Cot[c/
2]*Csc[c/2 + (d*x)/2]^2)/(20*d) - (3*(4*Cos[c/2] - Cos[(3*c)/2] + Cos[(5*c)
/2])*Cos[d*x]*Sec[2*c]*Sin[c/2])/(10*d*(-1 + 2*Cos[c])) - (7*Csc[c/2]*Csc[c
/2 + (d*x)/2]*Sin[(d*x)/2])/(10*d) + (Csc[c/2]*Csc[c/2 + (d*x)/2]^3*Sin[(d*
```

```
x)/2])/(20*d) - (3*(2 - 5*Cos[c] + 6*Cos[2*c] + Cos[3*c])*Sec[2*c]*Sin[d*x]
)/(20*d*(-1 + 2*Cos[c])))*Sin[c + d*x]^2*Tan[c + d*x]^2)/(e*Tan[c + d*x])^(
7/2) + ((E^((2*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*
I)*(c + d*x))]] - 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d
*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*
Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Tan[c +
 d*x]^(7/2))/(16*d*E^(I*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((
2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2*Cos[c])*(e*Tan[c + d*x]
)^(7/2)) + ((-(E^((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 +
E^((4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)
*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x
)))]])*Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*
Tan[c + d*x]^(7/2))/(16*d*E^((2*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x)))
)/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2*Cos[c])*(e*T
an[c + d*x])^(7/2)) - ((-(E^((6*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan
[Sqrt[-1 + E^((4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1
 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*
I)*(c + d*x)))]])*Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c
 + d*x])^2*Tan[c + d*x]^(7/2))/(16*d*E^((3*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*
(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2*C
os[c])*(e*Tan[c + d*x])^(7/2)) + ((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sq
rt[-1 + E^((4*I)*(c + d*x))]] - 2*E^((2*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))
]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1
+ E^((2*I)*(c + d*x)))]])*Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a +
 a*Sec[c + d*x])^2*Tan[c + d*x]^(7/2))/(16*d*E^(I*c)*Sqrt[((-I)*(-1 + E^((2
*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 +
 2*Cos[c])*(e*Tan[c + d*x])^(7/2)) - ((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTa
n[Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2*E^((4*I)*c)*Sqrt[-1 + E^((2*I)*(c + d
*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))
/(1 + E^((2*I)*(c + d*x)))]])*Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*
(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(7/2))/(16*d*E^((2*I)*c)*Sqrt[((-I)*(-1
 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)
))*(-1 + 2*Cos[c])*(e*Tan[c + d*x])^(7/2)) + ((Sqrt[-1 + E^((4*I)*(c + d*x)
)]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2*E^((6*I)*c)*Sqrt[-1 + E^((2*I
)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c
+ d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*
x)/2]^4*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(7/2))/(16*d*E^((3*I)*c)*Sqrt[(
(-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(
c + d*x)))*(-1 + 2*Cos[c])*(e*Tan[c + d*x])^(7/2)) - (Cos[c + d*x]^2*(3 - 3
*E^((4*I)*(c + d*x)) + E^((4*I)*(c + d*x))*(1 + E^((2*I)*c))*Sqrt[1 - E^((4
*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[
2*c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(7/2))/(20*d*
E^(I*(2*c + d*x))*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c +
 d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2*Cos[c])*(e*Tan[c + d*x])^(7/2))
- (Cos[c + d*x]^2*(3 - 3*E^((4*I)*(c + d*x)) + E^((2*I)*(c + 2*d*x))*(1 + E
```

```
^((2*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4,
E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*
Tan[c + d*x]^(7/2))/(20*d*E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/
(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2*Cos[c])*(e*Tan
[c + d*x])^(7/2)) + (E^(I*(c - d*x))*Cos[c + d*x]^2*(3 - 3*E^((4*I)*(c + d*
x)) + E^((4*I)*d*x)*(1 + E^((4*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hyperge
ometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c/2 + (d*x)/2]
^4*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(7/2))/(8*d*Sqrt[((-I)*(-1 + E^((2*I
)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2
*Cos[c])*(e*Tan[c + d*x])^(7/2)) - (Cos[c + d*x]^2*(3 - 3*E^((4*I)*(c + d*x
)) + E^((4*I)*(c + d*x))*(1 + E^((4*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hy
pergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c/2 + (d*
x)/2]^4*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(7/2))/(40*d*E^(I*(3*c + d*x))*
Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((
2*I)*(c + d*x)))*(-1 + 2*Cos[c])*(e*Tan[c + d*x])^(7/2)) - (Cos[c + d*x]^2*
(-3*E^((2*I)*c)*(-1 + E^((4*I)*(c + d*x))) + E^((4*I)*d*x)*(1 + E^((6*I)*c)
)*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(
c + d*x))])*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Tan[c + d*
x]^(7/2))/(10*d*E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2
*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2*Cos[c])*(e*Tan[c + d*x])
^(7/2))
```

Maple [C] time = 0.291, size = 1429, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(7/2),x)
```

```
[Out] 1/10*a^2/d*2^(1/2)*(5*I*cos(d*x+c)^2*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/
2*I,1/2*2^(1/2))-5*I*cos(d*x+c)^2*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I
,1/2*2^(1/2))-24*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2+1
2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2-5*((-1+cos(d*x+c
```

```
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-5*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-5*I*((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+5*I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/
2*I,1/2*2^(1/2))+24*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Elli
pticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-12*((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+5*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1
/2*I,1/2*2^(1/2))+5*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Elli
pticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+
26*cos(d*x+c)^2*2^(1/2)-22*cos(d*x+c)*2^(1/2))*sin(d*x+c)^3/(-1+cos(d*x+c))
/cos(d*x+c)^4/(e*sin(d*x+c)/cos(d*x+c))^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^2}{(e\tan(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/(e*tan(d*x + c))^(7/2), x)
```

3.118 $\int \frac{(e\tan(c+dx))^{11/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=330

$$\frac{5e^6\sqrt{\sin(2c+2dx)}\sec(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{21ad\sqrt{e\tan(c+dx)}}+\frac{e^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}-\frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+\right.}{\sqrt{2}ad}$$

```
[Out] (e^(11/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d)
 - (e^(11/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a
*d) + (e^(11/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*a*d) - (e^(11/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + S
qrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) + (5*e^6*EllipticF[c - Pi/4 +
 d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(21*a*d*Sqrt[e*Tan[c + d*x]])
 + (2*e^5*(21 - 5*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]])/(21*a*d) - (2*e^3*(7
- 5*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2))/(35*a*d)
```

Rubi [A] time = 0.419045, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.56, Rules used = {3888, 3881, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$\frac{e^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}-\frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad}+\frac{e^{11/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(11/2)/(a + a*Sec[c + d*x]),x]
```



```
[Out] (e^(11/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d)
 - (e^(11/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a
*d) + (e^(11/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*a*d) - (e^(11/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + S
qrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) + (5*e^6*EllipticF[c - Pi/4 +
 d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(21*a*d*Sqrt[e*Tan[c + d*x]])
 + (2*e^5*(21 - 5*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]])/(21*a*d) - (2*e^3*(7
- 5*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2))/(35*a*d)
```


Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{11/2}}{a+a\sec(c+dx)}\,dx = \frac{e^2\int(-a+a\sec(c+dx))(e\tan(c+dx))^{7/2}\,dx}{a^2}$$

$$= -\frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx))^{5/2}}{35ad} - \frac{\left(2e^4\right)\int\left(-\frac{7a}{2}+\frac{5}{2}a\sec(c+dx)\right)(e\tan(c+dx))^{3/2}}{7a^2}$$

$$= \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad} - \frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx))^{5/2}}{35ad} + \frac{\left(4e^6\right)\int}{}$$

$$= \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad} - \frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx))^{5/2}}{35ad} + \frac{\left(5e^6\right)\int}{2}$$

$$= \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad} - \frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx))^{5/2}}{35ad} - \frac{e^7\,\mathrm{Subst}}{}$$

$$= \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad} - \frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx))^{5/2}}{35ad} - \frac{\left(2e^7\right)\mathrm{Sub}}{}$$

$$= \frac{5e^6F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{21ad\sqrt{e\tan(c+dx)}} + \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad} - 2$$

$$= \frac{5e^6F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{21ad\sqrt{e\tan(c+dx)}} + \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad} - 2$$

$$= \frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} - \frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{}\right.}{2\sqrt{2}ad}$$

$$= \frac{e^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} - \frac{e^{11/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+\right.}{2\sqrt{2}}$$

Mathematica [C] time = 19.7406, size = 332, normalized size = 1.01

$$\frac{e^5\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\sqrt{\sec^2(c+dx)}+1\right)\sqrt{e\tan(c+dx)}\left(-320\sqrt{\tan(c+dx)}\mathrm{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Tan[c + d*x])^(11/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (e^5*Cos[(c + d*x)/2]^2*Sec[c + d*x]*(1 + Sqrt[Sec[c + d*x]^2])*Sqrt[e*Tan[
c + d*x]]*(70*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 70*Sqrt[2]*A
rcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + 35*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]] - 35*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
+ Tan[c + d*x]] + 280*Sqrt[Tan[c + d*x]] - 320*Hypergeometric2F1[-1/2, 1/4,
 5/4, -Tan[c + d*x]^2]*Sqrt[Tan[c + d*x]] + 280*Hypergeometric2F1[1/4, 1/2,
 5/4, -Tan[c + d*x]^2]*Sqrt[Tan[c + d*x]] + 40*Sqrt[Sec[c + d*x]^2]*Sqrt[Ta
n[c + d*x]] - 56*Tan[c + d*x]^(5/2) + 40*Sqrt[Sec[c + d*x]^2]*Tan[c + d*x]^
(5/2)))/(70*a*d*(1 + Sec[c + d*x])^2*Sqrt[Tan[c + d*x]])
```

Maple [C] time = 0.26, size = 734, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/210/a/d*2^(1/2)*(-1+cos(d*x+c))*(105*I*sin(d*x+c)*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-105*I*sin(d*x+c)*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi(((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-105*sin(d*x+c)*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi(((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-105*sin(d*x
+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*Ellipti
cPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+260
*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3
*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-252*2^
(1/2)*cos(d*x+c)^4+332*cos(d*x+c)^3*2^(1/2)-38*cos(d*x+c)^2*2^(1/2)-72*cos(
d*x+c)*2^(1/2)+30*2^(1/2))*cos(d*x+c)^2*(cos(d*x+c)+1)^2*(e*sin(d*x+c)/cos(
d*x+c))^(11/2)/sin(d*x+c)^9
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(11/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^{\frac{11}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^(11/2)/(a*sec(d*x + c) + a), x)
```

3.119 $\int \frac{(e\tan(c+dx))^{9/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=326

$$-\frac{e^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}+\frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad}+\frac{e^{9/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad}$$

```
[Out] -((e^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
)) + (e^(9/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
a*d) + (e^(9/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*a*d) - (e^(9/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sq
rt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) + (6*e^4*Cos[c + d*x]*Elliptic
E[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*a*d*Sqrt[Sin[2*c + 2*d*x]]) -
 (6*e^3*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(5*a*d) - (2*e^3*(5 - 3*Sec[c
+ d*x])*(e*Tan[c + d*x])^(3/2))/(15*a*d)
```

Rubi [A] time = 0.39367, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3888, 3881, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$-\frac{e^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}+\frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad}+\frac{e^{9/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(9/2)/(a + a*Sec[c + d*x]),x]
```


```
[Out] -((e^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
)) + (e^(9/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
a*d) + (e^(9/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*a*d) - (e^(9/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sq
rt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) + (6*e^4*Cos[c + d*x]*Elliptic
E[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*a*d*Sqrt[Sin[2*c + 2*d*x]]) -
 (6*e^3*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(5*a*d) - (2*e^3*(5 - 3*Sec[c
+ d*x])*(e*Tan[c + d*x])^(3/2))/(15*a*d)
```


Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{9/2}}{a+a\sec(c+dx)}\,dx = \frac{e^2\int(-a+a\sec(c+dx))(e\tan(c+dx))^{5/2}\,dx}{a^2}$$

$$= -\frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad} - \frac{\left(2e^4\right)\int\left(-\frac{5a}{2}+\frac{3}{2}a\sec(c+dx)\right)\sqrt{e\tan(c+dx)}\,dx}{5a^2}$$

$$= -\frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad} - \frac{\left(3e^4\right)\int\sec(c+dx)\sqrt{e\tan(c+dx)}\,dx}{5a} + \frac{e^4\int\sqrt{e\,\mathrm{ta}}}{}$$

$$= -\frac{6e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ad} - \frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad} + \frac{\left(6e^4\right)\int\cos(c+}{}$$

$$= -\frac{6e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ad} - \frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad} + \frac{\left(2e^5\right)\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{6e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ad} - \frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad} - \frac{e^5\,\mathrm{Subst}\left(\int\frac{e-}{e^2+}\right.}{}$$

$$= \frac{6e^4\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5ad\sqrt{\sin(2c+2dx)}} - \frac{6e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ad} - \frac{2e^3(5-}{}$$

$$= \frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} - \frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\,\mathrm{ta}}\right.}{2\sqrt{2}ad}$$

$$= -\frac{e^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{e^{9/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+d\right.}{2\sqrt{2}a}$$

Mathematica [C] time = 12.1554, size = 129, normalized size = 0.4

$$\frac{4e^3\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\sqrt{\sec^2(c+dx)}+1\right)(e\tan(c+dx))^{3/2}\left(\mathrm{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\tan^2(c+dx)\right)\right.}{3ad(\sec(c+dx)+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Tan[c + d*x])^(9/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (4*e^3*Cos[(c + d*x)/2]^2*(-1 + Hypergeometric2F1[-1/2, 3/4, 7/4, -Tan[c +
d*x]^2] - Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] + Hypergeometri
c2F1[3/4, 1, 7/4, -Tan[c + d*x]^2])*Sec[c + d*x]*(1 + Sqrt[Sec[c + d*x]^2])
*(e*Tan[c + d*x])^(3/2))/(3*a*d*(1 + Sec[c + d*x])^2)
```

Maple [C] time = 0.248, size = 1505, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/30/a/d*2^(1/2)*(-1+cos(d*x+c))^2*(15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1
/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^3-15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2
-1/2*I,1/2*2^(1/2))*cos(d*x+c)^3+15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1
/2*I,1/2*2^(1/2))*cos(d*x+c)^2-15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1
+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2
*I,1/2*2^(1/2))*cos(d*x+c)^2+15*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1
/2*2^(1/2))*cos(d*x+c)^3+15*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2
^(1/2))*cos(d*x+c)^3+36*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*
EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x
+c)^3-18*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^3+15*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*
```

```
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+15*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+36*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2-18*((-1+cos(d*x+c))/sin(d*x+c))^
(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
),1/2*2^(1/2))*cos(d*x+c)^2-28*cos(d*x+c)^3*2^(1/2)+24*cos(d*x+c)^2*2^(1/2)
+10*cos(d*x+c)*2^(1/2)-6*2^(1/2))*cos(d*x+c)^2*(cos(d*x+c)+1)^2*(e*sin(d*x+
c)/cos(d*x+c))^(9/2)/sin(d*x+c)^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^{\frac{9}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*tan(d*x + c))^(9/2)/(a*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(9/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan{(dx+c)})^{\frac{9}{2}}}{a\sec{(dx+c)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^(9/2)/(a*sec(d*x + c) + a), x)
```

3.120 $\int \frac{(e\tan(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=295

$$-\frac{e^4\sqrt{\sin(2c+2dx)}\sec(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3ad\sqrt{e\tan(c+dx)}}-\frac{e^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad}$$

```
[Out] -((e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
)) + (e^(7/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
a*d) - (e^(7/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*a*d) + (e^(7/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sq
rt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) - (e^4*EllipticF[c - Pi/4 + d*
x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*a*d*Sqrt[e*Tan[c + d*x]]) - (
2*e^3*(3 - Sec[c + d*x])*Sqrt[e*Tan[c + d*x]])/(3*a*d)
```

Rubi [A] time = 0.352503, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.56, Rules used = {3888, 3881, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$-\frac{e^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad}-\frac{e^{7/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad}+$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(7/2)/(a + a*Sec[c + d*x]),x]
```


```
[Out] -((e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
)) + (e^(7/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
a*d) - (e^(7/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*a*d) + (e^(7/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sq
rt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) - (e^4*EllipticF[c - Pi/4 + d*
x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*a*d*Sqrt[e*Tan[c + d*x]]) - (
2*e^3*(3 - Sec[c + d*x])*Sqrt[e*Tan[c + d*x]])/(3*a*d)
```


Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
```

```
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = \frac{e^2\int(-a+a\sec(c+dx))(e\tan(c+dx))^{3/2}\,dx}{a^2}$$

$$= -\frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad} - \frac{\left(2e^4\right)\int\frac{-\frac{3a}{2}+\frac{1}{2}a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3a^2}$$

$$= -\frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad} - \frac{e^4\int\frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3a} + \frac{e^4\int\frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a}$$

$$= -\frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad} + \frac{e^5\,\text{Subst}\left(\int\frac{1}{\sqrt{x}(e^2+x^2)}\,dx,x,e\tan(c+dx)\right)}{ad} - \frac{\left(e^4\sqrt{\sin\right.}{3a\ldots}$$

$$= -\frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad} + \frac{\left(2e^5\right)\text{Subst}\left(\int\frac{1}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{ad} - \frac{\left(e^4\sec\right.}{\ldots}$$

$$= -\frac{e^4F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad\sqrt{e\tan(c+dx)}} - \frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad} + \frac{e^4\,S\ldots}{\ldots}$$

$$= -\frac{e^4F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad\sqrt{e\tan(c+dx)}} - \frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad} - \frac{e^{7/2}\ldots}{\ldots}$$

$$= -\frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} + \frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{\ldots}\right.}{2\sqrt{2}ad}$$

$$= -\frac{e^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{e^{7/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} - \frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+\ldots\right.}{2\sqrt{2}\ldots}$$

Mathematica [C] time = 50.0332, size = 271, normalized size = 0.92

$$\frac{e^3\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\sqrt{\sec^2(c+dx)}+1\right)\sqrt{e\tan(c+dx)}\left(8\sqrt{\tan(c+dx)}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\text{t}\ldots\right.\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Tan[c + d*x])^(7/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (e^3*Cos[(c + d*x)/2]^2*Sec[c + d*x]*(1 + Sqrt[Sec[c + d*x]^2])*(-2*Sqrt[2]
*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt
[Tan[c + d*x]]] - Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]
```

```
] + Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - 8*Sqrt[Tan
[c + d*x]] + 8*Hypergeometric2F1[-1/2, 1/4, 5/4, -Tan[c + d*x]^2]*Sqrt[Tan[
c + d*x]] - 8*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c + d*x]^2]*Sqrt[Tan[c
+ d*x]])*Sqrt[e*Tan[c + d*x]])/(2*a*d*(1 + Sec[c + d*x])^2*Sqrt[Tan[c + d*x
]])
```

Maple [C] time = 0.259, size = 698, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/6/a/d*2^(1/2)*(-1+cos(d*x+c))*(3*I*sin(d*x+c)*cos(d*x+c)*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*I*sin(d*x+c)*cos(d*x+c)*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*sin(d*x+c)*cos(d*x+c)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+8*sin(d*x+c)*cos(d*x+c)*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-3*sin(d*x+c)*cos(d*x+c)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-6*cos(d*x+c)^2*2^(1/2)+8*cos(
d*x+c)*2^(1/2)-2*2^(1/2))*cos(d*x+c)^2*(cos(d*x+c)+1)^2*(e*sin(d*x+c)/cos(d
*x+c))^(7/2)/sin(d*x+c)^7
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^{\frac{7}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^(7/2)/(a*sec(d*x + c) + a), x)
```

3.121 $\int \frac{(e\tan(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=285

$$\frac{e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}-\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad}-\frac{e^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad}+\cdots$$

```
[Out] (e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d)
- (e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
) - (e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*
x]]])/(2*Sqrt[2]*a*d) + (e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[
2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) - (2*e^2*Cos[c + d*x]*EllipticE[c
 - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(a*d*Sqrt[Sin[2*c + 2*d*x]]) + (2*e
*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(a*d)
```

Rubi [A] time = 0.325608, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.56, Rules used = {3888, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$\frac{e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}-\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad}-\frac{e^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d)
- (e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
) - (e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*
x]]])/(2*Sqrt[2]*a*d) + (e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[
2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) - (2*e^2*Cos[c + d*x]*EllipticE[c
 - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(a*d*Sqrt[Sin[2*c + 2*d*x]]) + (2*e
*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(a*d)
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
```

```
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \frac{e^2\int(-a+a\sec(c+dx))\sqrt{e\tan(c+dx)}\,dx}{a^2}$$
$$= -\frac{e^2\int\sqrt{e\tan(c+dx)}\,dx}{a} + \frac{e^2\int\sec(c+dx)\sqrt{e\tan(c+dx)}\,dx}{a}$$
$$= \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{ad} - \frac{\left(2e^2\right)\int\cos(c+dx)\sqrt{e\tan(c+dx)}\,dx}{a} - \frac{e^3\operatorname{Subst}\left(\int\frac{\sqrt{x}}{e^2+x^2}\right.}{}$$
$$= \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{ad} - \frac{\left(2e^3\right)\operatorname{Subst}\left(\int\frac{x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{ad} - \frac{\left(2e^2\sqrt{\cos(c\right.}}{}$$
$$= \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{ad} + \frac{e^3\operatorname{Subst}\left(\int\frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{ad} - \frac{e^3\operatorname{Subst}\left(\int\frac{e+}{e^2+}\right.}{}$$
$$= -\frac{2e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{ad\sqrt{\sin(2c+2dx)}} + \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{ad} - \frac{e^{5/2}\mathrm{S}}{}$$
$$= -\frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} + \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{}\right.}{2\sqrt{2}ad}$$
$$= \frac{e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} - \frac{e^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} - \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+d\right.}{2\sqrt{2}a}$$

Mathematica [C] time = 5.13819, size = 105, normalized size = 0.37

$$\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\csc(c+dx)\left(\sqrt{\sec^2(c+dx)}+1\right)(e\tan(c+dx))^{5/2}\left(\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\tan^2(c+dx)\right)-\right.}{3ad(\sec(c+dx)+1)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Tan[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (4*Cos[(c + d*x)/2]^2*Csc[c + d*x]*(Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c
 + d*x]^2] - Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2])*(1 + Sqrt[Sec
[c + d*x]^2])*(e*Tan[c + d*x])^(5/2))/(3*a*d*(1 + Sec[c + d*x])^2)
```

Maple [C] time = 0.255, size = 1419, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/2/a/d*2^(1/2)*(-1+cos(d*x+c))^2*(I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1
/2*I,1/2*2^(1/2))*cos(d*x+c)-I*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1
/2+1/2*I,1/2*2^(1/2))-cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,
1/2*2^(1/2))-cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/
2))-4*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+2*cos(d*x+c)*(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+I*((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
),1/2-1/2*I,1/2*2^(1/2))-I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^
(1/2))-((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-4*((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),
1/2*2^(1/2))+2*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF
(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+2*cos(d*x+c)*2^(
1/2)-2*2^(1/2))*(e*sin(d*x+c)/cos(d*x+c))^(5/2)*cos(d*x+c)^2*(cos(d*x+c)+1)
^2/sin(d*x+c)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*tan(d*x + c))^(5/2)/(a*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^(5/2)/(a*sec(d*x + c) + a), x)
```

3.122 $\int \frac{(e\tan(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=257

$$\frac{e^2\sqrt{\sin(2c+2dx)}\sec(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{ad\sqrt{e\tan(c+dx)}}+\frac{e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}-\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad}$$

```
[Out] (e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d)
- (e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
) + (e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*
x]]])/(2*Sqrt[2]*a*d) - (e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[
2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) + (e^2*EllipticF[c - Pi/4 + d*x,
2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(a*d*Sqrt[e*Tan[c + d*x]])
```

Rubi [A] time = 0.291912, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {3888, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$\frac{e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}-\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad}+\frac{e^{3/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad}-$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```


```
[Out] (e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d)
- (e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
) + (e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*
x]]])/(2*Sqrt[2]*a*d) - (e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[
2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) + (e^2*EllipticF[c - Pi/4 + d*x,
2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(a*d*Sqrt[e*Tan[c + d*x]])
```


Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \frac{e^2\int \frac{-a+a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{a^2}$$

$$= -\frac{e^2\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a} + \frac{e^2\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{a}$$

$$= -\frac{e^3\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x, e\tan(c+dx)\right)}{ad} + \frac{\left(e^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{a\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$= -\frac{\left(2e^3\right)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} + \frac{\left(e^2\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,d}{a\sqrt{e\tan(c+dx)}}$$

$$= \frac{e^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{ad\sqrt{e\tan(c+dx)}} - \frac{e^2\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} - \frac{e^2}{}$$

$$= \frac{e^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{ad\sqrt{e\tan(c+dx)}} + \frac{e^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+d}\right.}{2\sqrt{2}ad}$$

$$= \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} - \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\,\text{ta}}\right.}{2\sqrt{2}ad}$$

$$= \frac{e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} - \frac{e^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)\right.}{2\sqrt{2}ad}$$

Mathematica [C] time = 12.5936, size = 1211, normalized size = 4.71

$$-\frac{4\sqrt[4]{-1}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right),-1\right)(e\tan(c+dx))^{3/2}\sec^4(c+dx)}{d(\sec(c+dx)a+a)\tan^{\frac{3}{2}}(c+dx)\left(\tan^2(c+dx)+1\right)^{3/2}} - \frac{2e^{-i(c+dx)}\sqrt{-\frac{i\left(-1+e^{2i(c}\right.}{1+e^{2i(c+}}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Tan[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[c/2 + (d*x)/2]^2*Csc[c + d*x]*((8*Cos[c]*Cos[d*x]*Sec[2*c]*Sin[c/2]^2)
/d - (16*Cos[c/2]*Sec[2*c]*Sin[c/2]^3*Sin[d*x])/d)*(e*Tan[c + d*x])^(3/2))/
(a + a*Sec[c + d*x]) - (2*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2
*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Se
c[c + d*x]*(e*Tan[c + d*x])^(3/2))/(d*E^(I*(c + d*x))*(a + a*Sec[c + d*x])*
Tan[c + d*x]^(3/2)) - (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)
```

```
*(c + d*x)))]*(E^((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 +
E^((4*I)*(c + d*x))]] + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*
(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)
))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*(e*Tan[c + d*x])^(3/2))/(2
*d*E^((2*I)*c)*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*Tan[c + d*x]
^(3/2)) - (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))
]*(Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] +
2*E^((4*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*
ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/
2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*(e*Tan[c + d*x])^(3/2))/(2*d*E^((2*I)*
c)*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*Tan[c + d*x]^(3/2)) + (S
qrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 +
(d*x)/2]^2*(3*(-1 + E^((4*I)*(c + d*x))) + E^((4*I)*(c + d*x))*(-1 + E^((2*
I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4
*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]*(e*Tan[c + d*x])^(3/2))/(3*d*E^(I*(2
*c + d*x))*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*Tan[c + d*x]^(3/
2)) - (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Co
s[c/2 + (d*x)/2]^2*(3 - 3*E^((4*I)*(c + d*x)) + E^((2*I)*(c + 2*d*x))*(-1 +
 E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4
, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]*(e*Tan[c + d*x])^(3/2))/(3*d*
E^(I*d*x)*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*Tan[c + d*x]^(3/2
)) - (4*(-1)^(1/4)*Cos[c/2 + (d*x)/2]^2*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt
[Tan[c + d*x]]], -1]*Sec[c + d*x]^4*(e*Tan[c + d*x])^(3/2))/(d*(a + a*Sec[c
 + d*x])*Tan[c + d*x]^(3/2)*(1 + Tan[c + d*x]^2)^(3/2))
```

Maple [C] time = 0.226, size = 319, normalized size = 1.2

$$\frac{\sqrt{2}\left(\cos\left(dx+c\right)+1\right)^{2}\left(-1+\cos\left(dx+c\right)\right)\cos\left(dx+c\right)}{2\,da\left(\sin\left(dx+c\right)\right)^{4}}\left(i\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos\left(dx+c\right)+\sin\left(dx+c\right)}{\sin\left(dx+c\right)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\mathrm{E}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/2/a/d*2^(1/2)*(cos(d*x+c)+1)^2*(I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+EllipticPi(((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+EllipticPi(((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-4*EllipticF(((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2)))*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(
```

```
d*x+c))/sin(d*x+c))^(1/2)*(-1+cos(d*x+c))*(e*sin(d*x+c)/cos(d*x+c))^(3/2)*c
os(d*x+c)/sin(d*x+c)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*tan(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{(e\tan(c+dx))^{\frac{3}{2}}}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral((e*tan(c + d*x))**(3/2)/(sec(c + d*x) + 1), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

3.123 $\int \frac{\sqrt{e\tan(c+dx)}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=315

$$-\frac{\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}+\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad}+\frac{\sqrt{e}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad}-$$

```
[Out] -((Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
)) + (Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
a*d) + (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*a*d) - (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sq
rt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) + (2*e*(1 - Sec[c + d*x]))/(a*
d*Sqrt[e*Tan[c + d*x]]) - (2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt
[e*Tan[c + d*x]])/(a*d*Sqrt[Sin[2*c + 2*d*x]]) + (2*Cos[c + d*x]*(e*Tan[c +
 d*x])^(3/2))/(a*d*e)
```

Rubi [A] time = 0.375458, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3888, 3882, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$-\frac{\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}+\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad}+\frac{\sqrt{e}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Tan[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
)) + (Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
a*d) + (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*a*d) - (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sq
rt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) + (2*e*(1 - Sec[c + d*x]))/(a*
d*Sqrt[e*Tan[c + d*x]]) - (2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt
[e*Tan[c + d*x]])/(a*d*Sqrt[Sin[2*c + 2*d*x]]) + (2*Cos[c + d*x]*(e*Tan[c +
 d*x])^(3/2))/(a*d*e)
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+a\sec(c+dx)}\,dx = \frac{e^2\int \frac{-a+a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx}{a^2}$$

$$= \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} + \frac{2\int \left(\frac{a}{2}+\frac{1}{2}a\sec(c+dx)\right)\sqrt{e\tan(c+dx)}\,dx}{a^2}$$

$$= \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} + \frac{\int \sqrt{e\tan(c+dx)}\,dx}{a} + \frac{\int \sec(c+dx)\sqrt{e\tan(c+dx)}\,dx}{a}$$

$$= \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} + \frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{ade} - \frac{2\int \cos(c+dx)\sqrt{e\tan(c+dx)}\,dx}{a} + \cdots$$

$$= \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} + \frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{ade} + \frac{(2e)\,\mathrm{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+\cdots}\right.}{ad}$$

$$= \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} + \frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{ade} - \frac{e\,\mathrm{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx}\right.}{ad}$$

$$= \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} - \frac{2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{ad\sqrt{\sin(2c+2dx)}} + \frac{2\cos(c+dx)(e\tan(c\cdots}{ade}$$

$$= \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} - \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\,\mathrm{t}\cdots}\right.}{2\sqrt{2}ad}$$

$$= -\frac{\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{\sqrt{e}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx\cdots\right.}{2\sqrt{2}a\cdots}$$

Mathematica [C] time = 7.70282, size = 2715, normalized size = 8.62

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Tan[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]*((-2*Cos[c/2]*Cos[d*x]*Sec[2*c]*(4*Sin[c
/2] + Sin[(3*c)/2] + Sin[(5*c)/2]))/(d*(1 + 2*Cos[c])) - (4*Sec[c/2]*Sec[c/
2 + (d*x)/2]*Sin[(d*x)/2])/d - ((-2 - 5*Cos[c] - 6*Cos[2*c] + Cos[3*c])*Sec
[2*c]*Sin[d*x])/(d*(1 + 2*Cos[c])) - (4*Tan[c/2])/d)*Sqrt[e*Tan[c + d*x]])/
(a + a*Sec[c + d*x]) + ((E^((2*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[
Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 +
 E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)
*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d
*x]])/(2*d*E^(I*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c
+ d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*Sqr
t[Tan[c + d*x]]) - ((-(E^((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sq
rt[-1 + E^((4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 +
E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*
(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d*
x]])/(2*d*E^((2*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*
(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*
Sqrt[Tan[c + d*x]]) - ((-(E^((6*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan
[Sqrt[-1 + E^((4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1
 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*
I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c +
 d*x]])/(2*d*E^((3*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*
I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x
])*Sqrt[Tan[c + d*x]]) + ((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 +
E^((4*I)*(c + d*x))]] - 2*E^((2*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1
 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*
I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c +
 d*x]])/(2*d*E^(I*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(
c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*S
qrt[Tan[c + d*x]]) + ((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((
4*I)*(c + d*x))]] - 2*E^((4*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E
^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(
c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d*x
]])/(2*d*E^((2*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(
c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*S
qrt[Tan[c + d*x]]) + ((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((
4*I)*(c + d*x))]] - 2*E^((6*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E
^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(
c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d*x
]])/(2*d*E^((3*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(
c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*S
```

```
qrt[Tan[c + d*x]]) + (Cos[c/2 + (d*x)/2]^2*(3 - 3*E^((4*I)*(c + d*x)) + E^(
(4*I)*(c + d*x))*(1 + E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeome
tric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]*Sqrt[e*T
an[c + d*x]])/(3*d*E^(I*(2*c + d*x))*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))
/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a
*Sec[c + d*x])*Sqrt[Tan[c + d*x]]) + (Cos[c/2 + (d*x)/2]^2*(3 - 3*E^((4*I)*
(c + d*x)) + E^((2*I)*(c + 2*d*x))*(1 + E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c +
 d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec
[c + d*x]*Sqrt[e*Tan[c + d*x]])/(3*d*E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*(c
 + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[
c])*(a + a*Sec[c + d*x])*Sqrt[Tan[c + d*x]]) + (5*E^(I*(c - d*x))*Cos[c/2 +
 (d*x)/2]^2*(3 - 3*E^((4*I)*(c + d*x)) + E^((4*I)*d*x)*(1 + E^((4*I)*c))*Sq
rt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c +
d*x))])*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d*x]])/(6*d*Sqrt[((-I)*(-1 + E
^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(
1 + 2*Cos[c])*(a + a*Sec[c + d*x])*Sqrt[Tan[c + d*x]]) - (Cos[c/2 + (d*x)/2
]^2*(3 - 3*E^((4*I)*(c + d*x)) + E^((4*I)*(c + d*x))*(1 + E^((4*I)*c))*Sqrt
[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*
x))])*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d*x]])/(6*d*E^(I*(3*c + d*x))*Sq
rt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*
I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*Sqrt[Tan[c + d*x]]) + (2
*Cos[c/2 + (d*x)/2]^2*(-3*E^((2*I)*c)*(-1 + E^((4*I)*(c + d*x))) + E^((4*I)
*d*x)*(1 + E^((6*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2
, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d*x]
])/(3*d*E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c +
 d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*Sqrt
[Tan[c + d*x]])
```

Maple [C] time = 0.239, size = 359, normalized size = 1.1

$$-\frac{\sqrt{2}\left(\cos\left(dx+c\right)+1\right)^{2}\left(-1+\cos\left(dx+c\right)\right)}{2\,da\left(\sin\left(dx+c\right)\right)^{3}}\sqrt{\frac{-1+\cos\left(dx+c\right)}{\sin\left(dx+c\right)}}\sqrt{\frac{-1+\cos\left(dx+c\right)+\sin\left(dx+c\right)}{\sin\left(dx+c\right)}}\sqrt{\frac{e\sin\left(dx+c\right)}{\cos\left(dx+c\right)}}\sqrt{-\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/2/a/d*2^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*(e*sin(d*x+c)/cos(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin
(d*x+c))/sin(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(I*EllipticPi((-(-1+cos(d*x+c)-
sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi((-(-1+cos
(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+4*EllipticE((-
```

```
(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-2*EllipticF((-(-1
+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-EllipticPi((-(-1+cos
(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-EllipticPi((-(
-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)))*(-1+cos
(d*x+c))/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\tan(dx+c)}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*tan(d*x + c))/(a*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{e\tan(c+dx)}}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(e*tan(c + d*x))/(sec(c + d*x) + 1), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\tan(dx+c)}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*tan(d*x + c))/(a*sec(d*x + c) + a), x)
```

3.124 $\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx$

Optimal. Leaf size=290

$$-\frac{\sqrt{\sin(2c+2dx)}\sec(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3ad\sqrt{e\tan(c+dx)}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad\sqrt{e}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad\sqrt{e}}-\frac{\log\left(\sqrt{\right.}}{}$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*Sqrt[e]))
 + ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*Sqrt[e])
 - Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]]/(2*Sq
rt[2]*a*d*Sqrt[e]) + Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Ta
n[c + d*x]]]/(2*Sqrt[2]*a*d*Sqrt[e]) + (2*e*(1 - Sec[c + d*x]))/(3*a*d*(e*T
an[c + d*x])^(3/2)) - (EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2
*c + 2*d*x]])/(3*a*d*Sqrt[e*Tan[c + d*x]])
```

Rubi [A] time = 0.345878, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.56, Rules used = {3888, 3882, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad\sqrt{e}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad\sqrt{e}}-\frac{\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad\sqrt{e}}+\frac{\log\left(\sqrt{e}\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]),x]
```


```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*Sqrt[e]))
 + ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*Sqrt[e])
 - Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]]/(2*Sq
rt[2]*a*d*Sqrt[e]) + Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Ta
n[c + d*x]]]/(2*Sqrt[2]*a*d*Sqrt[e]) + (2*e*(1 - Sec[c + d*x]))/(3*a*d*(e*T
an[c + d*x])^(3/2)) - (EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2
*c + 2*d*x]])/(3*a*d*Sqrt[e*Tan[c + d*x]])
```


Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
```

```
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx = \frac{e^2\int \frac{-a+a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx}{a^2}$$

$$= \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} + \frac{2\int \frac{\frac{3a}{2}-\frac{1}{2}a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3a^2}$$

$$= \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} - \frac{\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3a} + \frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a}$$

$$= \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} + \frac{e\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x, e\tan(c+dx)\right)}{ad} - \frac{\sqrt{\sin(c}}{3a\sqrt{c}}$$

$$= \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} + \frac{(2e)\,\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} - \frac{(\sec(c}{}$$

$$= \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} - \frac{F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad\sqrt{e\tan(c+dx)}} + \frac{\text{Sub}}{}$$

$$= \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} - \frac{F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad\sqrt{e\tan(c+dx)}} + \frac{\text{Sub}}{}$$

$$= -\frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad\sqrt{e}} + \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx\right.}{2\sqrt{2}a}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad\sqrt{e}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad\sqrt{e}} - \frac{\log\left(\sqrt{e}+\sqrt{e}\tan\right.}{}$$

Mathematica [C] time = 8.89735, size = 1253, normalized size = 4.32

$$\frac{4\sqrt[4]{-1}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right),-1\right)\sqrt{\tan(c+dx)}\sec^4(c+dx)}{3d(\sec(c+dx)a+a)\sqrt{e\tan(c+dx)}\left(\tan^2(c+dx)+1\right)^{3/2}} + \frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{2\sec^2\left(\frac{c}{2}+\right.}{3d}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]),x]
```

```
[Out] (2*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E
^((2*I)*(c + d*x)))*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[Tan[c +
 d*x]])/(3*d*E^(I*(c + d*x))*(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) + (
Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(E^((4*I)
*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] +
 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqr
t[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2
]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[Tan[c + d*x]])/(2*d*E^((2*I)*c)*(-1 + E^((2*
I)*(c + d*x)))*(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) + (Sqrt[((-I)*(-1
 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(Sqrt[-1 + E^((4*I)*(c
+ d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] + 2*E^((4*I)*c)*Sqrt[-1 + E
^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*
I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*S
ec[c + d*x]*Sqrt[Tan[c + d*x]])/(2*d*E^((2*I)*c)*(-1 + E^((2*I)*(c + d*x)))
*(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) - (Sqrt[((-I)*(-1 + E^((2*I)*(c
 + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d*x)/2]^2*(3*(-1 + E^((4*I
)*(c + d*x))) + E^((4*I)*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c
 + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*S
ec[c + d*x]*Sqrt[Tan[c + d*x]])/(3*d*E^(I*(2*c + d*x))*(-1 + E^((2*I)*(c +
d*x)))*(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) + (Sqrt[((-I)*(-1 + E^((2
*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d*x)/2]^2*(3 - 3*E^(
(4*I)*(c + d*x)) + E^((2*I)*(c + 2*d*x))*(-1 + E^((2*I)*c))*Sqrt[1 - E^((4*
I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2
*c]*Sec[c + d*x]*Sqrt[Tan[c + d*x]])/(3*d*E^(I*d*x)*(-1 + E^((2*I)*(c + d*x
)))*(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) + (Cos[c/2 + (d*x)/2]^2*Sec[
c + d*x]*(-4/(3*d) + (2*(3 - 2*Cos[c] + 3*Cos[2*c])*Cos[d*x]*Sec[2*c])/(3*d
) + (2*Sec[c/2 + (d*x)/2]^2)/(3*d) - (2*Sec[2*c]*(-2*Sin[c] + 3*Sin[2*c])*S
in[d*x])/(3*d))*Tan[c + d*x])/((a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) +
 (4*(-1)^(1/4)*Cos[c/2 + (d*x)/2]^2*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan
[c + d*x]]], -1]*Sec[c + d*x]^4*Sqrt[Tan[c + d*x]])/(3*d*(a + a*Sec[c + d*x
])*Sqrt[e*Tan[c + d*x]]*(1 + Tan[c + d*x]^2)^(3/2))
```

Maple [C] time = 0.254, size = 1269, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x)
```

```
[Out] 1/6/a/d*2^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*(3*I*EllipticPi(((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1
```

```
+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-3*I*EllipticPi(((
1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+3*I*Ellipti
cPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+3*sin(d*x+c)*cos
(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*sin(d*x+
c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*sin(d*x+c)*cos(
d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-8*sin(d*x+c)*
cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+3*sin(d*x+c)*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*sin(d*x+c)*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-8*sin(d*x+c)*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2),1/2*2^(1/2))+2*cos(d*x+c)^2*2^(1/2)-2*cos(d*x+c)*2^(1/2))/sin(d*x
+c)^5/cos(d*x+c)/(e*sin(d*x+c)/cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)\sqrt{e\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*sqrt(e*tan(d*x + c))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sqrt{e \tan{(c+dx)}} \sec{(c+dx)} + \sqrt{e \tan{(c+dx)}}}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*tan(c + d*x))*sec(c + d*x) + sqrt(e*tan(c + d*x))), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a \sec{(dx+c)} + a)\sqrt{e \tan{(dx+c)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*sqrt(e*tan(d*x + c))), x)
```

3.125 $\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=359

$$\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{3/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ade^{3/2}}-\frac{\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ade^{3/2}}+\frac{\log\left(\sqrt{e}\tan\right.}{}$$

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*e^(3/2)) -
ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*e^(3/2)) -
Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]]/(2*Sqrt[
2]*a*d*e^(3/2)) + Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c
 + d*x]]]/(2*Sqrt[2]*a*d*e^(3/2)) + (2*e*(1 - Sec[c + d*x]))/(5*a*d*(e*Tan[
c + d*x])^(5/2)) - (2*(5 - 3*Sec[c + d*x]))/(5*a*d*e*Sqrt[e*Tan[c + d*x]])
+ (6*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*a*d
*e^2*Sqrt[Sin[2*c + 2*d*x]]) - (6*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(5*a
*d*e^3)
```

Rubi [A] time = 0.452068, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3888, 3882, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{3/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ade^{3/2}}-\frac{\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ade^{3/2}}+\frac{\log\left(\sqrt{e}\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2)),x]
```


```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*e^(3/2)) -
ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*e^(3/2)) -
Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]]/(2*Sqrt[
2]*a*d*e^(3/2)) + Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c
 + d*x]]]/(2*Sqrt[2]*a*d*e^(3/2)) + (2*e*(1 - Sec[c + d*x]))/(5*a*d*(e*Tan[
c + d*x])^(5/2)) - (2*(5 - 3*Sec[c + d*x]))/(5*a*d*e*Sqrt[e*Tan[c + d*x]])
+ (6*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*a*d
*e^2*Sqrt[Sin[2*c + 2*d*x]]) - (6*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(5*a
*d*e^3)
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \frac{e^2\int \frac{-a+a\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx}{a^2}$$

$$= \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} + \frac{2\int \frac{\frac{5a}{2}-\frac{3}{2}a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx}{5a^2}$$

$$= \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} - \frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}} + \frac{4\int \left(-\frac{5a}{4}-\frac{3}{4}a\sec(c+dx)\right)\sqrt{\cdots}}{5a^2e^2}$$

$$= \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} - \frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}} - \frac{3\int \sec(c+dx)\sqrt{e\tan(c+d\cdots}}{5ae^2}$$

$$= \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} - \frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}} - \frac{6\cos(c+dx)(e\tan(c+dx))^{3\cdots}}{5ade^3}$$

$$= \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} - \frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}} - \frac{6\cos(c+dx)(e\tan(c+dx))^{3\cdots}}{5ade^3}$$

$$= \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} - \frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}} - \frac{6\cos(c+dx)(e\tan(c+dx))^{3\cdots}}{5ade^3}$$

$$= \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} - \frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}} + \frac{6\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\cdots\right.}{5ade^2\sqrt{\sin(2c+\cdots}}$$

$$= -\frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ade^{3/2}} + \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+d\cdots\right.}{2\sqrt{2}a\cdots}$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{3/2}} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{3/2}} - \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(\cdots\right.}{\cdots}$$

Mathematica [C] time = 12.8607, size = 180, normalized size = 0.5

$$-\frac{4\sin^2\left(\frac{1}{2}(c+dx)\right)\csc(c+dx)\left(\sqrt{\sec^2(c+dx)}+1\right)\sqrt{e\tan(c+dx)}\left(-5\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\tan^2(c+dx)\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2)),x]
```

```
[Out] (-4*Csc[c + d*x]*(15*Cot[c + d*x]^2 - 3*Cot[c + d*x]^4 + 3*Cot[c + d*x]^4*H
ypergeometric2F1[-5/4, -1/2, -1/4, -Tan[c + d*x]^2] - 15*Cot[c + d*x]^2*Hyp
ergeometric2F1[-1/2, -1/4, 3/4, -Tan[c + d*x]^2] - 5*Hypergeometric2F1[1/2,
 3/4, 7/4, -Tan[c + d*x]^2] + 5*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x
]^2])*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2*Sqrt[e*Tan[c + d*x]])/(
15*a*d*e^2)
```

Maple [C] time = 0.245, size = 2113, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x)
```

```
[Out] -1/10/a/d*2^(1/2)*(-1+cos(d*x+c))*(5*I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+
1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+10*I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/
2*I,1/2*2^(1/2))*cos(d*x+c)+5*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2
*2^(1/2))*cos(d*x+c)^2-10*I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2
^(1/2))*cos(d*x+c)+5*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*Ell
ipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))
*cos(d*x+c)^2-5*I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellipt
```

```
icPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*co
s(d*x+c)^2-12*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2+6*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2+10*cos(d*x+c)*((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-5*I*((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+10*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2),1/2-1/2*I,1/2*2^(1/2))+5*I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/
2*2^(1/2))-24*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+12*cos
(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+5*((1-cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+5*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2
*I,1/2*2^(1/2))-12*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellip
ticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+6*((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2),1/2*2^(1/2))-6*cos(d*x+c)^2*2^(1/2)-4*cos(d*x+c)*2^(1/
2))/sin(d*x+c)/cos(d*x+c)^2/(e*sin(d*x+c)/cos(d*x+c))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)(e\tan(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{(e\tan(c+dx))^{\frac{3}{2}}\sec(c+dx)+(e\tan(c+dx))^{\frac{3}{2}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((e*tan(c + d*x))**(3/2)*sec(c + d*x) + (e*tan(c + d*x))**(3/2))
, x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)(e\tan(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(3/2)), x)
```

3.126 $\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=328

$$\frac{5\sqrt{\sin(2c+2dx)}\sec(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{21ade^2\sqrt{e\tan(c+dx)}}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{5/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ade^{5/2}}+\frac{\log\left(\sqrt{\right.}}{}$$

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*e^(5/2)) -
ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*e^(5/2)) +
Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]]/(2*Sqrt[
2]*a*d*e^(5/2)) - Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c
 + d*x]]]/(2*Sqrt[2]*a*d*e^(5/2)) + (2*e*(1 - Sec[c + d*x]))/(7*a*d*(e*Tan[
c + d*x])^(7/2)) - (2*(7 - 5*Sec[c + d*x]))/(21*a*d*e*(e*Tan[c + d*x])^(3/2
)) + (5*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(
21*a*d*e^2*Sqrt[e*Tan[c + d*x]])
```

Rubi [A] time = 0.416543, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.56, Rules used = {3888, 3882, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{5/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ade^{5/2}}+\frac{\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ade^{5/2}}-\frac{\log\left(\sqrt{e}\tan(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)),x]
```


```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*e^(5/2)) -
ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*e^(5/2)) +
Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]]/(2*Sqrt[
2]*a*d*e^(5/2)) - Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c
 + d*x]]]/(2*Sqrt[2]*a*d*e^(5/2)) + (2*e*(1 - Sec[c + d*x]))/(7*a*d*(e*Tan[
c + d*x])^(7/2)) - (2*(7 - 5*Sec[c + d*x]))/(21*a*d*e*(e*Tan[c + d*x])^(3/2
)) + (5*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(
21*a*d*e^2*Sqrt[e*Tan[c + d*x]])
```


Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \frac{e^2\int \frac{-a+a\sec(c+dx)}{(e\tan(c+dx))^{9/2}}\,dx}{a^2}$$

$$= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} + \frac{2\int \frac{\frac{7a}{2}-\frac{5}{2}a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx}{7a^2}$$

$$= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} + \frac{4\int \frac{-\frac{21a}{4}+\frac{5}{4}a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{21a^2e^2}$$

$$= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} + \frac{5\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{21ae^2} - \int$$

$$= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x\right.}{ade}$$

$$= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} - \frac{2\,\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \right.}{ade}$$

$$= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} + \frac{5F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c}{21ade^2\sqrt{e\,\mathrm{ta}}}$$

$$= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} + \frac{5F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c}{21ade^2\sqrt{e\,\mathrm{ta}}}$$

$$= \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ade^{5/2}} - \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+d\right.}{2\sqrt{2}a}$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{5/2}} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{5/2}} + \frac{\log\left(\sqrt{e}+\sqrt{e}\tan\right.}{}$$

Mathematica [C] time = 9.17834, size = 1299, normalized size = 3.96

$$-\frac{20\sqrt[4]{-1}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right),-1\right)\tan^{\frac{5}{2}}(c+dx)\sec^4(c+dx)}{21d(\sec(c+dx)a+a)(e\tan(c+dx))^{5/2}\left(\tan^2(c+dx)+1\right)^{3/2}} + \frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{\sec^4\left(\frac{c}{2}+\right.}{14d}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)),x]
```

```
[Out] (-10*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 +
 E^((2*I)*(c + d*x)))*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Tan[c + d*
x]^(5/2))/(21*d*E^(I*(c + d*x))*(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)
) - (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(E^(
(4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x)
)]] + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTan
h[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d
*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Tan[c + d*x]^(5/2))/(2*d*E^((2*I)*c)*(-1 + E
^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)) - (Sqrt[((
-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(Sqrt[-1 + E^((4
*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] + 2*E^((4*I)*c)*Sqrt
[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 +
 E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec
[2*c]*Sec[c + d*x]*Tan[c + d*x]^(5/2))/(2*d*E^((2*I)*c)*(-1 + E^((2*I)*(c +
 d*x)))*(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)) + (Sqrt[((-I)*(-1 + E^
((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d*x)/2]^2*(3*(-1
+ E^((4*I)*(c + d*x))) + E^((4*I)*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 - E^
((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*S
ec[2*c]*Sec[c + d*x]*Tan[c + d*x]^(5/2))/(3*d*E^(I*(2*c + d*x))*(-1 + E^((2
*I)*(c + d*x)))*(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)) - (Sqrt[((-I)*
(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d*x)/2]^2
*(3 - 3*E^((4*I)*(c + d*x)) + E^((2*I)*(c + 2*d*x))*(-1 + E^((2*I)*c))*Sqrt
[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*
x))])*Sec[2*c]*Sec[c + d*x]*Tan[c + d*x]^(5/2))/(3*d*E^(I*d*x)*(-1 + E^((2*
I)*(c + d*x)))*(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)) + (Cos[c/2 + (d
*x)/2]^2*Sec[c + d*x]*(40/(21*d) - Csc[c/2 + (d*x)/2]^2/(6*d) - (2*(21 - 10
*Cos[c] + 21*Cos[2*c])*Cos[d*x]*Sec[2*c])/(21*d) - (13*Sec[c/2 + (d*x)/2]^2
)/(14*d) + Sec[c/2 + (d*x)/2]^4/(14*d) + (2*Sec[2*c]*(-10*Sin[c] + 21*Sin[2
*c])*Sin[d*x])/(21*d))*Tan[c + d*x]^3)/((a + a*Sec[c + d*x])*(e*Tan[c + d*x
])^(5/2)) - (20*(-1)^(1/4)*Cos[c/2 + (d*x)/2]^2*EllipticF[I*ArcSinh[(-1)^(1
/4)*Sqrt[Tan[c + d*x]]], -1]*Sec[c + d*x]^4*Tan[c + d*x]^(5/2))/(21*d*(a +
a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)*(1 + Tan[c + d*x]^2)^(3/2))
```

Maple [C] time = 0.257, size = 1896, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x)
```

```
[Out] -1/42/a/d*2^(1/2)*(-1+cos(d*x+c))^3*(21*I*sin(d*x+c)*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-21*I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1
/2*I,1/2*2^(1/2))*sin(d*x+c)*cos(d*x+c)^2+52*sin(d*x+c)*EllipticF(((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-21*sin(d*x+c)*EllipticPi(((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+21*I*((1-cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*cos(d*x+c)^2-21*sin(d*x+c)*Ellipti
cPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-42*I*EllipticP
i(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d
*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+104*sin
(d*x+c)*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*Ellip
ticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-42*sin(d*x+c
)*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-21*I*El
lipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-42*sin(d*x+
c)*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi
(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+42*I*E
llipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2
))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)
+52*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF
(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-21*sin(d*x+c)*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-21*sin(d*x+c)*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((
1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*
```

```
x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-20*cos(d*x+c)^3*2^(1/2)-4*co
s(d*x+c)^2*2^(1/2)+10*cos(d*x+c)*2^(1/2))*(cos(d*x+c)+1)^2/sin(d*x+c)^5/cos
(d*x+c)^3/(e*sin(d*x+c)/cos(d*x+c))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)(e\tan(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)(e\tan(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(5/2)), x)
```

3.127 $\int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=372

$$\frac{e^{13/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}-\frac{e^{13/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d}+\frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d}+\frac{2e^3(e\tan(c+dx))^{7/2}}{7a^2d}-\frac{e^{13/2}\log}{}$$

```
[Out] (e^(13/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2*
d) - (e^(13/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]
*a^2*d) - (e^(13/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan
[c + d*x]]])/(2*Sqrt[2]*a^2*d) + (e^(13/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*
x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (12*e^6*Cos[c + d*x
]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*a^2*d*Sqrt[Sin[2*c
+ 2*d*x]]) + (2*e^5*(e*Tan[c + d*x])^(3/2))/(3*a^2*d) + (12*e^5*Cos[c + d*x
]*(e*Tan[c + d*x])^(3/2))/(5*a^2*d) - (4*e^5*Sec[c + d*x]*(e*Tan[c + d*x])^
(3/2))/(5*a^2*d) + (2*e^3*(e*Tan[c + d*x])^(7/2))/(7*a^2*d)
```

Rubi [A] time = 0.491612, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 18, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.72, Rules used = {3888, 3886, 3473, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2611, 2613, 2615, 2572, 2639, 2607, 32}

$$\frac{e^{13/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}-\frac{e^{13/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d}+\frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d}+\frac{2e^3(e\tan(c+dx))^{7/2}}{7a^2d}-\frac{e^{13/2}\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(13/2)/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (e^(13/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2*
d) - (e^(13/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]
*a^2*d) - (e^(13/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan
[c + d*x]]])/(2*Sqrt[2]*a^2*d) + (e^(13/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*
x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (12*e^6*Cos[c + d*x
]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*a^2*d*Sqrt[Sin[2*c
+ 2*d*x]]) + (2*e^5*(e*Tan[c + d*x])^(3/2))/(3*a^2*d) + (12*e^5*Cos[c + d*x
]*(e*Tan[c + d*x])^(3/2))/(5*a^2*d) - (4*e^5*Sec[c + d*x]*(e*Tan[c + d*x])^
(3/2))/(5*a^2*d) + (2*e^3*(e*Tan[c + d*x])^(7/2))/(7*a^2*d)
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^4\int(-a+a\sec(c+dx))^2(e\tan(c+dx))^{5/2}\,dx}{a^4}$$

$$= \frac{e^4\int\left(a^2(e\tan(c+dx))^{5/2}-2a^2\sec(c+dx)(e\tan(c+dx))^{5/2}+a^2\sec^2(c+dx)(e\tan(c+dx)\right.}{a^4}$$

$$= \frac{e^4\int(e\tan(c+dx))^{5/2}\,dx}{a^2}+\frac{e^4\int\sec^2(c+dx)(e\tan(c+dx))^{5/2}\,dx}{a^2}-\frac{\left(2e^4\right)\int\sec(c+dx)(e\tan}{a^2}$$

$$= \frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d}-\frac{4e^5\sec(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d}+\frac{e^4\,\mathrm{Subst}\left(\int(ex)^{5/2}\,dx,x,\tan(c+\right.}{a^2d}$$

$$= \frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d}+\frac{12e^5\cos(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d}-\frac{4e^5\sec(c+dx)(e\tan(c+dx))^{3/}}{5a^2d}$$

$$= \frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d}+\frac{12e^5\cos(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d}-\frac{4e^5\sec(c+dx)(e\tan(c+dx))^{3/}}{5a^2d}$$

$$= \frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d}+\frac{12e^5\cos(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d}-\frac{4e^5\sec(c+dx)(e\tan(c+dx))^{3/}}{5a^2d}$$

$$= -\frac{12e^6\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5a^2d\sqrt{\sin(2c+2dx)}}+\frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d}+\frac{12e^5\cos(c+d}{}$$

$$= -\frac{e^{13/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d}+\frac{e^{13/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{}\right.}{2\sqrt{2}a^2d}$$

$$= \frac{e^{13/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}-\frac{e^{13/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}-\frac{e^{13/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c\right.}{2\sqrt{}}$$

Mathematica [F] time = 13.1493, size = 0, normalized size = 0.

$$\int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^(13/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] Integrate[(e*Tan[c + d*x])^(13/2)/(a + a*Sec[c + d*x])^2, x]
```

Maple [C] time = 0.281, size = 1518, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(13/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/210/a^2/d*2^(1/2)*(-1+cos(d*x+c))^2*(-105*I*EllipticPi(((1-cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^4+105*I*EllipticPi(((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3-105*EllipticPi(((1-cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^4-105*EllipticPi(((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2)*cos(d*x+c)^4-504*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*cos(d*x+c)^4+252*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+
c)^4-105*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*
I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c
)^3+105*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I
,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)
^4-105*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^3-
105*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^3-504
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^3+252*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^3+292*2^(1/2)*cos(d*x+c)^4-336*co
```

```
s(d*x+c)^3*2^(1/2)-10*cos(d*x+c)^2*2^(1/2)+84*cos(d*x+c)*2^(1/2)-30*2^(1/2)
)*cos(d*x+c)^3*(cos(d*x+c)+1)^2*(e*sin(d*x+c)/cos(d*x+c))^(13/2)/sin(d*x+c)
^11
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(13/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(13/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(13/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(13/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.128 $\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=339

$$\frac{2e^6\sqrt{\sin(2c+2dx)}\sec(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3a^2d\sqrt{e\tan(c+dx)}}+\frac{e^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}-\frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right.}{\sqrt{2}a^2d}$$

```
[Out] (e^(11/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2*
d) - (e^(11/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]
*a^2*d) + (e^(11/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan
[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (e^(11/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*
x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) + (2*e^6*EllipticF[c
- Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*a^2*d*Sqrt[e*Tan[c
 + d*x]]) + (2*e^5*Sqrt[e*Tan[c + d*x]])/(a^2*d) - (4*e^5*Sec[c + d*x]*Sqrt
[e*Tan[c + d*x]])/(3*a^2*d) + (2*e^3*(e*Tan[c + d*x])^(5/2))/(5*a^2*d)
```

Rubi [A] time = 0.458337, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.68, Rules used = {3888, 3886, 3473, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2611, 2614, 2573, 2641, 2607, 32}

$$\frac{e^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}-\frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d}+\frac{2e^5\sqrt{e\tan(c+dx)}}{a^2d}+\frac{2e^3(e\tan(c+dx))^{5/2}}{5a^2d}+\frac{e^{11/2}\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(11/2)/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (e^(11/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2*
d) - (e^(11/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]
*a^2*d) + (e^(11/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan
[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (e^(11/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*
x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) + (2*e^6*EllipticF[c
- Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*a^2*d*Sqrt[e*Tan[c
 + d*x]]) + (2*e^5*Sqrt[e*Tan[c + d*x]])/(a^2*d) - (4*e^5*Sec[c + d*x]*Sqrt
[e*Tan[c + d*x]])/(3*a^2*d) + (2*e^3*(e*Tan[c + d*x])^(5/2))/(5*a^2*d)
```


Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
```

```
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^4\int(-a+a\sec(c+dx))^2(e\tan(c+dx))^{3/2}\,dx}{a^4}$$

$$= \frac{e^4\int\left(a^2(e\tan(c+dx))^{3/2}-2a^2\sec(c+dx)(e\tan(c+dx))^{3/2}+a^2\sec^2(c+dx)(e\tan(c+dx)\right.}{a^4}$$

$$= \frac{e^4\int(e\tan(c+dx))^{3/2}\,dx}{a^2}+\frac{e^4\int\sec^2(c+dx)(e\tan(c+dx))^{3/2}\,dx}{a^2}-\frac{\left(2e^4\right)\int\sec(c+dx)(e\tan}{a^2}$$

$$= \frac{2e^5\sqrt{e\tan(c+dx)}}{a^2d}-\frac{4e^5\sec(c+dx)\sqrt{e\tan(c+dx)}}{3a^2d}+\frac{e^4\,\mathrm{Subst}\left(\int(ex)^{3/2}\,dx,x,\tan(c+dx)\right)}{a^2d}$$

$$= \frac{2e^5\sqrt{e\tan(c+dx)}}{a^2d}-\frac{4e^5\sec(c+dx)\sqrt{e\tan(c+dx)}}{3a^2d}+\frac{2e^3(e\tan(c+dx))^{5/2}}{5a^2d}-\frac{e^7\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{2e^5\sqrt{e\tan(c+dx)}}{a^2d}-\frac{4e^5\sec(c+dx)\sqrt{e\tan(c+dx)}}{3a^2d}+\frac{2e^3(e\tan(c+dx))^{5/2}}{5a^2d}-\frac{\left(2e^7\right)\mathrm{Subst}\left(\right.}{}$$

$$= \frac{2e^6F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3a^2d\sqrt{e\tan(c+dx)}}+\frac{2e^5\sqrt{e\tan(c+dx)}}{a^2d}-\frac{4e^5\sec(c+dx)\sqrt{e\tan}}{3a^2d}$$

$$= \frac{2e^6F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3a^2d\sqrt{e\tan(c+dx)}}+\frac{2e^5\sqrt{e\tan(c+dx)}}{a^2d}-\frac{4e^5\sec(c+dx)\sqrt{e\tan}}{3a^2d}$$

$$= \frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d}-\frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\right.}{2\sqrt{2}a^2d}$$

$$= \frac{e^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}-\frac{e^{11/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}+\frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c\right.}{2\sqrt{}}$$

Mathematica [F] time = 70.4309, size = 0, normalized size = 0.

$$\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^(11/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] Integrate[(e*Tan[c + d*x])^(11/2)/(a + a*Sec[c + d*x])^2, x]
```

Maple [C] time = 0.249, size = 721, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/30/a^2/d*2^(1/2)*(-1+cos(d*x+c))*(15*I*sin(d*x+c)*EllipticPi(((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-15*I*sin(d*x+c)*EllipticPi(((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+15*sin(d*x+c)*Ellipti
cPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+15*sin(d*x+c)*
EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/
2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-50*sin(
d*x+c)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+24*cos(d*x+
c)^3*2^(1/2)-44*cos(d*x+c)^2*2^(1/2)+26*cos(d*x+c)*2^(1/2)-6*2^(1/2))*cos(d
*x+c)^3*(cos(d*x+c)+1)^2*(e*sin(d*x+c)/cos(d*x+c))^(11/2)/sin(d*x+c)^9
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(11/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^{\frac{11}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^(11/2)/(a*sec(d*x + c) + a)^2, x)
```

3.129 $\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=312

$$-\frac{e^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}+\frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d}+\frac{2e^3(e\tan(c+dx))^{3/2}}{3a^2d}+\frac{e^{9/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\right.}{2\sqrt{2}a^2d}$$

```
[Out] -((e^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2
*d)) + (e^(9/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2
]*a^2*d) + (e^(9/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan
[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (e^(9/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x
] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) + (4*e^4*Cos[c + d*x]*
EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(a^2*d*Sqrt[Sin[2*c + 2*
d*x]]) + (2*e^3*(e*Tan[c + d*x])^(3/2))/(3*a^2*d) - (4*e^3*Cos[c + d*x]*(e*
Tan[c + d*x])^(3/2))/(a^2*d)
```

Rubi [A] time = 0.410649, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {3888, 3886, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639, 2607, 32}

$$-\frac{e^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}+\frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d}+\frac{2e^3(e\tan(c+dx))^{3/2}}{3a^2d}+\frac{e^{9/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\right.}{2\sqrt{2}a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(9/2)/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] -((e^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2
*d)) + (e^(9/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2
]*a^2*d) + (e^(9/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan
[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (e^(9/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x
] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) + (4*e^4*Cos[c + d*x]*
EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(a^2*d*Sqrt[Sin[2*c + 2*
d*x]]) + (2*e^3*(e*Tan[c + d*x])^(3/2))/(3*a^2*d) - (4*e^3*Cos[c + d*x]*(e*
Tan[c + d*x])^(3/2))/(a^2*d)
```


Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^4\int(-a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}\,dx}{a^4}$$

$$= \frac{e^4\int\left(a^2\sqrt{e\tan(c+dx)}-2a^2\sec(c+dx)\sqrt{e\tan(c+dx)}+a^2\sec^2(c+dx)\sqrt{e\tan(c+dx)}\right)d}{a^4}$$

$$= \frac{e^4\int\sqrt{e\tan(c+dx)}\,dx}{a^2}+\frac{e^4\int\sec^2(c+dx)\sqrt{e\tan(c+dx)}\,dx}{a^2}-\frac{\left(2e^4\right)\int\sec(c+dx)\sqrt{e\tan(c}{a^2}$$

$$= -\frac{4e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{a^2d}+\frac{\left(4e^4\right)\int\cos(c+dx)\sqrt{e\tan(c+dx)}\,dx}{a^2}+\frac{e^4\,\mathrm{Subst}\left(\int\sqrt{\right.}{}$$

$$= \frac{2e^3(e\tan(c+dx))^{3/2}}{3a^2d}-\frac{4e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{a^2d}+\frac{\left(2e^5\right)\mathrm{Subst}\left(\int\frac{x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan}\right.}{a^2d}$$

$$= \frac{2e^3(e\tan(c+dx))^{3/2}}{3a^2d}-\frac{4e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{a^2d}-\frac{e^5\,\mathrm{Subst}\left(\int\frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c}\right.}{a^2d}$$

$$= \frac{4e^4\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{a^2d\sqrt{\sin(2c+2dx)}}+\frac{2e^3(e\tan(c+dx))^{3/2}}{3a^2d}-\frac{4e^3\cos(c+dx)(e}{a^2d}$$

$$= \frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d}-\frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e}\right.}{2\sqrt{2}a^2d}$$

$$= -\frac{e^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}+\frac{e^{9/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}+\frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+\right.}{2\sqrt{}}$$

Mathematica [F] time = 4.30593, size = 0, normalized size = 0.

$$\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^(9/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] Integrate[(e*Tan[c + d*x])^(9/2)/(a + a*Sec[c + d*x])^2, x]
```

Maple [C] time = 0.251, size = 1504, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/a^2/d*2^(1/2)*(-1+cos(d*x+c))^2*(3*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c)
)/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/si
n(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c
))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/s
in(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c)
)/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/si
n(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin
(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+24*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2*2^(1/2))-12*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c)
)^(1/2)*cos(d*x+c)^2*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/
2),1/2*2^(1/2))+3*I*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c
))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*
I,1/2*2^(1/2))-3*I*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ell
```

```
ipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2
))*cos(d*x+c)+3*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellipt
icPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*
cos(d*x+c)+3*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticP
i((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos
(d*x+c)+24*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)-12*(-
(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos(d*x+
c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)-10*cos(d*x+c)^2*2^
(1/2)+12*cos(d*x+c)*2^(1/2)-2*2^(1/2))*cos(d*x+c)^3*(cos(d*x+c)+1)^2*(e*sin
(d*x+c)/cos(d*x+c))^(9/2)/sin(d*x+c)^9
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(9/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^{\frac{9}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^(9/2)/(a*sec(d*x + c) + a)^2, x)
```

3.130 $\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=281

$$-\frac{2e^4\sqrt{\sin(2c+2dx)}\sec(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{a^2d\sqrt{e\tan(c+dx)}}-\frac{e^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right.}{\sqrt{2}a^2d}$$

```
[Out] -((e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2
*d)) + (e^(7/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2
]*a^2*d) - (e^(7/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan
[c + d*x]]])/(2*Sqrt[2]*a^2*d) + (e^(7/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x
] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (2*e^4*EllipticF[c -
 Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(a^2*d*Sqrt[e*Tan[c +
d*x]]) + (2*e^3*Sqrt[e*Tan[c + d*x]])/(a^2*d)
```

Rubi [A] time = 0.378175, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3888, 3886, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641, 2607, 32}

$$-\frac{e^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d}+\frac{2e^3\sqrt{e\tan(c+dx)}}{a^2d}-\frac{e^{7/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e}\right.}{2\sqrt{2}a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(7/2)/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] -((e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2
*d)) + (e^(7/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2
]*a^2*d) - (e^(7/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan
[c + d*x]]])/(2*Sqrt[2]*a^2*d) + (e^(7/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x
] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (2*e^4*EllipticF[c -
 Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(a^2*d*Sqrt[e*Tan[c +
d*x]]) + (2*e^3*Sqrt[e*Tan[c + d*x]])/(a^2*d)
```


Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
```

```
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^4\int \frac{(-a+a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}\,dx}{a^4}$$

$$= \frac{e^4\int \left(\frac{a^2}{\sqrt{e\tan(c+dx)}} - \frac{2a^2\sec(c+dx)}{\sqrt{e\tan(c+dx)}} + \frac{a^2\sec^2(c+dx)}{\sqrt{e\tan(c+dx)}}\right)dx}{a^4}$$

$$= \frac{e^4\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a^2} + \frac{e^4\int \frac{\sec^2(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{a^2} - \frac{\left(2e^4\right)\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{a^2}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{1}{\sqrt{ex}}\,dx, x, \tan(c+dx)\right)}{a^2d} + \frac{e^5\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(e^2+x^2\right)}\,dx, x, e\tan(c+dx)\right)}{a^2d} - \frac{\left(2e^4\sqrt{s\right.}}{a}$$

$$= \frac{2e^3\sqrt{e\tan(c+dx)}}{a^2d} + \frac{\left(2e^5\right)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{a^2d} - \frac{\left(2e^4\sec(c+dx)\sqrt{\sin(2\right.}}{a^2\sqrt{e\,t}}$$

$$= -\frac{2e^4F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{a^2d\sqrt{e\tan(c+dx)}} + \frac{2e^3\sqrt{e\tan(c+dx)}}{a^2d} + \frac{e^4\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x}\right.}{}$$

$$= -\frac{2e^4F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{a^2d\sqrt{e\tan(c+dx)}} + \frac{2e^3\sqrt{e\tan(c+dx)}}{a^2d} - \frac{e^{7/2}\,\text{Subst}\left(\int \frac{}{-e-}\right.}{}$$

$$= -\frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} + \frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{\right.}}{2\sqrt{2}a^2d}$$

$$= -\frac{e^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} + \frac{e^{7/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} - \frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c\right.}{2}$$

Mathematica [F] time = 3.87051, size = 0, normalized size = 0.

$$\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^(7/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] Integrate[(e*Tan[c + d*x])^(7/2)/(a + a*Sec[c + d*x])^2, x]
```

Maple [C] time = 0.259, size = 653, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/2/a^2/d*2^(1/2)*(I*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)-I*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)+sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+sin
(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-6*sin(d*x+c)
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-2*cos(d*x+c)*2^(1/2)+2*2^(1/2)
)*(-1+cos(d*x+c))*cos(d*x+c)^3*(cos(d*x+c)+1)^2*(e*sin(d*x+c)/cos(d*x+c))^(
7/2)/sin(d*x+c)^7
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(7/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^{\frac{7}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^(7/2)/(a*sec(d*x + c) + a)^2, x)
```

3.131 $\int \frac{(e\tan(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=310

$$\frac{e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}-\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d}-\frac{4e^3}{a^2d\sqrt{e\tan(c+dx)}}-\frac{e^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right.}{2\sqrt{2}a^2d}$$

```
[Out] (e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2*d
) - (e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a
^2*d) - (e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c
+ d*x]]])/(2*Sqrt[2]*a^2*d) + (e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
 Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (4*e^3)/(a^2*d*Sqrt[e*T
an[c + d*x]]) + (4*e^3*Cos[c + d*x])/(a^2*d*Sqrt[e*Tan[c + d*x]]) + (4*e^2*
Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(a^2*d*Sqrt
[Sin[2*c + 2*d*x]])
```

Rubi [A] time = 0.465132, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.68, Rules used = {3888, 3886, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2608, 2615, 2572, 2639, 2607, 32}

$$\frac{e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}-\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d}-\frac{4e^3}{a^2d\sqrt{e\tan(c+dx)}}-\frac{e^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right.}{2\sqrt{2}a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2*d
) - (e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a
^2*d) - (e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c
+ d*x]]])/(2*Sqrt[2]*a^2*d) + (e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
 Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (4*e^3)/(a^2*d*Sqrt[e*T
an[c + d*x]]) + (4*e^3*Cos[c + d*x])/(a^2*d*Sqrt[e*Tan[c + d*x]]) + (4*e^2*
Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(a^2*d*Sqrt
[Sin[2*c + 2*d*x]])
```


Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2608

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(n + 1)), x] - Dist[(a^2*(m - 2))/(b^2*(n + 1)), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
```

```
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^4\int \frac{(-a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx}{a^4}$$

$$= \frac{e^4\int\left(\frac{a^2}{(e\tan(c+dx))^{3/2}} - \frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{3/2}} + \frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{3/2}}\right)dx}{a^4}$$

$$= \frac{e^4\int \frac{1}{(e\tan(c+dx))^{3/2}}\,dx}{a^2} + \frac{e^4\int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx}{a^2} - \frac{\left(2e^4\right)\int \frac{\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx}{a^2}$$

$$= -\frac{2e^3}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}} - \frac{e^2\int\sqrt{e\tan(c+dx)}\,dx}{a^2} + \frac{\left(4e^2\right)\int\cos(c+dx)\sqrt{}}{a^2}$$

$$= -\frac{4e^3}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}} - \frac{e^3\,\mathrm{Subst}\left(\int\frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\tan(c+dx)\right)}{a^2d} + \frac{\left(4e^2\sqrt{}\right.}{}$$

$$= -\frac{4e^3}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}} - \frac{\left(2e^3\right)\mathrm{Subst}\left(\int\frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{a^2d} + $$

$$= -\frac{4e^3}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx}}{a^2d\sqrt{\sin(2c+2dx)}}$$

$$= -\frac{4e^3}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx}}{a^2d\sqrt{\sin(2c+2dx)}}$$

$$= -\frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} + \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\right.}{2\sqrt{2}a^2d}$$

$$= \frac{e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} - \frac{e^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} - \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+d\right.}{2\sqrt{2}a}$$

Mathematica [C] time = 6.67189, size = 812, normalized size = 2.62

$$\frac{\csc^2(c+dx)\left(\frac{32\cos\left(\frac{c}{2}\right)\cos(dx)\sec(2c)\sin\left(\frac{c}{2}\right)}{d} + \frac{16\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{dx}{2}\right)}{d} - \frac{16\cos(c)\sec(2c)\sin(dx)}{d} + \frac{16\tan\left(\frac{c}{2}\right)}{d}\right)(e\tan(c+dx))^{5/2}\cos}{(\sec(c+dx)a+a)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Tan[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[c/2 + (d*x)/2]^4*Csc[c + d*x]^2*((32*Cos[c/2]*Cos[d*x]*Sec[2*c]*Sin[c/
2])/d + (16*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d - (16*Cos[c]*Sec[2*
c]*Sin[d*x])/d + (16*Tan[c/2])/d)*(e*Tan[c + d*x])^(5/2))/(a + a*Sec[c + d*
x])^2 + ((-(E^((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^(
(4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c
 + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))
]])*Cos[c/2 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*(e*Tan[c + d*x])^(5/2))/(d
*E^((2*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)
))]*(1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(5/2)) -
((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2
*E^((4*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*A
rcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2
 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*(e*Tan[c + d*x])^(5/2))/(d*E^((2*I)*c
)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^
((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(5/2)) - (8*E^(I*(c
- d*x))*Cos[c/2 + (d*x)/2]^4*(3 - 3*E^((4*I)*(c + d*x)) + E^((4*I)*d*x)*(1
+ E^((4*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/
4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]^2*(e*Tan[c + d*x])^(5/2))/(3
*d*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E
^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(5/2))
```

Maple [C] time = 0.251, size = 360, normalized size = 1.2

$$\frac{\sqrt{2}\,(\cos(dx+c)+1)^2\,(-1+\cos(dx+c))\,(\cos(dx+c))^2}{2\,da^2\,(\sin(dx+c))^5}\left(i\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/2/a^2/d*2^(1/2)*(cos(d*x+c)+1)^2*(I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi(((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+8*EllipticE(((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-4*EllipticF(((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-EllipticPi(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)))*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*(-1+cos(d*x+c))*(e*sin(d*x+c)/cos(d*x+c))^(5/2)*cos(
d*x+c)^2/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*tan(d*x + c))^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

3.132 $\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=316

$$\frac{2e^2\sqrt{\sin(2c+2dx)}\sec(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3a^2d\sqrt{e\tan(c+dx)}}+\frac{e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}-\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d}$$

```
[Out] (e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2*d
) - (e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a
^2*d) + (e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c
+ d*x]]])/(2*Sqrt[2]*a^2*d) - (e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
 Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (4*e^3)/(3*a^2*d*(e*Tan
[c + d*x])^(3/2)) + (4*e^3*Sec[c + d*x])/(3*a^2*d*(e*Tan[c + d*x])^(3/2)) +
 (2*e^2*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(
3*a^2*d*Sqrt[e*Tan[c + d*x]])
```

Rubi [A] time = 0.458754, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.68, Rules used = {3888, 3886, 3474, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2609, 2614, 2573, 2641, 2607, 32}

$$\frac{e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}-\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d}-\frac{4e^3}{3a^2d(e\tan(c+dx))^{3/2}}+\frac{e^{3/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{\ldots}\right.}{2\sqrt{2}a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2*d
) - (e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a
^2*d) + (e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c
+ d*x]]])/(2*Sqrt[2]*a^2*d) - (e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
 Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (4*e^3)/(3*a^2*d*(e*Tan
[c + d*x])^(3/2)) + (4*e^3*Sec[c + d*x])/(3*a^2*d*(e*Tan[c + d*x])^(3/2)) +
 (2*e^2*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(
3*a^2*d*Sqrt[e*Tan[c + d*x]])
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
```

```
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^4\int \frac{(-a+a\sec(c+dx))^2}{(e\tan(c+dx))^{5/2}}\,dx}{a^4}$$
$$= \frac{e^4\int\left(\frac{a^2}{(e\tan(c+dx))^{5/2}} - \frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{5/2}} + \frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{5/2}}\right)dx}{a^4}$$
$$= \frac{e^4\int \frac{1}{(e\tan(c+dx))^{5/2}}\,dx}{a^2} + \frac{e^4\int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx}{a^2} - \frac{\left(2e^4\right)\int \frac{\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx}{a^2}$$
$$= -\frac{2e^3}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{\left(2e^2\right)\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3a^2} - \frac{e^2\int \frac{1}{\sqrt{e\tan(c+dx)}}}{a^2}$$
$$= -\frac{4e^3}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}} - \frac{e^3\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(e^2+x^2\right)}\,dx, x, e\tan(c+dx\right.}{a^2d}$$
$$= -\frac{4e^3}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}} - \frac{\left(2e^3\right)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+d}\right.}{a^2d}$$
$$= -\frac{4e^3}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{2e^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c}}{3a^2d\sqrt{e\tan(c+dx)}}$$
$$= -\frac{4e^3}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{2e^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c}}{3a^2d\sqrt{e\tan(c+dx)}}$$
$$= \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} - \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e}\right.}{2\sqrt{2}a^2d}$$
$$= \frac{e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} - \frac{e^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} + \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+d\right.}{2\sqrt{2}a}$$

Mathematica [F] time = 15.7022, size = 0, normalized size = 0.

$$\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] Integrate[(e*Tan[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2, x]
```

Maple [C] time = 0.252, size = 1267, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/a^2/d*2^(1/2)*(-1+cos(d*x+c))^2*(3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-3*I*EllipticPi(((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+3*sin(d*x+c)*cos(d*x+c)*(
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*sin(d*x+c)*cos(d*x+
c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-10*sin(d*x+c)*cos
(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+3*I*EllipticPi(((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-3*I*sin(d*x+c)*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*sin(d*x+c)*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2),1/2+1/2*I,1/2*2^(1/2))-10*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/
2*2^(1/2))+4*cos(d*x+c)^2*2^(1/2)-4*cos(d*x+c)*2^(1/2))*cos(d*x+c)*(cos(d*x
+c)+1)^2*(e*sin(d*x+c)/cos(d*x+c))^(3/2)/sin(d*x+c)^7
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{(e\tan{(c+dx)})^{\frac{3}{2}}}{\sec^2{(c+dx)}+2\sec{(c+dx)}+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral((e*tan(c + d*x))**(3/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)
/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan{(dx+c)})^{\frac{3}{2}}}{(a\sec{(dx+c)}+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

3.133 $\int \frac{\sqrt{e\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=363

$$-\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}}+\frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}}-\frac{\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}+\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d}+\frac{}{a^2d\sqrt{}}$$

```
[Out] -((Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2
*d)) + (Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2
]*a^2*d) + (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan
[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x
] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (4*e^3)/(5*a^2*d*(e*
Tan[c + d*x])^(5/2)) + (4*e^3*Sec[c + d*x])/(5*a^2*d*(e*Tan[c + d*x])^(5/2)
) + (2*e)/(a^2*d*Sqrt[e*Tan[c + d*x]]) - (12*e*Cos[c + d*x])/(5*a^2*d*Sqrt[
e*Tan[c + d*x]]) - (12*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan
[c + d*x]])/(5*a^2*d*Sqrt[Sin[2*c + 2*d*x]])
```

Rubi [A] time = 0.524864, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 18, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.72, Rules used = {3888, 3886, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2609, 2608, 2615, 2572, 2639, 2607, 32}

$$-\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}}+\frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}}-\frac{\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}+\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d}+\frac{}{a^2d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Tan[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -((Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2
*d)) + (Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2
]*a^2*d) + (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan
[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x
] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (4*e^3)/(5*a^2*d*(e*
Tan[c + d*x])^(5/2)) + (4*e^3*Sec[c + d*x])/(5*a^2*d*(e*Tan[c + d*x])^(5/2)
) + (2*e)/(a^2*d*Sqrt[e*Tan[c + d*x]]) - (12*e*Cos[c + d*x])/(5*a^2*d*Sqrt[
e*Tan[c + d*x]]) - (12*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan
[c + d*x]])/(5*a^2*d*Sqrt[Sin[2*c + 2*d*x]])
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2608

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(n + 1)), x] - Dist[(a^2*(m - 2))/(b^2*(n + 1)), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{e\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^4\int \frac{(-a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx}{a^4}$$

$$= \frac{e^4\int \left(\frac{a^2}{(e\tan(c+dx))^{7/2}} - \frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{7/2}} + \frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{7/2}}\right)dx}{a^4}$$

$$= \frac{e^4\int \frac{1}{(e\tan(c+dx))^{7/2}}\,dx}{a^2} + \frac{e^4\int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx}{a^2} - \frac{\left(2e^4\right)\int \frac{\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx}{a^2}$$

$$= -\frac{2e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} - \frac{e^2\int \frac{1}{(e\tan(c+dx))^{3/2}}\,dx}{a^2} + \frac{\left(6e^2\right)\int \frac{\sec(c+d}{(e\tan(c+d}}{5a^2}$$

$$= -\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{2e}{a^2d\sqrt{e\tan(c+dx)}} - \frac{12e\cos(c+dx)}{5a^2d\sqrt{e\tan(c+d}}$$

$$= -\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{2e}{a^2d\sqrt{e\tan(c+dx)}} - \frac{12e\cos(c+dx)}{5a^2d\sqrt{e\tan(c+d}}$$

$$= -\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{2e}{a^2d\sqrt{e\tan(c+dx)}} - \frac{12e\cos(c+dx)}{5a^2d\sqrt{e\tan(c+d}}$$

$$= -\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{2e}{a^2d\sqrt{e\tan(c+dx)}} - \frac{12e\cos(c+dx)}{5a^2d\sqrt{e\tan(c+d}}$$

$$= -\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{2e}{a^2d\sqrt{e\tan(c+dx)}} - \frac{12e\cos(c+dx)}{5a^2d\sqrt{e\tan(c+d}}$$

$$= \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} - \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e}\right.}{2\sqrt{2}a^2d}$$

$$= -\frac{\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} + \frac{\sqrt{e}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} + \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+d\right.}{2\sqrt{2}a}$$

Mathematica [C] time = 8.30026, size = 2792, normalized size = 7.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Tan[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*((-24*Cos[c/2]*Cos[d*x]*Sec[2*c]*(4*Si
n[c/2] + Sin[(3*c)/2] + Sin[(5*c)/2]))/(5*d*(1 + 2*Cos[c])) - (56*Sec[c/2]*
```

```
Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*S
in[(d*x)/2])/(5*d) - (12*(-2 - 5*Cos[c] - 6*Cos[2*c] + Cos[3*c])*Sec[2*c]*S
in[d*x])/(5*d*(1 + 2*Cos[c])) - (56*Tan[c/2])/(5*d) + (4*Sec[c/2 + (d*x)/2]
^2*Tan[c/2])/(5*d))*Sqrt[e*Tan[c + d*x]])/(a + a*Sec[c + d*x])^2 + ((E^((2*
I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]]
 - 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[S
qrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x)
/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^(I*c)*Sqrt[((-I)*(
-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*
x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) - ((-(E^((4*
I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]]
) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[
Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x
)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^((2*I)*c)*Sqrt[((
-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c
 + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) - ((-(E
^((6*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*
x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Arc
Tanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 +
 (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^((3*I)*c)*Sq
rt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*
I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) +
((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2
*E^((2*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*A
rcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2
 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^(I*c)*Sqrt
[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)
*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) + ((
Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2*E
^((4*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Arc
Tanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 +
 (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^((2*I)*c)*Sq
rt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*
I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) +
((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2
*E^((6*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*A
rcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2
 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^((3*I)*c)*
Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((
2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]])
+ (4*Cos[c/2 + (d*x)/2]^4*(3 - 3*E^((4*I)*(c + d*x)) + E^((4*I)*(c + d*x))*
(1 + E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4,
 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(
5*d*E^(I*(2*c + d*x))*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*
(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^
```

```
2*Sqrt[Tan[c + d*x]]) + (4*Cos[c/2 + (d*x)/2]^4*(3 - 3*E^((4*I)*(c + d*x))
+ E^((2*I)*(c + 2*d*x))*(1 + E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hyp
ergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]^2
*Sqrt[e*Tan[c + d*x]])/(5*d*E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x)))
)/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a +
a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) + (2*E^(I*(c - d*x))*Cos[c/2 + (d*x)/
2]^4*(3 - 3*E^((4*I)*(c + d*x)) + E^((4*I)*d*x)*(1 + E^((4*I)*c))*Sqrt[1 -
E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])
*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*Sqrt[((-I)*(-1 + E^((2*I)
*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*C
os[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) - (2*Cos[c/2 + (d*x)/2]^4
*(3 - 3*E^((4*I)*(c + d*x)) + E^((4*I)*(c + d*x))*(1 + E^((4*I)*c))*Sqrt[1
- E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))
])*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(5*d*E^(I*(3*c + d*x))*Sqr
t[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I
)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) + (
8*Cos[c/2 + (d*x)/2]^4*(-3*E^((2*I)*c)*(-1 + E^((4*I)*(c + d*x))) + E^((4*I
)*d*x)*(1 + E^((6*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/
2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d
*x]])/(5*d*E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(
c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2
*Sqrt[Tan[c + d*x]])
```

Maple [C] time = 0.299, size = 2117, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/10/a^2/d*2^(1/2)*(e*sin(d*x+c)/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+co
s(d*x+c))^3*(-5*I*cos(d*x+c)^2*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/
2*2^(1/2))-10*I*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^
(1/2))+10*I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+
c)+5*I*cos(d*x+c)^2*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
```

```
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Elli
pticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-
5*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-5*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+24*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((1-cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2-12*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2+5*I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-
1/2*I,1/2*2^(1/2))-5*I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*E
llipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2
))-10*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-10*
cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((
1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+48*cos(d*
x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-24*cos(d*x+c)*((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2*2^(1/2))-5*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I
,1/2*2^(1/2))-5*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Elliptic
Pi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+24*(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-12*((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
),1/2*2^(1/2))+2*cos(d*x+c)^2*2^(1/2)-2*cos(d*x+c)*2^(1/2))/sin(d*x+c)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e \tan (dx + c)}}{(a \sec (dx + c) + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*tan(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{e \tan (c+dx)}}{\sec^2 (c+dx)+2 \sec (c+dx)+1}\, dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(1/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(e*tan(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a*
*2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e \tan (dx + c)}}{(a \sec (dx + c) + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*tan(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

3.134 $\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}\,dx$

Optimal. Leaf size=365

$$-\frac{10\sqrt{\sin(2c+2dx)}\sec(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{21a^2d\sqrt{e\tan(c+dx)}}-\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}}+\frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}}-\frac{\tan^{-1}\left(1-\right.}{\sqrt{2}}$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a^2*d*Sqrt[e]
)) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a^2*d*Sqrt
[e]) - Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]]/(
2*Sqrt[2]*a^2*d*Sqrt[e]) + Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqr
t[e*Tan[c + d*x]]]/(2*Sqrt[2]*a^2*d*Sqrt[e]) - (4*e^3)/(7*a^2*d*(e*Tan[c +
d*x])^(7/2)) + (4*e^3*Sec[c + d*x])/(7*a^2*d*(e*Tan[c + d*x])^(7/2)) + (2*e
)/(3*a^2*d*(e*Tan[c + d*x])^(3/2)) - (20*e*Sec[c + d*x])/(21*a^2*d*(e*Tan[c
 + d*x])^(3/2)) - (10*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*
c + 2*d*x]])/(21*a^2*d*Sqrt[e*Tan[c + d*x]])
```

Rubi [A] time = 0.530867, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.68, Rules used = {3888, 3886, 3474, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2609, 2614, 2573, 2641, 2607, 32}

$$-\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}}+\frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d\sqrt{e}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d\sqrt{e}}+\frac{2e}{3a^2d(e\tan(}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a^2*d*Sqrt[e]
)) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a^2*d*Sqrt
[e]) - Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]]/(
2*Sqrt[2]*a^2*d*Sqrt[e]) + Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqr
t[e*Tan[c + d*x]]]/(2*Sqrt[2]*a^2*d*Sqrt[e]) - (4*e^3)/(7*a^2*d*(e*Tan[c +
d*x])^(7/2)) + (4*e^3*Sec[c + d*x])/(7*a^2*d*(e*Tan[c + d*x])^(7/2)) + (2*e
)/(3*a^2*d*(e*Tan[c + d*x])^(3/2)) - (20*e*Sec[c + d*x])/(21*a^2*d*(e*Tan[c
 + d*x])^(3/2)) - (10*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*
c + 2*d*x]])/(21*a^2*d*Sqrt[e*Tan[c + d*x]])
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}\,dx = \frac{e^4\int \frac{(-a+a\sec(c+dx))^2}{(e\tan(c+dx))^{9/2}}\,dx}{a^4}$$

$$= \frac{e^4\int\left(\frac{a^2}{(e\tan(c+dx))^{9/2}} - \frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{9/2}} + \frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{9/2}}\right)dx}{a^4}$$

$$= \frac{e^4\int\frac{1}{(e\tan(c+dx))^{9/2}}\,dx}{a^2} + \frac{e^4\int\frac{\sec^2(c+dx)}{(e\tan(c+dx))^{9/2}}\,dx}{a^2} - \frac{\left(2e^4\right)\int\frac{\sec(c+dx)}{(e\tan(c+dx))^{9/2}}\,dx}{a^2}$$

$$= -\frac{2e^3}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} - \frac{e^2\int\frac{1}{(e\tan(c+dx))^{5/2}}\,dx}{a^2} + \frac{(1}{}$$

$$= -\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{2e}{3a^2d(e\tan(c+dx))^{3/2}} -$$

$$= -\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{2e}{3a^2d(e\tan(c+dx))^{3/2}} -$$

$$= -\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{2e}{3a^2d(e\tan(c+dx))^{3/2}} -$$

$$= -\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{2e}{3a^2d(e\tan(c+dx))^{3/2}} -$$

$$= -\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{2e}{3a^2d(e\tan(c+dx))^{3/2}} -$$

$$= -\frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d\sqrt{e}} + \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx\right.}{2\sqrt{2}a^2}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d\sqrt{e}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d\sqrt{e}} - \frac{\log\left(\sqrt{e}+\sqrt{e}\tan\right.}{}$$

Mathematica [C] time = 8.78683, size = 1281, normalized size = 3.51

$$\frac{80\sqrt[4]{-1}\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right),-1\right)\sqrt{\tan(c+dx)}\sec^5(c+dx)}{21d(\sec(c+dx)a+a)^2\sqrt{e\tan(c+dx)}\left(\tan^2(c+dx)+1\right)^{3/2}} + \frac{\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-\frac{2\sec^4\left(\frac{c}{2}+\right.}{7d}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]),x]
```

```
[Out] (40*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 +
E^((2*I)*(c + d*x)))*Cos[c/2 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[Tan[
c + d*x]])/(21*d*E^(I*(c + d*x))*(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]
]) + (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(E^
((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x
))]] + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTa
nh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (
d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[Tan[c + d*x]])/(d*E^((2*I)*c)*(-1 +
E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]) + (Sqrt[(
(-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(Sqrt[-1 + E^((
4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] + 2*E^((4*I)*c)*Sqr
t[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1
+ E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^4*Se
c[2*c]*Sec[c + d*x]^2*Sqrt[Tan[c + d*x]])/(d*E^((2*I)*c)*(-1 + E^((2*I)*(c
+ d*x)))*(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]) - (2*Sqrt[((-I)*(-1 +
 E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d*x)/2]^4*(3*(
-1 + E^((4*I)*(c + d*x))) + E^((4*I)*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 -
 E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))]
)*Sec[2*c]*Sec[c + d*x]^2*Sqrt[Tan[c + d*x]])/(3*d*E^(I*(2*c + d*x))*(-1 +
E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]) + (2*Sqrt
[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d*
x)/2]^4*(3 - 3*E^((4*I)*(c + d*x)) + E^((2*I)*(c + 2*d*x))*(-1 + E^((2*I)*c
))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*
(c + d*x))])*Sec[2*c]*Sec[c + d*x]^2*Sqrt[Tan[c + d*x]])/(3*d*E^(I*d*x)*(-1
 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]) + (Cos
[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*(-104/(21*d) + (4*(21 - 20*Cos[c] + 21*Cos
[2*c])*Cos[d*x]*Sec[2*c])/(21*d) + (64*Sec[c/2 + (d*x)/2]^2)/(21*d) - (2*Se
c[c/2 + (d*x)/2]^4)/(7*d) - (4*Sec[2*c]*(-20*Sin[c] + 21*Sin[2*c])*Sin[d*x]
)/(21*d))*Tan[c + d*x])/((a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]) + (80
*(-1)^(1/4)*Cos[c/2 + (d*x)/2]^4*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c
+ d*x]]], -1]*Sec[c + d*x]^5*Sqrt[Tan[c + d*x]])/(21*d*(a + a*Sec[c + d*x])
^2*Sqrt[e*Tan[c + d*x]]*(1 + Tan[c + d*x]^2)^(3/2))
```

Maple [C] time = 0.294, size = 1896, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x)
```

[Out] -1/42/a^2/d*2^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^3*(-21*I*EllipticPi(((
1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+21*I*sin(d*x+c)*cos(d*
x+c)^2*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+21*sin(d*x+c)
*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1
/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+21*sin
(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1
/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2
-62*sin(d*x+c)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2
^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-21*
I*sin(d*x+c)*cos(d*x+c)^2*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1
+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(
1/2))+42*I*cos(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
)*sin(d*x+c)+42*sin(d*x+c)*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1
/2*I,1/2*2^(1/2))+42*sin(d*x+c)*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),
1/2+1/2*I,1/2*2^(1/2))-124*sin(d*x+c)*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2),1/2*2^(1/2))+21*I*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)-42*I*cos(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*sin(d*x+c)+21*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*
I,1/2*2^(1/2))+21*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*
2^(1/2))-62*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*E

```
llipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+26*cos(d
*x+c)^3*2^(1/2)-6*cos(d*x+c)^2*2^(1/2)-20*cos(d*x+c)*2^(1/2))/sin(d*x+c)^7/
cos(d*x+c)/(e*sin(d*x+c)/cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^2\sqrt{e\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sqrt(e*tan(d*x + c))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}\sec^2(c+dx)+2\sqrt{e\tan(c+dx)}\sec(c+dx)+\sqrt{e\tan(c+dx)}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**2/(e*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*tan(c + d*x))*sec(c + d*x)**2 + 2*sqrt(e*tan(c + d*x))*s
ec(c + d*x) + sqrt(e*tan(c + d*x))), x)/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^2\sqrt{e\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sqrt(e*tan(d*x + c))), x)
```

3.135 $\int \sqrt{a+a\sec(c+dx)}\tan^5(c+dx)\,dx$

Optimal. Leaf size=147

$$\frac{2(a\sec(c+dx)+a)^{9/2}}{9a^4d}-\frac{6(a\sec(c+dx)+a)^{7/2}}{7a^3d}+\frac{2(a\sec(c+dx)+a)^{5/2}}{5a^2d}+\frac{2(a\sec(c+dx)+a)^{3/2}}{3ad}+\frac{2\sqrt{a\sec(c+dx)}}{d}$$

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*Sqrt[a + a*Se
c[c + d*x]])/d + (2*(a + a*Sec[c + d*x])^(3/2))/(3*a*d) + (2*(a + a*Sec[c +
 d*x])^(5/2))/(5*a^2*d) - (6*(a + a*Sec[c + d*x])^(7/2))/(7*a^3*d) + (2*(a
+ a*Sec[c + d*x])^(9/2))/(9*a^4*d)
```

Rubi [A] time = 0.114971, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3880, 88, 50, 63, 207}

$$\frac{2(a\sec(c+dx)+a)^{9/2}}{9a^4d}-\frac{6(a\sec(c+dx)+a)^{7/2}}{7a^3d}+\frac{2(a\sec(c+dx)+a)^{5/2}}{5a^2d}+\frac{2(a\sec(c+dx)+a)^{3/2}}{3ad}+\frac{2\sqrt{a\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^5,x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*Sqrt[a + a*Se
c[c + d*x]])/d + (2*(a + a*Sec[c + d*x])^(3/2))/(3*a*d) + (2*(a + a*Sec[c +
 d*x])^(5/2))/(5*a^2*d) - (6*(a + a*Sec[c + d*x])^(7/2))/(7*a^3*d) + (2*(a
+ a*Sec[c + d*x])^(9/2))/(9*a^4*d)
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)^2(a+ax)^{5/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^4d}$$

$$= \frac{\text{Subst}\left(\int \left(-3a^2(a+ax)^{5/2} + \frac{a^2(a+ax)^{5/2}}{x} + a(a+ax)^{7/2}\right)dx, x, \sec(c+dx)\right)}{a^4d}$$

$$= -\frac{6(a+a\sec(c+dx))^{7/2}}{7a^3d} + \frac{2(a+a\sec(c+dx))^{9/2}}{9a^4d} + \frac{\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{x}\,dx, x, \text{se}\right.}{a^2d}$$

$$= \frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d} - \frac{6(a+a\sec(c+dx))^{7/2}}{7a^3d} + \frac{2(a+a\sec(c+dx))^{9/2}}{9a^4d} + \frac{\text{S}}{}$$

$$= \frac{2(a+a\sec(c+dx))^{3/2}}{3ad} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d} - \frac{6(a+a\sec(c+dx))^{7/2}}{7a^3d} + \frac{2(}{}$$

$$= \frac{2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3ad} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d} - \frac{6(a}{}$$

$$= \frac{2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3ad} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d} - \frac{6(a}{}$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3ad}$$

Mathematica [A] time = 0.55141, size = 102, normalized size = 0.69

$$\frac{2\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{\sec(c+dx)+1}\left(35\sec^4(c+dx)+5\sec^3(c+dx)-132\sec^2(c+dx)-34\sec(c+dx)+383\right)-\right.}{315d\sqrt{\sec(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^5,x]
```

```
[Out] (2*Sqrt[a*(1 + Sec[c + d*x])]*(-315*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt[
1 + Sec[c + d*x]]*(383 - 34*Sec[c + d*x] - 132*Sec[c + d*x]^2 + 5*Sec[c + d
*x]^3 + 35*Sec[c + d*x]^4)))/(315*d*Sqrt[1 + Sec[c + d*x]])
```

Maple [B] time = 0.268, size = 359, normalized size = 2.4

$$\frac{1}{5040\,d\,(\cos(dx+c))^4}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(315\sqrt{2}(\cos(dx+c))^4\left(-2\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{9/2}\arctan\left(1/2\sqrt{2}\sqrt{-2\frac{\cos}{\cos(}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x)
```

```
[Out] 1/5040/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(315*2^(1/2)*cos(d*x+c)^4*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2))+1260*2^(1/2)*cos(d*x+c)^3*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/
2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+1890*2^(1/2)*co
s(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctan(1/2*2^(1/2)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2))+1260*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(9/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+31
5*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(9/2)+12256*cos(d*x+c)^4-1088*cos(d*x+c)^3-4224*cos(
d*x+c)^2+160*cos(d*x+c)+1120)/cos(d*x+c)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.42903, size = 810, normalized size = 5.51

$$\left[\frac{315\sqrt{a}\cos(dx+c)^4\log\left(-8\,a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos(dx+c)-a\right)}{630\,d\cos(dx+c)^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] [1/630*(315*sqrt(a)*cos(d*x + c)^4*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x +
 c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a
```

```
*cos(d*x + c) - a) + 4*(383*cos(d*x + c)^4 - 34*cos(d*x + c)^3 - 132*cos(d*
x + c)^2 + 5*cos(d*x + c) + 35)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d
*cos(d*x + c)^4), 1/315*(315*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^4 +
2*(383*cos(d*x + c)^4 - 34*cos(d*x + c)^3 - 132*cos(d*x + c)^2 + 5*cos(d*x
+ c) + 35)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\tan^{5}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c)**5,x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x)**5, x)
```

Giac [A] time = 5.0463, size = 261, normalized size = 1.78

$$\frac{\sqrt{2}\left(\frac{315\sqrt{2}a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\left(315\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)^{4}a-210\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)^{3}a^{2}+252\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)^{2}a^{3}+1080\left(a\tan\ldots\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)^{4}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}\ldots}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/315*sqrt(2)*(315*sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a)/sqrt(-a))/sqrt(-a) + 2*(315*(a*tan(1/2*d*x + 1/2*c)^2 - a)^4*a - 2
10*(a*tan(1/2*d*x + 1/2*c)^2 - a)^3*a^2 + 252*(a*tan(1/2*d*x + 1/2*c)^2 - a
)^2*a^3 + 1080*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a^4 + 560*a^5)/((a*tan(1/2*d*
x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))*sgn(cos(d*x + c))
/d
```

3.136 $\int \sqrt{a+a\sec(c+dx)}\tan^3(c+dx)\,dx$

Optimal. Leaf size=99

$$\frac{2(a\sec(c+dx)+a)^{5/2}}{5a^2d}-\frac{2(a\sec(c+dx)+a)^{3/2}}{3ad}-\frac{2\sqrt{a\sec(c+dx)+a}}{d}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[a + a*Sec
[c + d*x]])/d - (2*(a + a*Sec[c + d*x])^(3/2))/(3*a*d) + (2*(a + a*Sec[c +
d*x])^(5/2))/(5*a^2*d)
```

Rubi [A] time = 0.0837909, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3880, 80, 50, 63, 207}

$$\frac{2(a\sec(c+dx)+a)^{5/2}}{5a^2d}-\frac{2(a\sec(c+dx)+a)^{3/2}}{3ad}-\frac{2\sqrt{a\sec(c+dx)+a}}{d}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^3,x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[a + a*Sec
[c + d*x]])/d - (2*(a + a*Sec[c + d*x])^(3/2))/(3*a*d) + (2*(a + a*Sec[c +
d*x])^(5/2))/(5*a^2*d)
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)(a+ax)^{3/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^2d}$$

$$= \frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d} - \frac{\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{x}\,dx, x, \sec(c+dx)\right)}{ad}$$

$$= -\frac{2(a+a\sec(c+dx))^{3/2}}{3ad} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx, x, \sec\right.}{d}$$

$$= -\frac{2\sqrt{a+a\sec(c+dx)}}{d} - \frac{2(a+a\sec(c+dx))^{3/2}}{3ad} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d} - \frac{a\,\text{S}}{}$$

$$= -\frac{2\sqrt{a+a\sec(c+dx)}}{d} - \frac{2(a+a\sec(c+dx))^{3/2}}{3ad} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d} - \frac{2\,\text{S}}{}$$

$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{a+a\sec(c+dx)}}{d} - \frac{2(a+a\sec(c+dx))^{3/2}}{3ad} +$$

Mathematica [A] time = 0.156921, size = 80, normalized size = 0.81

$$\frac{2\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{\sec(c+dx)+1}\left(3\sec^2(c+dx)+\sec(c+dx)-17\right)+15\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)\right)}{15d\sqrt{\sec(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^3,x]
```

```
[Out] (2*Sqrt[a*(1 + Sec[c + d*x])]*(15*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt[1
+ Sec[c + d*x]]*(-17 + Sec[c + d*x] + 3*Sec[c + d*x]^2)))/(15*d*Sqrt[1 + Se
c[c + d*x]])
```

Maple [B] time = 0.235, size = 221, normalized size = 2.2

$$-\frac{1}{60\,d\,(\cos(dx+c))^2}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(15\,(\cos(dx+c))^2\sqrt{2}\arctan\left(1/2\sqrt{2}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\left(-2\frac{\cos(dx+}{\cos(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x)
```

```
[Out] -1/60/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(15*cos(d*x+c)^2*2^(1/2)*arctan
(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(5/2)+30*cos(d*x+c)*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)+15*2^(1/2)*arctan(1/2
*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(5/2)+136*cos(d*x+c)^2-8*cos(d*x+c)-24)/cos(d*x+c)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88456, size = 684, normalized size = 6.91

$$\left[\frac{15\sqrt{a}\cos(dx+c)^2\log\left(-8\,a\cos(dx+c)^2-4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos(dx+c)-a\right)}{30\,d\cos(dx+c)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(a)*cos(d*x + c)^2*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c
)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*c
os(d*x + c) - a) - 4*(17*cos(d*x + c)^2 - cos(d*x + c) - 3)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2), -1/15*(15*sqrt(-a)*arctan(2*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x +
 c) + a))*cos(d*x + c)^2 + 2*(17*cos(d*x + c)^2 - cos(d*x + c) - 3)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sec(c+dx)+1\right)}\tan^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c)**3,x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x)**3, x)
```

Giac [A] time = 5.126, size = 205, normalized size = 2.07

$$-\frac{\sqrt{2}\left(\frac{15\,\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\left(15\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^2a^2-10\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)a^3-12\,a^4\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}{15\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/15*sqrt(2)*(15*sqrt(2)*a^2*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)/sqrt(-a))/sqrt(-a) + 2*(15*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*a^2 -
 10*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a^3 - 12*a^4)/((a*tan(1/2*d*x + 1/2*c)^2
 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))*sgn(cos(d*x + c))/(a*d)
```

3.137 $\int \sqrt{a+a\sec(c+dx)}\tan(c+dx)\,dx$

Optimal. Leaf size=51

$$\frac{2\sqrt{a\sec(c+dx)+a}}{d}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}$$

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*Sqrt[a + a*Se
c[c + d*x]])/d
```

Rubi [A] time = 0.0445641, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3880, 50, 63, 207}

$$\frac{2\sqrt{a\sec(c+dx)+a}}{d}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*Sqrt[a + a*Se
c[c + d*x]])/d
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{d}$$
$$= \frac{2\sqrt{a+a\sec(c+dx)}}{d} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}$$
$$= \frac{2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d}$$
$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2\sqrt{a+a\sec(c+dx)}}{d}$$

Mathematica [A] time = 0.0458126, size = 60, normalized size = 1.18

$$\frac{\sqrt{a(\sec(c+dx)+1)}\left(2\sqrt{\sec(c+dx)+1}-2\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)\right)}{d\sqrt{\sec(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x],x]
```

```
[Out] (Sqrt[a*(1 + Sec[c + d*x])]*(-2*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + 2*Sqrt[1
+ Sec[c + d*x]]))/(d*Sqrt[1 + Sec[c + d*x]])
```

Maple [A] time = 0.049, size = 42, normalized size = 0.8

$$\frac{1}{d}\left(2\sqrt{a+a\sec\left(dx+c\right)}-2\sqrt{a}\,\mathrm{Artanh}\left(\frac{\sqrt{a+a\sec\left(dx+c\right)}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c),x)
```

```
[Out] 1/d*(2*(a+a*sec(d*x+c))^(1/2)-2*a^(1/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1
/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88467, size = 475, normalized size = 9.31

$$\left[\frac{\sqrt{a}\log\left(-8\,a\cos\left(dx+c\right)^{2}+4\left(2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos\left(dx+c\right)-a\right)+4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{2\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)^2 + cos(d*x + c))
*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) +
4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/d, (sqrt(-a)*arctan(2*sqrt(-a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))
```

```
+ 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\tan\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x), x)
```

Giac [A] time = 5.07202, size = 111, normalized size = 2.18

$$\frac{\sqrt{2}a^2\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{2}{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}a}\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="giac")
```

```
[Out] sqrt(2)*a^2*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)
/sqrt(-a))/(sqrt(-a)*a) + 2/(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a))*sgn(co
s(d*x + c))/d
```

3.138 $\int \cot(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (Sqrt[2]*Sqrt[a]*
ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
```

Rubi [A] time = 0.0713238, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3880, 86, 63, 207}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (Sqrt[2]*Sqrt[a]*
ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)\sqrt{a+a\sec(c+dx)}\,dx &= \frac{a^2\,\text{Subst}\left(\int \frac{1}{x(-a+ax)\sqrt{a+ax}}\,dx,x,\sec(c+dx)\right)}{d}\\ &= -\frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx,x,\sec(c+dx)\right)}{d}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a+ax}}\,dx,x,\sec(c+d\right.}{d}\\ &= -\frac{2\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx,x,\sqrt{a+a\sec(c+dx)}\right)}{d}+\frac{(2a)\,\text{Subst}\left(\int \frac{1}{-2a+x^2}\,dx,x,\sqrt{a+\right.}{d}\\ &= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.0495493, size = 72, normalized size = 0.99

$$\frac{\sqrt{a(\sec(c+dx)+1)}\left(2\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\sec(c+dx)+1}}{\sqrt{2}}\right)\right)}{d\sqrt{\sec(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((2*ArcTanh[Sqrt[1 + Sec[c + d*x]]] - Sqrt[2]*ArcTanh[Sqrt[1 + Sec[c + d*x]
]/Sqrt[2]])*Sqrt[a*(1 + Sec[c + d*x])])/(d*Sqrt[1 + Sec[c + d*x]])
```

Maple [A] time = 0.192, size = 98, normalized size = 1.3

$$-\frac{1}{d}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}\sqrt{-2\,\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-2\,\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\right)+\arctan\left(\frac{1}{\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+arctan
(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec\left(dx+c\right)+a}\cot\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*cot(d*x + c), x)
```

Fricas [B] time = 1.70235, size = 647, normalized size = 8.86

$$\left[\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)-3\,a\cos(dx+c)-a}{\cos(dx+c)-1}\right)+2\sqrt{a}\log\left(-2\,a\cos\left(dx+c\right)-2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos\left(dx+c\right)-\right.}{2\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + 2*sqr
t(a)*log(-2*a*cos(d*x + c) - 2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*cos(d*x + c) - a))/d, (sqrt(2)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 2*sqr
t(-a)*arctan(sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/
(a*cos(d*x + c) + a)))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\,(\sec{(c+dx)}+1)}\cot{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x), x)
```

Giac [A] time = 5.02681, size = 117, normalized size = 1.6

$$-\frac{\sqrt{2}a\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*a*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/
sqrt(-a))/sqrt(-a) - arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/s
qrt(-a))*sgn(cos(d*x + c))/d
```

3.139 $\int \cot^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=131

$$\frac{a}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a}{2d(1-\sec(c+dx))\sqrt{a\sec(c+dx)+a}}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{7\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}d}$$

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (7*Sqrt[a]*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(4*Sqrt[2]*d) + a/(4*d*Sqrt
[a + a*Sec[c + d*x]]) + a/(2*d*(1 - Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.118459, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3880, 103, 152, 156, 63, 207}

$$\frac{a}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a}{2d(1-\sec(c+dx))\sqrt{a\sec(c+dx)+a}}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{7\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```


```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (7*Sqrt[a]*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(4*Sqrt[2]*d) + a/(4*d*Sqrt
[a + a*Sec[c + d*x]]) + a/(2*d*(1 - Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]])
```


Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{a^4\,\text{Subst}\left(\int \frac{1}{x(-a+ax)^2(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{a}{2d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}} - \frac{a\,\text{Subst}\left(\int \frac{2a^2+\frac{3a^2x}{2}}{x(-a+ax)(a+ax)^{3/2}}\,dx, x, \sec(c\right.}{2d}$$

$$= \frac{a}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a}{2d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{}{x(-}\right.}{}$$

$$= \frac{a}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a}{2d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}} + \frac{a\,\text{Subst}\left(\int \frac{}{x}\right.}{}$$

$$= \frac{a}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a}{2d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{}{-}\right.}{}$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{7\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}d} + \frac{a}{4d\sqrt{a+a\sec(c}}$$

Mathematica [C] time = 0.274112, size = 87, normalized size = 0.66

$$\frac{\cot^2(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(-7(\sec(c+dx)-1)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(\sec(c+dx)+1)\right)+8(\sec(c+a\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Cot[c + d*x]^2*(-2 - 7*Hypergeometric2F1[-1/2, 1, 1/2, (1 + Sec[c + d*x])/
2]*(-1 + Sec[c + d*x]) + 8*Hypergeometric2F1[-1/2, 1, 1/2, 1 + Sec[c + d*x]
]*(-1 + Sec[c + d*x]))*Sqrt[a*(1 + Sec[c + d*x])])/(4*d)
```

Maple [B] time = 0.26, size = 267, normalized size = 2.

$$\frac{(\cos(dx+c))^2-1}{8\,d\,(\sin(dx+c))^4}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(8\,(\cos(dx+c))^2\sqrt{2}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(1/2\sqrt{2}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/8/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(8*cos(d*x+c)^2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))+7*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-8*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+6*cos(d*x+c)
^2-7*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2))-2*cos(d*x+c))/sin(d*x+c)^4*(cos(d*x+c)^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec(dx+c)+a}\cot(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*cot(d*x + c)^3, x)
```

Fricas [A] time = 2.54273, size = 1118, normalized size = 8.53

$$\left[\frac{8\left(\cos(dx+c)^2-1\right)\sqrt{a}\log\left(-8a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8a\cos(dx+c)-\right.}{16\left(d\,\mathrm{c}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*(cos(d*x + c)^2 - 1)*sqrt(a)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*
x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) -
8*a*cos(d*x + c) - a) + 7*(sqrt(2)*cos(d*x + c)^2 - sqrt(2))*sqrt(a)*log((2
```

```
*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) + 3*a
*cos(d*x + c) + a)/(cos(d*x + c) - 1)) + 4*(3*cos(d*x + c)^2 - cos(d*x + c)
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2 - d), -1/8*(7*
(sqrt(2)*cos(d*x + c)^2 - sqrt(2))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 8*(co
s(d*x + c)^2 - 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(3*cos(d*x + c)^2 - cos(
d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2 - d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\cot^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x)**3, x)
```

Giac [A] time = 5.07272, size = 189, normalized size = 1.44

$$\frac{\sqrt{2}\left(\frac{8\sqrt{2}a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{7a\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}-\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}\right)\mathrm{s}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(8*sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a)/sqrt(-a))/sqrt(-a) - 7*a*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sq
rt(-a))/sqrt(-a) + 2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a)/tan(1/2*d*x + 1/2*c)^2)*sgn(cos(d*x + c))/d
```

3.140 $\int \cot^5(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=193

$$\frac{43a^2}{96d(a\sec(c+dx)+a)^{3/2}}-\frac{15a^2}{16d(1-\sec(c+dx))(a\sec(c+dx)+a)^{3/2}}-\frac{a^2}{4d(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{3/2}}-\frac{}{64d}$$

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (107*Sqrt[a]*ArcT
anh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(64*Sqrt[2]*d) + (43*a^2)/
(96*d*(a + a*Sec[c + d*x])^(3/2)) - a^2/(4*d*(1 - Sec[c + d*x])^2*(a + a*Se
c[c + d*x])^(3/2)) - (15*a^2)/(16*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])
^(3/2)) - (21*a)/(64*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.157803, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3880, 103, 151, 152, 156, 63, 207}

$$\frac{43a^2}{96d(a\sec(c+dx)+a)^{3/2}}-\frac{15a^2}{16d(1-\sec(c+dx))(a\sec(c+dx)+a)^{3/2}}-\frac{a^2}{4d(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{3/2}}-\frac{}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Sqrt[a + a*Sec[c + d*x]],x]
```



```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (107*Sqrt[a]*ArcT
anh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(64*Sqrt[2]*d) + (43*a^2)/
(96*d*(a + a*Sec[c + d*x])^(3/2)) - a^2/(4*d*(1 - Sec[c + d*x])^2*(a + a*Se
c[c + d*x])^(3/2)) - (15*a^2)/(16*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])
^(3/2)) - (21*a)/(64*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^5(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{a^6\operatorname{Subst}\left(\int \frac{1}{x(-a+ax)^3(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{3/2}} - \frac{a^3\operatorname{Subst}\left(\int \frac{4a^2+\frac{7a^2x}{2}}{x(-a+ax)^2(a+ax)^{5/2}}\,dx, x,

```
[In] Integrate[Cot[c + d*x]^5*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Cot[c + d*x]^4*(-2*(57 + 32*Hypergeometric2F1[-3/2, 1, -1/2, 1 + Sec[c + d
*x]]*(-1 + Sec[c + d*x])^2 - 45*Sec[c + d*x]) + 107*Hypergeometric2F1[-3/2,
 1, -1/2, (1 + Sec[c + d*x])/2]*(-1 + Sec[c + d*x])^2)*Sqrt[a*(1 + Sec[c +
d*x])])/(96*d)
```

Maple [B] time = 0.335, size = 407, normalized size = 2.1

$$-\frac{(-1+\cos(dx+c))^2(\cos(dx+c)+1)^2}{384\,d(\sin(dx+c))^8}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(384\sqrt{2}(\cos(dx+c))^4\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(1/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/384/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(cos(d*x+c)+
1)^2*(384*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(
1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+321*cos(d*x+c)^4*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
-768*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+410*cos(d*x+c)^4-642*cos(d*x+c
)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2))+384*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^
(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-142*cos(d*x+c)^3-298*cos(d*x+c)
^2+321*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))+126*cos(d*x+c))/sin(d*x+c)^8
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.63913, size = 1438, normalized size = 7.45

$$\left[\frac{384\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1\right)\sqrt{a}\log\left(-8\,a\cos\left(dx+c\right)^{2}-4\left(2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(384*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sqrt(a)*log(-8*a*cos(d*
x + c)^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) + 321*(sqrt(2)*cos(d*x + c)^4 -
 2*sqrt(2)*cos(d*x + c)^2 + sqrt(2))*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos
(d*x + c) - 1)) - 4*(205*cos(d*x + c)^4 - 71*cos(d*x + c)^3 - 149*cos(d*x +
 c)^2 + 63*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*
x + c)^4 - 2*d*cos(d*x + c)^2 + d), 1/384*(321*(sqrt(2)*cos(d*x + c)^4 - 2*
sqrt(2)*cos(d*x + c)^2 + sqrt(2))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 384*(c
os(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(205
*cos(d*x + c)^4 - 71*cos(d*x + c)^3 - 149*cos(d*x + c)^2 + 63*cos(d*x + c))
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^4 - 2*d*cos(d*x +
 c)^2 + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 4.74632, size = 271, normalized size = 1.4

$$-\frac{\sqrt{2}\left(\frac{384\sqrt{2}a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{321\,a\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{8\left(\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}a^2+15\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+aa^3}\right.}{a^3}\right.}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/384*sqrt(2)*(384*sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)/sqrt(-a))/sqrt(-a) - 321*a*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a)/sqrt(-a))/sqrt(-a) + 8*((-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^2 + 15
*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^3)/a^3 + 3*(21*(-a*tan(1/2*d*x + 1/2
*c)^2 + a)^(3/2)*a - 19*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^2)/(a^2*tan(1
/2*d*x + 1/2*c)^4))*sgn(cos(d*x + c))/d
```

3.141 $\int \sqrt{a+a\sec(c+dx)}\tan^6(c+dx)\,dx$

Optimal. Leaf size=222

$$\frac{2a^6\tan^{11}(c+dx)}{11d(a\sec(c+dx)+a)^{11/2}}+\frac{10a^5\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{2a^4\tan^7(c+dx)}{d(a\sec(c+dx)+a)^{7/2}}+\frac{2a^3\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}-\frac{2a^2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (-2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) - (2*a^2*Tan[c + d*x]^3)/(3*d
*(a + a*Sec[c + d*x])^(3/2)) + (2*a^3*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d
*x])^(5/2)) + (2*a^4*Tan[c + d*x]^7)/(d*(a + a*Sec[c + d*x])^(7/2)) + (10*a
^5*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2)) + (2*a^6*Tan[c + d*x]^1
1)/(11*d*(a + a*Sec[c + d*x])^(11/2))
```

Rubi [A] time = 0.106012, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3887, 461, 203}

$$\frac{2a^6\tan^{11}(c+dx)}{11d(a\sec(c+dx)+a)^{11/2}}+\frac{10a^5\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{2a^4\tan^7(c+dx)}{d(a\sec(c+dx)+a)^{7/2}}+\frac{2a^3\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}-\frac{2a^2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^6,x]
```


```
[Out] (-2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) - (2*a^2*Tan[c + d*x]^3)/(3*d
*(a + a*Sec[c + d*x])^(3/2)) + (2*a^3*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d
*x])^(5/2)) + (2*a^4*Tan[c + d*x]^7)/(d*(a + a*Sec[c + d*x])^(7/2)) + (10*a
^5*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2)) + (2*a^6*Tan[c + d*x]^1
1)/(11*d*(a + a*Sec[c + d*x])^(11/2))
```


Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\tan^6(c+dx)\,dx = -\frac{\left(2a^4\right)\text{Subst}\left(\int \frac{x^6\left(2+ax^2\right)^3}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{\left(2a^4\right)\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}+7x^6+5ax^8+a^2x^{10}-\frac{1}{a^3\left(1+ax^2\right)}\right)dx, x, -\frac{\tan}{\sqrt{a+a\,s}}\right.}{d}$$

$$= \frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^3\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} + \frac{}{d}$$

$$= -\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}$$

Mathematica [A] time = 7.28604, size = 134, normalized size = 0.6

$$-\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(792\sin\left(\frac{1}{2}(c+dx)\right)-1386\sin\left(\frac{3}{2}(c+dx)\right)+495\sin\left(\frac{5}{2}(c+dx)\right)-6\right.}{3960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^6,x]
```

```
[Out] -(Sec[(c + d*x)/2]*Sec[c + d*x]^5*Sqrt[a*(1 + Sec[c + d*x])]*(3960*Sqrt[2]*
ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(11/2) + 792*Sin[(c + d*x)/2]
 - 1386*Sin[(3*(c + d*x))/2] + 495*Sin[(5*(c + d*x))/2] - 616*Sin[(7*(c + d
```

```
*x))/2] - 247*Sin[(11*(c + d*x))/2]))/(3960*d)
```

Maple [B] time = 0.287, size = 566, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^6,x)
```

```
[Out] -1/15840/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(495*arctanh(1/2*2^(1/2)*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*cos(d*x+c)^5*sin(d
*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)+2475*arctanh(1/2*2^(1/2
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*cos(d*x+c)^4*
sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)+4950*arctanh(1/2*2
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*cos(d*x+
c)^3*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)+4950*arctanh(
1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*cos
(d*x+c)^2*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)+2475*arc
tanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c)
)*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)+495*2
^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/
cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*sin(d*x+c)+31616*cos(d*x+
c)^6-15808*cos(d*x+c)^5-27712*cos(d*x+c)^4+1984*cos(d*x+c)^3+13120*cos(d*x+
c)^2-320*cos(d*x+c)-2880)/cos(d*x+c)^5/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.89183, size = 1007, normalized size = 4.54

$$\left[\frac{495\left(\cos(dx+c)^6+\cos(dx+c)^5\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(494\cos(d\right.}{495\left(d\cos(dx+c)^6\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] [1/495*(495*(cos(d*x + c)^6 + cos(d*x + c)^5)*sqrt(-a)*log((2*a*cos(d*x + c
)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d
*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(494*cos(d*x + c)^5 +
 247*cos(d*x + c)^4 - 186*cos(d*x + c)^3 - 155*cos(d*x + c)^2 + 50*cos(d*x
+ c) + 45)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c)^6 + d*cos(d*x + c)^5), 2/495*(495*(cos(d*x + c)^6 + cos(d*x + c)^5)*s
qrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)
*sin(d*x + c))) + (494*cos(d*x + c)^5 + 247*cos(d*x + c)^4 - 186*cos(d*x +
c)^3 - 155*cos(d*x + c)^2 + 50*cos(d*x + c) + 45)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec(c+dx)+1\right)}\tan^6(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c)**6,x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x)**6, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.142 $\int \sqrt{a+a\sec(c+dx)}\tan^4(c+dx)\,dx$

Optimal. Leaf size=160

$$\frac{2a^4\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{6a^3\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}+\frac{2a^2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}+\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{2a\tan\cdots}{d\sqrt{a\sec(\cdots}}$$

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (2*
a*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Tan[c + d*x]^3)/(3*d*
(a + a*Sec[c + d*x])^(3/2)) + (6*a^3*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*
x])^(5/2)) + (2*a^4*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2))
```

Rubi [A] time = 0.0947099, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3887, 461, 203}

$$\frac{2a^4\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{6a^3\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}+\frac{2a^2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}+\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{2a\tan\cdots}{d\sqrt{a\sec(\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^4,x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (2*
a*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Tan[c + d*x]^3)/(3*d*
(a + a*Sec[c + d*x])^(3/2)) + (6*a^3*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*
x])^(5/2)) + (2*a^4*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2))
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
```

```
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\tan^4(c+dx)\,dx = -\frac{(2a^3)\,\text{Subst}\left(\int \frac{x^4(2+ax^2)^2}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a^3)\,\text{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+3x^4+ax^6+\frac{1}{a^2(1+ax^2)}\right)dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{6a^3\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}}+$$

$$= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}-\frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+$$

Mathematica [A] time = 5.81185, size = 110, normalized size = 0.69

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(35\sin\left(\frac{1}{2}(c+dx)\right)-28\sin\left(\frac{3}{2}(c+dx)\right)-23\sin\left(\frac{7}{2}(c+dx)\right)+105\sqrt{2}\,\text{si}\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^4,x]
```

```
[Out] (Sec[(c + d*x)/2]*Sec[c + d*x]^3*Sqrt[a*(1 + Sec[c + d*x])]*(105*Sqrt[2]*Ar
cSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(7/2) + 35*Sin[(c + d*x)/2] - 2
8*Sin[(3*(c + d*x))/2] - 23*Sin[(7*(c + d*x))/2]))/(105*d)
```

Maple [B] time = 0.247, size = 317, normalized size = 2.

$$-\frac{1}{420\,d\sin(dx+c)(\cos(dx+c))^3}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(105\sqrt{2}\sin(dx+c)(\cos(dx+c))^3\operatorname{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+c}{\cos(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^4,x)
```

```
[Out] -1/420/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(105*2^(1/2)*sin(d*x+c)*cos(d*
x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/
cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)+210*arctanh(1/2*2^(1/2)*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(5/2)*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2+105*arctanh(1/2*2^(1/2)
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(5/2)*2^(1/2)*sin(d*x+c)*cos(d*x+c)-736*cos(d*x+c)^4+368*c
os(d*x+c)^3+512*cos(d*x+c)^2-24*cos(d*x+c)-120)/sin(d*x+c)/cos(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.80117, size = 890, normalized size = 5.56

$$\left[\frac{105\left(\cos(dx+c)^4+\cos(dx+c)^3\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-2\left(92\cos(dx\right.}{105\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] [1/105*(105*(cos(d*x + c)^4 + cos(d*x + c)^3)*sqrt(-a)*log((2*a*cos(d*x + c
)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d
*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(92*cos(d*x + c)^3 +
46*cos(d*x + c)^2 - 18*cos(d*x + c) - 15)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3), -2/105*(105*(co
s(d*x + c)^4 + cos(d*x + c)^3)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (92*cos(d*x + c)^3 + 46*c
os(d*x + c)^2 - 18*cos(d*x + c) - 15)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\tan^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c)**4,x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x)**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.143 $\int \sqrt{a+a\sec(c+dx)}\tan^2(c+dx)\,dx$

Optimal. Leaf size=96

$$\frac{2a^2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (-2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Tan[c + d*x]^3)/(3*d
*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.0769082, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3887, 459, 321, 203}

$$\frac{2a^2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^2,x]
```

```
[Out] (-2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Tan[c + d*x]^3)/(3*d
*(a + a*Sec[c + d*x])^(3/2))
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\tan^2(c+dx)\,dx = -\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{x^2\left(2+ax^2\right)}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} - \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{x^2}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{(2a)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}$$

Mathematica [C] time = 3.96644, size = 226, normalized size = 2.35

$$\frac{8\sqrt{2}\tan^3(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{7/2}\sqrt{a(\sec(c+dx)+1)}\left(-\frac{4}{7}\tan^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\text{Hypergeometric2F1}\left(2,\frac{7}{2},\frac{9}{2},-2\right.\right.}{3d\left(1-\text{ta}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^2,x]
```

```
[Out] (8*Sqrt[2]*((1 + Sec[c + d*x])^(-1))^(7/2)*Sqrt[a*(1 + Sec[c + d*x])]*(-(Co
s[c + d*x]*(7 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^4*Sec[(c + d*x)/2]^2*(-3*A
rcTanh[Sqrt[1 - Sec[c + d*x]]]*Cos[c + d*x] + (-1 + 4*Cos[c + d*x])*Sqrt[1
- Sec[c + d*x]]))/(24*Sqrt[1 - Sec[c + d*x]]) - (4*Hypergeometric2F1[2, 7/2
, 9/2, -2*Sec[c + d*x]*Sin[(c + d*x)/2]^2]*Sec[c + d*x]*Tan[(c + d*x)/2]^2)
/7)*Tan[c + d*x]^3)/(3*d*(1 - Tan[(c + d*x)/2]^2)^(5/2))
```

Maple [B] time = 0.197, size = 210, normalized size = 2.2

$$-\frac{1}{6\,d\sin\left(dx+c\right)\cos\left(dx+c\right)}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}\left(3\,\sqrt{2}\left(-2\,\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}\right)^{3/2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin\left(dx+c\right)}{\cos\left(dx+c\right)}\sqrt{-2\,\frac{c}{co}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x)
```

```
[Out] -1/6/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(3*2^(1/2)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)+3*2^(1/2)*arctanh(1/2*2^(1/2)*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(3/2)*sin(d*x+c)+8*cos(d*x+c)^2-4*cos(d*x+c)-4)/sin(d*x+c)/co
s(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.69962, size = 753, normalized size = 7.84

$$\left[\frac{3\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2+2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(2}{3\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] [1/3*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*log((2*a*cos(d*x + c)^2 +
2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c
) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*(2*cos(d*x + c) + 1)*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d
*x + c)), 2/3*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*(2*cos(d*x + c) + 1)*sin(d*x + c))/(d*cos(
d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\tan^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c)**2,x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.144 $\int \cot^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=109

$$-\frac{\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{d}-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{2}d}$$

```
[Out] (-2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (S
qrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(
Sqrt[2]*d) - (Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/d
```

Rubi [A] time = 0.0990509, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3887, 480, 522, 203}

$$-\frac{\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{d}-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```


```
[Out] (-2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (S
qrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(
Sqrt[2]*d) - (Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/d
```


Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
```

```
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d} - \frac{\mathrm{Subst}\left(\int \frac{-3a-a^2x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right.}{d}$$

$$= -\frac{\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{(2\ldots}{}$$

$$= -\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{2}d} - \frac{\cot(c+dx)\sqrt{a+a\ldots}}{d}$$

Mathematica [C] time = 23.8744, size = 5512, normalized size = 50.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.228, size = 188, normalized size = 1.7

$$\frac{1}{2\,d\sin\left(dx+c\right)}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}\left(2\sqrt{-2\,\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin\left(dx+c\right)}{\cos\left(dx+c\right)}\sqrt{-2\,\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\right)\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(2*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)/cos(d*x+c))*sin(d*x+c)+sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)-2*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sec\left(dx+c\right)+a}\cot\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*cot(d*x + c)^2, x)
```

Fricas [A] time = 2.4625, size = 1133, normalized size = 10.39

$$\left[\frac{\sqrt{2}\sqrt{-a}\log\left(-\frac{2\,\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3\,a\cos(dx+c)^{2}-2\,a\cos(dx+c)+a}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right)\sin\left(dx+c\right)+2\,\sqrt{-a}\log\left(-\frac{8\,a\cos(dx+c)^{3}+4\left(2\,c\right.}{}\right.}{4\,d\sin\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x +
 c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 2*sqrt(-a)*l
og(-(8*a*cos(d*x + c)^3 + 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(c
os(d*x + c) + 1))*sin(d*x + c) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c))/(d*sin(d*x + c)), -1/2*(sqrt(2)*sqrt(a)*arctan(sqrt(2)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(
d*x + c) + 2*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin
(d*x + c) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(d*sin(
d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\cot^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.145 $\int \cot^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=196

$$\frac{\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{12ad}+\frac{7\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{8d}+\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{9\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan}{\sqrt{2}\sqrt{a}\sec}\right.}{8\sqrt{2}d}$$

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (9*
Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/
(8*Sqrt[2]*d) + (7*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(8*d) + (Cot[c +
d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(12*a*d) - (Cos[c + d*x]*Cot[c + d*x]^3*
Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(3/2))/(4*a*d)
```

Rubi [A] time = 0.203387, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3887, 472, 583, 522, 203}

$$\frac{\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{12ad}+\frac{7\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{8d}+\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{9\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan}{\sqrt{2}\sqrt{a}\sec}\right.}{8\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (9*
Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/
(8*Sqrt[2]*d) + (7*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(8*d) + (Cot[c +
d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(12*a*d) - (Cos[c + d*x]*Cot[c + d*x]^3*
Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(3/2))/(4*a*d)
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad}$$

$$= -\frac{\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{4ad} - \frac{\text{Subst}\left(\int \frac{}{x^4(1}\right.}{}$$

$$= \frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12ad} - \frac{\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a}{4ad}$$

$$= \frac{7\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{8d} + \frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12ad} - \frac{\cos(c+}{}$$

$$= \frac{7\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{8d} + \frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12ad} - \frac{\cos(c+}{}$$

$$= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{9\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{8\sqrt{2}d} + \frac{7\cot(c+dx)\sqrt{a+a}}{8d}$$

Mathematica [C] time = 23.6937, size = 5562, normalized size = 28.38

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.278, size = 381, normalized size = 1.9

$$-\frac{(\cos(dx+c))^2-1}{48\,d\,(\sin(dx+c))^5}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(48\sqrt{2}(\cos(dx+c))^2\sin(dx+c)\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(}{\cos(d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/48/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(48*2^(1/2)*cos(d*x+c)^2*sin(d*
x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+27*cos(d*x+c)^2*sin(d*x+c)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-48*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)/cos(d*x+c))*sin(d*x+c)-62*cos(d*x+c)^3-27*sin(d*x+c)*ln(-(-(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)+4*cos(d*x+c)^2+42*cos(d*x+c))/sin(d*x+c)^5*(cos(
d*x+c)^2-1)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.60703, size = 1449, normalized size = 7.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(27*(sqrt(2)*cos(d*x + c)^2 - sqrt(2))*sqrt(-a)*log((2*sqrt(2)*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*
cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
))*sin(d*x + c) + 48*(cos(d*x + c)^2 - 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3
 - 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d
*x + c) + 4*(31*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 21*cos(d*x + c))*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c)^2 - d)*sin(d*x + c)), 1
/48*(48*(cos(d*x + c)^2 - 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*
x + c) - a))*sin(d*x + c) + 27*(sqrt(2)*cos(d*x + c)^2 - sqrt(2))*sqrt(a)*a
```

```
rctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)
*sin(d*x + c)))*sin(d*x + c) + 2*(31*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 21
*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c)^2
- d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**(1/2),x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.146 $\int \cot^6(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=280

$$\frac{87\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{160a^2d}-\frac{\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{16a^2d}-\frac{17\cos(c+dx}{}$$

```
[Out] (-2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (1
51*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/(128*Sqrt[2]*d) - (105*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(128*d) -
(23*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(192*a*d) + (87*Cot[c + d*x]
^5*(a + a*Sec[c + d*x])^(5/2))/(160*a^2*d) - (17*Cos[c + d*x]*Cot[c + d*x]^
5*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(5/2))/(32*a^2*d) - (Cos[c + d*x]
^2*Cot[c + d*x]^5*Sec[(c + d*x)/2]^4*(a + a*Sec[c + d*x])^(5/2))/(16*a^2*d)
```

Rubi [A] time = 0.271305, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3887, 472, 579, 583, 522, 203}

$$\frac{87\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{160a^2d}-\frac{\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{16a^2d}-\frac{17\cos(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Sqrt[a + a*Sec[c + d*x]],x]
```


```
[Out] (-2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (1
51*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/(128*Sqrt[2]*d) - (105*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(128*d) -
(23*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(192*a*d) + (87*Cot[c + d*x]
^5*(a + a*Sec[c + d*x])^(5/2))/(160*a^2*d) - (17*Cos[c + d*x]*Cot[c + d*x]^
5*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(5/2))/(32*a^2*d) - (Cos[c + d*x]
^2*Cot[c + d*x]^5*Sec[(c + d*x)/2]^4*(a + a*Sec[c + d*x])^(5/2))/(16*a^2*d)
```


Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
```

```
tegerQ[n - 1/2]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^6(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2d}$$

$$= -\frac{\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{16a^2d} - \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{17\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{32a^2d} - \frac{\cos^2(c+\cdots}{\cdots}$$

$$= \frac{87\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{160a^2d} - \frac{17\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+\cdots\right.}{32a^2d}$$

$$= -\frac{23\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{192ad} + \frac{87\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{160a^2d} - \cdots$$

$$= -\frac{105\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{128d} - \frac{23\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{192ad} + \cdots$$

$$= -\frac{105\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{128d} - \frac{23\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{192ad} + \cdots$$

$$= -\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{151\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{128\sqrt{2}d} - \frac{105\cot(c+dx\cdots}{\cdots}$$

Mathematica [C] time = 23.6775, size = 5604, normalized size = 20.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^6*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.371, size = 573, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/3840/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(cos(d*x+c)+
1)^2*(3840*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*cos(d*x+c)^4*sin(d*
x+c)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/co
s(d*x+c))+2265*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^4*sin(d*x+c)
*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*
x+c))-7680*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/co
s(d*x+c))-5642*cos(d*x+c)^5-4530*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+co
s(d*x+c)-1)/sin(d*x+c))+3840*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*a
rctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+
c))*sin(d*x+c)+556*cos(d*x+c)^4+7928*cos(d*x+c)^3+2265*sin(d*x+c)*ln(-(-(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-460*cos(d*x+c)^2-3150*cos(d*x+c))/sin(d*x+
c)^9
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.86668, size = 1781, normalized size = 6.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(2265*(sqrt(2)*cos(d*x + c)^4 - 2*sqrt(2)*cos(d*x + c)^2 + sqrt(2))
*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
```

```
cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos
(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 3840*(cos(d*x + c)^4 - 2*
cos(d*x + c)^2 + 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 + 4*(2*cos(d*x + c)^2
 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) - 4*(2821*cos(
d*x + c)^5 - 278*cos(d*x + c)^4 - 3964*cos(d*x + c)^3 + 230*cos(d*x + c)^2
+ 1575*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x +
 c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c)), -1/3840*(3840*(cos(d*x + c)^
4 - 2*cos(d*x + c)^2 + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x +
 c) - a))*sin(d*x + c) + 2265*(sqrt(2)*cos(d*x + c)^4 - 2*sqrt(2)*cos(d*x +
 c)^2 + sqrt(2))*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 2*(2821*cos(d*x +
c)^5 - 278*cos(d*x + c)^4 - 3964*cos(d*x + c)^3 + 230*cos(d*x + c)^2 + 1575
*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c)^4
- 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+a*sec(d*x+c))**(1/2),x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.147 $\int (a+a\sec(c+dx))^{3/2}\tan^5(c+dx)\,dx$

Optimal. Leaf size=169

$$\frac{2(a\sec(c+dx)+a)^{11/2}}{11a^4d}-\frac{2(a\sec(c+dx)+a)^{9/2}}{3a^3d}+\frac{2(a\sec(c+dx)+a)^{7/2}}{7a^2d}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{2(a\sec(c+a}{5aa}$$

```
[Out] (-2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*a*Sqrt[a + a*
Sec[c + d*x]])/d + (2*(a + a*Sec[c + d*x])^(3/2))/(3*d) + (2*(a + a*Sec[c +
 d*x])^(5/2))/(5*a*d) + (2*(a + a*Sec[c + d*x])^(7/2))/(7*a^2*d) - (2*(a +
a*Sec[c + d*x])^(9/2))/(3*a^3*d) + (2*(a + a*Sec[c + d*x])^(11/2))/(11*a^4*
d)
```

Rubi [A] time = 0.136147, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3880, 88, 50, 63, 207}

$$\frac{2(a\sec(c+dx)+a)^{11/2}}{11a^4d}-\frac{2(a\sec(c+dx)+a)^{9/2}}{3a^3d}+\frac{2(a\sec(c+dx)+a)^{7/2}}{7a^2d}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{2(a\sec(c+a}{5aa}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^5,x]
```


```
[Out] (-2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*a*Sqrt[a + a*
Sec[c + d*x]])/d + (2*(a + a*Sec[c + d*x])^(3/2))/(3*d) + (2*(a + a*Sec[c +
 d*x])^(5/2))/(5*a*d) + (2*(a + a*Sec[c + d*x])^(7/2))/(7*a^2*d) - (2*(a +
a*Sec[c + d*x])^(9/2))/(3*a^3*d) + (2*(a + a*Sec[c + d*x])^(11/2))/(11*a^4*
d)
```


Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)^2(a+ax)^{7/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^4d}$$

$$= \frac{\text{Subst}\left(\int \left(-3a^2(a+ax)^{7/2}+\frac{a^2(a+ax)^{7/2}}{x}+a(a+ax)^{9/2}\right)dx, x, \sec(c+dx)\right)}{a^4d}$$

$$= -\frac{2(a+a\sec(c+dx))^{9/2}}{3a^3d}+\frac{2(a+a\sec(c+dx))^{11/2}}{11a^4d}+\frac{\text{Subst}\left(\int \frac{(a+ax)^{7/2}}{x}\,dx, x, \text{s}\right.}{a^2d}$$

$$= \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2d}-\frac{2(a+a\sec(c+dx))^{9/2}}{3a^3d}+\frac{2(a+a\sec(c+dx))^{11/2}}{11a^4d}+$$

$$= \frac{2(a+a\sec(c+dx))^{5/2}}{5ad}+\frac{2(a+a\sec(c+dx))^{7/2}}{7a^2d}-\frac{2(a+a\sec(c+dx))^{9/2}}{3a^3d}+$$

$$= \frac{2(a+a\sec(c+dx))^{3/2}}{3d}+\frac{2(a+a\sec(c+dx))^{5/2}}{5ad}+\frac{2(a+a\sec(c+dx))^{7/2}}{7a^2d}-$$

$$= \frac{2a\sqrt{a+a\sec(c+dx)}}{d}+\frac{2(a+a\sec(c+dx))^{3/2}}{3d}+\frac{2(a+a\sec(c+dx))^{5/2}}{5ad}+$$

$$= \frac{2a\sqrt{a+a\sec(c+dx)}}{d}+\frac{2(a+a\sec(c+dx))^{3/2}}{3d}+\frac{2(a+a\sec(c+dx))^{5/2}}{5ad}+$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{2a\sqrt{a+a\sec(c+dx)}}{d}+\frac{2(a+a\sec(c+dx))^{3}}{3d}$$

Mathematica [A] time = 0.48923, size = 112, normalized size = 0.66

$$\frac{2(a(\sec(c+dx)+1))^{3/2}\left(\sqrt{\sec(c+dx)+1}\left(105\sec^5(c+dx)+140\sec^4(c+dx)-325\sec^3(c+dx)-534\sec^2(c+dx)+3\right.\right.}{1155d(\sec(c+dx)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^5,x]
```

```
[Out] (2*(a*(1 + Sec[c + d*x]))^(3/2)*(-1155*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sq
rt[1 + Sec[c + d*x]]*(1656 + 327*Sec[c + d*x] - 534*Sec[c + d*x]^2 - 325*Se
c[c + d*x]^3 + 140*Sec[c + d*x]^4 + 105*Sec[c + d*x]^5)))/(1155*d*(1 + Sec[
c + d*x])^(3/2))
```

Maple [B] time = 0.237, size = 429, normalized size = 2.5

$$-\frac{a}{36960\,d\left(\cos\left(dx+c\right)\right)^{5}}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}\left(1155\,\sqrt{2}\left(\cos\left(dx+c\right)\right)^{5}\arctan\left(1/2\,\sqrt{2}\sqrt{-2\,\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\right)\left(-2\,\frac{\cos}{\cos}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^5,x)
```

```
[Out] -1/36960/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(1155*2^(1/2)*cos(d*x+c)^5
*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(11/2)+5775*2^(1/2)*cos(d*x+c)^4*arctan(1/2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)+11550*2^
(1/2)*cos(d*x+c)^3*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)+11550*2^(1/2)*cos(d*x+c)^2*arctan(1/
2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(11/2)+5775*2^(1/2)*cos(d*x+c)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)+1155*2^(1/2)*arctan(
1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(11/2)-105984*cos(d*x+c)^5-20928*cos(d*x+c)^4+34176*cos(d*x+c)^3+2080
0*cos(d*x+c)^2-8960*cos(d*x+c)-6720)/cos(d*x+c)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07939, size = 921, normalized size = 5.45

$$\left[\frac{1155\,a^{\frac{3}{2}}\cos\left(dx+c\right)^{5}\log\left(-8\,a\cos\left(dx+c\right)^{2}+4\left(2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos\left(dx+c\right)-a\right.}{23}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] [1/2310*(1155*a^(3/2)*cos(d*x + c)^5*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x
 + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8
*a*cos(d*x + c) - a) + 4*(1656*a*cos(d*x + c)^5 + 327*a*cos(d*x + c)^4 - 53
4*a*cos(d*x + c)^3 - 325*a*cos(d*x + c)^2 + 140*a*cos(d*x + c) + 105*a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^5), 1/1155*(1155*sqrt
(-a)*a*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^5 + 2*(1656*a*cos(d*x + c)^5 + 327*
a*cos(d*x + c)^4 - 534*a*cos(d*x + c)^3 - 325*a*cos(d*x + c)^2 + 140*a*cos(
d*x + c) + 105*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^
5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 4.87729, size = 298, normalized size = 1.76

$$\frac{\sqrt{2}\left(\frac{1155\,\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\left(1155\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^5a^2-770\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^4a^3+924\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^3a^4-1320\cdots\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^5\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\cdots}}\right.}{1155\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/1155*sqrt(2)*(1155*sqrt(2)*a^2*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a)/sqrt(-a))/sqrt(-a) + 2*(1155*(a*tan(1/2*d*x + 1/2*c)^2 - a)^5*
a^2 - 770*(a*tan(1/2*d*x + 1/2*c)^2 - a)^4*a^3 + 924*(a*tan(1/2*d*x + 1/2*c
)^2 - a)^3*a^4 - 1320*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*a^5 - 6160*(a*tan(1/
2*d*x + 1/2*c)^2 - a)*a^6 - 3360*a^7)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^5*sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a)))*sgn(cos(d*x + c))/d
```

3.148 $\int (a+a\sec(c+dx))^{3/2}\tan^3(c+dx)\,dx$

Optimal. Leaf size=121

$$\frac{2(a\sec(c+dx)+a)^{7/2}}{7a^2d}+\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{2(a\sec(c+dx)+a)^{5/2}}{5ad}-\frac{2(a\sec(c+dx)+a)^{3/2}}{3d}-\frac{2a\sqrt{a\sec(c+}}{d}$$

```
[Out] (2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (2*a*Sqrt[a + a*S
ec[c + d*x]])/d - (2*(a + a*Sec[c + d*x])^(3/2))/(3*d) - (2*(a + a*Sec[c +
d*x])^(5/2))/(5*a*d) + (2*(a + a*Sec[c + d*x])^(7/2))/(7*a^2*d)
```

Rubi [A] time = 0.10358, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3880, 80, 50, 63, 207}

$$\frac{2(a\sec(c+dx)+a)^{7/2}}{7a^2d}+\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{2(a\sec(c+dx)+a)^{5/2}}{5ad}-\frac{2(a\sec(c+dx)+a)^{3/2}}{3d}-\frac{2a\sqrt{a\sec(c+}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^3,x]
```

```
[Out] (2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (2*a*Sqrt[a + a*S
ec[c + d*x]])/d - (2*(a + a*Sec[c + d*x])^(3/2))/(3*d) - (2*(a + a*Sec[c +
d*x])^(5/2))/(5*a*d) + (2*(a + a*Sec[c + d*x])^(7/2))/(7*a^2*d)
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)(a+ax)^{5/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^2 d}$$

$$= \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2 d} - \frac{\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{x}\,dx, x, \sec(c+dx)\right)}{ad}$$

$$= -\frac{2(a+a\sec(c+dx))^{5/2}}{5ad} + \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2 d} - \frac{\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{x}\,dx, x, \text{se}\right.}{d}$$

$$= -\frac{2(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\sec(c+dx))^{5/2}}{5ad} + \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2 d} -$$

$$= -\frac{2a\sqrt{a+a\sec(c+dx)}}{d} - \frac{2(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\sec(c+dx))^{5/2}}{5ad} + \frac{2}{}$$

$$= -\frac{2a\sqrt{a+a\sec(c+dx)}}{d} - \frac{2(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\sec(c+dx))^{5/2}}{5ad} + \frac{2}{}$$

$$= \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2a\sqrt{a+a\sec(c+dx)}}{d} - \frac{2(a+a\sec(c+dx))^{3/2}}{3d}$$

Mathematica [A] time = 0.247848, size = 92, normalized size = 0.76

$$\frac{2(a(\sec(c+dx)+1))^{3/2}\left(\sqrt{\sec(c+dx)+1}\left(15\sec^3(c+dx)+24\sec^2(c+dx)-32\sec(c+dx)-146\right)+105\tanh^{-1}\left(\sqrt{\text{sec}}\right.\right.}{105d(\sec(c+dx)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^3,x]
```

```
[Out] (2*(a*(1 + Sec[c + d*x]))^(3/2)*(105*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt
[1 + Sec[c + d*x]]*(-146 - 32*Sec[c + d*x] + 24*Sec[c + d*x]^2 + 15*Sec[c +
 d*x]^3)))/(105*d*(1 + Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.204, size = 291, normalized size = 2.4

$$\frac{a}{840\,d\,(\cos(dx+c))^3}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(105\,(\cos(dx+c))^3\sqrt{2}\arctan\left(1/2\sqrt{2}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\left(-2\frac{\cos(dx-}{\cos(dx+c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^3,x)
```

```
[Out] 1/840/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(105*cos(d*x+c)^3*2^(1/2)*arc
tan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(7/2)+315*cos(d*x+c)^2*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)+315*cos(d*x+c)*
2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(7/2)+105*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)-2336*cos(d*x+c)^3
-512*cos(d*x+c)^2+384*cos(d*x+c)+240)/cos(d*x+c)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01883, size = 782, normalized size = 6.46

$$\left[\frac{105\, a^{\frac{3}{2}} \cos\left(dx+c\right)^{3} \log\left(-8\, a \cos\left(dx+c\right)^{2} - 4\left(2\, \cos\left(dx+c\right)^{2} + \cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}} - 8\, a\cos\left(dx+c\right) - a\right)}{210\, d\cos\left(dx+c\right)^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] [1/210*(105*a^(3/2)*cos(d*x + c)^3*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x +
 c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a
*cos(d*x + c) - a) - 4*(146*a*cos(d*x + c)^3 + 32*a*cos(d*x + c)^2 - 24*a*c
```

```
os(d*x + c) - 15*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c
)^3), -1/105*(105*sqrt(-a)*a*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^3 + 2*(146*a*
cos(d*x + c)^3 + 32*a*cos(d*x + c)^2 - 24*a*cos(d*x + c) - 15*a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 4.98603, size = 238, normalized size = 1.97

$$-\frac{\sqrt{2}\left(\frac{105\,\sqrt{2}a^3\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\left(105\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^3a^3-70\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^2a^4+84\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)a^5+120\,a^6\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^3\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}\right)\mathrm{sg}}{105\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/105*sqrt(2)*(105*sqrt(2)*a^3*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a)/sqrt(-a))/sqrt(-a) + 2*(105*(a*tan(1/2*d*x + 1/2*c)^2 - a)^3*a^
3 - 70*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*a^4 + 84*(a*tan(1/2*d*x + 1/2*c)^2
- a)*a^5 + 120*a^6)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a)))*sgn(cos(d*x + c))/(a*d)
```

3.149 $\int (a + a\sec(c + dx))^{3/2} \tan(c + dx)\, dx$

Optimal. Leaf size=73

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{2a\sqrt{a\sec(c+dx)+a}}{d}+\frac{2(a\sec(c+dx)+a)^{3/2}}{3d}$$

```
[Out] (-2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*a*Sqrt[a + a*
Sec[c + d*x]])/d + (2*(a + a*Sec[c + d*x])^(3/2))/(3*d)
```

Rubi [A] time = 0.0552833, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3880, 50, 63, 207}

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{2a\sqrt{a\sec(c+dx)+a}}{d}+\frac{2(a\sec(c+dx)+a)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x],x]
```

```
[Out] (-2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*a*Sqrt[a + a*
Sec[c + d*x]])/d + (2*(a + a*Sec[c + d*x])^(3/2))/(3*d)
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{x}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{2(a+a\sec(c+dx))^{3/2}}{3d} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{2a\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3d} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec\right.}{d}$$

$$= \frac{2a\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3d} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{\right.}{d}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2a\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3d}$$

Mathematica [A] time = 0.137176, size = 70, normalized size = 0.96

$$\frac{2(a(\sec(c+dx)+1))^{3/2}\left(\sqrt{\sec(c+dx)+1}(\sec(c+dx)+4)-3\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)\right)}{3d(\sec(c+dx)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x],x]
```

```
[Out] (2*(a*(1 + Sec[c + d*x]))^(3/2)*(-3*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt[
1 + Sec[c + d*x]]*(4 + Sec[c + d*x])))/(3*d*(1 + Sec[c + d*x])^(3/2))
```

Maple [A] time = 0.031, size = 57, normalized size = 0.8

$$\frac{1}{d}\left(\frac{2}{3}\left(a+a\sec\left(dx+c\right)\right)^{\frac{3}{2}}+2a\sqrt{a+a\sec\left(dx+c\right)}-2a^{3/2}\operatorname{Artanh}\left(\frac{\sqrt{a+a\sec\left(dx+c\right)}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c),x)
```

```
[Out] 1/d*(2/3*(a+a*sec(d*x+c))^(3/2)+2*a*(a+a*sec(d*x+c))^(1/2)-2*a^(3/2)*arctan
h((a+a*sec(d*x+c))^(1/2)/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.9428, size = 626, normalized size = 8.58

$$\left[\frac{3\,a^{\frac{3}{2}}\cos\left(dx+c\right)\log\left(-8\,a\cos\left(dx+c\right)^{2}+4\left(2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos\left(dx+c\right)-a\right)+}{6\,d\cos\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^(3/2)*cos(d*x + c)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)^2
+ cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d
```

```
*x + c) - a) + 4*(4*a*cos(d*x + c) + a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c)))/(d*cos(d*x + c)), 1/3*(3*sqrt(-a)*a*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)
+ 2*(4*a*cos(d*x + c) + a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(
d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a(\sec(c+dx)+1))^{\frac{3}{2}} \tan(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x), x)
```

Giac [A] time = 4.69551, size = 163, normalized size = 2.23

$$\frac{\sqrt{2}a^4\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{2\left(3a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-5a\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}a^2}\right)\operatorname{sgn}(\cos(dx+c))}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*a^4*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + 2*(3*a*tan(1/2*d*x + 1/2*c)^2 - 5*a)/((a*
tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^2))*sgn(c
os(d*x + c))/d
```

3.150 $\int \cot(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=73

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

```
[Out] (2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[2]*a^(3/2
)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
```

Rubi [A] time = 0.0788902, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3880, 83, 63, 207}

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[2]*a^(3/2
)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^2\,\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x(-a+ax)}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} + \frac{\left(2a^3\right)\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a+ax}}\,dx, x, \text{se}\right.}{d}$$

$$= -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d} + \frac{\left(4a^2\right)\text{Subst}\left(\int \frac{1}{-2a+x^2}\,dx, x,\right.}{d}$$

$$= \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

Mathematica [A] time = 0.0572098, size = 72, normalized size = 0.99

$$\frac{(a(\sec(c+dx)+1))^{3/2}\left(2\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\sec(c+dx)+1}}{\sqrt{2}}\right)\right)}{d(\sec(c+dx)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((2*ArcTanh[Sqrt[1 + Sec[c + d*x]]] - 2*Sqrt[2]*ArcTanh[Sqrt[1 + Sec[c + d*
x]]/Sqrt[2]])*(a*(1 + Sec[c + d*x]))^(3/2))/(d*(1 + Sec[c + d*x])^(3/2))
```

Maple [A] time = 0.141, size = 101, normalized size = 1.4

$$-\frac{a}{d}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}\sqrt{-2\,\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-2\,\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\right)+2\,\arctan\left(\frac{1}{\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+2*ar
ctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cot\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*cot(d*x + c), x)
```

Fricas [B] time = 1.67245, size = 644, normalized size = 8.82

$$\left[\frac{\sqrt{2}a^{\frac{3}{2}}\log\left(-\frac{2\,\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)-3\,a\cos(dx+c)-a}{\cos(dx+c)-1}\right)+a^{\frac{3}{2}}\log\left(-2\,a\cos\left(dx+c\right)-2\,\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos\left(dx+c\right)-a\right)}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*a^(3/2)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + a^(3/2)*l
og(-2*a*cos(d*x + c) - 2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c) - a))/d, 2*(sqrt(2)*sqrt(-a)*a*arctan(sqrt(2)*sqrt(-a)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - sqrt(-a
)*a*arctan(sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a
*cos(d*x + c) + a)))/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 4.71027, size = 128, normalized size = 1.75

$$-\frac{\sqrt{2}a^{3}\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}a}-\frac{2\,\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}\right)\operatorname{sgn}\left(\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*a^3*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
)/sqrt(-a))/(sqrt(-a)*a) - 2*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqr
t(-a))/(sqrt(-a)*a))*sgn(cos(d*x + c))/d
```

3.151 $\int \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=109

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}d}+\frac{a\sqrt{a\sec(c+dx)+a}}{2d(1-\sec(c+dx))}$$

```
[Out] (-2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (5*a^(3/2)*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(2*Sqrt[2]*d) + (a*Sqrt[a +
 a*Sec[c + d*x]])/(2*d*(1 - Sec[c + d*x]))
```

Rubi [A] time = 0.107463, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3880, 103, 156, 63, 207}

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}d}+\frac{a\sqrt{a\sec(c+dx)+a}}{2d(1-\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] (-2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (5*a^(3/2)*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(2*Sqrt[2]*d) + (a*Sqrt[a +
 a*Sec[c + d*x]])/(2*d*(1 - Sec[c + d*x]))
```


Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^4\,\text{Subst}\left(\int \frac{1}{x(-a+ax)^2\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{a\sqrt{a+a\sec(c+dx)}}{2d(1-\sec(c+dx))} - \frac{a\,\text{Subst}\left(\int \frac{2a^2+\frac{a^2x}{2}}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{2d}$$

$$= \frac{a\sqrt{a+a\sec(c+dx)}}{2d(1-\sec(c+dx))} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} - \frac{\left(5a^3\right)\text{Subst}\left(\right.}{}$$

$$= \frac{a\sqrt{a+a\sec(c+dx)}}{2d(1-\sec(c+dx))} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d} - \frac{\left(5a^2\right)}{}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}d} + \frac{a\sqrt{a+a\sec(c}}{2d(1-\sec(c+}$$

Mathematica [A] time = 0.339515, size = 99, normalized size = 0.91

$$\frac{(a(\sec(c+dx)+1))^{3/2}\left(-\frac{2\sqrt{\sec(c+dx)+1}}{\sec(c+dx)-1}-8\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)+5\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\sec(c+dx)+1}}{\sqrt{2}}\right)\right)}{4d(\sec(c+dx)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(3/2)*(-8*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + 5*Sqrt[
2]*ArcTanh[Sqrt[1 + Sec[c + d*x]]/Sqrt[2]] - (2*Sqrt[1 + Sec[c + d*x]])/(-1
 + Sec[c + d*x])))/(4*d*(1 + Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.213, size = 258, normalized size = 2.4

$$-\frac{a}{4\,d\,(\sin(dx+c))^2}\sqrt{\frac{a\,(\cos(dx+c)+1)}{\cos(dx+c)}}\left(4\,(\cos(dx+c))^2\sqrt{2}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(1/2\,\sqrt{2}\sqrt{-2\,\frac{\cos(dx+c}{\cos(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(4*cos(d*x+c)^2*2^(1/2)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2))+5*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-4*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+2*cos(d*x
+c)^2-5*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2))+2*cos(d*x+c))/sin(d*x+c)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

[Out] Timed out

Fricas [B] time = 1.76525, size = 1003, normalized size = 9.2

$$\left[\frac{4\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos\left(dx+c\right)+8\left(a\cos\left(dx+c\right)-a\right)\sqrt{a}\log\left(-2\,a\cos\left(dx+c\right)+2\,\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos\left(dx+c\right)-a\right)+5}{8\left(d\cos\left(dx+c\right)-d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) + 8*(a*cos(d
*x + c) - a)*sqrt(a)*log(-2*a*cos(d*x + c) + 2*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*cos(d*x + c) - a) + 5*(sqrt(2)*a*cos(d*x + c) - sqrt(2)
*a)*sqrt(a)*log((2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c) + 3*a*cos(d*x + c) + a)/(cos(d*x + c) - 1)))/(d*cos(d*x + c) -
 d), -1/4*(5*(sqrt(2)*a*cos(d*x + c) - sqrt(2)*a)*sqrt(-a)*arctan(sqrt(2)*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c
) + a)) - 8*(a*cos(d*x + c) - a)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 2*a*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(d*cos(d*x + c) - d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 5.00076, size = 167, normalized size = 1.53

$$\frac{\sqrt{2}a^2\left(\frac{4\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{5\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}\right)\operatorname{sgn}\left(\cos\left(dx+c\right)\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*a^2*(4*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a)/sqrt(-a))/sqrt(-a) - 5*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sq
rt(-a))/sqrt(-a) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/(a*tan(1/2*d*x + 1/2
*c)^2))*sgn(cos(d*x + c))/d
```

3.152 $\int \cot^5(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=171

$$\frac{7a^2}{32d\sqrt{a\sec(c+dx)+a}}-\frac{13a^2}{16d(1-\sec(c+dx))\sqrt{a\sec(c+dx)+a}}-\frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}\tanh}{}$$

```
[Out] (2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (71*a^(3/2)*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(32*Sqrt[2]*d) + (7*a^2)/(3
2*d*Sqrt[a + a*Sec[c + d*x]]) - a^2/(4*d*(1 - Sec[c + d*x])^2*Sqrt[a + a*Se
c[c + d*x]]) - (13*a^2)/(16*d*(1 - Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.143066, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3880, 103, 151, 152, 156, 63, 207}

$$\frac{7a^2}{32d\sqrt{a\sec(c+dx)+a}}-\frac{13a^2}{16d(1-\sec(c+dx))\sqrt{a\sec(c+dx)+a}}-\frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}\tanh}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] (2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (71*a^(3/2)*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(32*Sqrt[2]*d) + (7*a^2)/(3
2*d*Sqrt[a + a*Sec[c + d*x]]) - a^2/(4*d*(1 - Sec[c + d*x])^2*Sqrt[a + a*Se
c[c + d*x]]) - (13*a^2)/(16*d*(1 - Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]])
```


Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
```

```
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
```

```
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^6\,\text{Subst}\left(\int \frac{1}{x(-a+ax)^3(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a+a\sec(c+dx)}} - \frac{a^3\,\text{Subst}\left(\int \frac{4a^2+\frac{5a^2x}{2}}{x(-a+ax)^2(a+ax)^{3/2}}\,dx, x, s\right.}{4d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a+a\sec(c+dx)}} - \frac{13a^2}{16d(1-\sec(c+dx))\sqrt{a+a\sec(c+}}$$

$$= \frac{7a^2}{32d\sqrt{a+a\sec(c+dx)}} - \frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a+a\sec(c+dx)}} - \frac{}{16d(1-\text{se}}$$

$$= \frac{7a^2}{32d\sqrt{a+a\sec(c+dx)}} - \frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a+a\sec(c+dx)}} - \frac{}{16d(1-\text{se}}$$

$$= \frac{7a^2}{32d\sqrt{a+a\sec(c+dx)}} - \frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a+a\sec(c+dx)}} - \frac{}{16d(1-\text{se}}$$

$$= \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{71a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}d} + \frac{7a^2}{32d\sqrt{a+a\sec}}$$

Mathematica [C] time = 0.272561, size = 104, normalized size = 0.61

$$\frac{a^2\left(71(\sec(c+dx)-1)^2\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(\sec(c+dx)+1)\right)-64(\sec(c+dx)-1)^2\text{Hypergeometric2F1}\right.}{32d(\sec(c+dx)-1)^2\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a^2*(-34 + 71*Hypergeometric2F1[-1/2, 1, 1/2, (1 + Sec[c + d*x])/2]*(-1 +
Sec[c + d*x])^2 - 64*Hypergeometric2F1[-1/2, 1, 1/2, 1 + Sec[c + d*x]]*(-1
```

```
+ Sec[c + d*x])^2 + 26*Sec[c + d*x]))/(32*d*(-1 + Sec[c + d*x])^2*Sqrt[a*(1
 + Sec[c + d*x])])
```

Maple [B] time = 0.283, size = 502, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/64/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(cos(d*x+c)+1)
^2*(64*2^(1/2)*cos(d*x+c)^3*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2
*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-64*cos(d*x+c)^2*2^(1/2)*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2))+71*cos(d*x+c)^3*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(
1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-64*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))-71*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2))+64*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+54*cos(d*x+c)^
3-71*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2))-24*cos(d*x+c)^2+71*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-14*cos(d*x+c))/sin(d*x
+c)^6
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.58474, size = 1555, normalized size = 9.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/128*(64*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*sqrt(
a)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) + 71*(sqrt(2)
*a*cos(d*x + c)^3 - sqrt(2)*a*cos(d*x + c)^2 - sqrt(2)*a*cos(d*x + c) + sqr
t(2)*a)*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) - 4*(27*a*cos
(d*x + c)^3 - 12*a*cos(d*x + c)^2 - 7*a*cos(d*x + c))*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c)))/(d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x + c) +
 d), 1/64*(71*(sqrt(2)*a*cos(d*x + c)^3 - sqrt(2)*a*cos(d*x + c)^2 - sqrt(2
)*a*cos(d*x + c) + sqrt(2)*a)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 64*(a*cos(
d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*sqrt(-a)*arctan(2*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c)
+ a)) - 2*(27*a*cos(d*x + c)^3 - 12*a*cos(d*x + c)^2 - 7*a*cos(d*x + c))*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^3 - d*cos(d*x + c)^2
 - d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 5.04413, size = 230, normalized size = 1.35

$$-\frac{\sqrt{2}\left(\frac{64\sqrt{2}a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{71a\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+8\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}+\frac{17\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\cdots\right.}{a^2\tan\cdots}\right.}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/64*sqrt(2)*(64*sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a)/sqrt(-a))/sqrt(-a) - 71*a*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
)/sqrt(-a))/sqrt(-a) + 8*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a) + (17*(-a*tan(
1/2*d*x + 1/2*c)^2 + a)^(3/2)*a - 15*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^
2)/(a^2*tan(1/2*d*x + 1/2*c)^4))*a*sgn(cos(d*x + c))/d
```

3.153 $\int (a + a\sec(c + dx))^{3/2} \tan^6(c + dx)\,dx$

Optimal. Leaf size=258

$$\frac{2a^8\tan^{13}(c+dx)}{13d(a\sec(c+dx)+a)^{13/2}}+\frac{14a^7\tan^{11}(c+dx)}{11d(a\sec(c+dx)+a)^{11/2}}+\frac{34a^6\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{30a^5\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{2a^4\tan}{5d(a\sec(}$$

```
[Out] (-2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) - (2*a^3*Tan[c + d*x]^3)/(3
*d*(a + a*Sec[c + d*x])^(3/2)) + (2*a^4*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c +
 d*x])^(5/2)) + (30*a^5*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2)) +
(34*a^6*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2)) + (14*a^7*Tan[c +
d*x]^11)/(11*d*(a + a*Sec[c + d*x])^(11/2)) + (2*a^8*Tan[c + d*x]^13)/(13*d
*(a + a*Sec[c + d*x])^(13/2))
```

Rubi [A] time = 0.121799, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3887, 461, 203}

$$\frac{2a^8\tan^{13}(c+dx)}{13d(a\sec(c+dx)+a)^{13/2}}+\frac{14a^7\tan^{11}(c+dx)}{11d(a\sec(c+dx)+a)^{11/2}}+\frac{34a^6\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{30a^5\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{2a^4\tan}{5d(a\sec(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^6,x]
```


```
[Out] (-2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) - (2*a^3*Tan[c + d*x]^3)/(3
*d*(a + a*Sec[c + d*x])^(3/2)) + (2*a^4*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c +
 d*x])^(5/2)) + (30*a^5*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2)) +
(34*a^6*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2)) + (14*a^7*Tan[c +
d*x]^11)/(11*d*(a + a*Sec[c + d*x])^(11/2)) + (2*a^8*Tan[c + d*x]^13)/(13*d
*(a + a*Sec[c + d*x])^(13/2))
```


Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
```

```
tegerQ[n - 1/2]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}\tan^6(c+dx)\,dx = -\frac{\left(2a^5\right)\text{Subst}\left(\int \frac{x^6\left(2+ax^2\right)^4}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{\left(2a^5\right)\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}+15x^6+17ax^8+7a^2x^{10}+a^3x^{12}-\frac{1}{a^3\left(1+ax^2\right)}\right)\right.}{d}$$

$$= \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a^3\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^4\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} +$$

$$= -\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a^3\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3}}$$

Mathematica [A] time = 8.41438, size = 147, normalized size = 0.57

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(164736\sin\left(\frac{1}{2}(c+dx)\right)+81081\sin\left(\frac{3}{2}(c+dx)\right)+134849\sin\left(\frac{5}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^6,x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]^6*Sqrt[a*(1 + Sec[c + d*x])]*(-1441440*Sqr
t[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(13/2) + 164736*Sin[(c +
```

```
d*x)/2] + 81081*Sin[(3*(c + d*x))/2] + 134849*Sin[(5*(c + d*x))/2] + 98176
*Sin[(9*(c + d*x))/2] + 45045*Sin[(11*(c + d*x))/2] + 32429*Sin[(13*(c + d*
x))/2]))/(1441440*d)
```

Maple [B] time = 0.259, size = 656, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^6,x)
```

```
[Out] 1/2882880/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(45045*2^(1/2)*sin(d*x+c)
*cos(d*x+c)^6*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(13/2)+270270*2^(1/2)*sin
(d*x+c)*cos(d*x+c)^5*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(13/2)+675675*2^(1
/2)*sin(d*x+c)*cos(d*x+c)^4*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(13/2)+9009
00*2^(1/2)*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(13/
2)+675675*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(13/2)+270270*2^(1/2)*sin(d*x+c)*cos(d*x+c)*arctanh(1/2*2^(1/2)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(13/2)+45045*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(13/2)*sin
(d*x+c)-4150912*cos(d*x+c)^7-807424*cos(d*x+c)^6+5563904*cos(d*x+c)^5+27819
52*cos(d*x+c)^4-2585600*cos(d*x+c)^3-1809920*cos(d*x+c)^2+564480*cos(d*x+c)
+443520)/cos(d*x+c)^6/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^6,x, algorithm="maxima")
```

[Out] Timed out

Fricas [A] time = 1.98966, size = 1156, normalized size = 4.48

$$\left[\frac{45045\left(a\cos\left(dx+c\right)^{7}+a\cos\left(dx+c\right)^{6}\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^{2}+2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(3242\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] [1/45045*(45045*(a*cos(d*x + c)^7 + a*cos(d*x + c)^6)*sqrt(-a)*log((2*a*cos
(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(32429*a*cos(
d*x + c)^6 + 38737*a*cos(d*x + c)^5 - 4731*a*cos(d*x + c)^4 - 26465*a*cos(d
*x + c)^3 - 6265*a*cos(d*x + c)^2 + 7875*a*cos(d*x + c) + 3465*a)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^7 + d*cos(d*x
+ c)^6), 2/45045*(45045*(a*cos(d*x + c)^7 + a*cos(d*x + c)^6)*sqrt(a)*arcta
n(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c
))) + (32429*a*cos(d*x + c)^6 + 38737*a*cos(d*x + c)^5 - 4731*a*cos(d*x + c
)^4 - 26465*a*cos(d*x + c)^3 - 6265*a*cos(d*x + c)^2 + 7875*a*cos(d*x + c)
+ 3465*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
+ c)^7 + d*cos(d*x + c)^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.154 $\int (a+a\sec(c+dx))^{3/2}\tan^4(c+dx)\,dx$

Optimal. Leaf size=194

$$\frac{2a^6\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{10a^5\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{14a^4\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}+\frac{2a^3\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}+\frac{2a^{3/2}\tan^{-1}(}{}$$

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (2*
a^2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*Tan[c + d*x]^3)/(3*
d*(a + a*Sec[c + d*x])^(3/2)) + (14*a^4*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c +
 d*x])^(5/2)) + (10*a^5*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2)) +
(2*a^6*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2))
```

Rubi [A] time = 0.109926, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3887, 461, 203}

$$\frac{2a^6\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{10a^5\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{14a^4\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}+\frac{2a^3\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}+\frac{2a^{3/2}\tan^{-1}(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^4,x]
```


```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (2*
a^2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*Tan[c + d*x]^3)/(3*
d*(a + a*Sec[c + d*x])^(3/2)) + (14*a^4*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c +
 d*x])^(5/2)) + (10*a^5*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2)) +
(2*a^6*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2))
```


Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}\tan^4(c+dx)\,dx = -\frac{\left(2a^4\right)\text{Subst}\left(\int \frac{x^4\left(2+ax^2\right)^3}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{\left(2a^4\right)\text{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+7x^4+5ax^6+a^2x^8+\frac{1}{a^2\left(1+ax^2\right)}\right) dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2a^3\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{14a^4\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}}+$$

$$= \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}-\frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2a^3\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}$$

Mathematica [A] time = 6.4642, size = 123, normalized size = 0.63

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(126\sin\left(\frac{1}{2}(c+dx)\right)-288\sin\left(\frac{5}{2}(c+dx)\right)-315\sin\left(\frac{7}{2}(c+dx)\right)-169\right.}{2520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^4,x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]^4*Sqrt[a*(1 + Sec[c + d*x])]*(2520*Sqrt[2]
*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(9/2) + 126*Sin[(c + d*x)/2]
 - 288*Sin[(5*(c + d*x))/2] - 315*Sin[(7*(c + d*x))/2] - 169*Sin[(9*(c + d*
x))/2]))/(2520*d)
```

Maple [B] time = 0.222, size = 407, normalized size = 2.1

$$\frac{a}{2520\,d\left(\cos\left(dx+c\right)\right)^{4}\sin\left(dx+c\right)}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}\left(315\sqrt{2}\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^{4}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin\left(dx+c\right.}{\cos\left(dx+c\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^4,x)
```

```
[Out] 1/2520/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(315*2^(1/2)*sin(d*x+c)*cos(
d*x+c)^4*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)+945*2^(1/2)*sin(d*x+c)*c
os(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)+945*2^(1/2)*sin(d*x+c
)*cos(d*x+c)^2*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)+315*2^(1/2)*sin(d*
x+c)*cos(d*x+c)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)+2704*cos(d*x+c)^5
+1168*cos(d*x+c)^4-3488*cos(d*x+c)^3-1744*cos(d*x+c)^2+800*cos(d*x+c)+560)/
cos(d*x+c)^4/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.82924, size = 987, normalized size = 5.09

$$\left[\frac{315\left(a\cos\left(dx+c\right)^{5}+a\cos\left(dx+c\right)^{4}\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^{2}-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-2\left(169\,a\cos\right.}{315\left(d\cos\left(dx+c\right)^{5}+d\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] [1/315*(315*(a*cos(d*x + c)^5 + a*cos(d*x + c)^4)*sqrt(-a)*log((2*a*cos(d*x
 + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*s
in(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(169*a*cos(d*x +
c)^4 + 242*a*cos(d*x + c)^3 + 24*a*cos(d*x + c)^2 - 85*a*cos(d*x + c) - 35*
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5
+ d*cos(d*x + c)^4), -2/315*(315*(a*cos(d*x + c)^5 + a*cos(d*x + c)^4)*sqrt
(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*si
n(d*x + c))) + (169*a*cos(d*x + c)^4 + 242*a*cos(d*x + c)^3 + 24*a*cos(d*x
+ c)^2 - 85*a*cos(d*x + c) - 35*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sin(d*x + c))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.155 $\int (a+a\sec(c+dx))^{3/2}\tan^2(c+dx)\,dx$

Optimal. Leaf size=128

$$\frac{2a^4\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}+\frac{2a^3\tan^3(c+dx)}{d(a\sec(c+dx)+a)^{3/2}}-\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (-2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*Tan[c + d*x]^3)/(d
*(a + a*Sec[c + d*x])^(3/2)) + (2*a^4*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d
*x])^(5/2))
```

Rubi [A] time = 0.0927509, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3887, 461, 203}

$$\frac{2a^4\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}+\frac{2a^3\tan^3(c+dx)}{d(a\sec(c+dx)+a)^{3/2}}-\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^2,x]
```

```
[Out] (-2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*Tan[c + d*x]^3)/(d
*(a + a*Sec[c + d*x])^(3/2)) + (2*a^4*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d
*x])^(5/2))
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
```

```
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}\tan^2(c+dx)\,dx = -\frac{\left(2a^3\right)\text{Subst}\left(\int \frac{x^2\left(2+ax^2\right)^2}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{\left(2a^3\right)\text{Subst}\left(\int \left(\frac{1}{a}+3x^2+ax^4-\frac{1}{a\left(1+ax^2\right)}\right)dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^3\tan^3(c+dx)}{d(a+a\sec(c+dx))^{3/2}} + \frac{2a^4\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} + \cdots$$

$$= -\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^3\tan^3(c+dx)}{d(a+a\sec(c+dx))^{3/2}} \cdots$$

Mathematica [A] time = 5.53601, size = 97, normalized size = 0.76

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(5\sin\left(\frac{3}{2}(c+dx)\right)+\sin\left(\frac{5}{2}(c+dx)\right)-10\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cdots\right.}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^2,x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]^2*Sqrt[a*(1 + Sec[c + d*x])]*(-10*Sqrt[2]*
ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(5/2) + 5*Sin[(3*(c + d*x))/2
] + Sin[(5*(c + d*x))/2]))/(10*d)
```

Maple [B] time = 0.183, size = 300, normalized size = 2.3

$$\frac{a}{20\,d\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^{2}}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}\left(5\,\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin\left(dx+c\right)}{\cos\left(dx+c\right)}\sqrt{-2\,\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\right)\left(-2\,\frac{\cos}{\cos(d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^2,x)
```

```
[Out] 1/20/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(5*arctanh(1/2*2^(1/2)*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(5/2)*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2+10*arctanh(1/2*2^(1/2)*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(5/2)*2^(1/2)*sin(d*x+c)*cos(d*x+c)+5*2^(1/2)*arctanh(1/2*2^(1/2
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)-8*cos(d*x+c)^3-16*cos(d*x+c)^2+16*cos(d*
x+c)+8)/sin(d*x+c)/cos(d*x+c)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.74454, size = 832, normalized size = 6.5

$$\left[\frac{5\left(a\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)^{2}\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^{2}+2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(a\cos\left(dx\right.\right.}{5\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] [1/5*(5*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(-a)*log((2*a*cos(d*x + c
)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d
*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(a*cos(d*x + c)^2 + 3
*a*cos(d*x + c) + a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/
(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 2/5*(5*(a*cos(d*x + c)^3 + a*cos(d*x
 + c)^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c
)/(sqrt(a)*sin(d*x + c))) + (a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(
d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a(\sec(c+dx)+1))^{\frac{3}{2}}\tan^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c)**2,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.156 $\int \cot^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=64

$$-\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{2a\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{d}$$

```
[Out] (-2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (2
*a*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/d
```

Rubi [A] time = 0.0709612, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3887, 325, 203}

$$-\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{2a\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (2
*a*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/d
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$
$$= -\frac{2a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$
$$= -\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{2a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d}$$

Mathematica [A] time = 0.362542, size = 102, normalized size = 1.59

$$-\frac{2\cot(c+dx)\sqrt{\frac{1}{\sec(c+dx)+1}}(a(\sec(c+dx)+1))^{3/2}\left(\sqrt{\cos(c+dx)}\sqrt{\frac{1}{\cos(c+dx)+1}}+\tan\left(\frac{1}{2}(c+dx)\right)\sin^{-1}\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{1}{\cos(c+dx)+1}}}\right)\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cot[c + d*x]*Sqrt[(1 + Sec[c + d*x])^(-1)]*(a*(1 + Sec[c + d*x]))^(3/2)
*(Sqrt[Cos[c + d*x]]*Sqrt[(1 + Cos[c + d*x])^(-1)] + ArcSin[Tan[(c + d*x)/2
]/Sqrt[(1 + Cos[c + d*x])^(-1)]]*Tan[(c + d*x)/2]))/d
```

Maple [B] time = 0.197, size = 115, normalized size = 1.8

$$-\frac{a}{d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(-\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}\sin(dx+c)}{2\cos(dx+c)}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sin(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)/cos(d*x+c))*sin(d*x+c)+2*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*cot(d*x + c)^2, x)
```

Fricas [B] time = 1.96566, size = 684, normalized size = 10.69

$$\left[\frac{\sqrt{-a}a\log\left(-\frac{8\,a\cos(dx+c)^3+4\left(2\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)-7\,a\cos(dx+c)+a}{\cos(dx+c)+1}\right)\sin(dx+c)-4\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\mathrm{co}}{2\,d\sin(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a)*a*log(-(8*a*cos(d*x + c)^3 + 4*(2*cos(d*x + c)^2 - cos(d*x +
 c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*co
s(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) - 4*a*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*cos(d*x + c))/(d*sin(d*x + c)), -(a^(3/2)*arctan(2*sqrt
(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*
cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2*a*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*cos(d*x + c))/(d*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.157 $\int \cot^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=144

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}d}-\frac{\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{3d}+\frac{3a\cot(c+dx)\sqrt{a\sec(}}{2d}$$

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (a^
(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2
*Sqrt[2]*d) + (3*a*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(2*d) - (Cot[c +
d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(3*d)
```

Rubi [A] time = 0.148464, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3887, 480, 583, 522, 203}

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}d}-\frac{\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{3d}+\frac{3a\cot(c+dx)\sqrt{a\sec(}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (a^
(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2
*Sqrt[2]*d) + (3*a*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(2*d) - (Cot[c +
d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(3*d)
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
```

```
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{3d} - \frac{\text{Subst}\left(\int \frac{-9a-3a^2x^2}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(\cdots}{\sqrt{a+a\,\text{se}\cdots}}\right.}{3d}$$

$$= \frac{3a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{2d} - \frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{3d} + \frac{\text{Sub}\cdots}{\cdots}$$

$$= \frac{3a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{2d} - \frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{3d} + \frac{a^2\,\text{S}\cdots}{\cdots}$$

$$= \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}d} + \frac{3a\cot(c+dx)\sqrt{\cdots}}{2\cdots}$$

Mathematica [C] time = 23.7178, size = 5552, normalized size = 38.56

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.234, size = 372, normalized size = 2.6

$$\frac{a}{12\,d\,(\sin(dx+c))^3}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(12\sqrt{2}(\cos(dx+c))^2\sin(dx+c)\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{Artanh}\left(1/2\frac{\sqrt{2}\sin(\cdots}{\cos(d\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/12/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(12*2^(1/2)*cos(d*x+c)^2*sin(d
*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+3*cos(d*x+c)^2*sin(d*x+c)*(
```

```
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-12*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)/cos(d*x+c))*sin(d*x+c)-22*cos(d*x+c)^3-3*sin(d*x+c)*ln(-(-(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)-4*cos(d*x+c)^2+18*cos(d*x+c))/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.5628, size = 1403, normalized size = 9.74

$$\left[\frac{3\left(\sqrt{2}a\cos\left(dx+c\right)-\sqrt{2}a\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)\sin\left(dx+c\right)+12\left(a\,\mathrm{c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(sqrt(2)*a*cos(d*x + c) - sqrt(2)*a)*sqrt(-a)*log((2*sqrt(2)*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a
*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
1))*sin(d*x + c) + 12*(a*cos(d*x + c) - a)*sqrt(-a)*log(-(8*a*cos(d*x + c)^
3 - 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(
d*x + c) + 4*(11*a*cos(d*x + c)^2 - 9*a*cos(d*x + c))*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c)))/((d*cos(d*x + c) - d)*sin(d*x + c)), 1/12*(12*(a*cos(d*
x + c) - a)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
```

```
)*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(
d*x + c) + 3*(sqrt(2)*a*cos(d*x + c) - sqrt(2)*a)*sqrt(a)*arctan(sqrt(2)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*
sin(d*x + c) + 2*(11*a*cos(d*x + c)^2 - 9*a*cos(d*x + c))*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c)))/((d*cos(d*x + c) - d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.36641, size = 501, normalized size = 3.48

$$-\frac{3\sqrt{2}\sqrt{-a}a\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+24\sqrt{-a}a\log\left(\left|\left(\sqrt{-a}\tan\left(\cdots\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*(3*sqrt(2)*sqrt(-a)*a*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2)*sgn(cos(d*x + c)) + 24*sqrt(-a)*a*log(abs((sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2
*sqrt(2) + 3)))*sgn(cos(d*x + c)) - 24*sqrt(-a)*a*log(abs((sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))
)*sgn(cos(d*x + c)) + 8*sqrt(2)*(6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 9*(sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^
3*sgn(cos(d*x + c)) + 5*sqrt(-a)*a^4*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*
d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^3)/d
```

3.158 $\int \cot^6(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=226

$$-\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{11a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}d}+\frac{3\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{20ad}+\frac{5\cot^3(c+dx)(a\,\mathrm{se}}{24}$$

```
[Out] (-2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (1
1*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]
)/(16*Sqrt[2]*d) - (21*a*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(16*d) + (5
*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(24*d) + (3*Cot[c + d*x]^5*(a +
 a*Sec[c + d*x])^(5/2))/(20*a*d) - (Cos[c + d*x]*Cot[c + d*x]^5*Sec[(c + d*
x)/2]^2*(a + a*Sec[c + d*x])^(5/2))/(4*a*d)
```

Rubi [A] time = 0.230955, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3887, 472, 583, 522, 203}

$$-\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{11a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}d}+\frac{3\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{20ad}+\frac{5\cot^3(c+dx)(a\,\mathrm{se}}{24}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] (-2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (1
1*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]
)/(16*Sqrt[2]*d) - (21*a*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(16*d) + (5
*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(24*d) + (3*Cot[c + d*x]^5*(a +
 a*Sec[c + d*x])^(5/2))/(20*a*d) - (Cos[c + d*x]*Cot[c + d*x]^5*Sec[(c + d*
x)/2]^2*(a + a*Sec[c + d*x])^(5/2))/(4*a*d)
```


Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad}$$

$$= -\frac{\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{4ad} - \frac{\text{Subst}\left(\int \frac{}{x}\right.}{}$$

$$= \frac{3\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{20ad} - \frac{\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right.}{4ad}$$

$$= \frac{5\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{24d} + \frac{3\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{20ad} - \frac{\text{co}}{}$$

$$= -\frac{21a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{16d} + \frac{5\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{24d} + \frac{3}{}$$

$$= -\frac{21a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{16d} + \frac{5\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{24d} + \frac{3}{}$$

$$= -\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{11a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}d} - \frac{21a\cot(c+dx}{}$$

Mathematica [C] time = 23.6675, size = 5592, normalized size = 24.74

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.327, size = 720, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/480/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(cos(d*x+c)+
1)^2*(480*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos
(d*x+c))-480*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/
cos(d*x+c))+165*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d
*x+c))-480*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(
d*x+c))-165*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln
(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c
))+480*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)-898*cos
(d*x+c)^4-165*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln
(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c
))+702*cos(d*x+c)^3+165*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)+730*cos(d*x+c)^2-630*cos(d*x+c))/sin(d*x+c)^7
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.73812, size = 1899, normalized size = 8.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/960*(165*(sqrt(2)*a*cos(d*x + c)^3 - sqrt(2)*a*cos(d*x + c)^2 - sqrt(2)*
a*cos(d*x + c) + sqrt(2)*a)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 -
```

```
 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c)
+ 480*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*sqrt(-a)*l
og(-(8*a*cos(d*x + c)^3 + 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(c
os(d*x + c) + 1))*sin(d*x + c) - 4*(449*a*cos(d*x + c)^4 - 351*a*cos(d*x +
c)^3 - 365*a*cos(d*x + c)^2 + 315*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c)))/((d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x + c) + d)
*sin(d*x + c)), -1/480*(480*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*
x + c) + a)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(
d*x + c) + 165*(sqrt(2)*a*cos(d*x + c)^3 - sqrt(2)*a*cos(d*x + c)^2 - sqrt(
2)*a*cos(d*x + c) + sqrt(2)*a)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 2*(4
49*a*cos(d*x + c)^4 - 351*a*cos(d*x + c)^3 - 365*a*cos(d*x + c)^2 + 315*a*c
os(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c)^3 -
d*cos(d*x + c)^2 - d*cos(d*x + c) + d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.91471, size = 703, normalized size = 3.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/960*(165*sqrt(2)*sqrt(-a)*a*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a))^2)*sgn(cos(d*x + c)) + 30*sqrt(2)*sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a)*a*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c) + 960*sqrt
(-a)*a*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
```

```
)^2 + a))^2 - a*(2*sqrt(2) + 3)))*sgn(cos(d*x + c)) - 960*sqrt(-a)*a*log(ab
s((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 +
 a*(2*sqrt(2) - 3)))*sgn(cos(d*x + c)) + 32*sqrt(2)*(60*(sqrt(-a)*tan(1/2*d
*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*sqrt(-a)*a^2*sgn(cos(d
*x + c)) - 195*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^6*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 275*(sqrt(-a)*tan(1/2*d*x + 1/
2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a^4*sgn(cos(d*x + c)
) - 175*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^2*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 47*sqrt(-a)*a^6*sgn(cos(d*x + c)))/((
sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)
^5)/d
```

3.159 $\int (a + a\sec(c + dx))^{5/2}\tan^5(c + dx)\,dx$

Optimal. Leaf size=193

$$\frac{2(a\sec(c+dx)+a)^{13/2}}{13a^4d}-\frac{6(a\sec(c+dx)+a)^{11/2}}{11a^3d}+\frac{2(a\sec(c+dx)+a)^{9/2}}{9a^2d}+\frac{2a^2\sqrt{a\sec(c+dx)+a}}{d}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}}{}\right.}{d}$$

```
[Out] (-2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*a^2*Sqrt[a +
a*Sec[c + d*x]])/d + (2*a*(a + a*Sec[c + d*x])^(3/2))/(3*d) + (2*(a + a*Sec
[c + d*x])^(5/2))/(5*d) + (2*(a + a*Sec[c + d*x])^(7/2))/(7*a*d) + (2*(a +
a*Sec[c + d*x])^(9/2))/(9*a^2*d) - (6*(a + a*Sec[c + d*x])^(11/2))/(11*a^3*
d) + (2*(a + a*Sec[c + d*x])^(13/2))/(13*a^4*d)
```

Rubi [A] time = 0.145893, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3880, 88, 50, 63, 207}

$$\frac{2(a\sec(c+dx)+a)^{13/2}}{13a^4d}-\frac{6(a\sec(c+dx)+a)^{11/2}}{11a^3d}+\frac{2(a\sec(c+dx)+a)^{9/2}}{9a^2d}+\frac{2a^2\sqrt{a\sec(c+dx)+a}}{d}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}}{}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^5,x]
```

```
[Out] (-2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*a^2*Sqrt[a +
a*Sec[c + d*x]])/d + (2*a*(a + a*Sec[c + d*x])^(3/2))/(3*d) + (2*(a + a*Sec
[c + d*x])^(5/2))/(5*d) + (2*(a + a*Sec[c + d*x])^(7/2))/(7*a*d) + (2*(a +
a*Sec[c + d*x])^(9/2))/(9*a^2*d) - (6*(a + a*Sec[c + d*x])^(11/2))/(11*a^3*
d) + (2*(a + a*Sec[c + d*x])^(13/2))/(13*a^4*d)
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^{5/2}\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)^2(a+ax)^{9/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^4d}$$
$$= \frac{\text{Subst}\left(\int \left(-3a^2(a+ax)^{9/2}+\frac{a^2(a+ax)^{9/2}}{x}+a(a+ax)^{11/2}\right)dx, x, \sec(c+dx)\right)}{a^4d}$$
$$= -\frac{6(a+a\sec(c+dx))^{11/2}}{11a^3d}+\frac{2(a+a\sec(c+dx))^{13/2}}{13a^4d}+\frac{\text{Subst}\left(\int \frac{(a+ax)^{9/2}}{x}\,dx, x,\right.}{a^2d}$$
$$= \frac{2(a+a\sec(c+dx))^{9/2}}{9a^2d}-\frac{6(a+a\sec(c+dx))^{11/2}}{11a^3d}+\frac{2(a+a\sec(c+dx))^{13/2}}{13a^4d}+$$
$$= \frac{2(a+a\sec(c+dx))^{7/2}}{7ad}+\frac{2(a+a\sec(c+dx))^{9/2}}{9a^2d}-\frac{6(a+a\sec(c+dx))^{11/2}}{11a^3d}+$$
$$= \frac{2(a+a\sec(c+dx))^{5/2}}{5d}+\frac{2(a+a\sec(c+dx))^{7/2}}{7ad}+\frac{2(a+a\sec(c+dx))^{9/2}}{9a^2d}-$$
$$= \frac{2a(a+a\sec(c+dx))^{3/2}}{3d}+\frac{2(a+a\sec(c+dx))^{5/2}}{5d}+\frac{2(a+a\sec(c+dx))^{7/2}}{7ad}+$$
$$= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d}+\frac{2a(a+a\sec(c+dx))^{3/2}}{3d}+\frac{2(a+a\sec(c+dx))^{5/2}}{5d}+$$
$$= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d}+\frac{2a(a+a\sec(c+dx))^{3/2}}{3d}+\frac{2(a+a\sec(c+dx))^{5/2}}{5d}+$$
$$= -\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{2a^2\sqrt{a+a\sec(c+dx)}}{d}+\frac{2a(a+a\sec(c+dx)}{3d}$$

Mathematica [A] time = 0.734564, size = 156, normalized size = 0.81

$$\frac{(a(\sec(c+dx)+1))^{5/2}\left(\frac{2}{13}(\sec(c+dx)+1)^{13/2}-\frac{6}{11}(\sec(c+dx)+1)^{11/2}+\frac{2}{9}(\sec(c+dx)+1)^{9/2}+\frac{2}{7}(\sec(c+dx)+1)^{7/2}+\right.}{d(\sec(c+dx)+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^5,x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(5/2)*(-2*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + 2*Sqrt[
1 + Sec[c + d*x]] + (2*(1 + Sec[c + d*x])^(3/2))/3 + (2*(1 + Sec[c + d*x])^
(5/2))/5 + (2*(1 + Sec[c + d*x])^(7/2))/7 + (2*(1 + Sec[c + d*x])^(9/2))/9
```

```
- (6*(1 + Sec[c + d*x])^(11/2))/11 + (2*(1 + Sec[c + d*x])^(13/2))/13))/(d*
(1 + Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.316, size = 500, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^5,x)
```

```
[Out] 1/2882880/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(45045*cos(d*x+c)^6*2^(
1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(13/2)+270270*cos(d*x+c)^5*2^(1/2)*arctan(1/2*2^(1/2)*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(13/2)+67
5675*cos(d*x+c)^4*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(13/2)+900900*cos(d*x+c)^3*2^(1/2)*a
rctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(13/2)+675675*cos(d*x+c)^2*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(13/2)+270270*c
os(d*x+c)*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(13/2)+45045*2^(1/2)*arctan(1/2*2^(1/2)*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(13/2)+917
6192*cos(d*x+c)^6+4060544*cos(d*x+c)^5-1603968*cos(d*x+c)^4-3468160*cos(d*x
+c)^3-568960*cos(d*x+c)^2+1088640*cos(d*x+c)+443520)/cos(d*x+c)^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.24114, size = 1056, normalized size = 5.47

$$\left[\frac{45045\,a^{\frac{5}{2}}\cos\left(dx+c\right)^{6}\log\left(-8\,a\cos\left(dx+c\right)^{2}+4\left(2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos\left(dx+c\right)-a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] [1/90090*(45045*a^(5/2)*cos(d*x + c)^6*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d
*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) -
 8*a*cos(d*x + c) - a) + 4*(71689*a^2*cos(d*x + c)^6 + 31723*a^2*cos(d*x +
c)^5 - 12531*a^2*cos(d*x + c)^4 - 27095*a^2*cos(d*x + c)^3 - 4445*a^2*cos(d
*x + c)^2 + 8505*a^2*cos(d*x + c) + 3465*a^2)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c)))/(d*cos(d*x + c)^6), 1/45045*(45045*sqrt(-a)*a^2*arctan(2*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) +
 a))*cos(d*x + c)^6 + 2*(71689*a^2*cos(d*x + c)^6 + 31723*a^2*cos(d*x + c)^
5 - 12531*a^2*cos(d*x + c)^4 - 27095*a^2*cos(d*x + c)^3 - 4445*a^2*cos(d*x
+ c)^2 + 8505*a^2*cos(d*x + c) + 3465*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c)))/(d*cos(d*x + c)^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 4.85917, size = 332, normalized size = 1.72

$$\frac{\sqrt{2}\left(\frac{45045\,\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\left(45045\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^6a^2-30030\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^5a^3+36036\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^4a^4 \cdots}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) \cdots}\right.}{45045\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/45045*sqrt(2)*(45045*sqrt(2)*a^2*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a)/sqrt(-a))/sqrt(-a) + 2*(45045*(a*tan(1/2*d*x + 1/2*c)^2 - a)
^6*a^2 - 30030*(a*tan(1/2*d*x + 1/2*c)^2 - a)^5*a^3 + 36036*(a*tan(1/2*d*x
+ 1/2*c)^2 - a)^4*a^4 - 51480*(a*tan(1/2*d*x + 1/2*c)^2 - a)^3*a^5 + 80080*
(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*a^6 + 393120*(a*tan(1/2*d*x + 1/2*c)^2 - a
)*a^7 + 221760*a^8)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^6*sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a)))*a*sgn(cos(d*x + c))/d
```

3.160 $\int (a+a\sec(c+dx))^{5/2}\tan^3(c+dx)\,dx$

Optimal. Leaf size=145

$$\frac{2(a\sec(c+dx)+a)^{9/2}}{9a^2d}-\frac{2a^2\sqrt{a\sec(c+dx)+a}}{d}+\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{2(a\sec(c+dx)+a)^{7/2}}{7ad}-\frac{2(a\sec(c+d\cdots}{5d}$$

```
[Out] (2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (2*a^2*Sqrt[a + a
*Sec[c + d*x]])/d - (2*a*(a + a*Sec[c + d*x])^(3/2))/(3*d) - (2*(a + a*Sec[
c + d*x])^(5/2))/(5*d) - (2*(a + a*Sec[c + d*x])^(7/2))/(7*a*d) + (2*(a + a
*Sec[c + d*x])^(9/2))/(9*a^2*d)
```

Rubi [A] time = 0.118375, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3880, 80, 50, 63, 207}

$$\frac{2(a\sec(c+dx)+a)^{9/2}}{9a^2d}-\frac{2a^2\sqrt{a\sec(c+dx)+a}}{d}+\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{2(a\sec(c+dx)+a)^{7/2}}{7ad}-\frac{2(a\sec(c+d\cdots}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^3,x]
```

```
[Out] (2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (2*a^2*Sqrt[a + a
*Sec[c + d*x]])/d - (2*a*(a + a*Sec[c + d*x])^(3/2))/(3*d) - (2*(a + a*Sec[
c + d*x])^(5/2))/(5*d) - (2*(a + a*Sec[c + d*x])^(7/2))/(7*a*d) + (2*(a + a
*Sec[c + d*x])^(9/2))/(9*a^2*d)
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a + a\sec(c+dx))^{5/2}\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)(a+ax)^{7/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^2 d}$$
$$= \frac{2(a+a\sec(c+dx))^{9/2}}{9a^2 d} - \frac{\text{Subst}\left(\int \frac{(a+ax)^{7/2}}{x}\,dx, x, \sec(c+dx)\right)}{ad}$$
$$= -\frac{2(a+a\sec(c+dx))^{7/2}}{7ad} + \frac{2(a+a\sec(c+dx))^{9/2}}{9a^2 d} - \frac{\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{x}\,dx, x, \text{se}\right.}{d}$$
$$= -\frac{2(a+a\sec(c+dx))^{5/2}}{5d} - \frac{2(a+a\sec(c+dx))^{7/2}}{7ad} + \frac{2(a+a\sec(c+dx))^{9/2}}{9a^2 d} -$$
$$= -\frac{2a(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\sec(c+dx))^{5/2}}{5d} - \frac{2(a+a\sec(c+dx))^{7/2}}{7ad} +$$
$$= -\frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} - \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\sec(c+dx))^{5/2}}{5d} -$$
$$= -\frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} - \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\sec(c+dx))^{5/2}}{5d} -$$
$$= \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} - \frac{2a(a+a\sec(c+dx))^{3}}{3d}$$

Mathematica [A] time = 0.583184, size = 102, normalized size = 0.7

$$\frac{2(a(\sec(c+dx)+1))^{5/2}\left(\sqrt{\sec(c+dx)+1}\left(35\sec^4(c+dx)+95\sec^3(c+dx)+12\sec^2(c+dx)-226\sec(c+dx)-493\right)\right.}{315d(\sec(c+dx)+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^3,x]
```

```
[Out] (2*(a*(1 + Sec[c + d*x]))^(5/2)*(315*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt
[1 + Sec[c + d*x]]*(-493 - 226*Sec[c + d*x] + 12*Sec[c + d*x]^2 + 95*Sec[c
+ d*x]^3 + 35*Sec[c + d*x]^4)))/(315*d*(1 + Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.225, size = 362, normalized size = 2.5

$$-\frac{a^2}{5040\,d\,(\cos(dx+c))^4}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(315\sqrt{2}(\cos(dx+c))^4\left(-2\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{9/2}\arctan\left(1/2\sqrt{2}\sqrt{-2\frac{\cos}{\cos(}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^3,x)
```

```
[Out] -1/5040/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(315*2^(1/2)*cos(d*x+c)^4
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2))+1260*2^(1/2)*cos(d*x+c)^3*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(9/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+1890*2^(1/
2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctan(1/2*2^(1/2)*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+1260*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(9/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
))+315*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)+15776*cos(d*x+c)^4+7232*cos(d*x+c)^3-384*
cos(d*x+c)^2-3040*cos(d*x+c)-1120)/cos(d*x+c)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01509, size = 873, normalized size = 6.02

$$\left[\frac{315\,a^{\frac{5}{2}}\cos\left(dx+c\right)^{4}\log\left(-8\,a\cos\left(dx+c\right)^{2}-4\left(2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos\left(dx+c\right)-a\right)}{630\,d\cos\left(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] [1/630*(315*a^(5/2)*cos(d*x + c)^4*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x +
 c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a
```

```
*cos(d*x + c) - a) - 4*(493*a^2*cos(d*x + c)^4 + 226*a^2*cos(d*x + c)^3 - 1
2*a^2*cos(d*x + c)^2 - 95*a^2*cos(d*x + c) - 35*a^2)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c)))/(d*cos(d*x + c)^4), -1/315*(315*sqrt(-a)*a^2*arctan(2*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x +
c) + a))*cos(d*x + c)^4 + 2*(493*a^2*cos(d*x + c)^4 + 226*a^2*cos(d*x + c)^
3 - 12*a^2*cos(d*x + c)^2 - 95*a^2*cos(d*x + c) - 35*a^2)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 4.8191, size = 266, normalized size = 1.83

$$-\frac{\sqrt{2}\left(\frac{315\sqrt{2}a^3\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\left(315\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^4a^3-210\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^3a^4+252\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2a^5-360\left(a\,\mathrm{t}\cdots\right.\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^4\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}\cdots\right.}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/315*sqrt(2)*(315*sqrt(2)*a^3*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a)/sqrt(-a))/sqrt(-a) + 2*(315*(a*tan(1/2*d*x + 1/2*c)^2 - a)^4*a^
3 - 210*(a*tan(1/2*d*x + 1/2*c)^2 - a)^3*a^4 + 252*(a*tan(1/2*d*x + 1/2*c)^
2 - a)^2*a^5 - 360*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a^6 - 560*a^7)/((a*tan(1/
2*d*x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))*sgn(cos(d*x +
 c))/d
```

3.161 $\int (a+a\sec(c+dx))^{5/2}\tan(c+dx)\,dx$

Optimal. Leaf size=97

$$\frac{2a^2\sqrt{a\sec(c+dx)+a}}{d}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{2a(a\sec(c+dx)+a)^{3/2}}{3d}+\frac{2(a\sec(c+dx)+a)^{5/2}}{5d}$$

```
[Out] (-2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*a^2*Sqrt[a +
a*Sec[c + d*x]])/d + (2*a*(a + a*Sec[c + d*x])^(3/2))/(3*d) + (2*(a + a*Sec
[c + d*x])^(5/2))/(5*d)
```

Rubi [A] time = 0.0650882, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3880, 50, 63, 207}

$$\frac{2a^2\sqrt{a\sec(c+dx)+a}}{d}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{2a(a\sec(c+dx)+a)^{3/2}}{3d}+\frac{2(a\sec(c+dx)+a)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x],x]
```

```
[Out] (-2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*a^2*Sqrt[a +
a*Sec[c + d*x]])/d + (2*a*(a + a*Sec[c + d*x])^(3/2))/(3*d) + (2*(a + a*Sec
[c + d*x])^(5/2))/(5*d)
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^{5/2}\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{x}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{2(a+a\sec(c+dx))^{5/2}}{5d} + \frac{a\,\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{x}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5d} + \frac{a^2\,\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx, x, \text{se}\right.}{d}$$

$$= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5d} + \frac{a}{}$$

$$= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5d} + \frac{(2}{}$$

$$= -\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2a(a+a\sec(c+dx))}{3d}$$

Mathematica [A] time = 0.20706, size = 82, normalized size = 0.85

$$\frac{2(a(\sec(c+dx)+1))^{5/2}\left(\sqrt{\sec(c+dx)+1}\left(3\sec^2(c+dx)+11\sec(c+dx)+23\right)-15\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)\right)}{15d(\sec(c+dx)+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x],x]
```

```
[Out] (2*(a*(1 + Sec[c + d*x]))^(5/2)*(-15*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt
[1 + Sec[c + d*x]]*(23 + 11*Sec[c + d*x] + 3*Sec[c + d*x]^2)))/(15*d*(1 + S
ec[c + d*x])^(5/2))
```

Maple [A] time = 0.029, size = 74, normalized size = 0.8

$$\frac{1}{d}\left(\frac{2}{5}\left(a+a\sec\left(dx+c\right)\right)^{\frac{5}{2}}+\frac{2a}{3}\left(a+a\sec\left(dx+c\right)\right)^{\frac{3}{2}}+2a^{2}\sqrt{a+a\sec\left(dx+c\right)}-2a^{5/2}\mathrm{Artanh}\left(\frac{\sqrt{a+a\sec\left(dx+c\right)}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c),x)
```

```
[Out] 1/d*(2/5*(a+a*sec(d*x+c))^(5/2)+2/3*(a+a*sec(d*x+c))^(3/2)*a+2*a^2*(a+a*sec
(d*x+c))^(1/2)-2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.98756, size = 729, normalized size = 7.52

$$\left[\frac{15\,a^{\frac{5}{2}}\cos\left(dx+c\right)^{2}\log\left(-8\,a\cos\left(dx+c\right)^{2}+4\left(2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos\left(dx+c\right)-a\right)}{30\,d\cos\left(dx+c\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] [1/30*(15*a^(5/2)*cos(d*x + c)^2*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c
)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*c
os(d*x + c) - a) + 4*(23*a^2*cos(d*x + c)^2 + 11*a^2*cos(d*x + c) + 3*a^2)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2), 1/15*(15*sqrt(
-a)*a^2*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^2 + 2*(23*a^2*cos(d*x + c)^2 + 11*
a^2*cos(d*x + c) + 3*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d
*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 4.60844, size = 203, normalized size = 2.09

$$\frac{\sqrt{2}a^6\left(\frac{15\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^3}+\frac{2\left(15\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2-10\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)a+12a^2\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}a^3}\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c),x, algorithm="giac")
```

```
[Out] 1/15*sqrt(2)*a^6*(15*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a)/sqrt(-a))/(sqrt(-a)*a^3) + 2*(15*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2
```

```
- 10*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a + 12*a^2)/((a*tan(1/2*d*x + 1/2*c)^2
- a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^3))*sgn(cos(d*x + c))/d
```

3.162 $\int \cot(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=95

$$\frac{2a^2\sqrt{a\sec(c+dx)+a}}{d}+\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{4\sqrt{2}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

```
[Out] (2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (4*Sqrt[2]*a^(5/2
)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d + (2*a^2*Sqrt[a +
a*Sec[c + d*x]])/d
```

Rubi [A] time = 0.0891302, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3880, 84, 156, 63, 207}

$$\frac{2a^2\sqrt{a\sec(c+dx)+a}}{d}+\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{4\sqrt{2}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```


```
[Out] (2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (4*Sqrt[2]*a^(5/2
)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d + (2*a^2*Sqrt[a +
a*Sec[c + d*x]])/d
```


Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\,\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{x(-a+ax)}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} + \frac{a\,\text{Subst}\left(\int \frac{a^3+3a^3x}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} - \frac{a^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} + \frac{\left(4a^4\right)\text{Subs}}{}$$

$$= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} - \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d} + \frac{(}{}$$

$$= \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{4\sqrt{2}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{2a^2\sqrt{a+a\,\text{se}}}{d}$$

Mathematica [A] time = 0.09055, size = 83, normalized size = 0.87

$$\frac{2(a(\sec(c+dx)+1))^{5/2}\left(\sqrt{\sec(c+dx)+1}+\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\sec(c+dx)+1}}{\sqrt{2}}\right)\right)}{d(\sec(c+dx)+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(a*(1 + Sec[c + d*x]))^(5/2)*(ArcTanh[Sqrt[1 + Sec[c + d*x]]] - 2*Sqrt[2
]*ArcTanh[Sqrt[1 + Sec[c + d*x]]/Sqrt[2]] + Sqrt[1 + Sec[c + d*x]]))/(d*(1
+ Sec[c + d*x])^(5/2))
```

Maple [A] time = 0.149, size = 124, normalized size = 1.3

$$-\frac{a^2}{d}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(\sqrt{2}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+4\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{arcta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(2^(1/2)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+4*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2))-2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*cot(d*x + c), x)
```

Fricas [A] time = 1.6934, size = 782, normalized size = 8.23

$$\left[\frac{2\sqrt{2}a^{\frac{5}{2}}\log\left(-\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)-3\,a\cos(dx+c)-a}{\cos(dx+c)-1}\right)+a^{\frac{5}{2}}\log\left(-2\,a\cos\left(dx+c\right)-2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos\left(dx+c\right)-a\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [(2*sqrt(2)*a^(5/2)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + a^(5/2)
*log(-2*a*cos(d*x + c) - 2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c) - a) + 2*a^2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/d, 2*(2*
sqrt(2)*sqrt(-a)*a^2*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - sqrt(-a)*a^2*arctan(sqrt(-a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a))
 + a^2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 4.64374, size = 159, normalized size = 1.67

$$-\frac{\sqrt{2}a^5\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^2}-\frac{4\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}-\frac{2}{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}a^2}\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*a^5*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
)/sqrt(-a))/(sqrt(-a)*a^2) - 4*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/s
qrt(-a))/(sqrt(-a)*a^2) - 2/(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^2))*sgn(
cos(d*x + c))/d
```

3.163 $\int \cot^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=106

$$\frac{a^2\sqrt{a\sec(c+dx)+a}}{d(1-\sec(c+dx))}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d}$$

```
[Out] (-2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (3*a^(5/2)*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*d) + (a^2*Sqrt[a +
 a*Sec[c + d*x]])/(d*(1 - Sec[c + d*x]))
```

Rubi [A] time = 0.102885, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3880, 99, 156, 63, 207}

$$\frac{a^2\sqrt{a\sec(c+dx)+a}}{d(1-\sec(c+dx))}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (3*a^(5/2)*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*d) + (a^2*Sqrt[a +
 a*Sec[c + d*x]])/(d*(1 - Sec[c + d*x]))
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
```

```
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^4\,\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x(-a+ax)^2}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{a^2\sqrt{a+a\sec(c+dx)}}{d(1-\sec(c+dx))} + \frac{a^3\,\text{Subst}\left(\int \frac{-a-\frac{ax}{2}}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{a^2\sqrt{a+a\sec(c+dx)}}{d(1-\sec(c+dx))} + \frac{a^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} - \frac{\left(3a^4\right)\text{Subst}}{}$$

$$= \frac{a^2\sqrt{a+a\sec(c+dx)}}{d(1-\sec(c+dx))} + \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d} - \frac{(3a}{}$$

$$= -\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d} + \frac{a^2\sqrt{a+a\sec(c}}{d(1-\sec(c+}$$

Mathematica [A] time = 0.265232, size = 115, normalized size = 1.08

$$-\frac{(a(\sec(c+dx)+1))^{5/2}\left(2\sqrt{\sec(c+dx)+1}+4(\sec(c+dx)-1)\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)-3\sqrt{2}(\sec(c+dx)-1)\tan\right.}{2d(\sec(c+dx)-1)(\sec(c+dx)+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] -((a*(1 + Sec[c + d*x]))^(5/2)*(4*ArcTanh[Sqrt[1 + Sec[c + d*x]]]*(-1 + Sec
[c + d*x]) - 3*Sqrt[2]*ArcTanh[Sqrt[1 + Sec[c + d*x]]/Sqrt[2]]*(-1 + Sec[c
+ d*x]) + 2*Sqrt[1 + Sec[c + d*x]]))/(2*d*(-1 + Sec[c + d*x])*(1 + Sec[c +
d*x])^(5/2))
```

Maple [B] time = 0.204, size = 248, normalized size = 2.3

$$\frac{a^2}{2\,d\,(-1+\cos(dx+c))}\sqrt{\frac{a\,(\cos(dx+c)+1)}{\cos(dx+c)}}\left(2\,\sqrt{2}\cos(dx+c)\,\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(1/2\,\sqrt{2}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/2/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(2*2^(1/2)*cos(d*x+c)*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2))-2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/
2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+3*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-3*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+2
*cos(d*x+c))/(-1+cos(d*x+c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.77089, size = 1029, normalized size = 9.71

$$\left[\frac{4\,a^2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos\left(dx+c\right)+4\left(a^2\cos\left(dx+c\right)-a^2\right)\sqrt{a}\log\left(-2\,a\cos\left(dx+c\right)+2\,\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos\left(dx+c\right)-a\right)}{4\left(d\cos\left(dx+c\right)-d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) + 4*(a^2*c
os(d*x + c) - a^2)*sqrt(a)*log(-2*a*cos(d*x + c) + 2*sqrt(a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*cos(d*x + c) - a) + 3*(sqrt(2)*a^2*cos(d*x + c) -
 sqrt(2)*a^2)*sqrt(a)*log((2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*cos(d*x + c) + 3*a*cos(d*x + c) + a)/(cos(d*x + c) - 1)))/(d*cos(
d*x + c) - d), 1/2*(2*a^2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c) - 3*(sqrt(2)*a^2*cos(d*x + c) - sqrt(2)*a^2)*sqrt(-a)*arctan(sqrt(2)*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c)
 + a)) + 4*(a^2*cos(d*x + c) - a^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)))/(d*cos(d*x +
c) - d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 5.70923, size = 176, normalized size = 1.66

$$\frac{\sqrt{2}a^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a}-\frac{3\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}-\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}\right)\operatorname{sgn}\left(\cos\left(dx+c\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*a^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a)/sqrt(-a))/(sqrt(-a)*a) - 3*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
)/sqrt(-a))/(sqrt(-a)*a) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/(a^2*tan(1/2
*d*x + 1/2*c)^2))*sgn(cos(d*x + c))/d
```

3.164 $\int \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=147

$$-\frac{11a^2\sqrt{a\sec(c+dx)+a}}{16d(1-\sec(c+dx))}-\frac{a^2\sqrt{a\sec(c+dx)+a}}{4d(1-\sec(c+dx))^2}+\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{43a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}d}$$

```
[Out] (2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (43*a^(5/2)*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(16*Sqrt[2]*d) - (a^2*Sqrt[
a + a*Sec[c + d*x]])/(4*d*(1 - Sec[c + d*x])^2) - (11*a^2*Sqrt[a + a*Sec[c
+ d*x]])/(16*d*(1 - Sec[c + d*x]))
```

Rubi [A] time = 0.126855, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3880, 103, 151, 156, 63, 207}

$$-\frac{11a^2\sqrt{a\sec(c+dx)+a}}{16d(1-\sec(c+dx))}-\frac{a^2\sqrt{a\sec(c+dx)+a}}{4d(1-\sec(c+dx))^2}+\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{43a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2),x]
```


```
[Out] (2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (43*a^(5/2)*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(16*Sqrt[2]*d) - (a^2*Sqrt[
a + a*Sec[c + d*x]])/(4*d*(1 - Sec[c + d*x])^2) - (11*a^2*Sqrt[a + a*Sec[c
+ d*x]])/(16*d*(1 - Sec[c + d*x]))
```


Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
```

```
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^6\,\text{Subst}\left(\int \frac{1}{x(-a+ax)^3\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{a^2\sqrt{a+a\sec(c+dx)}}{4d(1-\sec(c+dx))^2} - \frac{a^3\,\text{Subst}\left(\int \frac{4a^2+\frac{3a^2x}{2}}{x(-a+ax)^2\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{4d}$$

$$= -\frac{a^2\sqrt{a+a\sec(c+dx)}}{4d(1-\sec(c+dx))^2} - \frac{11a^2\sqrt{a+a\sec(c+dx)}}{16d(1-\sec(c+dx))} + \frac{\text{Subst}\left(\int \frac{8a^4+\frac{11a^4x}{4}}{x(-a+ax)\sqrt{a+ax}}\,dx,\right.}{8d}$$

$$= -\frac{a^2\sqrt{a+a\sec(c+dx)}}{4d(1-\sec(c+dx))^2} - \frac{11a^2\sqrt{a+a\sec(c+dx)}}{16d(1-\sec(c+dx))} - \frac{a^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \text{s}\right.}{d}$$

$$= -\frac{a^2\sqrt{a+a\sec(c+dx)}}{4d(1-\sec(c+dx))^2} - \frac{11a^2\sqrt{a+a\sec(c+dx)}}{16d(1-\sec(c+dx))} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x\right.}{d}$$

$$= \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{43a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}d} - \frac{a^2\sqrt{a+a\sec(c}}{4d(1-\sec(c+}$$

Mathematica [A] time = 1.29451, size = 138, normalized size = 0.94

$$\frac{(a(\sec(c+dx)+1))^{5/2}\left(\sqrt{\sec(c+dx)+1}(11\sec(c+dx)-15)+32(\sec(c+dx)-1)^2\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)-86\sqrt{2}\,\text{s}\right.}{16d(\sec(c+dx)-1)^2(\sec(c+dx)+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(5/2)*(32*ArcTanh[Sqrt[1 + Sec[c + d*x]]]*(-1 + Sec
[c + d*x])^2 + Sqrt[1 + Sec[c + d*x]]*(-15 + 11*Sec[c + d*x]) - 86*Sqrt[2]*
ArcTanh[Sqrt[1 + Sec[c + d*x]]/Sqrt[2]]*Sec[c + d*x]^2*Sin[(c + d*x)/2]^4))
/(16*d*(-1 + Sec[c + d*x])^2*(1 + Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.262, size = 376, normalized size = 2.6

$$-\frac{a^2(\cos(dx+c)+1)^2}{32\,d\,(\sin(dx+c))^4}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(32\,(\cos(dx+c))^2\sqrt{2}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(1/2\sqrt{2}\sqrt{-2\frac{\cos(dx}{\cos(dx+}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(32*cos(d*
x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-64*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+
43*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2))+32*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ar
ctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-86*cos(d*x+c)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
))+43*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2))+30*cos(d*x+c)^2-22*cos(d*x+c))/sin(d*x+c)^4
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.79357, size = 1319, normalized size = 8.97

$$\left[\frac{64\left(a^2\cos(dx+c)^2-2a^2\cos(dx+c)+a^2\right)\sqrt{a}\log\left(-2a\cos(dx+c)-2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)-a\right)+43\left(\sqrt{2}a\right.}{64}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(64*(a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c) + a^2)*sqrt(a)*log(-2*a*
cos(d*x + c) - 2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
```

```
c) - a) + 43*(sqrt(2)*a^2*cos(d*x + c)^2 - 2*sqrt(2)*a^2*cos(d*x + c) + sqr
t(2)*a^2)*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) - 4*(15*a^2
*cos(d*x + c)^2 - 11*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c)))/(d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d), 1/32*(43*(sqrt(2)*a^2*cos(d
*x + c)^2 - 2*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*sqrt(-a)*arctan(sqrt(
2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x
 + c) + a)) - 64*(a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c) + a^2)*sqrt(-a)*a
rctan(sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(
d*x + c) + a)) - 2*(15*a^2*cos(d*x + c)^2 - 11*a^2*cos(d*x + c))*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 6.68825, size = 198, normalized size = 1.35

$$-\frac{\sqrt{2}a^3\left(\frac{32\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{43\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{13\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}-11\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}\right)\mathrm{sgn}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/32*sqrt(2)*a^3*(32*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)/sqrt(-a))/sqrt(-a) - 43*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
)/sqrt(-a))/sqrt(-a) + (13*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) - 11*sqrt(
```

```
-a*tan(1/2*d*x + 1/2*c)^2 + a)*a)/(a^2*tan(1/2*d*x + 1/2*c)^4))*sgn(cos(d*x
 + c))/d
```

3.165 $\int (a+a\sec(c+dx))^{5/2}\tan^6(c+dx)\,dx$

Optimal. Leaf size=290

$$\frac{2a^{10}\tan^{15}(c+dx)}{15d(a\sec(c+dx)+a)^{15/2}}+\frac{18a^9\tan^{13}(c+dx)}{13d(a\sec(c+dx)+a)^{13/2}}+\frac{62a^8\tan^{11}(c+dx)}{11d(a\sec(c+dx)+a)^{11/2}}+\frac{98a^7\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{62a^6}{7d(a\sec}$$

```
[Out] (-2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^3*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) - (2*a^4*Tan[c + d*x]^3)/(3
*d*(a + a*Sec[c + d*x])^(3/2)) + (2*a^5*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c +
 d*x])^(5/2)) + (62*a^6*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2)) +
(98*a^7*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2)) + (62*a^8*Tan[c +
d*x]^11)/(11*d*(a + a*Sec[c + d*x])^(11/2)) + (18*a^9*Tan[c + d*x]^13)/(13*
d*(a + a*Sec[c + d*x])^(13/2)) + (2*a^10*Tan[c + d*x]^15)/(15*d*(a + a*Sec[
c + d*x])^(15/2))
```

Rubi [A] time = 0.127909, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3887, 461, 203}

$$\frac{2a^{10}\tan^{15}(c+dx)}{15d(a\sec(c+dx)+a)^{15/2}}+\frac{18a^9\tan^{13}(c+dx)}{13d(a\sec(c+dx)+a)^{13/2}}+\frac{62a^8\tan^{11}(c+dx)}{11d(a\sec(c+dx)+a)^{11/2}}+\frac{98a^7\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{62a^6}{7d(a\sec}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^6,x]
```


```
[Out] (-2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^3*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) - (2*a^4*Tan[c + d*x]^3)/(3
*d*(a + a*Sec[c + d*x])^(3/2)) + (2*a^5*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c +
 d*x])^(5/2)) + (62*a^6*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2)) +
(98*a^7*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2)) + (62*a^8*Tan[c +
d*x]^11)/(11*d*(a + a*Sec[c + d*x])^(11/2)) + (18*a^9*Tan[c + d*x]^13)/(13*
d*(a + a*Sec[c + d*x])^(13/2)) + (2*a^10*Tan[c + d*x]^15)/(15*d*(a + a*Sec[
c + d*x])^(15/2))
```


Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
```

```
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^{5/2}\tan^6(c+dx)\,dx = -\frac{\left(2a^6\right)\text{Subst}\left(\int \frac{x^6\left(2+ax^2\right)^5}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{\left(2a^6\right)\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}+31x^6+49ax^8+31a^2x^{10}+9a^3x^{12}+a^4x^{14}-\right.\right.}{d}$$

$$= \frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^5\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} +$$

$$= -\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3}}$$

Mathematica [A] time = 9.97692, size = 173, normalized size = 0.6

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^7(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(604890\sin\left(\frac{1}{2}(c+dx)\right)-87230\sin\left(\frac{3}{2}(c+dx)\right)+450450\sin\left(\frac{5}{2}(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^6,x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sec[c + d*x]^7*Sqrt[a*(1 + Sec[c + d*x])]*(-2882880*S
qrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(15/2) + 604890*Sin[(c
 + d*x)/2] - 87230*Sin[(3*(c + d*x))/2] + 450450*Sin[(5*(c + d*x))/2] - 137
670*Sin[(7*(c + d*x))/2] + 210210*Sin[(9*(c + d*x))/2] + 75450*Sin[(11*(c +
 d*x))/2] + 90090*Sin[(13*(c + d*x))/2] + 16066*Sin[(15*(c + d*x))/2]))/(28
82880*d)
```

Maple [B] time = 0.286, size = 747, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^6,x)
```

```
[Out] -1/5765760/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(45045*2^(1/2)*sin(d*x
+c)*cos(d*x+c)^7*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(15/2)*arctanh(1/2*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+315315*2^(1/2)*
sin(d*x+c)*cos(d*x+c)^6*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(15/2)*arctanh(1/2*2
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+945945*2
^(1/2)*sin(d*x+c)*cos(d*x+c)^5*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(15/2)*arctan
h(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+1
576575*2^(1/2)*sin(d*x+c)*cos(d*x+c)^4*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(15/2
)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d
*x+c))+1576575*2^(1/2)*sin(d*x+c)*cos(d*x+c)^3*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(15/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)/cos(d*x+c))+945945*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(15/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)/cos(d*x+c))+315315*2^(1/2)*sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(15/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)/cos(d*x+c))+45045*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(15/2)*sin(d*x+c)+4112896*cos(d*x+c)^8+9475072*cos(d*x+c)^7-9162752*cos(d
*x+c)^6-12269056*cos(d*x+c)^5+980480*cos(d*x+c)^4+7605248*cos(d*x+c)^3+1860
096*cos(d*x+c)^2-1833216*cos(d*x+c)-768768)/cos(d*x+c)^7/sin(d*x+c)
```

Maximа [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.09329, size = 1283, normalized size = 4.42

$$\left[\frac{45045\left(a^2\cos(dx+c)^8+a^2\cos(dx+c)^7\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(16\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] [1/45045*(45045*(a^2*cos(d*x + c)^8 + a^2*cos(d*x + c)^7)*sqrt(-a)*log((2*a
*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(16066*a^
2*cos(d*x + c)^7 + 53078*a^2*cos(d*x + c)^6 + 17286*a^2*cos(d*x + c)^5 - 30
640*a^2*cos(d*x + c)^4 - 26810*a^2*cos(d*x + c)^3 + 2898*a^2*cos(d*x + c)^2
 + 10164*a^2*cos(d*x + c) + 3003*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c)^8 + d*cos(d*x + c)^7), 2/45045*(45045*(a^2
*cos(d*x + c)^8 + a^2*cos(d*x + c)^7)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (16066*a^2*cos(d*x
 + c)^7 + 53078*a^2*cos(d*x + c)^6 + 17286*a^2*cos(d*x + c)^5 - 30640*a^2*c
os(d*x + c)^4 - 26810*a^2*cos(d*x + c)^3 + 2898*a^2*cos(d*x + c)^2 + 10164*
a^2*cos(d*x + c) + 3003*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*
x + c))/(d*cos(d*x + c)^8 + d*cos(d*x + c)^7)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.166 $\int (a + a\sec(c + dx))^{5/2}\tan^4(c + dx)\,dx$

Optimal. Leaf size=224

$$\frac{2a^8\tan^{11}(c+dx)}{11d(a\sec(c+dx)+a)^{11/2}} + \frac{14a^7\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}} + \frac{34a^6\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}} + \frac{6a^5\tan^5(c+dx)}{d(a\sec(c+dx)+a)^{5/2}} + \frac{2a^4\tan}{3d(a\sec(c}$$

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (2*
a^3*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^4*Tan[c + d*x]^3)/(3*
d*(a + a*Sec[c + d*x])^(3/2)) + (6*a^5*Tan[c + d*x]^5)/(d*(a + a*Sec[c + d*
x])^(5/2)) + (34*a^6*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2)) + (14
*a^7*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2)) + (2*a^8*Tan[c + d*x]
^11)/(11*d*(a + a*Sec[c + d*x])^(11/2))
```

Rubi [A] time = 0.111037, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3887, 461, 203}

$$\frac{2a^8\tan^{11}(c+dx)}{11d(a\sec(c+dx)+a)^{11/2}} + \frac{14a^7\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}} + \frac{34a^6\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}} + \frac{6a^5\tan^5(c+dx)}{d(a\sec(c+dx)+a)^{5/2}} + \frac{2a^4\tan}{3d(a\sec(c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^4,x]
```


```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (2*
a^3*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^4*Tan[c + d*x]^3)/(3*
d*(a + a*Sec[c + d*x])^(3/2)) + (6*a^5*Tan[c + d*x]^5)/(d*(a + a*Sec[c + d*
x])^(5/2)) + (34*a^6*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2)) + (14
*a^7*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2)) + (2*a^8*Tan[c + d*x]
^11)/(11*d*(a + a*Sec[c + d*x])^(11/2))
```


Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^{5/2}\tan^4(c+dx)\,dx = -\frac{\left(2a^5\right)\text{Subst}\left(\int \frac{x^4\left(2+ax^2\right)^4}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{\left(2a^5\right)\text{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+15x^4+17ax^6+7a^2x^8+a^3x^{10}+\frac{1}{a^2\left(1+ax^2\right)}\right)dx, x\right.}{d}$$

$$= -\frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{6a^5\tan^5(c+dx)}{d(a+a\sec(c+dx))^{5/2}}+$$

$$= \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}-\frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}$$

Mathematica [A] time = 7.34645, size = 149, normalized size = 0.67

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(-1386\sin\left(\frac{1}{2}(c+dx)\right)+1584\sin\left(\frac{3}{2}(c+dx)\right)-1386\sin\left(\frac{5}{2}(c+dx)\right)\right.}{5544}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^4,x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sec[c + d*x]^5*Sqrt[a*(1 + Sec[c + d*x])]*(5544*Sqrt[
2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(11/2) - 1386*Sin[(c + d*x
)/2] + 1584*Sin[(3*(c + d*x))/2] - 1386*Sin[(5*(c + d*x))/2] - 143*Sin[(7*(
c + d*x))/2] - 693*Sin[(9*(c + d*x))/2] - 26*Sin[(11*(c + d*x))/2]))/(5544*
```

d)

Maple [B] time = 0.23, size = 498, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^4,x)
```

```
[Out] -1/11088/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(693*2^(1/2)*sin(d*x+c)*
cos(d*x+c)^5*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)+2772*2^(1/2)*sin(d*x
+c)*cos(d*x+c)^4*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)+4158*2^(1/2)*sin
(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)+2772*2^(1/2)
*sin(d*x+c)*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+693*2^(1
/2)*sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*
2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))-1664*co
s(d*x+c)^6-21344*cos(d*x+c)^5+11296*cos(d*x+c)^4+16736*cos(d*x+c)^3+2144*co
s(d*x+c)^2-5152*cos(d*x+c)-2016)/cos(d*x+c)^5/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.92927, size = 1095, normalized size = 4.89

$$\left[\frac{693\left(a^2\cos(dx+c)^6+a^2\cos(dx+c)^5\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-2\left(52\,a^2\,\mathrm{c}\right.}{693\left(d\cos(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] [1/693*(693*(a^2*cos(d*x + c)^6 + a^2*cos(d*x + c)^5)*sqrt(-a)*log((2*a*cos
(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(52*a^2*cos(d
*x + c)^5 + 719*a^2*cos(d*x + c)^4 + 366*a^2*cos(d*x + c)^3 - 157*a^2*cos(d
*x + c)^2 - 224*a^2*cos(d*x + c) - 63*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5), -2/693*(693*(a
^2*cos(d*x + c)^6 + a^2*cos(d*x + c)^5)*sqrt(a)*arctan(sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (52*a^2*cos(d*x
+ c)^5 + 719*a^2*cos(d*x + c)^4 + 366*a^2*cos(d*x + c)^3 - 157*a^2*cos(d*x
+ c)^2 - 224*a^2*cos(d*x + c) - 63*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sin(d*x + c))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.167 $\int (a + a\sec(c + dx))^{5/2} \tan^2(c + dx)\, dx$

Optimal. Leaf size=160

$$\frac{2a^6 \tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}} + \frac{2a^5 \tan^5(c+dx)}{d(a\sec(c+dx)+a)^{5/2}} + \frac{14a^4 \tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}} - \frac{2a^{5/2} \tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d} + \frac{2a^3 \tan(c}{d\sqrt{a\sec(c+}}$$

```
[Out] (-2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^3*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (14*a^4*Tan[c + d*x]^3)/(
3*d*(a + a*Sec[c + d*x])^(3/2)) + (2*a^5*Tan[c + d*x]^5)/(d*(a + a*Sec[c +
d*x])^(5/2)) + (2*a^6*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2))
```

Rubi [A] time = 0.0987673, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3887, 461, 203}

$$\frac{2a^6 \tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}} + \frac{2a^5 \tan^5(c+dx)}{d(a\sec(c+dx)+a)^{5/2}} + \frac{14a^4 \tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}} - \frac{2a^{5/2} \tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d} + \frac{2a^3 \tan(c}{d\sqrt{a\sec(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^2,x]
```

```
[Out] (-2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^3*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (14*a^4*Tan[c + d*x]^3)/(
3*d*(a + a*Sec[c + d*x])^(3/2)) + (2*a^5*Tan[c + d*x]^5)/(d*(a + a*Sec[c +
d*x])^(5/2)) + (2*a^6*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2))
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
```

```
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^{5/2}\tan^2(c+dx)\,dx = -\frac{(2a^4)\,\text{Subst}\left(\int \frac{x^2(2+ax^2)^3}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a^4)\,\text{Subst}\left(\int \left(\frac{1}{a}+7x^2+5ax^4+a^2x^6-\frac{1}{a(1+ax^2)}\right)dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{14a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{2a^5\tan^5(c+dx)}{d(a+a\sec(c+dx))^{5/2}}+\frac{}{7}$$

$$= -\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{14a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^3}$$

Mathematica [A] time = 5.75748, size = 125, normalized size = 0.78

$$-\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(-35\sin\left(\frac{1}{2}(c+dx)\right)+7\sin\left(\frac{3}{2}(c+dx)\right)-21\sin\left(\frac{5}{2}(c+dx)\right)+5\,\text{si}\right.}{42d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^2,x]
```

```
[Out] -(a^2*Sec[(c + d*x)/2]*Sec[c + d*x]^3*Sqrt[a*(1 + Sec[c + d*x])]*(42*Sqrt[2
]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(7/2) - 35*Sin[(c + d*x)/2]
 + 7*Sin[(3*(c + d*x))/2] - 21*Sin[(5*(c + d*x))/2] + 5*Sin[(7*(c + d*x))/2
]))/(42*d)
```

Maple [B] time = 0.189, size = 391, normalized size = 2.4

$$-\frac{a^2}{168\,d\sin(dx+c)(\cos(dx+c))^3}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(21\sqrt{2}\sin(dx+c)(\cos(dx+c))^3\operatorname{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^2,x)
```

```
[Out] -1/168/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(21*2^(1/2)*sin(d*x+c)*cos
(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)+63*2^(1/2)*sin(d*x+c)*c
os(d*x+c)^2*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)+63*2^(1/2)*sin(d*x+c)
*cos(d*x+c)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)+21*2^(1/2)*arctanh(1/
2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(7/2)*sin(d*x+c)-160*cos(d*x+c)^4+416*cos(d*x+c)^
3-64*cos(d*x+c)^2-144*cos(d*x+c)-48)/sin(d*x+c)/cos(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.82668, size = 945, normalized size = 5.91

$$\left[\frac{21\left(a^2\cos(dx+c)^4+a^2\cos(dx+c)^3\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2+2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-2\left(10\,a^2\cos\right.}{21\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] [1/21*(21*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(-a)*log((2*a*cos(d
*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(10*a^2*cos(d*x
 + c)^3 - 16*a^2*cos(d*x + c)^2 - 12*a^2*cos(d*x + c) - 3*a^2)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c
)^3), 2/21*(21*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(a)*arctan(sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) -
 (10*a^2*cos(d*x + c)^3 - 16*a^2*cos(d*x + c)^2 - 12*a^2*cos(d*x + c) - 3*a
^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4
 + d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.168 $\int \cot^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=66

$$-\frac{4a^2\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{d}-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] (-2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (4
*a^2*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/d
```

Rubi [A] time = 0.0710611, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3887, 453, 203}

$$-\frac{4a^2\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{d}-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (4
*a^2*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/d
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = -\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{2+ax^2}{x^2\left(1+ax^2\right)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{4a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d} + \frac{\left(2a^3\right)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{4a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d}$$

Mathematica [A] time = 0.714181, size = 124, normalized size = 1.88

$$-\frac{\sqrt{2}\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{\sec(c+dx)+1}\right)^{3/2}(a(\sec(c+dx)+1))^{5/2}\left(2\cos(c+dx)-\frac{(\cos(c+dx)-1)\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)}{\sqrt{1-\sec(c+dx)}}\right)}{d\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] -((Sqrt[2]*Cot[c + d*x]*Sec[(c + d*x)/2]^2*(2*Cos[c + d*x] - (ArcTanh[Sqrt[
1 - Sec[c + d*x]]]*(-1 + Cos[c + d*x]))/Sqrt[1 - Sec[c + d*x]])*((1 + Sec[c
 + d*x])^(-1))^(3/2)*(a*(1 + Sec[c + d*x]))^(5/2))/(d*Sqrt[1 - Tan[(c + d*x
)/2]^2]))
```

Maple [B] time = 0.166, size = 192, normalized size = 2.9

$$\frac{a^2}{d\left((\cos(dx+c))^2-1\right)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}(\cos(dx+c))^2\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}\sin(dx+c)}{2\cos(dx+c)}\sqrt{-2\frac{}{\text{co}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*((-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*cos(d*x+c)^2*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)/cos(d*x+c))-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/
2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(
d*x+c))+4*cos(d*x+c)*sin(d*x+c))/(cos(d*x+c)^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*cot(d*x + c)^2, x)
```

Fricas [B] time = 1.99467, size = 693, normalized size = 10.5

$$\left[\frac{\sqrt{-a}a^2\log\left(-\frac{8\,a\cos(dx+c)^3+4\left(2\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)-7\,a\cos(dx+c)+a}{\cos(dx+c)+1}\right)\sin(dx+c)-8\,a^2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\mathrm{co}}{2\,d\sin(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a)*a^2*log(-(8*a*cos(d*x + c)^3 + 4*(2*cos(d*x + c)^2 - cos(d*x
 + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*
cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) - 8*a^2*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*cos(d*x + c))/(d*sin(d*x + c)), -(a^(5/2)*arctan(2*
sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(
2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 4*a^2*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(d*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.169 $\int \cot^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=96

$$\frac{2a^2\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{d}+\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{2a\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{3d}$$

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2*
a^2*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/d - (2*a*Cot[c + d*x]^3*(a + a*S
ec[c + d*x])^(3/2))/(3*d)
```

Rubi [A] time = 0.0760531, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3887, 325, 203}

$$\frac{2a^2\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{d}+\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{2a\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```


```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2*
a^2*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/d - (2*a*Cot[c + d*x]^3*(a + a*S
ec[c + d*x])^(3/2))/(3*d)
```


Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = -\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{x^4(1+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{2a\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{3d} + \frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{x^2(1+ax^2)}\,dx, x, -\frac{\mathrm{ta}}{\sqrt{a+}}\right.}{d}$$

$$= \frac{2a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d} - \frac{2a\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{3d} -$$

$$= \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d} - \frac{2a\cot^3(c}{}$$

Mathematica [C] time = 0.229109, size = 81, normalized size = 0.84

$$-\frac{2\left(\frac{1}{\cos(c+dx)+1}\right)^{3/2}\cot^3(c+dx)(a(\sec(c+dx)+1))^{5/2}\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},2\sin^2\left(\frac{1}{2}(c+dx)\right)\right)}{3d\sqrt{\frac{1}{\sec(c+dx)+1}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*((1 + Cos[c + d*x])^(-1))^(3/2)*Cot[c + d*x]^3*Hypergeometric2F1[-3/2,
-3/2, -1/2, 2*Sin[(c + d*x)/2]^2]*(a*(1 + Sec[c + d*x]))^(5/2))/(3*d*Sqrt[(
1 + Sec[c + d*x])^(-1)])
```

Maple [B] time = 0.279, size = 214, normalized size = 2.2

$$-\frac{a^2}{3\,d\sin(dx+c)(-1+\cos(dx+c))}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(3\sqrt{2}\cos(dx+c)\sin(dx+c)\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{Artanh}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/3/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(3*2^(1/2)*cos(d*x+c)*sin(d*
x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))-3*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)-8*cos(d*x+c)^2+6*cos(d*x+c))/sin(d*x+c)/
(-1+cos(d*x+c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.99897, size = 888, normalized size = 9.25

$$\left[\frac{3\left(a^2\cos(dx+c)-a^2\right)\sqrt{-a}\log\left(-\frac{8\,a\cos(dx+c)^3-4\left(2\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)-7\,a\cos(dx+c)+a}{\cos(dx+c)+1}\right)\sin(dx+c)+}{6\left(d\cos(dx+c)-d\right)\sin(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^2*cos(d*x + c) - a^2)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 - 4*(2*c
os(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) +
 4*(4*a^2*cos(d*x + c)^2 - 3*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c)))/((d*cos(d*x + c) - d)*sin(d*x + c)), 1/3*(3*(a^2*cos(d*x + c)
- a^2)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos
(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x +
 c) + 2*(4*a^2*cos(d*x + c)^2 - 3*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c)))/((d*cos(d*x + c) - d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.170 $\int \cot^6(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=176

$$-\frac{7a^2\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{2}d}-\frac{\cot^5(c+dx)(a\sec(c-}{5d}$$

```
[Out] (-2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a
^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(
4*Sqrt[2]*d) - (7*a^2*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(4*d) + (a*Cot
[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(2*d) - (Cot[c + d*x]^5*(a + a*Sec[
c + d*x])^(5/2))/(5*d)
```

Rubi [A] time = 0.183707, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3887, 480, 583, 522, 203}

$$-\frac{7a^2\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{2}d}-\frac{\cot^5(c+dx)(a\sec(c-}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a
^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(
4*Sqrt[2]*d) - (7*a^2*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(4*d) + (a*Cot
[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(2*d) - (Cot[c + d*x]^5*(a + a*Sec[
c + d*x])^(5/2))/(5*d)
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{5d} - \frac{\text{Subst}\left(\int \frac{-15a-5a^2x^2}{x^4(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+}{\sqrt{a+a\sec(}}\right.}{5d}$$

$$= \frac{a\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{2d} - \frac{\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{5d} + \frac{\text{Sul}}{}$$

$$= -\frac{7a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4d} + \frac{a\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{2d} - \frac{\text{c}}{}$$

$$= -\frac{7a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4d} + \frac{a\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{2d} - \frac{\text{c}}{}$$

$$= -\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{2}d} - \frac{7a^2\cot(c+dx)\sqrt{}}{4}$$

Mathematica [C] time = 23.7503, size = 5572, normalized size = 31.66

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.285, size = 542, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/40/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(40*2^(1/2)
*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2
```

```
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))-80*2^(1/
2)*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+5*cos(d*
x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+40*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)-10*cos(d*x+c)*sin(d*x+c)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-98*cos(d*x+c)^3+5*sin(d*x+c)*ln(-(
-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+160*cos(d*x+c)^2-70*cos(d*x+c))/sin(d*
x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.67018, size = 1708, normalized size = 9.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/80*(5*(sqrt(2)*a^2*cos(d*x + c)^2 - 2*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)
*a^2)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 40*(a^2*cos(d*x + c)
^2 - 2*a^2*cos(d*x + c) + a^2)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 + 4*(2*cos
(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) - 4
*(49*a^2*cos(d*x + c)^3 - 80*a^2*cos(d*x + c)^2 + 35*a^2*cos(d*x + c))*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c)^2 - 2*d*cos(d*x + c)
```

```
+ d)*sin(d*x + c)), -1/40*(40*(a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c) + a^
2)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c)
+ 5*(sqrt(2)*a^2*cos(d*x + c)^2 - 2*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)
*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c
)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 2*(49*a^2*cos(d*x + c)^3 - 80*a^2*
cos(d*x + c)^2 + 35*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
)))/((d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 10.6204, size = 657, normalized size = 3.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/80*(5*sqrt(2)*sqrt(-a)*a^2*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^2)*sgn(cos(d*x + c)) + 80*sqrt(-a)*a^2*log(abs(
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a
*(2*sqrt(2) + 3)))*sgn(cos(d*x + c)) - 80*sqrt(-a)*a^2*log(abs((sqrt(-a)*ta
n(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2)
- 3)))*sgn(cos(d*x + c)) + 4*sqrt(2)*(55*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 170*
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqr
t(-a)*a^4*sgn(cos(d*x + c)) + 240*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 150*(sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a
^6*sgn(cos(d*x + c)) + 41*sqrt(-a)*a^7*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^5)/d
```

3.171 $\int \frac{\tan^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=126

$$\frac{2(a\sec(c+dx)+a)^{7/2}}{7a^4d}-\frac{6(a\sec(c+dx)+a)^{5/2}}{5a^3d}+\frac{2(a\sec(c+dx)+a)^{3/2}}{3a^2d}+\frac{2\sqrt{a\sec(c+dx)+a}}{ad}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+d}}{\sqrt{a}}\right.}{\sqrt{a}d}$$

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) + (2*Sqrt[a + a*
Sec[c + d*x]])/(a*d) + (2*(a + a*Sec[c + d*x])^(3/2))/(3*a^2*d) - (6*(a + a
*Sec[c + d*x])^(5/2))/(5*a^3*d) + (2*(a + a*Sec[c + d*x])^(7/2))/(7*a^4*d)
```

Rubi [A] time = 0.101379, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3880, 88, 50, 63, 207}

$$\frac{2(a\sec(c+dx)+a)^{7/2}}{7a^4d}-\frac{6(a\sec(c+dx)+a)^{5/2}}{5a^3d}+\frac{2(a\sec(c+dx)+a)^{3/2}}{3a^2d}+\frac{2\sqrt{a\sec(c+dx)+a}}{ad}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+d}}{\sqrt{a}}\right.}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/Sqrt[a + a*Sec[c + d*x]],x]
```


```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) + (2*Sqrt[a + a*
Sec[c + d*x]])/(a*d) + (2*(a + a*Sec[c + d*x])^(3/2))/(3*a^2*d) - (6*(a + a
*Sec[c + d*x])^(5/2))/(5*a^3*d) + (2*(a + a*Sec[c + d*x])^(7/2))/(7*a^4*d)
```


Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)^2(a+ax)^{3/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^4d}$$

$$= \frac{\text{Subst}\left(\int \left(-3a^2(a+ax)^{3/2} + \frac{a^2(a+ax)^{3/2}}{x} + a(a+ax)^{5/2}\right)dx, x, \sec(c+dx)\right)}{a^4d}$$

$$= -\frac{6(a+a\sec(c+dx))^{5/2}}{5a^3d} + \frac{2(a+a\sec(c+dx))^{7/2}}{7a^4d} + \frac{\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^2d}$$

$$= \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2d} - \frac{6(a+a\sec(c+dx))^{5/2}}{5a^3d} + \frac{2(a+a\sec(c+dx))^{7/2}}{7a^4d} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+a}}{x}\right.}{}$$

$$= \frac{2\sqrt{a+a\sec(c+dx)}}{ad} + \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2d} - \frac{6(a+a\sec(c+dx))^{5/2}}{5a^3d} + \frac{2(a+a\sec(c+d}{7a^4d}$$

$$= \frac{2\sqrt{a+a\sec(c+dx)}}{ad} + \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2d} - \frac{6(a+a\sec(c+dx))^{5/2}}{5a^3d} + \frac{2(a+a\sec(c+d}{7a^4d}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{2\sqrt{a+a\sec(c+dx)}}{ad} + \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2d} - \frac{6(a+a\sec(c}{5a^3d}$$

Mathematica [A] time = 0.18377, size = 88, normalized size = 0.7

$$\frac{2\left(15\sec^4(c+dx) - 3\sec^3(c+dx) - 64\sec^2(c+dx) + 46\sec(c+dx) - 105\sqrt{\sec(c+dx)+1}\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right.\right.}{105d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*(92 + 46*Sec[c + d*x] - 64*Sec[c + d*x]^2 - 3*Sec[c + d*x]^3 + 15*Sec[c
+ d*x]^4 - 105*ArcTanh[Sqrt[1 + Sec[c + d*x]]]*Sqrt[1 + Sec[c + d*x]]))/(10
5*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.239, size = 293, normalized size = 2.3

$$-\frac{1}{840\,ad\left(\cos(dx+c)\right)^3}\sqrt{\frac{a\left(\cos(dx+c)+1\right)}{\cos(dx+c)}}\left(105\left(\cos(dx+c)\right)^3\sqrt{2}\arctan\left(1/2\sqrt{2}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\left(-2\frac{\cos(}{\cos(d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/840/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(105*cos(d*x+c)^3*2^(1/2)*ar
ctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(7/2)+315*cos(d*x+c)^2*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)+315*cos(d*x+c)
*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(7/2)+105*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)-1472*cos(d*x+c)^
3+736*cos(d*x+c)^2+288*cos(d*x+c)-240)/cos(d*x+c)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.95474, size = 759, normalized size = 6.02

$$\left[\frac{105\,\sqrt{a}\cos\left(dx+c\right)^{3}\log\left(-8\,a\cos\left(dx+c\right)^{2}+4\left(2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos\left(dx+c\right)-a\right)}{210\,ad\cos\left(dx+c\right)^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*sqrt(a)*cos(d*x + c)^3*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x +
 c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a
*cos(d*x + c) - a) + 4*(92*cos(d*x + c)^3 - 46*cos(d*x + c)^2 - 18*cos(d*x
```

```
+ c) + 15)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a*d*cos(d*x + c)^3), 1
/105*(105*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^3 + 2*(92*cos(d*x + c)^
3 - 46*cos(d*x + c)^2 - 18*cos(d*x + c) + 15)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c)))/(a*d*cos(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^5(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**5/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [A] time = 12.2559, size = 257, normalized size = 2.04

$$-\frac{\sqrt{2}\left(\frac{105\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{2\left(105\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^3-70\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2a-252\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)a^2-120a^3\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^3\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/105*sqrt(2)*(105*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a)/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 2*(105*(a*ta
n(1/2*d*x + 1/2*c)^2 - a)^3 - 70*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*a - 252*(
a*tan(1/2*d*x + 1/2*c)^2 - a)*a^2 - 120*a^3)/((a*tan(1/2*d*x + 1/2*c)^2 - a
)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.172 $\int \frac{\tan^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=78

$$\frac{2(a\sec(c+dx)+a)^{3/2}}{3a^2d}-\frac{2\sqrt{a\sec(c+dx)+a}}{ad}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - (2*Sqrt[a + a*S
ec[c + d*x]])/(a*d) + (2*(a + a*Sec[c + d*x])^(3/2))/(3*a^2*d)
```

Rubi [A] time = 0.0740956, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3880, 80, 50, 63, 207}

$$\frac{2(a\sec(c+dx)+a)^{3/2}}{3a^2d}-\frac{2\sqrt{a\sec(c+dx)+a}}{ad}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - (2*Sqrt[a + a*S
ec[c + d*x]])/(a*d) + (2*(a + a*Sec[c + d*x])^(3/2))/(3*a^2*d)
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{(-a+ax)\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{a^2d}\\
&= \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2d} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{ad}\\
&= -\frac{2\sqrt{a+a\sec(c+dx)}}{ad} + \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2d} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}\\
&= -\frac{2\sqrt{a+a\sec(c+dx)}}{ad} + \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2d} - \frac{2\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+d
\end{aligned}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{2\sqrt{a+a\sec(c+dx)}}{ad} + \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2d}$$

Mathematica [A] time = 0.0828739, size = 66, normalized size = 0.85

$$\frac{2\left(\sec^2(c+dx)-\sec(c+dx)+3\sqrt{\sec(c+dx)+1}\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)-2\right)}{3d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*(-2 - Sec[c + d*x] + Sec[c + d*x]^2 + 3*ArcTanh[Sqrt[1 + Sec[c + d*x]]]*
Sqrt[1 + Sec[c + d*x]]))/(3*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.207, size = 155, normalized size = 2.

$$\frac{1}{6\,ad\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(3\sqrt{2}\arctan\left(1/2\sqrt{2}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\left(-2\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{3/2}\cos(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/6/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(3*2^(1/2)*arctan(1/2*2^(1/2)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*c
os(d*x+c)+3*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)-8*cos(d*x+c)+4)/cos(d*x+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.92287, size = 625, normalized size = 8.01

$$\left[\frac{3\sqrt{a}\cos(dx+c)\log\left(-8\,a\cos(dx+c)^2-4\left(2\,\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos(dx+c)-a\right)-}{6\,ad\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(a)*cos(d*x + c)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2
+ cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d
*x + c) - a) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(2*cos(d*x + c) -
1))/(a*d*cos(d*x + c)), -1/3*(3*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c) +
 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(2*cos(d*x + c) - 1))/(a*d*cos(d
*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**3/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [B] time = 10.243, size = 201, normalized size = 2.58

$$\frac{\sqrt{2}\left(\frac{3\sqrt{2}a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{2\left(3\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)a+2\,a^2\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(3*sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a)/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 2*(3*(a*tan(1/2
*d*x + 1/2*c)^2 - a)*a + 2*a^2)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/(a*d)
```

3.173 $\int \frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=31

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d)
```

Rubi [A] time = 0.0392376, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3880, 63, 207}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d)
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{ad}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

Mathematica [A] time = 0.0366614, size = 44, normalized size = 1.42

$$-\frac{2\sqrt{\sec(c+dx)+1}\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)}{d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[1 + Sec[c + d*x]]]*Sqrt[1 + Sec[c + d*x]])/(d*Sqrt[a*(1 +
Sec[c + d*x])])
```

Maple [A] time = 0.04, size = 26, normalized size = 0.8

$$-2\,\frac{1}{d\sqrt{a}}\text{Artanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.8784, size = 359, normalized size = 11.58

$$\left[\frac{\log\left(-8\,a\cos\left(dx+c\right)^{2}+4\left(2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos\left(dx+c\right)-a\right)}{2\sqrt{a}d},\ \frac{\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\cdots}{\cdots}\right)}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a)/(sqrt(a)*d)
, sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c)/(2*a*cos(d*x + c) + a))/(a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [B] time = 9.69199, size = 74, normalized size = 2.39

$$-\frac{2\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}d\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-
a)*d*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))
```

3.174 $\int \frac{\cot(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=92

$$-\frac{1}{d\sqrt{a\sec(c+dx)+a}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d}$$

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - ArcTanh[Sqrt[a
+ a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(Sqrt[2]*Sqrt[a]*d) - 1/(d*Sqrt[a + a*
Sec[c + d*x]])
```

Rubi [A] time = 0.0864493, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3880, 85, 156, 63, 207}

$$-\frac{1}{d\sqrt{a\sec(c+dx)+a}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - ArcTanh[Sqrt[a
+ a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(Sqrt[2]*Sqrt[a]*d) - 1/(d*Sqrt[a + a*
Sec[c + d*x]])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
```

```
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{a^2\,\text{Subst}\left(\int \frac{1}{x(-a+ax)(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{1}{d\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{2a^2-a^2x}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{2ad}$$

$$= -\frac{1}{d\sqrt{a+a\sec(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a+ax}}\,dx, x,\right.}{2d}$$

$$= -\frac{1}{d\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{-2a+x^2}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d} - \frac{2\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx,\right.}{a}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d} - \frac{1}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [C] time = 0.0511051, size = 57, normalized size = 0.62

$$\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(\sec(c+dx)+1)\right)-2\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\sec(c+dx)+1\right)}{d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Hypergeometric2F1[-1/2, 1, 1/2, (1 + Sec[c + d*x])/2] - 2*Hypergeometric2F
1[-1/2, 1, 1/2, 1 + Sec[c + d*x]])/(d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.215, size = 259, normalized size = 2.8

$$\frac{1}{2\,ad\,(\sin(dx+c))^2}\left(2\,(\cos(dx+c))^2\sqrt{2}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(1/2\,\sqrt{2}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+(\cos(dx+c))^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d/a*(2*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan
(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+cos(d*x+c)^2*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-2*
2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2))+2*cos(d*x+c)^2-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-2*cos(d*x+c))*(a*(cos(d*
x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(dx+c)}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [B] time = 9.35424, size = 203, normalized size = 2.21

$$\frac{\sqrt{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
a)/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - arctan(sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 -
1)) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/(a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1
)))/d
```

3.175 $\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=152

$$-\frac{a}{12d(a\sec(c+dx)+a)^{3/2}}+\frac{a}{2d(1-\sec(c+dx))(a\sec(c+dx)+a)^{3/2}}+\frac{7}{8d\sqrt{a\sec(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) + (9*ArcTanh[Sqr
t[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(8*Sqrt[2]*Sqrt[a]*d) - a/(12*d*(
a + a*Sec[c + d*x])^(3/2)) + a/(2*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])
^(3/2)) + 7/(8*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.135295, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3880, 103, 152, 156, 63, 207}

$$-\frac{a}{12d(a\sec(c+dx)+a)^{3/2}}+\frac{a}{2d(1-\sec(c+dx))(a\sec(c+dx)+a)^{3/2}}+\frac{7}{8d\sqrt{a\sec(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```



```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) + (9*ArcTanh[Sqr
t[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(8*Sqrt[2]*Sqrt[a]*d) - a/(12*d*(
a + a*Sec[c + d*x])^(3/2)) + a/(2*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])
^(3/2)) + 7/(8*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{a^4\,\mathrm{Subst}\left(\int \frac{1}{x(-a+ax)^2(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} - \frac{a\,\mathrm{Subst}\left(\int \frac{2a^2+\frac{5a^2x}{2}}{x(-a+ax)(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{2d}$$

$$= -\frac{a}{12d(a+a\sec(c+dx))^{3/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} + \frac{\mathrm{Subst}\left(\int \frac{-6a^4-3a}{x(-a+ax)(a+}\right.}{6}$$

$$= -\frac{a}{12d(a+a\sec(c+dx))^{3/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} + \frac{7}{8d\sqrt{a+a\sec(c+dx}}$$

$$= -\frac{a}{12d(a+a\sec(c+dx))^{3/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} + \frac{7}{8d\sqrt{a+a\sec(c+dx}}$$

$$= -\frac{a}{12d(a+a\sec(c+dx))^{3/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} + \frac{7}{8d\sqrt{a+a\sec(c+dx}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{9\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}d} - \frac{a}{12d(a+a\sec(c+dx))^{3/2}} + \frac{}{2d(1-\mathrm{s}}$$

<u>Mathematica</u> [C] time = 0.166507, size = 90, normalized size = 0.59

$$\frac{a\left(-9(\sec(c+dx)-1)\mathrm{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{1}{2}(\sec(c+dx)+1)\right)+8(\sec(c+dx)-1)\mathrm{Hypergeometric2F1}\left(\right.\right.}{12d(\sec(c+dx)-1)(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (a*(-6 - 9*Hypergeometric2F1[-3/2, 1, -1/2, (1 + Sec[c + d*x])/2]*(-1 + Sec
[c + d*x]) + 8*Hypergeometric2F1[-3/2, 1, -1/2, 1 + Sec[c + d*x]]*(-1 + Sec
[c + d*x])))/(12*d*(-1 + Sec[c + d*x])*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.274, size = 504, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/48/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)*(-1+cos(d*x+c)
)^2*(48*2^(1/2)*cos(d*x+c)^3*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/
2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+48*cos(d*x+c)^2*2^(1/2)*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))+27*cos(d*x+c)^3*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan
(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-48*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2))+27*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2))-48*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+62*cos(d*x+c)
^3-27*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2))+4*cos(d*x+c)^2-27*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-42*cos(d*x+c))/sin(d*x
+c)^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(dx+c)^3}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^3/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**3/(a+a*sec(d*x+c))**(1/2),x)

[Out] Integral(cot(c + d*x)**3/sqrt(a*(sec(c + d*x) + 1)), x)

Giac [A] time = 9.8231, size = 309, normalized size = 2.03

$$-\frac{\sqrt{2}\left(\frac{48\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{27\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{3\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}+\frac{2\left(\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 \cdots\right.\right.}{a^6\mathrm{sg}\cdots}\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] -1/48*sqrt(2)*(48*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 27*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/(a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*tan(1/2*d*x + 1/2*c)^2) + 2*((-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^4 + 12*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^5)/(a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d

3.176 $\int \frac{\cot^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=214

$$\frac{87a^2}{160d(a\sec(c+dx)+a)^{5/2}}-\frac{17a^2}{16d(1-\sec(c+dx))(a\sec(c+dx)+a)^{5/2}}-\frac{a^2}{4d(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{5/2}}+1$$

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - (151*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(128*Sqrt[2]*Sqrt[a]*d) + (87*a^
2)/(160*d*(a + a*Sec[c + d*x])^(5/2)) - a^2/(4*d*(1 - Sec[c + d*x])^2*(a +
a*Sec[c + d*x])^(5/2)) - (17*a^2)/(16*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d
*x])^(5/2)) + (23*a)/(192*d*(a + a*Sec[c + d*x])^(3/2)) - 105/(128*d*Sqrt[a
 + a*Sec[c + d*x]])
```

Rubi [A] time = 0.177853, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3880, 103, 151, 152, 156, 63, 207}

$$\frac{87a^2}{160d(a\sec(c+dx)+a)^{5/2}}-\frac{17a^2}{16d(1-\sec(c+dx))(a\sec(c+dx)+a)^{5/2}}-\frac{a^2}{4d(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{5/2}}+1$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/Sqrt[a + a*Sec[c + d*x]],x]
```


```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - (151*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(128*Sqrt[2]*Sqrt[a]*d) + (87*a^
2)/(160*d*(a + a*Sec[c + d*x])^(5/2)) - a^2/(4*d*(1 - Sec[c + d*x])^2*(a +
a*Sec[c + d*x])^(5/2)) - (17*a^2)/(16*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d
*x])^(5/2)) + (23*a)/(192*d*(a + a*Sec[c + d*x])^(3/2)) - 105/(128*d*Sqrt[a
 + a*Sec[c + d*x]])
```


Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{a^6\,\text{Subst}\left(\int \frac{1}{x(-a+ax)^3(a+ax)^{7/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{a^3\,\text{Subst}\left(\int \frac{4a^2+\frac{9a^2x}{2}}{x(-a+ax)^2(a+ax)^{7/2}}\,dx, x, \sec(c+dx\right.}{4d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{17a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \cdots$$

$$= \frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{}{16d(1-\sec(c+d}$$

$$= \frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{}{16d(1-\sec(c+d}$$

$$= \frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{}{16d(1-\sec(c+d}$$

$$= \frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{}{16d(1-\sec(c+d}$$

$$= \frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{}{16d(1-\sec(c+d}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{151\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}d} + \frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}} - \frac{}{4d(1}$$

Mathematica [C] time = 0.255118, size = 102, normalized size = 0.48

$$\frac{\cot^4(c+dx)\left(151(\sec(c+dx)-1)^2\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{1}{2}(\sec(c+dx)+1)\right)-2\left(32(\sec(c+dx)-1)^2\text{Hyper}\right.\right.}{160d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Cot[c + d*x]^4*(-2*(105 + 32*Hypergeometric2F1[-5/2, 1, -3/2, 1 + Sec[c +
d*x]]*(-1 + Sec[c + d*x])^2 - 85*Sec[c + d*x]) + 151*Hypergeometric2F1[-5/2
, 1, -3/2, (1 + Sec[c + d*x])/2]*(-1 + Sec[c + d*x])^2))/(160*d*Sqrt[a*(1 +
 Sec[c + d*x])])
```

Maple [B] time = 0.382, size = 746, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/3840/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x
+c))^3*(3840*cos(d*x+c)^5*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arct
an(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+2265*cos(d*x+c)^5*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2))+3840*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(
1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+2265*cos(d*x+c)^4*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
)-7680*2^(1/2)*cos(d*x+c)^3*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2
*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+5642*cos(d*x+c)^5-4530*cos(d
*x+c)^3*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2))-7680*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+556*cos(d*x+c
)^4-4530*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2))+3840*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-7
928*cos(d*x+c)^3+2265*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arcta
n(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+3840*2^(1/2)*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-4
60*cos(d*x+c)^2+2265*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(
```

```
d*x+c)/(cos(d*x+c)+1))^(1/2))+3150*cos(d*x+c))/sin(d*x+c)^10
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 10.1823, size = 400, normalized size = 1.87

$$\frac{\sqrt{2}\left(\frac{3840\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{2265\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{15\left(25\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}-23\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}a\right)}{a^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}+\right.}{3840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*sqrt(2)*(3840*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a)/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 2265*arctan
(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x +
1/2*c)^2 - 1)) + 15*(25*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) - 23*sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a)*a)/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*tan(1/2
*d*x + 1/2*c)^4) + 8*(3*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a)*a^12 + 25*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^13 + 24
0*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^14)/(a^15*sgn(tan(1/2*d*x + 1/2*c)^
2 - 1)))/d
```

3.177 $\int \frac{\tan^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=189

$$\frac{2a^4\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{6a^3\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{2a^2\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}-\frac{2a\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{a\,s}}\right.}{\sqrt{a}}$$

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) +
(2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) - (2*a*Tan[c + d*x]^3)/(3*d*(
a + a*Sec[c + d*x])^(3/2)) + (2*a^2*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*x
])^(5/2)) + (6*a^3*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2)) + (2*a^
4*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2))
```

Rubi [A] time = 0.100538, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3887, 461, 203}

$$\frac{2a^4\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{6a^3\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{2a^2\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}-\frac{2a\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{a\,s}}\right.}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) +
(2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) - (2*a*Tan[c + d*x]^3)/(3*d*(
a + a*Sec[c + d*x])^(3/2)) + (2*a^2*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*x
])^(5/2)) + (6*a^3*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2)) + (2*a^
4*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2))
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\left(2a^3\right)\text{Subst}\left(\int \frac{x^6\left(2+ax^2\right)^2}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{\left(2a^3\right)\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}+3x^6+ax^8-\frac{1}{a^3\left(1+ax^2\right)}\right)dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} + \frac{6a^3\tan^7(c+}{7d(a+a\sec(c+}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^2\tan^5(c}{5d(a+a\sec(c}$$

Mathematica [C] time = 19.3146, size = 469, normalized size = 2.48

$$16\left(-3-2\sqrt{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\left(10-7\sqrt{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)-5\sqrt{2}+7}{\cos\left(\frac{1}{2}(c+dx)\right)+1}}\sqrt{\frac{-\left(\sqrt{2}-2\right)\cos\left(\frac{1}{2}(c+dx)\right)+\sqrt{2}-1}{\cos\left(\frac{1}{2}(c+dx)\right)+1}}\left(\left(\sqrt{2}-2\right)\cos\left(\frac{1}{2}(c+dx)\right)-\sqrt{2}+1\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^6/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]*((1532*Sin[(c + d*x)/2])/315 + (136*Sec[c +
d*x]*Sin[(c + d*x)/2])/315 - (176*Sec[c + d*x]^2*Sin[(c + d*x)/2])/105 - (4
*Sec[c + d*x]^3*Sin[(c + d*x)/2])/63 + (4*Sec[c + d*x]^4*Sin[(c + d*x)/2])/
9))/(d*Sqrt[a*(1 + Sec[c + d*x])]) + (16*(-3 - 2*Sqrt[2])*Cos[(c + d*x)/4]^
```

```
4*Cos[(c + d*x)/2]*Sqrt[(7 - 5*Sqrt[2] + (10 - 7*Sqrt[2])*Cos[(c + d*x)/2])
/(1 + Cos[(c + d*x)/2])]*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(c + d*x)/
2])/(1 + Cos[(c + d*x)/2])]*(1 - Sqrt[2] + (-2 + Sqrt[2])*Cos[(c + d*x)/2])
*(EllipticF[ArcSin[Tan[(c + d*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]
+ 2*EllipticPi[-3 + 2*Sqrt[2], -ArcSin[Tan[(c + d*x)/4]/Sqrt[3 - 2*Sqrt[2]]
], 17 - 12*Sqrt[2]])*Sqrt[(-1 - Sqrt[2] + (2 + Sqrt[2])*Cos[(c + d*x)/2])*S
ec[(c + d*x)/4]^2]*Sec[c + d*x]^2*Sqrt[3 - 2*Sqrt[2] - Tan[(c + d*x)/4]^2])
/(d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.261, size = 480, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/5040/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(315*2^(1/2)*sin(d*x+c)*cos(
d*x+c)^4*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)+1260*2^(1/2)*sin(d*x+c)*
cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)+1890*2^(1/2)*sin(d*x
+c)*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/2)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+1260*2^(1/2)*sin
(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/2)
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+315*2^(1/2)*ar
ctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c
))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*sin(d*x+c)-12256*cos(d*x+c)^5+11168
*cos(d*x+c)^4+5312*cos(d*x+c)^3-4064*cos(d*x+c)^2-1280*cos(d*x+c)+1120)/sin
(d*x+c)/cos(d*x+c)^4
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

[Out] Timed out

Fricas [A] time = 1.81951, size = 957, normalized size = 5.06

$$\left[-\frac{315 \left(\cos \left(dx+c\right)^{5}+\cos \left(dx+c\right)^{4}\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^{2}-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-2\left(383\,\cos(d\right.}{315\left(ad\cos\left(dx+c\right)^{5}+ad\cos(d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/315*(315*(cos(d*x + c)^5 + cos(d*x + c)^4)*sqrt(-a)*log((2*a*cos(d*x +
c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(
d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(383*cos(d*x + c)^4
+ 34*cos(d*x + c)^3 - 132*cos(d*x + c)^2 - 5*cos(d*x + c) + 35)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^5 + a*d*cos(d*
x + c)^4), 2/315*(315*(cos(d*x + c)^5 + cos(d*x + c)^4)*sqrt(a)*arctan(sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) +
(383*cos(d*x + c)^4 + 34*cos(d*x + c)^3 - 132*cos(d*x + c)^2 - 5*cos(d*x +
c) + 35)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x
 + c)^5 + a*d*cos(d*x + c)^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^{6}(c+dx)}{\sqrt{a\left(\sec(c+dx)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**6/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.178 $\int \frac{\tan^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=125

$$\frac{2a^2\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}+\frac{2a\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}+\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) - (
2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Tan[c + d*x]^3)/(3*d*(a
 + a*Sec[c + d*x])^(3/2)) + (2*a^2*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*x]
)^(5/2))
```

Rubi [A] time = 0.0838666, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3887, 459, 302, 203}

$$\frac{2a^2\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}+\frac{2a\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}+\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```


```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) - (
2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Tan[c + d*x]^3)/(3*d*(a
 + a*Sec[c + d*x])^(3/2)) + (2*a^2*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*x]
)^(5/2))
```


Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
```

```
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{(2a^2)\,\text{Subst}\left(\int \frac{x^4(2+ax^2)}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2a^2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{x^4}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2a^2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} - \frac{(2a^2)\,\text{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+\frac{1}{a^2(1+ax^2)}\right)dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} - \frac{2\,\text{Subst}\left(\int \frac{}{1}\right.}{}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^2\tan^5(c}{5d(a+a\sec(c}$$

Mathematica [C] time = 2.62243, size = 238, normalized size = 1.9

$$\frac{16\sqrt{2}\tan^5(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{9/2}\left(-\frac{4}{9}\tan^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\text{Hypergeometric2F1}\left(2,\frac{9}{2},\frac{11}{2},-2\sin^2\left(\frac{1}{2}(c+dx)\right)\right.\right.}{5d\left(1-\tan^2\left(\frac{1}{2}(c+\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (16*Sqrt[2]*((1 + Sec[c + d*x])^(-1))^(9/2)*(-(Cos[c + d*x]*(9 + 5*Cos[c +
d*x])*Csc[(c + d*x)/2]^6*Sec[(c + d*x)/2]^2*(30*ArcTanh[Sqrt[1 - Sec[c + d*
x]]]*Cos[c + d*x]^2 + (-29 + 22*Cos[c + d*x] - 23*Cos[2*(c + d*x)])*Sqrt[1
- Sec[c + d*x]]))/(480*Sqrt[1 - Sec[c + d*x]]) - (4*Hypergeometric2F1[2, 9/
2, 11/2, -2*Sec[c + d*x]*Sin[(c + d*x)/2]^2]*Sec[c + d*x]*Tan[(c + d*x)/2]^
2)/9)*Tan[c + d*x]^5)/(5*d*Sqrt[a*(1 + Sec[c + d*x])]*(1 - Tan[(c + d*x)/2]
^2)^(7/2))
```

Maple [B] time = 0.237, size = 231, normalized size = 1.9

$$\frac{1}{30\,ad\left(\cos\left(dx+c\right)\right)^{2}\sin\left(dx+c\right)}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}\left(15\,\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin\left(dx+c\right)}{\cos\left(dx+c\right)}\sqrt{-2\,\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\right)\left(-2\,\frac{c}{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/30/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(15*2^(1/2)*arctanh(1/2*2^(1/2
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)^2*sin(d*x+c)+15*2^(1/2)*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)+68*cos(d*x+c)^3-64*cos(d*x+
c)^2-16*cos(d*x+c)+12)/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.76342, size = 832, normalized size = 6.66

$$\left[-\frac{15\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^{2}+2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(17\,\cos\left(dx\right.\right.}{15\left(ad\cos\left(dx+c\right)^{3}+ad\cos\left(dx+c\right)^{2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/15*(15*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(-a)*log((2*a*cos(d*x + c)
^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*
x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(17*cos(d*x + c)^2 + c
os(d*x + c) - 3)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d
*cos(d*x + c)^3 + a*d*cos(d*x + c)^2), -2/15*(15*(cos(d*x + c)^3 + cos(d*x
+ c)^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
/(sqrt(a)*sin(d*x + c))) + (17*cos(d*x + c)^2 + cos(d*x + c) - 3)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(
d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan^{4}\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**4/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.179 $\int \frac{\tan^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=63

$$\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) +
(2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.0626098, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3887, 321, 203}

$$\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) +
(2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx &= -\frac{(2a)\,\mathrm{Subst}\left(\int \frac{x^2}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\ &= \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2\,\mathrm{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\ &= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.717519, size = 119, normalized size = 1.89

$$\frac{16\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{5/2}\left(\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{\frac{1}{\cos(c+dx)+1}} - \cos(c+dx)\sin^{-1}\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{1}{\cos(c+dx)+1}}}\right)\right)}{d\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (16*Cos[(c + d*x)/2]^6*Sec[c + d*x]^4*((1 + Sec[c + d*x])^(-1))^(5/2)*(-(Ar
cSin[Tan[(c + d*x)/2]/Sqrt[(1 + Cos[c + d*x])^(-1)]]*Cos[c + d*x]) + Sqrt[C
os[c + d*x]]*Sqrt[(1 + Cos[c + d*x])^(-1)]*Sin[c + d*x]))/(d*Sqrt[a*(1 + Se
c[c + d*x])])
```

Maple [B] time = 0.162, size = 116, normalized size = 1.8

$$\frac{1}{ad\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}\sin(dx+c)}{2\,\cos(dx+c)}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sin(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*((-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)/cos(d*x+c))*sin(d*x+c)-2*cos(d*x+c)+2)/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.67118, size = 620, normalized size = 9.84

$$\left[-\frac{\sqrt{-a}(\cos(dx+c)+1)\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-2\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{ad\cos(dx+c)+ad},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-(sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c)
 - a)/(cos(d*x + c) + 1)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d
*x + c))/(a*d*cos(d*x + c) + a*d), 2*(sqrt(a)*(cos(d*x + c) + 1)*arctan(sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) +
 sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) +
a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.180 $\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=165

$$-\frac{\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4ad}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{7\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{2}\sqrt{a}d}-\frac{\cos(c+dx)\cot(c+dx)\sec^2(}{4a}$$

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) +
(7*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(4*Sq
rt[2]*Sqrt[a]*d) - (Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(4*a*d) - (Cos[c
 + d*x]*Cot[c + d*x]*Sec[(c + d*x)/2]^2*Sqrt[a + a*Sec[c + d*x]])/(4*a*d)
```

Rubi [A] time = 0.141488, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3887, 472, 583, 522, 203}

$$-\frac{\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4ad}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{7\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{2}\sqrt{a}d}-\frac{\cos(c+dx)\cot(c+dx)\sec^2(}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) +
(7*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(4*Sq
rt[2]*Sqrt[a]*d) - (Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(4*a*d) - (Cos[c
 + d*x]*Cot[c + d*x]*Sec[(c + d*x)/2]^2*Sqrt[a + a*Sec[c + d*x]])/(4*a*d)
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
```

```
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad}$$

$$= -\frac{\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{4ad} - \frac{\text{Subst}\left(\int \frac{a-3a^2x^2}{x^2(1+ax^2)(2+ax^2)}\,dx\right.}{2a^2d}$$

$$= -\frac{\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4ad} - \frac{\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+}}{4ad}$$

$$= -\frac{\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4ad} - \frac{\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+}}{4ad}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{7\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{2}\sqrt{a}d} - \frac{\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4ad} - \frac{\text{co}}{}$$

Mathematica [C] time = 23.7217, size = 5544, normalized size = 33.6

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.232, size = 374, normalized size = 2.3

$$-\frac{1}{8\,ad\,(\sin(dx+c))^3}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(8\sqrt{2}(\cos(dx+c))^2\sin(dx+c)\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{Artanh}\left(1/2\frac{\sqrt{2}\sin(}{\cos(d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/8/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(8*2^(1/2)*cos(d*x+c)^2*sin(d*
x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c
```

```
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+7*cos(d*x+c)^2*sin(d*x+c)*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-8*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)/cos(d*x+c))*sin(d*x+c)-6*cos(d*x+c)^3-7*sin(d*x+c)*ln(-(-(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)+4*cos(d*x+c)^2+2*cos(d*x+c))/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(dx+c)^2}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^2/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [A] time = 9.87269, size = 166, normalized size = 1.01

$$-\frac{\sqrt{2}\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}+\frac{4\sqrt{-a}}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a*s
gn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 4*sqrt(-a)/(((sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)*sgn(tan(1/2*d*x + 1/2*c)^2
 - 1)))/d
```

3.181 $\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=251

$$\frac{43\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{96a^2d}-\frac{\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{3/2}}{16a^2d}-\frac{15\cos(c+dx)}{}$$

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) - (
107*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(64*
Sqrt[2]*Sqrt[a]*d) + (21*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(64*a*d) +
(43*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(96*a^2*d) - (15*Cos[c + d*x
]*Cot[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(3/2))/(32*a^2*d)
- (Cos[c + d*x]^2*Cot[c + d*x]^3*Sec[(c + d*x)/2]^4*(a + a*Sec[c + d*x])^(3
/2))/(16*a^2*d)
```

Rubi [A] time = 0.225917, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3887, 472, 579, 583, 522, 203}

$$\frac{43\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{96a^2d}-\frac{\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{3/2}}{16a^2d}-\frac{15\cos(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) - (
107*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(64*
Sqrt[2]*Sqrt[a]*d) + (21*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(64*a*d) +
(43*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(96*a^2*d) - (15*Cos[c + d*x
]*Cot[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(3/2))/(32*a^2*d)
- (Cos[c + d*x]^2*Cot[c + d*x]^3*Sec[(c + d*x)/2]^4*(a + a*Sec[c + d*x])^(3
/2))/(16*a^2*d)
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
```

```
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2d}$$

$$= -\frac{\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{16a^2d} - \frac{\text{Subst}\left(\int \frac{a-7a^2x^2}{x^4(1+ax^2)(2+ax^2)}\right.}{4a^3}$$

$$= -\frac{15\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{32a^2d} - \frac{\cos^2(c+dx)\cot^3(c+}{}$$

$$= \frac{43\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{96a^2d} - \frac{15\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\text{se}}{32a^2d}$$

$$= \frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{64ad} + \frac{43\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{96a^2d} - \frac{15\cos(c+dx)\,\text{co}}{}$$

$$= \frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{64ad} + \frac{43\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{96a^2d} - \frac{15\cos(c+dx)\,\text{co}}{}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{107\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{64\sqrt{2}\sqrt{a}d} + \frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{64ad} +$$

Mathematica [C] time = 23.7249, size = 5584, normalized size = 22.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.304, size = 722, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/384/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)*(-1+cos(d*x+c)
)^2*(384*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(
d*x+c))+384*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/c
os(d*x+c))+321*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*
x+c))-384*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d
*x+c))+321*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(
-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c)
)-384*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)-410*cos(
d*x+c)^4-321*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(
-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c)
)-142*cos(d*x+c)^3-321*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
+298*cos(d*x+c)^2+126*cos(d*x+c))/sin(d*x+c)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(dx+c)^4}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^4/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**4/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [A] time = 9.98379, size = 346, normalized size = 1.38

$$-\frac{\sqrt{2}\left(3\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}-\frac{21}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{32\left(9\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\cdots\right.\right.}{\left(\left(\sqrt{\cdots}\right.\right.}\right.}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/384*sqrt(2)*(3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*tan(1/2*d*x + 1/2*
c)^2/(a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 21/(a*sgn(tan(1/2*d*x + 1/2*c)^2
 - 1)))*tan(1/2*d*x + 1/2*c) - 32*(9*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a) - 15*(sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a + 8*sqrt(-a)*a^2)/(
((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 -
a)^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.182 $\int \frac{\cot^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=335

$$\frac{579\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{640a^3d}-\frac{323\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{768a^2d}-\frac{\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+\right.}{48a^3d}$$

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) +
(835*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(51
2*Sqrt[2]*Sqrt[a]*d) - (189*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(512*a*d
) - (323*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(768*a^2*d) + (579*Cot[
c + d*x]^5*(a + a*Sec[c + d*x])^(5/2))/(640*a^3*d) - (101*Cos[c + d*x]*Cot[
c + d*x]^5*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(5/2))/(128*a^3*d) - (23
*Cos[c + d*x]^2*Cot[c + d*x]^5*Sec[(c + d*x)/2]^4*(a + a*Sec[c + d*x])^(5/2
))/(192*a^3*d) - (Cos[c + d*x]^3*Cot[c + d*x]^5*Sec[(c + d*x)/2]^6*(a + a*S
ec[c + d*x])^(5/2))/(48*a^3*d)
```

Rubi [A] time = 0.321512, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3887, 472, 579, 583, 522, 203}

$$\frac{579\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{640a^3d}-\frac{323\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{768a^2d}-\frac{\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+\right.}{48a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/Sqrt[a + a*Sec[c + d*x]],x]
```



```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) +
(835*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(51
2*Sqrt[2]*Sqrt[a]*d) - (189*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(512*a*d
) - (323*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(768*a^2*d) + (579*Cot[
c + d*x]^5*(a + a*Sec[c + d*x])^(5/2))/(640*a^3*d) - (101*Cos[c + d*x]*Cot[
c + d*x]^5*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(5/2))/(128*a^3*d) - (23
*Cos[c + d*x]^2*Cot[c + d*x]^5*Sec[(c + d*x)/2]^4*(a + a*Sec[c + d*x])^(5/2
))/(192*a^3*d) - (Cos[c + d*x]^3*Cot[c + d*x]^5*Sec[(c + d*x)/2]^6*(a + a*S
ec[c + d*x])^(5/2))/(48*a^3*d)
```


Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^6\left(1+ax^2\right)\left(2+ax^2\right)^4}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^3 d}$$

$$= -\frac{\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{48a^3 d} - \frac{\text{Subst}\left(\int \frac{a-11a^2x^2}{x^6\left(1+ax^2\right)\left(2+ax\right.}\right.}{6\ldots}$$

$$= -\frac{23\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{192a^3 d} - \frac{\cos^3(c+dx)\cot^5(c\ldots}{\ldots}$$

$$= -\frac{101\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{128a^3 d} - \frac{23\cos^2(c+dx)\cot^5\ldots}{\ldots}$$

$$= \frac{579\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{640a^3 d} - \frac{101\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+\ldots}{128a^3 d}$$

$$= -\frac{323\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{768a^2 d} + \frac{579\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{640a^3 d} - \frac{101\cos\ldots}{\ldots}$$

$$= -\frac{189\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{512ad} - \frac{323\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{768a^2 d} + \frac{579\cot^5(c\ldots}{\ldots}$$

$$= -\frac{189\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{512ad} - \frac{323\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{768a^2 d} + \frac{579\cot^5(c\ldots}{\ldots}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{835\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{512\sqrt{2}\sqrt{a}d} - \frac{189\cot(c+dx)\sqrt{a+a\sec(c+d\ldots}}{512ad}$$

Mathematica [C] time = 23.5984, size = 5628, normalized size = 16.8

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[c + d*x]^6/Sqrt[a + a*Sec[c + d*x]],x]

[Out] Result too large to show

Maple [B] time = 0.292, size = 1068, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x)

[Out] 1/15360/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^3*(-15360*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^5*sin(d*x+c)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))-12525*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^5*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-15360*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*cos(d*x+c)^4*sin(d*x+c)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))-12525*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+30720*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+19474*cos(d*x+c)^6+25050*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+30720*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+6902*cos(d*x+c)^5+25050*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-15360*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))-28788*cos(d*x+c)^4-12525*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-15360*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)-12316*cos(d*x+c)^3-12525*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+12130*cos(d*x+c)^2+5670*cos(d*x+c))/sin(d*x+c)^11

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 10.0916, size = 522, normalized size = 1.56

$$-\frac{\sqrt{2}\left(5\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\left(2\left(\frac{4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}-\frac{43}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\frac{567}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/15360*sqrt(2)*(5*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*(4*tan(1/2*d*x +
 1/2*c)^2/(a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 43/(a*sgn(tan(1/2*d*x + 1/2
*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)^2 + 567/(a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1
)))*tan(1/2*d*x + 1/2*c) + 96*(145*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^8*sqrt(-a) - 500*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(-a)*a + 710*(sqrt(-a)*tan(
1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a^2 - 46
0*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*s
qrt(-a)*a^3 + 121*sqrt(-a)*a^4)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^5*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.183 $\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{2(a\sec(c+dx)+a)^{5/2}}{5a^4d}-\frac{2(a\sec(c+dx)+a)^{3/2}}{a^3d}+\frac{2\sqrt{a\sec(c+dx)+a}}{a^2d}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}$$

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (2*Sqrt[a + a*
Sec[c + d*x]])/(a^2*d) - (2*(a + a*Sec[c + d*x])^(3/2))/(a^3*d) + (2*(a + a
*Sec[c + d*x])^(5/2))/(5*a^4*d)
```

Rubi [A] time = 0.0972202, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3880, 88, 50, 63, 207}

$$\frac{2(a\sec(c+dx)+a)^{5/2}}{5a^4d}-\frac{2(a\sec(c+dx)+a)^{3/2}}{a^3d}+\frac{2\sqrt{a\sec(c+dx)+a}}{a^2d}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (2*Sqrt[a + a*
Sec[c + d*x]])/(a^2*d) - (2*(a + a*Sec[c + d*x])^(3/2))/(a^3*d) + (2*(a + a
*Sec[c + d*x])^(5/2))/(5*a^4*d)
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{(-a+ax)^2\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{a^4 d}\\
&= \frac{\text{Subst}\left(\int \left(-3a^2\sqrt{a+ax}+\frac{a^2\sqrt{a+ax}}{x}+a(a+ax)^{3/2}\right)dx, x, \sec(c+dx)\right)}{a^4 d}\\
&= -\frac{2(a+a\sec(c+dx))^{3/2}}{a^3 d}+\frac{2(a+a\sec(c+dx))^{5/2}}{5a^4 d}+\frac{\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{a^2 d}\\
&= \frac{2\sqrt{a+a\sec(c+dx)}}{a^2 d}-\frac{2(a+a\sec(c+dx))^{3/2}}{a^3 d}+\frac{2(a+a\sec(c+dx))^{5/2}}{5a^4 d}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}} \right.}{}\\
&= \frac{2\sqrt{a+a\sec(c+dx)}}{a^2 d}-\frac{2(a+a\sec(c+dx))^{3/2}}{a^3 d}+\frac{2(a+a\sec(c+dx))^{5/2}}{5a^4 d}+\frac{2\,\text{Subst}\left(\int \frac{1}{-1+} \right.}{}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2} d}+\frac{2\sqrt{a+a\sec(c+dx)}}{a^2 d}-\frac{2(a+a\sec(c+dx))^{3/2}}{a^3 d}+\frac{2(a+a\sec(}{5a^4}
\end{aligned}$$

Mathematica [A] time = 0.14375, size = 79, normalized size = 0.79

$$\frac{2\left(\sec^3(c+dx)-2\sec^2(c+dx)-2\sec(c+dx)-5\sqrt{\sec(c+dx)+1}\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)+1\right)}{5ad\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(1 - 2*Sec[c + d*x] - 2*Sec[c + d*x]^2 + Sec[c + d*x]^3 - 5*ArcTanh[Sqrt
[1 + Sec[c + d*x]]]*Sqrt[1 + Sec[c + d*x]]))/(5*a*d*Sqrt[a*(1 + Sec[c + d*x
])])
```

Maple [B] time = 0.202, size = 224, normalized size = 2.2

$$\frac{1}{20\,da^2\,(\cos(dx+c))^2}\sqrt{\frac{a\,(\cos(dx+c)+1)}{\cos(dx+c)}}\left(5\,(\cos(dx+c))^2\sqrt{2}\arctan\left(1/2\sqrt{2}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\left(-2\frac{\cos(dx+}{\cos(dx+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/20/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(5*cos(d*x+c)^2*2^(1/2)*arct
an(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(5/2)+10*cos(d*x+c)*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)+5*2^(1/2)*arctan(1/
2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(5/2)+8*cos(d*x+c)^2-24*cos(d*x+c)+8)/cos(d*x+c)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94421, size = 687, normalized size = 6.87

$$\left[\frac{5\sqrt{a}\cos(dx+c)^2\log\left(-8\,a\cos(dx+c)^2+4\left(2\,\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos(dx+c)-a\right)+}{10\,a^2d\cos(dx+c)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/10*(5*sqrt(a)*cos(d*x + c)^2*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)
^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*co
s(d*x + c) - a) + 4*(cos(d*x + c)^2 - 3*cos(d*x + c) + 1)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c)))/(a^2*d*cos(d*x + c)^2), 1/5*(5*sqrt(-a)*arctan(2*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x +
c) + a))*cos(d*x + c)^2 + 2*(cos(d*x + c)^2 - 3*cos(d*x + c) + 1)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c)))/(a^2*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^5(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**5/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [A] time = 12.0769, size = 227, normalized size = 2.27

$$-\frac{2\left(\frac{5\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{\sqrt{2}\left(5\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2+10\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)a+4a^2\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -2/5*(5*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(s
qrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + sqrt(2)*(5*(a*tan(1/2*d*x + 1/
2*c)^2 - a)^2 + 10*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a + 4*a^2)/((a*tan(1/2*d*
x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a*sgn(tan(1/2*d*x +
 1/2*c)^2 - 1)))/d
```

3.184 $\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=54

$$\frac{2\sqrt{a\sec(c+dx)+a}}{a^2d}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}$$

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (2*Sqrt[a + a*S
ec[c + d*x]])/(a^2*d)
```

Rubi [A] time = 0.0749947, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3880, 80, 63, 207}

$$\frac{2\sqrt{a\sec(c+dx)+a}}{a^2d}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (2*Sqrt[a + a*S
ec[c + d*x]])/(a^2*d)
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{-a+ax}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{a^2 d}\\
&= \frac{2\sqrt{a+a\sec(c+dx)}}{a^2 d} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{ad}\\
&= \frac{2\sqrt{a+a\sec(c+dx)}}{a^2 d} - \frac{2\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{a^2 d}\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2} d} + \frac{2\sqrt{a+a\sec(c+dx)}}{a^2 d}
\end{aligned}$$

Mathematica [A] time = 0.069861, size = 56, normalized size = 1.04

$$\frac{2\left(\sec(c+dx)+\sqrt{\sec(c+dx)+1}\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)+1\right)}{ad\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(1 + Sec[c + d*x] + ArcTanh[Sqrt[1 + Sec[c + d*x]]]*Sqrt[1 + Sec[c + d*x
]]))/(a*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.147, size = 81, normalized size = 1.5

$$-\frac{1}{da^2}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}\left(\sqrt{2}\sqrt{-2\,\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-2\,\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\right)-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(2^(1/2)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83357, size = 493, normalized size = 9.13

$$\left[\frac{\sqrt{a}\log\left(-8\,a\cos\left(dx+c\right)^2-4\left(2\,\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos\left(dx+c\right)-a\right)+4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{2\,a^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))
*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) +
4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^2*d), -(sqrt(-a)*arctan(2*sqr
```

```
t(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c
) + a)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**3/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [B] time = 10.1773, size = 134, normalized size = 2.48

$$\frac{2\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}-\frac{\sqrt{2}}{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 2*(arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-
a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - sqrt(2)/(sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/(a*d)
```

3.185 $\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=54

$$\frac{2}{ad\sqrt{a\sec(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}$$

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + 2/(a*d*Sqrt[a
+ a*Sec[c + d*x]])
```

Rubi [A] time = 0.0492996, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3880, 51, 63, 207}

$$\frac{2}{ad\sqrt{a\sec(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + 2/(a*d*Sqrt[a
+ a*Sec[c + d*x]])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{d}\\
&= \frac{2}{ad\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{ad}\\
&= \frac{2}{ad\sqrt{a+a\sec(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{a^2d}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2}{ad\sqrt{a+a\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 0.0265715, size = 38, normalized size = 0.7

$$\frac{2\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\sec(c+dx)+1\right)}{ad\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + Sec[c + d*x]])/(a*d*Sqrt[a*(1 + Sec[
c + d*x])])
```

Maple [A] time = 0.031, size = 45, normalized size = 0.8

$$\frac{1}{d}\left(-2\frac{1}{a^{3/2}}\mathrm{Artanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)+2\frac{1}{a\sqrt{a+a\sec(dx+c)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d*(-2/a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))+2/a/(a+a*sec(d*x+c)
)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.87431, size = 635, normalized size = 11.76

$$\left[\frac{\sqrt{a}(\cos(dx+c)+1)\log\left(-8\,a\cos(dx+c)^2+4\left(2\,\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos(dx+c)-a\right)}{2\left(a^2d\cos(dx+c)+a^2d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*(cos(d*x + c) + 1)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c
)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*c
```

```
os(d*x + c) - a) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/
(a^2*d*cos(d*x + c) + a^2*d), (sqrt(-a)*(cos(d*x + c) + 1)*arctan(2*sqrt(-a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) +
a)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(a^2*d*cos(d*
x + c) + a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [B] time = 9.82059, size = 138, normalized size = 2.56

$$-\frac{\frac{2\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -(2*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(
-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.186 $\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=120

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{1}{3d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - ArcTanh[Sqrt[a
+ a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(2*Sqrt[2]*a^(3/2)*d) - 1/(3*d*(a + a*
Sec[c + d*x])^(3/2)) - 3/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.106695, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3880, 85, 152, 156, 63, 207}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{1}{3d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - ArcTanh[Sqrt[a
+ a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(2*Sqrt[2]*a^(3/2)*d) - 1/(3*d*(a + a*
Sec[c + d*x])^(3/2)) - 3/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
```

```
, f}, x] && LtQ[p, -1]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{a^2\,\text{Subst}\left(\int \frac{1}{x(-a+ax)(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{d}$$
$$= -\frac{1}{3d(a+a\sec(c+dx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{2a^2-a^2x}{x(-a+ax)(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{2ad}$$
$$= -\frac{1}{3d(a+a\sec(c+dx))^{3/2}} - \frac{3}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{\text{Subst}\left(\int \frac{-2a^4+\frac{3a^4x}{2}}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+\right.}{2a^4d}$$
$$= -\frac{1}{3d(a+a\sec(c+dx))^{3/2}} - \frac{3}{2ad\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+\right.}{4d}$$
$$= -\frac{1}{3d(a+a\sec(c+dx))^{3/2}} - \frac{3}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{2\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c\right.}{a^2d}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{1}{3d(a+a\sec(c+dx))^{3/2}} - \frac{}{2ad\sqrt{a+a}}$$

Mathematica [C] time = 0.0589617, size = 60, normalized size = 0.5

$$\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{1}{2}(\sec(c+dx)+1)\right) - 2\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\sec(c+dx)+1\right)}{3d(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Hypergeometric2F1[-3/2, 1, -1/2, (1 + Sec[c + d*x])/2] - 2*Hypergeometric2
F1[-3/2, 1, -1/2, 1 + Sec[c + d*x]])/(3*d*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.194, size = 376, normalized size = 3.1

$$-\frac{(-1+\cos(dx+c))^2}{12\,da^2(\sin(dx+c))^4}\left(12\,(\cos(dx+c))^2\sqrt{2}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(1/2\sqrt{2}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+3\,(\cos(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/12/d/a^2*(-1+cos(d*x+c))^2*(12*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+3
*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2))+24*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+6*cos(d*x+c
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2))+12*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/
2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+22*cos(d*x+c)^2+3*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+18*cos(
d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(dx+c)}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [A] time = 9.61937, size = 254, normalized size = 2.12

$$-\frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{24\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{\sqrt{2}\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}a^6+9\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a^7}{a^9\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/12*(3*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt
(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 24*arctan(1/2*sqrt(2)*sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 -
 1)) - (sqrt(2)*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^6 + 9*sqrt(2)*sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^7)/(a^9*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/
d
```

3.187 $\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{11\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}d}-\frac{3a}{20d(a\sec(c+dx)+a)^{5/2}}+\frac{a}{2d(1-\sec(c+dx))(a\sec(c+dx)}$$

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (11*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(16*Sqrt[2]*a^(3/2)*d) - (3*a)/(
20*d*(a + a*Sec[c + d*x])^(5/2)) + a/(2*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
 d*x])^(5/2)) + 5/(24*d*(a + a*Sec[c + d*x])^(3/2)) + 21/(16*a*d*Sqrt[a + a
*Sec[c + d*x]])
```

Rubi [A] time = 0.162197, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3880, 103, 152, 156, 63, 207}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{11\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}d}-\frac{3a}{20d(a\sec(c+dx)+a)^{5/2}}+\frac{a}{2d(1-\sec(c+dx))(a\sec(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (11*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(16*Sqrt[2]*a^(3/2)*d) - (3*a)/(
20*d*(a + a*Sec[c + d*x])^(5/2)) + a/(2*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
 d*x])^(5/2)) + 5/(24*d*(a + a*Sec[c + d*x])^(3/2)) + 21/(16*a*d*Sqrt[a + a
*Sec[c + d*x]])
```


Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{a^4\,\text{Subst}\left(\int \frac{1}{x(-a+ax)^2(a+ax)^{7/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} - \frac{a\,\text{Subst}\left(\int \frac{2a^2+\frac{7a^2x}{2}}{x(-a+ax)(a+ax)^{7/2}}\,dx, x, \sec(c+dx)\right)}{2d}$$

$$= -\frac{3a}{20d(a+a\sec(c+dx))^{5/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{\text{Subst}\left(\int \frac{-10a}{x(-a+a}\right.}{}$$

$$= -\frac{3a}{20d(a+a\sec(c+dx))^{5/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{5}{24d(a+a\sec(c}$$

$$= -\frac{3a}{20d(a+a\sec(c+dx))^{5/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{5}{24d(a+a\sec(c}$$

$$= -\frac{3a}{20d(a+a\sec(c+dx))^{5/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{5}{24d(a+a\sec(c}$$

$$= -\frac{3a}{20d(a+a\sec(c+dx))^{5/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{5}{24d(a+a\sec(c}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{11\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}d} - \frac{3a}{20d(a+a\sec(c+dx))^{5/2}} + \frac{}{2d(}$$

Mathematica [C] time = 0.139785, size = 90, normalized size = 0.51

$$\frac{a\left(-11(\sec(c+dx)-1)\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{1}{2}(\sec(c+dx)+1)\right)+8(\sec(c+dx)-1)\text{Hypergeometric2F1}\right.}{20d(\sec(c+dx)-1)(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*(-10 - 11*Hypergeometric2F1[-5/2, 1, -3/2, (1 + Sec[c + d*x])/2]*(-1 + S
ec[c + d*x]) + 8*Hypergeometric2F1[-5/2, 1, -3/2, 1 + Sec[c + d*x]]*(-1 + S
ec[c + d*x])))/(20*d*(-1 + Sec[c + d*x])*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] time = 0.265, size = 514, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/480/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)*(-1+cos(d*x+
c))^3*(480*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan
(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+960*2^(1/2)*cos(d*x+c)^3
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2))+165*cos(d*x+c)^4*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a
rctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+330*cos(d*x+c)^3*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-960
*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+898*cos(d*x+c)^4-480*2^(1/2)*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2))-330*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+702*cos(d*x+c)^3-165*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-730*cos(d
*x+c)^2-630*cos(d*x+c))/sin(d*x+c)^8
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot{(dx+c)}^3}{(a\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^3/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)**3/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [B] time = 9.82777, size = 387, normalized size = 2.2

$$\frac{\frac{165\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{960\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{15\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}-\frac{2\left(3\sqrt{2}\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.\right.}{}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/480*(165*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sq
rt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 960*arctan(1/2*sqrt(2)*sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^
2 - 1)) + 15*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/(a^2*sgn(tan(1/2*d
*x + 1/2*c)^2 - 1)*tan(1/2*d*x + 1/2*c)^2) - 2*(3*sqrt(2)*(a*tan(1/2*d*x +
1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^16 + 20*sqrt(2)*(-a*t
an(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^17 + 165*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a)*a^18)/(a^20*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.188 $\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=238

$$\frac{139a^2}{224d(a\sec(c+dx)+a)^{7/2}}-\frac{19a^2}{16d(1-\sec(c+dx))(a\sec(c+dx)+a)^{7/2}}-\frac{a^2}{4d(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{7/2}}+\frac{2\,\mathrm{t}}{}$$

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - (203*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(256*Sqrt[2]*a^(3/2)*d) + (139*a
^2)/(224*d*(a + a*Sec[c + d*x])^(7/2)) - a^2/(4*d*(1 - Sec[c + d*x])^2*(a +
 a*Sec[c + d*x])^(7/2)) - (19*a^2)/(16*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
d*x])^(7/2)) + (15*a)/(64*d*(a + a*Sec[c + d*x])^(5/2)) - 53/(384*d*(a + a*
Sec[c + d*x])^(3/2)) - 309/(256*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.204777, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3880, 103, 151, 152, 156, 63, 207}

$$\frac{139a^2}{224d(a\sec(c+dx)+a)^{7/2}}-\frac{19a^2}{16d(1-\sec(c+dx))(a\sec(c+dx)+a)^{7/2}}-\frac{a^2}{4d(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{7/2}}+\frac{2\,\mathrm{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - (203*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(256*Sqrt[2]*a^(3/2)*d) + (139*a
^2)/(224*d*(a + a*Sec[c + d*x])^(7/2)) - a^2/(4*d*(1 - Sec[c + d*x])^2*(a +
 a*Sec[c + d*x])^(7/2)) - (19*a^2)/(16*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
d*x])^(7/2)) + (15*a)/(64*d*(a + a*Sec[c + d*x])^(5/2)) - 53/(384*d*(a + a*
Sec[c + d*x])^(3/2)) - 309/(256*a*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{a^6\,\text{Subst}\left(\int \frac{1}{x(-a+ax)^3(a+ax)^{9/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{a^3\,\text{Subst}\left(\int \frac{4a^2+\frac{11a^2x}{2}}{x(-a+ax)^2(a+ax)^{9/2}}\,dx, x, \sec(c+a\right.}{4d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{19a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}}$$

$$= \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{}{16d(1-\sec(c+}$$

$$= \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{}{16d(1-\sec(c+}$$

$$= \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{}{16d(1-\sec(c+}$$

$$= \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{}{16d(1-\sec(c+}$$

$$= \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{}{16d(1-\sec(c+}$$

$$= \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{}{16d(1-\sec(c+}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{203\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}a^{3/2}d} + \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{}{4a}$$

Mathematica [C] time = 0.265213, size = 99, normalized size = 0.42

$$\frac{\cot^4(c+dx)\left(203(\sec(c+dx)-1)^2\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},\frac{1}{2}(\sec(c+dx)+1)\right)-64(\sec(c+dx)-1)^2\text{Hyper}\right.}{224d(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Cot[c + d*x]^4*(-322 + 203*Hypergeometric2F1[-7/2, 1, -5/2, (1 + Sec[c + d
*x])/2]*(-1 + Sec[c + d*x])^2 - 64*Hypergeometric2F1[-7/2, 1, -5/2, 1 + Sec
[c + d*x]]*(-1 + Sec[c + d*x])^2 + 266*Sec[c + d*x]))/(224*d*(a*(1 + Sec[c
+ d*x]))^(3/2))
```

Maple [B] time = 0.276, size = 866, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/10752/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos
(d*x+c))^4*(10752*2^(1/2)*cos(d*x+c)^6*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+21504*cos(d*x+c)^
5*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2))+4263*cos(d*x+c)^6*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-10752*2^(1/2)*cos(d
*x+c)^4*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2))+8526*cos(d*x+c)^5*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-43008*2^(1/2)*cos(d*x
+c)^3*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2))+20726*cos(d*x+c)^6-4263*cos(d*x+c)^4*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-1075
2*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1
/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+16074*cos(d*x+c)^5-17052*cos(d*x+
c)^3*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2))+21504*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-33076*cos(d*x+c)^
4-4263*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2))+10752*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-28476*cos(d*x
+c)^3+8526*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2))+14462*cos(d*x+c)^2+4263*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+12978*cos(d
*x+c))/sin(d*x+c)^12
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 10.0635, size = 481, normalized size = 2.02

$$-\frac{\frac{4263\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}-\frac{21504\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}-\frac{21\sqrt{2}\left(29\left(-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)^{\frac{3}{2}}-27\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}a\right)}{a^{3}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/10752*(4263*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))
/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 21504*arctan(1/2*sqrt(2)*sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1
/2*c)^2 - 1)) - 21*sqrt(2)*(29*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) - 27*s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a)/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*
tan(1/2*d*x + 1/2*c)^4) + 8*(3*sqrt(2)*(a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^30 - 21*sqrt(2)*(a*tan(1/2*d*x + 1/2*c)^
2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^31 - 112*sqrt(2)*(-a*tan(1/2
*d*x + 1/2*c)^2 + a)^(3/2)*a^32 - 882*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a)*a^33)/(a^35*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.189 $\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=157

$$\frac{2a^2\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{2a\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}-\frac{2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}+\frac{2\tan(c+}{ad\sqrt{a\sec(c+}}$$

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
(2*Tan[c + d*x])/(a*d*Sqrt[a + a*Sec[c + d*x]]) - (2*Tan[c + d*x]^3)/(3*d*(
a + a*Sec[c + d*x])^(3/2)) + (2*a*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*x])
^(5/2)) + (2*a^2*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2))
```

Rubi [A] time = 0.0970172, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3887, 459, 302, 203}

$$\frac{2a^2\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{2a\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}-\frac{2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}+\frac{2\tan(c+}{ad\sqrt{a\sec(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
(2*Tan[c + d*x])/(a*d*Sqrt[a + a*Sec[c + d*x]]) - (2*Tan[c + d*x]^3)/(3*d*(
a + a*Sec[c + d*x])^(3/2)) + (2*a*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*x])
^(5/2)) + (2*a^2*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2))
```


Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
```

```
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{x^6\left(2+ax^2\right)}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2a^2\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}} - \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{x^6}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2a^2\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}} - \frac{\left(2a^2\right)\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}-\frac{1}{a^3\left(1+ax^2\right)}\right)dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} - \frac{2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} + \frac{2a^2\tan^7}{7d(a+a\sec}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} - \frac{2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a\tan^5}{5d(a+a\sec}$$

Mathematica [C] time = 2.44781, size = 248, normalized size = 1.58

$$\frac{32\sqrt{2}\tan^7(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{11/2}\left(\frac{\cos(c+dx)(7\cos(c+dx)+11)\csc^8\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\left((-198\cos(c+dx)+61\cos(2(c+dx))-44\cos(3(c+dx))+\right.}{3360\sqrt{1-\sec(c+dx)}}\right.}{7d\left(1-\tan^2\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (32*Sqrt[2]*((1 + Sec[c + d*x])^(-1))^(11/2)*((Cos[c + d*x]*(11 + 7*Cos[c +
 d*x])*Csc[(c + d*x)/2]^8*Sec[(c + d*x)/2]^2*(105*ArcTanh[Sqrt[1 - Sec[c +
d*x]]]*Cos[c + d*x]^3 + (76 - 198*Cos[c + d*x] + 61*Cos[2*(c + d*x)] - 44*C
os[3*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]))/(3360*Sqrt[1 - Sec[c + d*x]]) - (
4*Hypergeometric2F1[2, 11/2, 13/2, -2*Sec[c + d*x]*Sin[(c + d*x)/2]^2]*Sec[
c + d*x]*Tan[(c + d*x)/2]^2)/11)*Tan[c + d*x]^7)/(7*d*(a*(1 + Sec[c + d*x])
)^(3/2)*(1 - Tan[(c + d*x)/2]^2)^(9/2))
```

Maple [B] time = 0.229, size = 391, normalized size = 2.5

$$-\frac{1}{840\,da^2\sin(dx+c)(\cos(dx+c))^3}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(105\sqrt{2}\sin(dx+c)(\cos(dx+c))^3\,\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+}{\cos(dx+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/840/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(105*2^(1/2)*sin(d*x+c)*co
s(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)+315*2^(1/2)*sin(d*x+c)
*cos(d*x+c)^2*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)+315*2^(1/2)*sin(d*x
+c)*cos(d*x+c)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)+105*2^(1/2)*arctan
h(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*sin(d*x+c)+2336*cos(d*x+c)^4-2848*cos(d
*x+c)^3+128*cos(d*x+c)^2+624*cos(d*x+c)-240)/sin(d*x+c)/cos(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

[Out] Timed out

Fricas [A] time = 1.82474, size = 914, normalized size = 5.82

$$\left[-\frac{105\left(\cos\left(dx+c\right)^{4}+\cos\left(dx+c\right)^{3}\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^{2}-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-2\left(146\,\cos\left(dx\right.\right.}{105\left(a^{2}d\cos\left(dx+c\right)^{4}+a^{2}d\cos\left(dx+c\right)^{3}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/105*(105*(cos(d*x + c)^4 + cos(d*x + c)^3)*sqrt(-a)*log((2*a*cos(d*x +
c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(
d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(146*cos(d*x + c)^3
- 32*cos(d*x + c)^2 - 24*cos(d*x + c) + 15)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x + c)^3), 2/105
*(105*(cos(d*x + c)^4 + cos(d*x + c)^3)*sqrt(a)*arctan(sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (146*cos(d*x + c
)^3 - 32*cos(d*x + c)^2 - 24*cos(d*x + c) + 15)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^{6}\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**6/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [B] time = 15.6055, size = 456, normalized size = 2.9

$$-\frac{105\sqrt{-a}\left(\frac{\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+a\left(2\sqrt{2}-3\right)\right|\right)}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\cdots}{105\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/105*(105*sqrt(-a)*(log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(a^2*sgn(tan(1/2*d*x + 1/2
*c)^2 - 1)) - log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 -
1))) + 2*(((139*sqrt(2)*a^2*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c)
^2 - 1) - 539*sqrt(2)*a^2/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/
2*c)^2 + 385*sqrt(2)*a^2/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2
*c)^2 - 105*sqrt(2)*a^2/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*
c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/
d
```

3.190 $\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}-\frac{2\tan(c+dx)}{ad\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) - (
2*Tan[c + d*x])/(a*d*Sqrt[a + a*Sec[c + d*x]]) + (2*Tan[c + d*x]^3)/(3*d*(a
 + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.0795231, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3887, 302, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}-\frac{2\tan(c+dx)}{ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) - (
2*Tan[c + d*x])/(a*d*Sqrt[a + a*Sec[c + d*x]]) + (2*Tan[c + d*x]^3)/(3*d*(a
 + a*Sec[c + d*x])^(3/2))
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(2a)\,\text{Subst}\left(\int \frac{x^4}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$
$$= -\frac{(2a)\,\text{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+\frac{1}{a^2(1+ax^2)}\right)dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$
$$= -\frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}}+\frac{2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}-\frac{2\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad}$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}-\frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}}+\frac{2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}$$

Mathematica [A] time = 4.40266, size = 162, normalized size = 1.71

$$\frac{64\cos^6\left(\frac{1}{2}(c+dx)\right)\cot^4\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{7/2}\left((\sin(c+dx)-2\sin(2(c+dx)))\sqrt{\frac{1}{\cos(c+dx)+1}}\sqrt{\cos(c+\right.}{3d\left(\cot^2\left(\frac{1}{2}(c+dx)\right)-1\right)^2(a(\sec(c+dx)+1))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (64*Cos[(c + d*x)/2]^6*Cot[(c + d*x)/2]^4*Sec[c + d*x]^5*((1 + Sec[c + d*x]
)^(-1))^(7/2)*(3*ArcSin[Tan[(c + d*x)/2]/Sqrt[(1 + Cos[c + d*x])^(-1)]]*Cos
[c + d*x]^2 + Sqrt[Cos[c + d*x]]*Sqrt[(1 + Cos[c + d*x])^(-1)]*(Sin[c + d*x
] - 2*Sin[2*(c + d*x)])))/(3*d*(-1 + Cot[(c + d*x)/2]^2)^2*(a*(1 + Sec[c +
d*x]))^(3/2))
```

Maple [A] time = 0.202, size = 142, normalized size = 1.5

$$-\frac{1}{3\,da^{2}\sin(dx+c)\cos(dx+c)}\left(3\sqrt{2}\cos(dx+c)\sin(dx+c)\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-2\,\frac{\cos}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/3/d/a^2*(3*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/c
os(d*x+c))-8*cos(d*x+c)^2+10*cos(d*x+c)-2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1
/2)/sin(d*x+c)/cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.7873, size = 778, normalized size = 8.19

$$\left[-\frac{3\left(\cos(dx+c)^{2}+\cos(dx+c)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^{2}+2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,(}{3\left(a^{2}d\cos(dx+c)^{2}+a^{2}d\cos(dx+c)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*log((2*a*cos(d*x + c)^2 +
 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
```

```
c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*(4*cos(d*x + c) - 1)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + a^
2*d*cos(d*x + c)), -2/3*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
 + sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(4*cos(d*x + c) - 1)*sin(d*x + c
))/(a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^4(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**4/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [B] time = 12.6768, size = 348, normalized size = 3.66

$$\frac{3\sqrt{-a}\left(\frac{\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{a^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+a\left(2\sqrt{2}-3\right)\right|\right)}{a^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)+\frac{2\cdots}{\cdots}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*sqrt(-a)*(log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2
 - 1)) - log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))
+ 2*(5*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 3*s
qrt(2)/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*
x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

3.191 $\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}$$

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
(2*Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/(a^(3/2)*d)
```

Rubi [A] time = 0.0861983, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3887, 481, 203}

$$\frac{2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
(2*Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/(a^(3/2)*d)
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
```

```
2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx &= -\frac{2\,\mathrm{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\ &= \frac{2\,\mathrm{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad} - \frac{4\,\mathrm{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad}\\ &= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}\end{aligned}$$

Mathematica [C] time = 23.4534, size = 4752, normalized size = 55.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-32*Cos[(c + d*x)/4]^2*Cos[(c + d*x)/2]^2*(-2/Sqrt[Sec[c + d*x]] + 2*Sqrt[
Sec[c + d*x]])*Sec[c + d*x]^2*((((2 + Sqrt[2])*EllipticF[ArcSin[2^(1/4)*Sqr
t[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]],
1/2] - (2 + Sqrt[2])*EllipticPi[-(1/Sqrt[2]), ArcSin[2^(1/4)*Sqrt[(1 + Sqrt
[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] + (-2
+ Sqrt[2])*EllipticPi[1/Sqrt[2], ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c
+ d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2])*Sqrt[(-1 - Sqrt[2]
 + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*Sqrt[(1 - Sqrt[2] +
 Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*(-1 + Sqrt[2] + Tan[(
c + d*x)/4])^2*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c
 + d*x)/4])])/4 - EllipticPi[-3 + 2*Sqrt[2], -ArcSin[Tan[(c + d*x)/4]/Sqrt[
3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]*Sqrt[3 + 2*Sqrt[2] - Tan[(c + d*x)/4]^2]*
Sqrt[(-3 + 2*Sqrt[2])*(-3 + 2*Sqrt[2] + Tan[(c + d*x)/4]^2)]))/(d*(a*(1 + S
ec[c + d*x]))^(3/2)*(16*Cos[(c + d*x)/4]*Sqrt[Sec[c + d*x]]*Sin[(c + d*x)/4
```

]*((((2 + Sqrt[2])*EllipticF[ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] - (2 + Sqrt[2])*EllipticPi[-(1/Sqrt[2]), ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] + (-2 + Sqrt[2])*EllipticPi[1/Sqrt[2], ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2])*Sqrt[(-1 - Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*Sqrt[(1 - Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*(-1 + Sqrt[2] + Tan[(c + d*x)/4])^2*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])])/4 - EllipticPi[-3 + 2*Sqrt[2], -ArcSin[Tan[(c + d*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]*Sqrt[3 + 2*Sqrt[2] - Tan[(c + d*x)/4]^2]*Sqrt[(-3 + 2*Sqrt[2])*(-3 + 2*Sqrt[2] + Tan[(c + d*x)/4]^2)]) - 16*Cos[(c + d*x)/4]^2*Sec[c + d*x]^(3/2)*Sin[c + d*x]*((((2 + Sqrt[2])*EllipticF[ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] - (2 + Sqrt[2])*EllipticPi[-(1/Sqrt[2]), ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] + (-2 + Sqrt[2])*EllipticPi[1/Sqrt[2], ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2])*Sqrt[(-1 - Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*Sqrt[(1 - Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*(-1 + Sqrt[2] + Tan[(c + d*x)/4])^2*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])])/4 - EllipticPi[-3 + 2*Sqrt[2], -ArcSin[Tan[(c + d*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]*Sqrt[3 + 2*Sqrt[2] - Tan[(c + d*x)/4]^2]*Sqrt[(-3 + 2*Sqrt[2])*(-3 + 2*Sqrt[2] + Tan[(c + d*x)/4]^2)]) - 32*Cos[(c + d*x)/4]^2*Sqrt[Sec[c + d*x]]*((((2 + Sqrt[2])*EllipticF[ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] - (2 + Sqrt[2])*EllipticPi[-(1/Sqrt[2]), ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] + (-2 + Sqrt[2])*EllipticPi[1/Sqrt[2], ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2])*Sec[(c + d*x)/4]^2*Sqrt[(-1 - Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*Sqrt[(1 - Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*(-1 + Sqrt[2] + Tan[(c + d*x)/4])*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])])/8 - ((-3 + 2*Sqrt[2])*EllipticPi[-3 + 2*Sqrt[2], -ArcSin[Tan[(c + d*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]*Sec[(c + d*x)/4]^2*Tan[(c + d*x)/4]*Sqrt[3 + 2*Sqrt[2] - Tan[(c + d*x)/4]^2])/(4*Sqrt[(-3 + 2*Sqrt[2])*(-3 + 2*Sqrt[2] + Tan[(c + d*x)/4]^2)]) + (EllipticPi[-3 + 2*Sqrt[2], -ArcSin[Tan[(c + d*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]*Sec[(c + d*x)/4]^2*Tan[(c + d*x)/4]*Sqrt[(-3 + 2*Sqrt[2])*(-3 + 2*Sqrt[2] + Tan[(c + d*x)/4]^2)])/(4*Sqrt[3 + 2*Sqrt[2] - Tan[(c + d*x)/4]^2]) + (Sec[(c + d*x)/4]^2*Sqrt[3 + 2*Sqrt[2] - Tan[(c + d*x)/4]^2]*Sqrt[(-3 + 2*Sqrt[2])*(-3 + 2*Sqrt[2] + Tan[(c + d*x)/4]^2)])/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(c + d*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(c + d*x)/4]^2)/(3 - 2*Sqrt[2])]*(1 - ((-3 + 2*Sqrt[2])*Tan[(c + d*x)/4]^2)/(3 - 2*Sqrt[2]))) + (((2 + Sqrt[2])*EllipticF[ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c +

```
d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] - (2 + Sqrt[2])*Ellip
ticPi[-(1/Sqrt[2]), ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1
 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] + (-2 + Sqrt[2])*EllipticPi[1/Sq
rt[2], ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[
2])*Tan[(c + d*x)/4])]], 1/2])*Sqrt[(1 - Sqrt[2] + Tan[(c + d*x)/4])/(-1 +
Sqrt[2] + Tan[(c + d*x)/4])]*(-1 + Sqrt[2] + Tan[(c + d*x)/4])^2*Sqrt[(1 +
Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*(-(Sec[(c +
d*x)/4]^2*(-1 - Sqrt[2] + Tan[(c + d*x)/4]))/(4*(-1 + Sqrt[2] + Tan[(c + d*
x)/4])^2) + Sec[(c + d*x)/4]^2/(4*(-1 + Sqrt[2] + Tan[(c + d*x)/4]))))/(8*S
qrt[(-1 - Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]) +
 (((2 + Sqrt[2])*EllipticF[ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)
/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] - (2 + Sqrt[2])*EllipticP
i[-(1/Sqrt[2]), ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (
1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] + (-2 + Sqrt[2])*EllipticPi[1/Sqrt[2
], ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*
Tan[(c + d*x)/4])]], 1/2])*Sqrt[(-1 - Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqr
t[2] + Tan[(c + d*x)/4])]*(-1 + Sqrt[2] + Tan[(c + d*x)/4])^2*Sqrt[(1 + Sqr
t[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*(-(Sec[(c + d*x
)/4]^2*(1 - Sqrt[2] + Tan[(c + d*x)/4]))/(4*(-1 + Sqrt[2] + Tan[(c + d*x)/4
])^2) + Sec[(c + d*x)/4]^2/(4*(-1 + Sqrt[2] + Tan[(c + d*x)/4]))))/(8*Sqrt[
(1 - Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]) + (((2
 + Sqrt[2])*EllipticF[ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/
(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] - (2 + Sqrt[2])*EllipticPi[-(1
/Sqrt[2]), ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + S
qrt[2])*Tan[(c + d*x)/4])]], 1/2] + (-2 + Sqrt[2])*EllipticPi[1/Sqrt[2], Ar
cSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(
c + d*x)/4])]], 1/2])*Sqrt[(-1 - Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2]
+ Tan[(c + d*x)/4])]*Sqrt[(1 - Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] +
Tan[(c + d*x)/4])]*(-1 + Sqrt[2] + Tan[(c + d*x)/4])^2*(Sec[(c + d*x)/4]^2/
(4*(-1 + Sqrt[2] + Tan[(c + d*x)/4])) - (Sec[(c + d*x)/4]^2*(1 + Sqrt[2] +
Tan[(c + d*x)/4]))/(4*(-1 + Sqrt[2] + Tan[(c + d*x)/4])^2)))/(8*Sqrt[(1 + S
qrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]) + (Sqrt[(-1
- Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*Sqrt[(1 -
Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*(-1 + Sqrt[2
] + Tan[(c + d*x)/4])^2*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2]
 + Tan[(c + d*x)/4])]*(((2 + Sqrt[2])*(-((1 + Sqrt[2])*Sec[(c + d*x)/4]^2*(
1 + Sqrt[2] + Tan[(c + d*x)/4]))/(4*(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])^2)
 + Sec[(c + d*x)/4]^2/(4*(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4]))))/(2^(3/4)*S
qrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]*
Sqrt[1 - (1 + Sqrt[2] + Tan[(c + d*x)/4])/(Sqrt[2]*(1 + (1 + Sqrt[2])*Tan[(
c + d*x)/4]))]*Sqrt[1 - (Sqrt[2]*(1 + Sqrt[2] + Tan[(c + d*x)/4]))/(1 + (1
+ Sqrt[2])*Tan[(c + d*x)/4])]) + ((-2 + Sqrt[2])*(-((1 + Sqrt[2])*Sec[(c +
d*x)/4]^2*(1 + Sqrt[2] + Tan[(c + d*x)/4]))/(4*(1 + (1 + Sqrt[2])*Tan[(c +
d*x)/4])^2) + Sec[(c + d*x)/4]^2/(4*(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4]))))
/(2^(3/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c +
```

```
d*x)/4])]*(1 - (1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c
+ d*x)/4]))*Sqrt[1 - (1 + Sqrt[2] + Tan[(c + d*x)/4])/(Sqrt[2]*(1 + (1 + Sq
rt[2])*Tan[(c + d*x)/4]))]*Sqrt[1 - (Sqrt[2]*(1 + Sqrt[2] + Tan[(c + d*x)/4
]))/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]) - ((2 + Sqrt[2])*(-((1 + Sqrt[2]
)*Sec[(c + d*x)/4]^2*(1 + Sqrt[2] + Tan[(c + d*x)/4]))/(4*(1 + (1 + Sqrt[2]
)*Tan[(c + d*x)/4])^2) + Sec[(c + d*x)/4]^2/(4*(1 + (1 + Sqrt[2])*Tan[(c +
d*x)/4]))))/(2^(3/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2
])*Tan[(c + d*x)/4])]*(1 + (1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[
2])*Tan[(c + d*x)/4]))*Sqrt[1 - (1 + Sqrt[2] + Tan[(c + d*x)/4])/(Sqrt[2]*(
1 + (1 + Sqrt[2])*Tan[(c + d*x)/4]))]*Sqrt[1 - (Sqrt[2]*(1 + Sqrt[2] + Tan[
(c + d*x)/4]))/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]))/4)))
```

Maple [B] time = 0.127, size = 142, normalized size = 1.7

$$\frac{1}{da^2}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}\sin(dx+c)}{2\cos(dx+c)}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+2\ln\left(-\frac{1}{\sin(dx+c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)/cos(d*x+c))+2*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
+cos(d*x+c)-1)/sin(d*x+c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 2.0362, size = 784, normalized size = 9.22

$$\left[\frac{\sqrt{2}a\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,c}{\cos(dx+}\right.}{a^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*a*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c
) + 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - sqrt(-a)*log((2*a*cos(d*x +
 c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)))/(a^2*d), -2*(sqrt(2)*s
qrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/
(sqrt(a)*sin(d*x + c))) - sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [B] time = 12.5451, size = 302, normalized size = 3.55

$$\frac{\frac{\sqrt{2}\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{\sqrt{-a}a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.\right.}{\sqrt{-a}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] (sqrt(2)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a))^2)/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + log(abs((sqrt(-a)
*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(
2) + 3)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - log(abs((sqrt(-a)*
tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2
) - 3)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.192 $\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=215

$$\frac{7\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{32a^2d}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{71\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{32\sqrt{2}a^{3/2}d}-\frac{\cos^2(c+dx)\cot(c+dx)\sec}{1}$$

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
(71*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(32*
Sqrt[2]*a^(3/2)*d) + (7*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(32*a^2*d) -
 (13*Cos[c + d*x]*Cot[c + d*x]*Sec[(c + d*x)/2]^2*Sqrt[a + a*Sec[c + d*x]])
/(32*a^2*d) - (Cos[c + d*x]^2*Cot[c + d*x]*Sec[(c + d*x)/2]^4*Sqrt[a + a*Se
c[c + d*x]])/(16*a^2*d)
```

Rubi [A] time = 0.195117, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3887, 472, 579, 583, 522, 203}

$$\frac{7\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{32a^2d}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{71\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{32\sqrt{2}a^{3/2}d}-\frac{\cos^2(c+dx)\cot(c+dx)\sec}{1}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
(71*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(32*
Sqrt[2]*a^(3/2)*d) + (7*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(32*a^2*d) -
 (13*Cos[c + d*x]*Cot[c + d*x]*Sec[(c + d*x)/2]^2*Sqrt[a + a*Sec[c + d*x]])
/(32*a^2*d) - (Cos[c + d*x]^2*Cot[c + d*x]*Sec[(c + d*x)/2]^4*Sqrt[a + a*Se
c[c + d*x]])/(16*a^2*d)
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2 d}$$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{16a^2 d} - \frac{\text{Subst}\left(\int \frac{3a-5a^2x^2}{x^2(1+ax^2)(2+ax^2}\right.}{4a}$$

$$= -\frac{13\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{32a^2 d} - \frac{\cos^2(c+dx)\cot(c+d}{}$$

$$= \frac{7\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{32a^2 d} - \frac{13\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec}}{32a^2 d}$$

$$= \frac{7\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{32a^2 d} - \frac{13\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec}}{32a^2 d}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2} d} + \frac{71\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{32\sqrt{2}a^{3/2} d} + \frac{7\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{32a^2 d}$$

Mathematica [C] time = 23.5864, size = 5588, normalized size = 25.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.238, size = 542, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/64/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(-64*2^(1
/2)*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/
```

```
2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))-71*co
s(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-128*2^(1/2)
*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(
1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))-142*cos(d*
x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-64*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+54*cos(d*x+c)^3-71*sin(d*x+c
)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d
*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+24*cos(d*x+c)^2-14*cos(d*x+c))/
sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [A] time = 9.45942, size = 221, normalized size = 1.03

$$\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\,\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}-\frac{17\,\sqrt{2}}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\right.}\right.\right.}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2
/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 17*sqrt(2)/(a^2*sgn(tan(1/2*d*x +
1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c) + 16*sqrt(2)/(((sqrt(-a)*tan(1/2*d*x +
 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)*sqrt(-a)*sgn(tan(1/2*
d*x + 1/2*c)^2 - 1)))/d
```

3.193 $\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=303

$$\frac{277\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{384a^3d}-\frac{21\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{256a^2d}+\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{533\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{256\sqrt{2}a^{3/2}d}$$

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) - (
533*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(256
*Sqrt[2]*a^(3/2)*d) - (21*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(256*a^2*d
) + (277*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(384*a^3*d) - (81*Cos[c
 + d*x]*Cot[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(3/2))/(128*
a^3*d) - (7*Cos[c + d*x]^2*Cot[c + d*x]^3*Sec[(c + d*x)/2]^4*(a + a*Sec[c +
 d*x])^(3/2))/(64*a^3*d) - (Cos[c + d*x]^3*Cot[c + d*x]^3*Sec[(c + d*x)/2]^
6*(a + a*Sec[c + d*x])^(3/2))/(48*a^3*d)
```

Rubi [A] time = 0.28155, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3887, 472, 579, 583, 522, 203}

$$\frac{277\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{384a^3d}-\frac{21\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{256a^2d}+\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{533\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{256\sqrt{2}a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) - (
533*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(256
*Sqrt[2]*a^(3/2)*d) - (21*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(256*a^2*d
) + (277*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(384*a^3*d) - (81*Cos[c
 + d*x]*Cot[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(3/2))/(128*
a^3*d) - (7*Cos[c + d*x]^2*Cot[c + d*x]^3*Sec[(c + d*x)/2]^4*(a + a*Sec[c +
 d*x])^(3/2))/(64*a^3*d) - (Cos[c + d*x]^3*Cot[c + d*x]^3*Sec[(c + d*x)/2]^
6*(a + a*Sec[c + d*x])^(3/2))/(48*a^3*d)
```


Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
```

```
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)^4}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^3d}$$

$$= -\frac{\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{48a^3d} - \frac{\text{Subst}\left(\int \frac{3a-9a^2x^2}{x^4(1+ax^2)(2+a}\right.}{6}$$

$$= -\frac{7\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{64a^3d} - \frac{\cos^3(c+dx)\cot^3(c-}{}$$

$$= -\frac{81\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{128a^3d} - \frac{7\cos^2(c+dx)\cot^3(c}{}$$

$$= \frac{277\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{384a^3d} - \frac{81\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a}{128a^3d}$$

$$= -\frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{256a^2d} + \frac{277\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{384a^3d} - \frac{81\cos(c+d}{}$$

$$= -\frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{256a^2d} + \frac{277\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{384a^3d} - \frac{81\cos(c+d}{}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{533\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{256\sqrt{2}a^{3/2}d} - \frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{256a^2d}$$

Mathematica [C] time = 23.679, size = 5630, normalized size = 18.58

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.326, size = 732, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/1536/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)*(-1+cos(d*x
+c))^3*(-1536*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*cos(d*x+c)^4*sin
(d*x+c)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
/cos(d*x+c))-1599*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^4*sin(d*x
+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin
(d*x+c))-3072*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
/cos(d*x+c))-3198*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin
(d*x+c))+1638*cos(d*x+c)^5+3072*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)/cos(d*x+c))+984*cos(d*x+c)^4+3198*cos(d*x+c)*sin(d*x+c)*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+1536*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)/cos(d*x+c))*sin(d*x+c)-1380*cos(d*x+c)^3+1599*sin(d*x+c)*ln(-(-(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)-856*cos(d*x+c)^2+126*cos(d*x+c))/sin(d*x+c)
^9
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)**4/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [A] time = 10.0694, size = 390, normalized size = 1.29

$$\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(2\left(\frac{4\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}-\frac{37\sqrt{2}}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{417\sqrt{2}}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)\mathrm{t}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/1536*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*(4*sqrt(2)*tan(1/2*d*x + 1/2
*c)^2/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 37*sqrt(2)/(a^2*sgn(tan(1/2*d
*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)^2 + 417*sqrt(2)/(a^2*sgn(tan(1/2*
d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c) - 32*sqrt(2)*(21*(sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 36*(sqrt(-a)*tan
(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + 19*a^2)/(((s
qrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^
3*sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.194 $\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=387

$$\frac{12267\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{10240a^4d}-\frac{8171\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{12288a^3d}-\frac{21\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{8192a^2d}$$

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
(16363*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(
8192*Sqrt[2]*a^(3/2)*d) - (21*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(8192*
a^2*d) - (8171*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(12288*a^3*d) + (
12267*Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2))/(10240*a^4*d) - (2045*Cos[
c + d*x]*Cot[c + d*x]^5*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(5/2))/(204
8*a^4*d) - (511*Cos[c + d*x]^2*Cot[c + d*x]^5*Sec[(c + d*x)/2]^4*(a + a*Sec
[c + d*x])^(5/2))/(3072*a^4*d) - (29*Cos[c + d*x]^3*Cot[c + d*x]^5*Sec[(c +
 d*x)/2]^6*(a + a*Sec[c + d*x])^(5/2))/(768*a^4*d) - (Cos[c + d*x]^4*Cot[c
+ d*x]^5*Sec[(c + d*x)/2]^8*(a + a*Sec[c + d*x])^(5/2))/(128*a^4*d)
```

Rubi [A] time = 0.3678, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3887, 472, 579, 583, 522, 203}

$$\frac{12267\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{10240a^4d}-\frac{8171\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{12288a^3d}-\frac{21\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{8192a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
(16363*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(
8192*Sqrt[2]*a^(3/2)*d) - (21*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(8192*
a^2*d) - (8171*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(12288*a^3*d) + (
12267*Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2))/(10240*a^4*d) - (2045*Cos[
c + d*x]*Cot[c + d*x]^5*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(5/2))/(204
8*a^4*d) - (511*Cos[c + d*x]^2*Cot[c + d*x]^5*Sec[(c + d*x)/2]^4*(a + a*Sec
[c + d*x])^(5/2))/(3072*a^4*d) - (29*Cos[c + d*x]^3*Cot[c + d*x]^5*Sec[(c +
 d*x)/2]^6*(a + a*Sec[c + d*x])^(5/2))/(768*a^4*d) - (Cos[c + d*x]^4*Cot[c
+ d*x]^5*Sec[(c + d*x)/2]^8*(a + a*Sec[c + d*x])^(5/2))/(128*a^4*d)
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)^5}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^4d}$$

$$= -\frac{\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{128a^4d} - \frac{\mathrm{Subst}\left(\int \frac{3a-13a^2x}{x^6(1+ax^2)(2+}\right.}{}$$

$$= -\frac{29\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{768a^4d} - \frac{\cos^4(c+dx)\cot^5}{}$$

$$= -\frac{511\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{3072a^4d} - \frac{29\cos^3(c+dx)\,c}{}$$

$$= -\frac{2045\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{2048a^4d} - \frac{511\cos^2(c+dx)}{}$$

$$= \frac{12267\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{10240a^4d} - \frac{2045\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right.}{2048a^4d}$$

$$= -\frac{8171\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12288a^3d} + \frac{12267\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{10240a^4d} - \frac{20}{}$$

$$= -\frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{8192a^2d} - \frac{8171\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12288a^3d} + \frac{12267\,\mathrm{co}}{}$$

$$= -\frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{8192a^2d} - \frac{8171\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12288a^3d} + \frac{12267\,\mathrm{co}}{}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{16363\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{8192\sqrt{2}a^{3/2}d} - \frac{21\cot(c+dx)\sqrt{a+a\sec(c}}{8192a^2d}$$

Mathematica [C] time = 23.6231, size = 5672, normalized size = 14.66

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.379, size = 1240, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/245760/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+co
s(d*x+c))^4*(-245760*cos(d*x+c)^6*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)/cos(d*x+c))-245445*cos(d*x+c)^6*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+
c)-1)/sin(d*x+c))-491520*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^5*
sin(d*x+c)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)/cos(d*x+c))-490890*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x
+c)^5*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(
d*x+c)-1)/sin(d*x+c))+245760*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*c
os(d*x+c)^4*sin(d*x+c)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)/cos(d*x+c))+302082*cos(d*x+c)^7+245445*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+983040*2^(1/2)*cos(d*x+c)^3*s
in(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+207048*cos(d*x+c)^6+981
780*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+24576
0*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arct
anh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))
-457998*cos(d*x+c)^5+245445*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x
+c)-1)/sin(d*x+c))-491520*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)/cos(d*x+c))-362512*cos(d*x+c)^4-490890*cos(d*x+c)*sin(d*x+c)*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-245760*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*si
```

```
n(d*x+c)/cos(d*x+c))*sin(d*x+c)+195222*cos(d*x+c)^3-245445*sin(d*x+c)*ln(-(
-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+164680*cos(d*x+c)^2+630*cos(d*x+c))/si
n(d*x+c)^13
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 10.2234, size = 559, normalized size = 1.44

$$5\left(2\left(4\left(\frac{6\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}-\frac{65\sqrt{2}}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1451\sqrt{2}}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\frac{}{a^2\mathrm{sg}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/245760*(5*(2*(4*(6*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a^2*sgn(tan(1/2*d*x +
1/2*c)^2 - 1)) - 65*sqrt(2)/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*
d*x + 1/2*c)^2 + 1451*sqrt(2)/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/
2*d*x + 1/2*c)^2 - 13503*sqrt(2)/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c) + 256*sqrt(2)*(555*(s
qrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8 - 195
0*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*a
 + 2780*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^4*a^2 - 1810*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a))^2*a^3 + 473*a^4)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2 - a)^5*sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))
)/d
```

3.195 $\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=78

$$\frac{2(a\sec(c+dx)+a)^{3/2}}{3a^4d}-\frac{6\sqrt{a\sec(c+dx)+a}}{a^3d}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}$$

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) - (6*Sqrt[a + a*
Sec[c + d*x]])/(a^3*d) + (2*(a + a*Sec[c + d*x])^(3/2))/(3*a^4*d)
```

Rubi [A] time = 0.090066, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3880, 88, 63, 207}

$$\frac{2(a\sec(c+dx)+a)^{3/2}}{3a^4d}-\frac{6\sqrt{a\sec(c+dx)+a}}{a^3d}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) - (6*Sqrt[a + a*
Sec[c + d*x]])/(a^3*d) + (2*(a + a*Sec[c + d*x])^(3/2))/(3*a^4*d)
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{(-a+ax)^2}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{a^4d}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{3a^2}{\sqrt{a+ax}}+\frac{a^2}{x\sqrt{a+ax}}+a\sqrt{a+ax}\right)dx, x, \sec(c+dx)\right)}{a^4d}\\ &= -\frac{6\sqrt{a+a\sec(c+dx)}}{a^3d}+\frac{2(a+a\sec(c+dx))^{3/2}}{3a^4d}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{a^2d}\\ &= -\frac{6\sqrt{a+a\sec(c+dx)}}{a^3d}+\frac{2(a+a\sec(c+dx))^{3/2}}{3a^4d}+\frac{2\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{a^3d}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d}-\frac{6\sqrt{a+a\sec(c+dx)}}{a^3d}+\frac{2(a+a\sec(c+dx))^{3/2}}{3a^4d}\end{aligned}$$

Mathematica [A] time = 0.110246, size = 69, normalized size = 0.88

$$\frac{2\left(\sec^2(c+dx)-7\sec(c+dx)-3\sqrt{\sec(c+dx)+1}\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)-8\right)}{3a^2d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(-8 - 7*Sec[c + d*x] + Sec[c + d*x]^2 - 3*ArcTanh[Sqrt[1 + Sec[c + d*x]]
]*Sqrt[1 + Sec[c + d*x]]))/(3*a^2*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.199, size = 155, normalized size = 2.

$$-\frac{1}{6\,da^{3}\cos\left(dx+c\right)}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}\left(3\sqrt{2}\arctan\left(1/2\sqrt{2}\sqrt{-2\,\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\right)\left(-2\,\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}\right)^{3/2}\cos\left(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/6/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(3*2^(1/2)*arctan(1/2*2^(1/2
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2
)*cos(d*x+c)+3*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)+32*cos(d*x+c)-4)/cos(d*x+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.90927, size = 629, normalized size = 8.06

$$\left[\frac{3\sqrt{a}\cos\left(dx+c\right)\log\left(-8\,a\cos\left(dx+c\right)^{2}+4\left(2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos\left(dx+c\right)-a\right)-}{6\,a^{3}d\cos\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(a)*cos(d*x + c)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)^2
+ cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d
*x + c) - a) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(8*cos(d*x + c) -
1))/(a^3*d*cos(d*x + c)), 1/3*(3*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)
- 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(8*cos(d*x + c) - 1))/(a^3*d*co
s(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^5(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**5/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [B] time = 11.3755, size = 189, normalized size = 2.42

$$-\frac{2\left(\frac{3\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{\sqrt{2}\left(9a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-7a\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(s
qrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - sqrt(2)*(9*a*tan(1/2*d*x + 1
/2*c)^2 - 7*a)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.196 $\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=54

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}-\frac{4}{a^2d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) - 4/(a^2*d*Sqrt[a
 + a*Sec[c + d*x]])
```

Rubi [A] time = 0.072335, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3880, 78, 63, 207}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}-\frac{4}{a^2d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) - 4/(a^2*d*Sqrt[a
 + a*Sec[c + d*x]])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{-a+ax}{x(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{a^2d}\\
&= -\frac{4}{a^2d\sqrt{a+a\sec(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{a^2d}\\
&= -\frac{4}{a^2d\sqrt{a+a\sec(c+dx)}} - \frac{2\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{a^3d}\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} - \frac{4}{a^2d\sqrt{a+a\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0525905, size = 50, normalized size = 0.93

$$\frac{2\left(\sqrt{\sec(c+dx)+1}\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)-2\right)}{a^2d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(-2 + ArcTanh[Sqrt[1 + Sec[c + d*x]]]*Sqrt[1 + Sec[c + d*x]]))/(a^2*d*Sq
rt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.136, size = 154, normalized size = 2.9

$$-\frac{1}{d a^{3}\left(\cos \left(d x+c\right)+1\right)}\sqrt{\frac{a\left(\cos \left(d x+c\right)+1\right)}{\cos \left(d x+c\right)}}\left(\sqrt{2}\cos \left(d x+c\right)\sqrt{-2\frac{\cos \left(d x+c\right)}{\cos \left(d x+c\right)+1}}\arctan \left(\frac{\sqrt{2}}{2}\sqrt{-2\frac{\cos \left(d x+c\right)}{\cos \left(d x+c\right)+1}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(2^(1/2)*cos(d*x+c)*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2))+2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+4*cos(d*x+c))/(cos(d*x+c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.9204, size = 636, normalized size = 11.78

$$\left[\frac{\sqrt{a}(\cos \left(d x+c\right)+1)\log \left(-8\,a\cos \left(d x+c\right)^{2}-4\left(2\,\cos \left(d x+c\right)^{2}+\cos \left(d x+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos \left(d x+c\right)-a\right.}{2\left(a^{3}d\cos \left(d x+c\right)+a^{3}d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*(cos(d*x + c) + 1)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c
)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*c
os(d*x + c) - a) - 8*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/
(a^3*d*cos(d*x + c) + a^3*d), -(sqrt(-a)*(cos(d*x + c) + 1)*arctan(2*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) +
 a)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(a^3*d*cos(d
*x + c) + a^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**3/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [B] time = 9.96519, size = 140, normalized size = 2.59

$$\frac{2\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a^2\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 2*(arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-
a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a)/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/(a*d)
```

3.197 $\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=78

$$\frac{2}{a^2d\sqrt{a\sec(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{2}{3ad(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) + 2/(3*a*d*(a +
a*Sec[c + d*x])^(3/2)) + 2/(a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.0582467, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3880, 51, 63, 207}

$$\frac{2}{a^2d\sqrt{a\sec(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{2}{3ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) + 2/(3*a*d*(a +
a*Sec[c + d*x])^(3/2)) + 2/(a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{d}\\
&= \frac{2}{3ad(a+a\sec(c+dx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{ad}\\
&= \frac{2}{3ad(a+a\sec(c+dx))^{3/2}} + \frac{2}{a^2d\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{a^2d}\\
&= \frac{2}{3ad(a+a\sec(c+dx))^{3/2}} + \frac{2}{a^2d\sqrt{a+a\sec(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c
\end{aligned}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{2}{3ad(a+a\sec(c+dx))^{3/2}} + \frac{2}{a^2d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [C] time = 0.0507304, size = 40, normalized size = 0.51

$$\frac{2\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\sec(c+dx)+1\right)}{3ad(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

[Out] (2*Hypergeometric2F1[-3/2, 1, -1/2, 1 + Sec[c + d*x]])/(3*a*d*(a*(1 + Sec[c + d*x]))^(3/2))

Maple [A] time = 0.035, size = 62, normalized size = 0.8

$$\frac{1}{d}\left(-2\,\frac{1}{a^{5/2}}\operatorname{Artanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)+2\,\frac{1}{a^{2}\sqrt{a+a\sec(dx+c)}}+\frac{2}{3\,a}\,(a+a\sec(dx+c))^{-\frac{3}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)/(a+a*sec(d*x+c))^(5/2),x)

[Out] 1/d*(-2/a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))+2/a^2/(a+a*sec(d*x+c))^(1/2)+2/3/a/(a+a*sec(d*x+c))^(3/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.92842, size = 826, normalized size = 10.59

$$\left[\frac{3\left(\cos(dx+c)^{2}+2\,\cos(dx+c)+1\right)\sqrt{a}\log\left(-8\,a\cos(dx+c)^{2}+4\left(2\,\cos(dx+c)^{2}+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-\right.}{6\left(a^{3}d\cos(dx+c)^{2}+2\,a^{3}d\cos(dx+c)+a^{3}d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")

```
[Out] [1/6*(3*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-8*a*cos(d*x + c)
^2 + 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c)) - 8*a*cos(d*x + c) - a) + 4*(4*cos(d*x + c)^2 + 3*cos(d*x + c
))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^3*d*cos(d*x + c)^2 + 2*a^3*d
*cos(d*x + c) + a^3*d), 1/3*(3*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-
a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
2*a*cos(d*x + c) + a)) + 2*(4*cos(d*x + c)^2 + 3*cos(d*x + c))*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c)))/(a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) +
 a^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [B] time = 9.57922, size = 180, normalized size = 2.31

$$-\frac{\frac{12\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)}+\frac{\sqrt{2}\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a\right)^{\frac{3}{2}}a^{8}+6\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}a^{9}}{a^{12}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*(12*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(
sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + (sqrt(2)*(-a*tan(1/2*d*x +
1/2*c)^2 + a)^(3/2)*a^8 + 6*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^9
)/(a^12*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.198 $\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{7}{4a^2d\sqrt{a\sec(c+dx)+a}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d}-\frac{1}{2ad(a\sec(c+dx)+a)^{3/2}}-\frac{}{5d(a\sec(}$$

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) - ArcTanh[Sqrt[a
+ a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(4*Sqrt[2]*a^(5/2)*d) - 1/(5*d*(a + a*
Sec[c + d*x])^(5/2)) - 1/(2*a*d*(a + a*Sec[c + d*x])^(3/2)) - 7/(4*a^2*d*Sq
rt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.125346, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3880, 85, 152, 156, 63, 207}

$$-\frac{7}{4a^2d\sqrt{a\sec(c+dx)+a}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d}-\frac{1}{2ad(a\sec(c+dx)+a)^{3/2}}-\frac{}{5d(a\sec(}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) - ArcTanh[Sqrt[a
+ a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(4*Sqrt[2]*a^(5/2)*d) - 1/(5*d*(a + a*
Sec[c + d*x])^(5/2)) - 1/(2*a*d*(a + a*Sec[c + d*x])^(3/2)) - 7/(4*a^2*d*Sq
rt[a + a*Sec[c + d*x]])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
```

```
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{a^2\,\text{Subst}\left(\int \frac{1}{x(-a+ax)(a+ax)^{7/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{1}{5d(a+a\sec(c+dx))^{5/2}} + \frac{\text{Subst}\left(\int \frac{2a^2-a^2x}{x(-a+ax)(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{2ad}$$

$$= -\frac{1}{5d(a+a\sec(c+dx))^{5/2}} - \frac{1}{2ad(a+a\sec(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{-6a^4+\frac{9a^4x}{2}}{x(-a+ax)(a+ax)^{3/2}}\,dx, x, \text{se}\right.}{6a^4d}$$

$$= -\frac{1}{5d(a+a\sec(c+dx))^{5/2}} - \frac{1}{2ad(a+a\sec(c+dx))^{3/2}} - \frac{7}{4a^2d\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}}{}$$

$$= -\frac{1}{5d(a+a\sec(c+dx))^{5/2}} - \frac{1}{2ad(a+a\sec(c+dx))^{3/2}} - \frac{7}{4a^2d\sqrt{a+a\sec(c+dx)}} - \frac{\text{Subst}}{}$$

$$= -\frac{1}{5d(a+a\sec(c+dx))^{5/2}} - \frac{1}{2ad(a+a\sec(c+dx))^{3/2}} - \frac{7}{4a^2d\sqrt{a+a\sec(c+dx)}} - \frac{2\,\text{Sub}}{}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} - \frac{1}{5d(a+a\sec(c+dx))^{5/2}} - \frac{}{2ad(a+}$$

Mathematica [C] time = 0.0617843, size = 60, normalized size = 0.42

$$\frac{\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{1}{2}(\sec(c+dx)+1)\right) - 2\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\sec(c+dx)+1\right)}{5d(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Hypergeometric2F1[-5/2, 1, -3/2, (1 + Sec[c + d*x])/2] - 2*Hypergeometric2
F1[-5/2, 1, -3/2, 1 + Sec[c + d*x]])/(5*d*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] time = 0.224, size = 496, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/40/d/a^3*(-1+cos(d*x+c))^3*(40*2^(1/2)*cos(d*x+c)^3*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+5*
cos(d*x+c)^3*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2))+120*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+15*cos(d*
x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))+120*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+98*cos(d*x+c)^3+1
5*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2))+40*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arcta
n(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+160*cos(d*x+c)^2+5*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2))+70*cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)/(a*sec(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 9.0779, size = 315, normalized size = 2.19

$$-\frac{\frac{5\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{80\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{\sqrt{2}\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}a^{20}+5\sqrt{2}\left(-a\tan\left(\frac{1}{2}dx\right.\right.}{a^{25}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\right.}}{40d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/40*(5*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt
(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 80*arctan(1/2*sqrt(2)*sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)
^2 - 1)) - (sqrt(2)*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a)*a^20 + 5*sqrt(2)*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^21 +
 35*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^22)/(a^25*sgn(tan(1/2*d*x
 + 1/2*c)^2 - 1)))/d
```

3.199 $\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=200

$$\frac{51}{32a^2d\sqrt{a\sec(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{13\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}a^{5/2}d}-\frac{5a}{28d(a\sec(c+dx)+a)^{7/2}}+\frac{}{2d(1-\text{se}}$$

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) + (13*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(32*Sqrt[2]*a^(5/2)*d) - (5*a)/(
28*d*(a + a*Sec[c + d*x])^(7/2)) + a/(2*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
 d*x])^(7/2)) + 3/(40*d*(a + a*Sec[c + d*x])^(5/2)) + 19/(48*a*d*(a + a*Sec
[c + d*x])^(3/2)) + 51/(32*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.174887, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3880, 103, 152, 156, 63, 207}

$$\frac{51}{32a^2d\sqrt{a\sec(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{13\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}a^{5/2}d}-\frac{5a}{28d(a\sec(c+dx)+a)^{7/2}}+\frac{}{2d(1-\text{se}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) + (13*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(32*Sqrt[2]*a^(5/2)*d) - (5*a)/(
28*d*(a + a*Sec[c + d*x])^(7/2)) + a/(2*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
 d*x])^(7/2)) + 3/(40*d*(a + a*Sec[c + d*x])^(5/2)) + 19/(48*a*d*(a + a*Sec
[c + d*x])^(3/2)) + 51/(32*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{a^4\,\text{Subst}\left(\int \frac{1}{x(-a+ax)^2(a+ax)^{9/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} - \frac{a\,\text{Subst}\left(\int \frac{2a^2+\frac{9a^2x}{2}}{x(-a+ax)(a+ax)^{9/2}}\,dx, x, \sec(c+dx)\right)}{2d}$$

$$= -\frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{\text{Subst}\left(\int \frac{-14a^4\ldots}{x(-a+ax)\ldots}\right.}{}$$

$$= -\frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{3}{40d(a+a\sec(c+\ldots}$$

$$= -\frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{3}{40d(a+a\sec(c+\ldots}$$

$$= -\frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{3}{40d(a+a\sec(c+\ldots}$$

$$= -\frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{3}{40d(a+a\sec(c+\ldots}$$

$$= -\frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{3}{40d(a+a\sec(c+\ldots}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{13\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}a^{5/2}d} - \frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{\ldots}{2d(1\ldots}$$

Mathematica [C] time = 0.18076, size = 90, normalized size = 0.45

$$\frac{a\left(-13(\sec(c+dx)-1)\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},\frac{1}{2}(\sec(c+dx)+1)\right)+8(\sec(c+dx)-1)\text{Hypergeometric2F1}\ldots\right.}{28d(\sec(c+dx)-1)(a(\sec(c+dx)+1))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a*(-14 - 13*Hypergeometric2F1[-7/2, 1, -5/2, (1 + Sec[c + d*x])/2]*(-1 + S
ec[c + d*x]) + 8*Hypergeometric2F1[-7/2, 1, -5/2, 1 + Sec[c + d*x]]*(-1 + S
ec[c + d*x])))/(28*d*(-1 + Sec[c + d*x])*(a*(1 + Sec[c + d*x]))^(7/2))
```

Maple [B] time = 0.318, size = 744, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/6720/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)*(-1+cos(d*
x+c))^4*(6720*cos(d*x+c)^5*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arc
tan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+1365*cos(d*x+c)^5*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))+20160*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arcta
n(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+4095*cos(d*x+c)^4*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2))+13440*2^(1/2)*cos(d*x+c)^3*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(
1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+16034*cos(d*x+c)^5+2730*c
os(d*x+c)^3*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2))-13440*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+25280*co
s(d*x+c)^4-2730*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-20160*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2))-3164*cos(d*x+c)^3-4095*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-6720*2^(1/2)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2))-24080*cos(d*x+c)^2-1365*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(
1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-10710*cos(d*x+c))/sin(d*x+c)^10
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

[Out] Timed out

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**3/(a+a*sec(d*x+c))**(5/2),x)

[Out] Timed out

Giac [B] time = 10.1288, size = 448, normalized size = 2.24

$$\frac{\frac{1365\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{13440\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{105\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}+\frac{2\left(15\sqrt{2}\left(a\tan\left(\frac{1}{2}dx+\right.\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] 1/6720*(1365*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(
sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 13440*arctan(1/2*sqrt(2)*sq

```
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x +
 1/2*c)^2 - 1)) + 105*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/(a^3*sgn(
tan(1/2*d*x + 1/2*c)^2 - 1)*tan(1/2*d*x + 1/2*c)^2) + 2*(15*sqrt(2)*(a*tan(
1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^36 - 84*sqr
t(2)*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a
^37 - 385*sqrt(2)*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^38 - 2730*sqrt(2)
*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^39)/(a^42*sgn(tan(1/2*d*x + 1/2*c)^2
 - 1)))/d
```

3.200 $\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=262

$$\frac{199a^2}{288d(a\sec(c+dx)+a)^{9/2}}-\frac{21a^2}{16d(1-\sec(c+dx))(a\sec(c+dx)+a)^{9/2}}-\frac{a^2}{4d(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{9/2}}-\frac{}{512\ldots}$$

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) - (263*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(512*Sqrt[2]*a^(5/2)*d) + (199*a
^2)/(288*d*(a + a*Sec[c + d*x])^(9/2)) - a^2/(4*d*(1 - Sec[c + d*x])^2*(a +
 a*Sec[c + d*x])^(9/2)) - (21*a^2)/(16*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
d*x])^(9/2)) + (135*a)/(448*d*(a + a*Sec[c + d*x])^(7/2)) + 7/(640*d*(a + a
*Sec[c + d*x])^(5/2)) - 83/(256*a*d*(a + a*Sec[c + d*x])^(3/2)) - 761/(512*
a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.226566, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3880, 103, 151, 152, 156, 63, 207}

$$\frac{199a^2}{288d(a\sec(c+dx)+a)^{9/2}}-\frac{21a^2}{16d(1-\sec(c+dx))(a\sec(c+dx)+a)^{9/2}}-\frac{a^2}{4d(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{9/2}}-\frac{}{512\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) - (263*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(512*Sqrt[2]*a^(5/2)*d) + (199*a
^2)/(288*d*(a + a*Sec[c + d*x])^(9/2)) - a^2/(4*d*(1 - Sec[c + d*x])^2*(a +
 a*Sec[c + d*x])^(9/2)) - (21*a^2)/(16*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
d*x])^(9/2)) + (135*a)/(448*d*(a + a*Sec[c + d*x])^(7/2)) + 7/(640*d*(a + a
*Sec[c + d*x])^(5/2)) - 83/(256*a*d*(a + a*Sec[c + d*x])^(3/2)) - 761/(512*
a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
```

```
d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{a^6\,\mathrm{Subst}\left(\int \frac{1}{x(-a+ax)^3(a+ax)^{11/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{a^3\,\mathrm{Subst}\left(\int \frac{4a^2+\frac{13a^2x}{2}}{x(-a+ax)^2(a+ax)^{11/2}}\,dx, x, \sec(c+\right.}{4d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{21a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{9/2}} -$$

$$= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{}{16d(1-\sec(c+}$$

$$= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{}{16d(1-\sec(c+}$$

$$= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{}{16d(1-\sec(c+}$$

$$= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{}{16d(1-\sec(c+}$$

$$= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{}{16d(1-\sec(c+}$$

$$= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{}{16d(1-\sec(c+}$$

$$= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{}{16d(1-\sec(c+}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} - \frac{263\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}a^{5/2}d} + \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{}{4a}$$

Mathematica [C] time = 0.290965, size = 99, normalized size = 0.38

$$\frac{\cot^4(c+dx)\left(263(\sec(c+dx)-1)^2\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},\frac{1}{2}(\sec(c+dx)+1)\right)-64(\sec(c+dx)-1)^2\text{Hyperge}\right.}{288d(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Cot[c + d*x]^4*(-450 + 263*Hypergeometric2F1[-9/2, 1, -7/2, (1 + Sec[c + d
*x])/2]*(-1 + Sec[c + d*x])^2 - 64*Hypergeometric2F1[-9/2, 1, -7/2, 1 + Sec
[c + d*x]]*(-1 + Sec[c + d*x])^2 + 378*Sec[c + d*x]))/(288*d*(a*(1 + Sec[c
+ d*x]))^(5/2))
```

Maple [B] time = 0.342, size = 986, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/322560/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos
(d*x+c))^5*(322560*cos(d*x+c)^7*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+82845*cos(d*x+c)
^7*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2))+967680*2^(1/2)*cos(d*x+c)^6*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+248535*cos(d*x+c
)^6*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2))+322560*cos(d*x+c)^5*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+764402*cos(d*x+
c)^7+82845*cos(d*x+c)^5*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2))-1612800*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))+1183040*cos(d*x+c)^6-414225*cos(d*x+c)^4*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-1612800*2^(1/2)*cos
(d*x+c)^3*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2))-807214*cos(d*x+c)^5-414225*cos(d*x+c)^3*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
))+322560*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(
1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-2224080*cos(d*x+c)^4+8284
```

```
5*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2))+967680*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-378378
*cos(d*x+c)^3+248535*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan
(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+322560*2^(1/2)*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+
1063440*cos(d*x+c)^2+82845*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+479430*cos(d*x+c))/sin(d*x+c)^14
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 9.52446, size = 541, normalized size = 2.06

$$-\frac{\frac{82845\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{645120\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{315\sqrt{2}\left(33\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}-31\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a}{a^4\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/322560*(82845*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a
))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 645120*arctan(1/2*sqrt(
2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*
d*x + 1/2*c)^2 - 1)) - 315*sqrt(2)*(33*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2
) - 31*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a)/(a^4*sgn(tan(1/2*d*x + 1/2*c)
^2 - 1)*tan(1/2*d*x + 1/2*c)^4) - 8*(35*sqrt(2)*(a*tan(1/2*d*x + 1/2*c)^2 -
 a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^56 - 225*sqrt(2)*(a*tan(1/2*d*x
 + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^57 + 1008*sqrt(2)*
(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^58 +
 4410*sqrt(2)*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^59 + 31185*sqrt(2)*sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^60)/(a^63*sgn(tan(1/2*d*x + 1/2*c)^2 -
1)))/d
```

3.201 $\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{2\tan(c+dx)}{a^2d\sqrt{a\sec(c+dx)+a}}+\frac{2\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}-\frac{2\tan^3(c+dx)}{3ad(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
(2*Tan[c + d*x])/(a^2*d*Sqrt[a + a*Sec[c + d*x]]) - (2*Tan[c + d*x]^3)/(3*a
*d*(a + a*Sec[c + d*x])^(3/2)) + (2*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*x
])^(5/2))
```

Rubi [A] time = 0.0865185, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3887, 302, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{2\tan(c+dx)}{a^2d\sqrt{a\sec(c+dx)+a}}+\frac{2\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}-\frac{2\tan^3(c+dx)}{3ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
(2*Tan[c + d*x])/(a^2*d*Sqrt[a + a*Sec[c + d*x]]) - (2*Tan[c + d*x]^3)/(3*a
*d*(a + a*Sec[c + d*x])^(3/2)) + (2*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*x
])^(5/2))
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(2a)\,\text{Subst}\left(\int \frac{x^6}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a)\,\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}-\frac{1}{a^3(1+ax^2)}\right)dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2\tan(c+dx)}{a^2d\sqrt{a+a\sec(c+dx)}} - \frac{2\tan^3(c+dx)}{3ad(a+a\sec(c+dx))^{3/2}} + \frac{2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} + \frac{2\,\text{Subst}\left(\ldots\right.}{}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{2\tan(c+dx)}{a^2d\sqrt{a+a\sec(c+dx)}} - \frac{2\tan^3(c+dx)}{3ad(a+a\sec(c+dx))^{3/2}} + \frac{2\tan\ldots}{5d(a+a\ldots}$$

Mathematica [B] time = 6.06771, size = 447, normalized size = 3.52

$$\frac{\sqrt{2}\sqrt{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\left(\frac{2\tan^2\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}-1\right)^3\tan^7(c+dx)\cot^8\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{\sec(c+dx)+1}\right)^{9/2}\left(\frac{8\tan^6\left(\frac{1}{2}(c+dx)\right)}{5\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^3\left(\frac{2\tan^2\left(\frac{1}{2}(c\ldots\right.}{\tan^2\left(\frac{1}{2}(c+\ldots\right.}\right.}\right.\ldots}{d\left(1-\frac{2\tan^2\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^{5/2}(\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[2]*Cot[(c + d*x)/2]^8*((1 + Sec[c + d*x])^(-1))^(9/2)*Sqrt[1 + Tan[(c
 + d*x)/2]^2]*(-1 + (2*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2))^3*((Sq
rt[2]*ArcSin[(Sqrt[2]*Tan[(c + d*x)/2])/Sqrt[1 + Tan[(c + d*x)/2]^2]]*Tan[(
```

```
c + d*x)/2])/(Sqrt[1 + Tan[(c + d*x)/2]^2]*Sqrt[1 - (2*Tan[(c + d*x)/2]^2)/
(1 + Tan[(c + d*x)/2]^2)]) + (8*Tan[(c + d*x)/2]^6)/(5*(1 + Tan[(c + d*x)/2
]^2)^3*(-1 + (2*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2))^3) + (4*Tan[(
c + d*x)/2]^4)/(3*(1 + Tan[(c + d*x)/2]^2)^2*(-1 + (2*Tan[(c + d*x)/2]^2)/(
1 + Tan[(c + d*x)/2]^2))^2) + (2*Tan[(c + d*x)/2]^2)/((1 + Tan[(c + d*x)/2]
^2)*(-1 + (2*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2))))*Tan[c + d*x]^7
)/(d*(a*(1 + Sec[c + d*x]))^(5/2)*(1 - (2*Tan[(c + d*x)/2]^2)/(1 + Tan[(c +
 d*x)/2]^2))^(5/2))
```

Maple [B] time = 0.214, size = 302, normalized size = 2.4

$$\frac{1}{60\,da^{3}\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^{2}}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}\left(15\,\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin\left(dx+c\right)}{\cos\left(dx+c\right)}\sqrt{-2\,\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\right)\left(-2\,\frac{c}{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/60/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(15*arctanh(1/2*2^(1/2)*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(5/2)*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2+30*arctanh(1/2*2^(1/2)*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(5/2)*2^(1/2)*sin(d*x+c)*cos(d*x+c)+15*2^(1/2)*arctanh(1/2*2^
(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)-184*cos(d*x+c)^3+272*cos(d*x+c)^2-11
2*cos(d*x+c)+24)/sin(d*x+c)/cos(d*x+c)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.03001, size = 849, normalized size = 6.69

$$\left[-\frac{15\left(\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-2\left(23\,\cos\left(dx+\right.\right.}{15\left(a^3d\cos\left(dx+c\right)^3+a^3d\cos\left(dx+c\right)^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/15*(15*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(-a)*log((2*a*cos(d*x + c)
^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*
x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(23*cos(d*x + c)^2 - 1
1*cos(d*x + c) + 3)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(
a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2), 2/15*(15*(cos(d*x + c)^3 + co
s(d*x + c)^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c)/(sqrt(a)*sin(d*x + c))) + (23*cos(d*x + c)^2 - 11*cos(d*x + c) + 3)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3
 + a^3*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^6\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**6/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [B] time = 16.5845, size = 394, normalized size = 3.1

$$-\frac{15\sqrt{-a}\left(\frac{\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+a\left(2\sqrt{2}-3\right)\right|\right)}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)+}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/15*(15*sqrt(-a)*(log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(a^3*sgn(tan(1/2*d*x + 1/2*c
)^2 - 1)) - log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
)) + 2*((37*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
- 40*sqrt(2)/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2 + 15*s
qrt(2)/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*
x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

3.202 $\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=113

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{4\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{2\tan(c+dx)}{a^2d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) - (
4*Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]
)/(a^(5/2)*d) + (2*Tan[c + d*x])/(a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.106845, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3887, 479, 522, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{4\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{2\tan(c+dx)}{a^2d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) - (
4*Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]
)/(a^(5/2)*d) + (2*Tan[c + d*x])/(a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
```

```
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{x^4}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\
&= \frac{2\tan(c+dx)}{a^2d\sqrt{a+a\sec(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{2+3ax^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2d}\\
&= \frac{2\tan(c+dx)}{a^2d\sqrt{a+a\sec(c+dx)}} - \frac{2\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2d} + \frac{8\,\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x\right.}{a^2d}\\
&= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{4\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{2\tan(c+dx)}{a^2d\sqrt{a+a\sec(c+dx)}}
\end{aligned}
$$

Mathematica [C] time = 23.4441, size = 5501, normalized size = 48.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

[Out] Result too large to show

Maple [B] time = 0.207, size = 327, normalized size = 2.9

$$\frac{1}{da^3\left(\cos\left(dx+c\right)+1\right)}\left(-\cos\left(dx+c\right)\sqrt{2}\sqrt{-2\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\mathrm{Artanh}\left(\frac{\sqrt{2}\sin\left(dx+c\right)}{2\cos\left(dx+c\right)}\sqrt{-2\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\right)-4\cos\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/d/a^3*(-cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1
/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))-4*co
s(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)/cos(d*x+c))-4*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+2*sin
(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/(cos(d*x+c)+1)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.90603, size = 1102, normalized size = 9.75

$$\left[\frac{2\sqrt{2}(a\cos(dx+c)+a)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-\sqrt{-a}(\cos(dx+c)+1}{a^3d\cos(dx+c)+a^3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [(2*sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) + 3*cos(d*x +
c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - sqrt(-a
)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos
(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(
a^3*d*cos(d*x + c) + a^3*d), -2*(sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*sq
rt(2)*(a*cos(d*x + c) + a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a) - sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^4(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**4/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [B] time = 13.0947, size = 398, normalized size = 3.52

$$\frac{\frac{2\sqrt{2}\sqrt{-a}\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{\log(|(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan(}}{\sqrt{-a}a^2\mathrm{sgn}(\tan(\frac{1}{2}a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] (2*sqrt(2)*sqrt(-a)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^2)/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 2*sqrt(2)*sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2
*c)^2 - a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - log(abs((sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))
)/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + log(abs((sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)
))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.203 $\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=127

$$\frac{\sin(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}{2a^2d\sqrt{a\sec(c+dx)+a}}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{2}a^{5/2}d}$$

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
(3*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt
[2]*a^(5/2)*d) + (Sec[(c + d*x)/2]^2*Sin[c + d*x])/(2*a^2*d*Sqrt[a + a*Sec[
c + d*x]])
```

Rubi [A] time = 0.106109, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3887, 471, 522, 203}

$$\frac{\sin(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}{2a^2d\sqrt{a\sec(c+dx)+a}}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{2}a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```



```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
(3*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt
[2]*a^(5/2)*d) + (Sec[(c + d*x)/2]^2*Sin[c + d*x])/(2*a^2*d*Sqrt[a + a*Sec[
c + d*x]])
```


Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad}\\
&= \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{2a^2d\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1-ax^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2d}\\
&= \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{2a^2d\sqrt{a+a\sec(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2d} - \frac{3\,\text{Subst}\left(\int \frac{1}{2+ax^2}\right.}{}\\
&= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{2}a^{5/2}d} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{2a^2d\sqrt{a+a\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 23.4534, size = 5531, normalized size = 43.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.14, size = 370, normalized size = 2.9

$$\frac{1}{2\,da^{3}\left(\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}\left(2\sqrt{2}\cos\left(dx+c\right)\sin\left(dx+c\right)\sqrt{-2\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\mathrm{Artanh}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/2/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(2*2^(1/2)*cos(d*x+c)*sin(d*x
+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+2*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+3*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+c
os(d*x+c)-1)/sin(d*x+c))+3*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)-2*cos(d*x+c)^2+2*cos(d*x+c))/(cos(d*x+c)+1)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(dx+c\right)^{2}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^2/(a*sec(d*x + c) + a)^(5/2), x)
```

Fricas [A] time = 3.28473, size = 1319, normalized size = 10.39

$$\left[-\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\sqrt{-a}\log\left(\frac{2\,\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3\,a\cos(dx+c)^{2}+2\,a\cos(dx+c)-a}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right)+4\left(\mathrm{c}\cdots\right.}{4\left(a^{3}d\cos\left(\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*sqrt
(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
 c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*
x + c) + 1)) + 4*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*a*co
s(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 4*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c))/(a^3*d*cos(d*x + c)^
2 + 2*a^3*d*cos(d*x + c) + a^3*d), -1/2*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 4*(cos(d*x + c)^2 + 2*cos(d*x + c)
 + 1)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
sqrt(a)*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)*sin(d*x + c))/(a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^{2}\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [B] time = 12.5877, size = 375, normalized size = 2.95

$$\frac{\frac{3\sqrt{2}\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{4\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx\cdots\right.\right.\right.}{\sqrt{-a}a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\cdots\right.\right.}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*sqrt(2)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^2)/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 2*sqrt(2)
*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^3*sgn(tan(1/2*
d*x + 1/2*c)^2 - 1)) + 4*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*a^2*sgn(tan(1
/2*d*x + 1/2*c)^2 - 1)) - 4*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*a^2*sgn(ta
n(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.204 $\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=265

$$\frac{63\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{128a^3d}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{319\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{128\sqrt{2}a^{5/2}d}-\frac{\cos^3(c+dx)\cot(c+dx)\sec}{4}$$

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
(319*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(12
8*Sqrt[2]*a^(5/2)*d) + (63*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(128*a^3*
d) - (191*Cos[c + d*x]*Cot[c + d*x]*Sec[(c + d*x)/2]^2*Sqrt[a + a*Sec[c + d
*x]])/(384*a^3*d) - (19*Cos[c + d*x]^2*Cot[c + d*x]*Sec[(c + d*x)/2]^4*Sqrt
[a + a*Sec[c + d*x]])/(192*a^3*d) - (Cos[c + d*x]^3*Cot[c + d*x]*Sec[(c + d
*x)/2]^6*Sqrt[a + a*Sec[c + d*x]])/(48*a^3*d)
```

Rubi [A] time = 0.23319, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3887, 472, 579, 583, 522, 203}

$$\frac{63\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{128a^3d}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{319\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{128\sqrt{2}a^{5/2}d}-\frac{\cos^3(c+dx)\cot(c+dx)\sec}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
(319*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(12
8*Sqrt[2]*a^(5/2)*d) + (63*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(128*a^3*
d) - (191*Cos[c + d*x]*Cot[c + d*x]*Sec[(c + d*x)/2]^2*Sqrt[a + a*Sec[c + d
*x]])/(384*a^3*d) - (19*Cos[c + d*x]^2*Cot[c + d*x]*Sec[(c + d*x)/2]^4*Sqrt
[a + a*Sec[c + d*x]])/(192*a^3*d) - (Cos[c + d*x]^3*Cot[c + d*x]*Sec[(c + d
*x)/2]^6*Sqrt[a + a*Sec[c + d*x]])/(48*a^3*d)
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
```

```
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)^4}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^3d}$$

$$= -\frac{\cos^3(c+dx)\cot(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{48a^3d} - \frac{\text{Subst}\left(\int \frac{5a-7a^2x^2}{x^2(1+ax^2)(2+ax^2)^3}\right.}{6a^4d}$$

$$= -\frac{19\cos^2(c+dx)\cot(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{192a^3d} - \frac{\cos^3(c+dx)\cot(c+d}{}$$

$$= -\frac{191\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{384a^3d} - \frac{19\cos^2(c+dx)\cot(c+}{}$$

$$= \frac{63\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{128a^3d} - \frac{191\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\,\text{se}}}{384a^3d}$$

$$= \frac{63\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{128a^3d} - \frac{191\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\,\text{se}}}{384a^3d}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{319\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{128\sqrt{2}a^{5/2}d} + \frac{63\cot(c+dx)\sqrt{a+a\sec(c+d}}{128a^3d}$$

Mathematica [C] time = 23.627, size = 5614, normalized size = 21.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.259, size = 714, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/768/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^3*(768*2^(
1/2)*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1
/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+2304
*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arcta
nh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+
957*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+2304*
2^(1/2)*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(
1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+287
1*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+768*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)-818*cos(d*x+c)^4+
2871*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-698*co
s(d*x+c)^3+957*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+370*cos
(d*x+c)^2+378*cos(d*x+c))/sin(d*x+c)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^2/(a*sec(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 9.28286, size = 277, normalized size = 1.05

$$-\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(2\left(\frac{4\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}-\frac{31\sqrt{2}}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{291\sqrt{2}}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/768*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*(4*sqrt(2)*tan(1/2*d*x + 1/2
*c)^2/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 31*sqrt(2)/(a^3*sgn(tan(1/2*d
*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)^2 + 291*sqrt(2)/(a^3*sgn(tan(1/2*
d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c) - 96*sqrt(2)/(((sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)*sqrt(-a)*a*sgn(
tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.205 $\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=355

$$\frac{5587\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{6144a^4d}-\frac{1491\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4096a^3d}+\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{9683\tan^{-1}\left(\right.}{4096}$$

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) - (
9683*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(40
96*Sqrt[2]*a^(5/2)*d) - (1491*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(4096*
a^3*d) + (5587*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(6144*a^4*d) - (1
527*Cos[c + d*x]*Cot[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(3/
2))/(2048*a^4*d) - (145*Cos[c + d*x]^2*Cot[c + d*x]^3*Sec[(c + d*x)/2]^4*(a
 + a*Sec[c + d*x])^(3/2))/(1024*a^4*d) - (9*Cos[c + d*x]^3*Cot[c + d*x]^3*S
ec[(c + d*x)/2]^6*(a + a*Sec[c + d*x])^(3/2))/(256*a^4*d) - (Cos[c + d*x]^4
*Cot[c + d*x]^3*Sec[(c + d*x)/2]^8*(a + a*Sec[c + d*x])^(3/2))/(128*a^4*d)
```

Rubi [A] time = 0.329425, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3887, 472, 579, 583, 522, 203}

$$\frac{5587\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{6144a^4d}-\frac{1491\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4096a^3d}+\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{9683\tan^{-1}\left(\right.}{4096}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```


```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) - (
9683*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(40
96*Sqrt[2]*a^(5/2)*d) - (1491*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(4096*
a^3*d) + (5587*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(6144*a^4*d) - (1
527*Cos[c + d*x]*Cot[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(3/
2))/(2048*a^4*d) - (145*Cos[c + d*x]^2*Cot[c + d*x]^3*Sec[(c + d*x)/2]^4*(a
 + a*Sec[c + d*x])^(3/2))/(1024*a^4*d) - (9*Cos[c + d*x]^3*Cot[c + d*x]^3*S
ec[(c + d*x)/2]^6*(a + a*Sec[c + d*x])^(3/2))/(256*a^4*d) - (Cos[c + d*x]^4
*Cot[c + d*x]^3*Sec[(c + d*x)/2]^8*(a + a*Sec[c + d*x])^(3/2))/(128*a^4*d)
```


Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)^5}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^4 d}$$

$$= -\frac{\cos^4(c+dx)\cot^3(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{128a^4 d} - \frac{\text{Subst}\left(\int \frac{5a-11a^2x}{x^4(1+ax^2)(2+}\right.}{}$$

$$= -\frac{9\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{256a^4 d} - \frac{\cos^4(c+dx)\cot^3(c}{}$$

$$= -\frac{145\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{1024a^4 d} - \frac{9\cos^3(c+dx)\,\text{co}}{}$$

$$= -\frac{1527\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{2048a^4 d} - \frac{145\cos^2(c+dx)}{}$$

$$= \frac{5587\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{6144a^4 d} - \frac{1527\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}{2048a^4 d}$$

$$= -\frac{1491\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4096a^3 d} + \frac{5587\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{6144a^4 d} - \frac{1527\,\text{c}}{}$$

$$= -\frac{1491\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4096a^3 d} + \frac{5587\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{6144a^4 d} - \frac{1527\,\text{c}}{}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{9683\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{4096\sqrt{2}a^{5/2}d} - \frac{1491\cot(c+dx)\sqrt{a+a\sec(c+}}{4096a^3 d}$$

Mathematica [C] time = 23.6848, size = 5656, normalized size = 15.93

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.356, size = 1066, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/24576/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)*(-1+cos(d
*x+c))^4*(-24576*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^5*sin(d*x+
c)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)/cos(d*x+c))-29049*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^5*sin
(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)
/sin(d*x+c))-73728*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*cos(d*x+c)^
4*sin(d*x+c)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)/cos(d*x+c))-87147*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^4*s
in(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-
1)/sin(d*x+c))-49152*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)/cos(d*x+c))+29258*cos(d*x+c)^6-58098*cos(d*x+c)^3*sin(d*x+c)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+49152*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+28466*cos(d*x+c)^5+58098*cos(d*x+c
)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+73728*2^(1/2)*cos(
d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))-28116*cos(d*x+c
)^4+87147*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+2
4576*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)-34852*cos
(d*x+c)^3+29049*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+4490*c
os(d*x+c)^2+8946*cos(d*x+c))/sin(d*x+c)^11
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 10.1262, size = 447, normalized size = 1.26

$$-\frac{3\left(2\left(4\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{19\sqrt{2}}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{369\sqrt{2}}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/24576*(3*(2*(4*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a^3*sgn(tan(1/2*d*x +
1/2*c)^2 - 1)) - 19*sqrt(2)/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*
d*x + 1/2*c)^2 + 369*sqrt(2)/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2
*d*x + 1/2*c)^2 - 2989*sqrt(2)/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c) + 512*sqrt(2)*(12*(sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 21*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + 11
*a^2)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^2 - a)^3*sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.206 $\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=439

$$\frac{58077\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{40960a^5d}-\frac{41693\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{49152a^4d}+\frac{8925\cot(c+dx)\sqrt{a\sec(c+dx)}}{32768a^3d}$$

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
(74461*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(
32768*Sqrt[2]*a^(5/2)*d) + (8925*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(32
768*a^3*d) - (41693*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(49152*a^4*d
) + (58077*Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2))/(40960*a^5*d) - (9467
*Cos[c + d*x]*Cot[c + d*x]^5*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(5/2))
/(8192*a^5*d) - (2473*Cos[c + d*x]^2*Cot[c + d*x]^5*Sec[(c + d*x)/2]^4*(a +
 a*Sec[c + d*x])^(5/2))/(12288*a^5*d) - (155*Cos[c + d*x]^3*Cot[c + d*x]^5*
Sec[(c + d*x)/2]^6*(a + a*Sec[c + d*x])^(5/2))/(3072*a^5*d) - (7*Cos[c + d*
x]^4*Cot[c + d*x]^5*Sec[(c + d*x)/2]^8*(a + a*Sec[c + d*x])^(5/2))/(512*a^5
*d) - (Cos[c + d*x]^5*Cot[c + d*x]^5*Sec[(c + d*x)/2]^10*(a + a*Sec[c + d*x
])^(5/2))/(320*a^5*d)
```

Rubi [A] time = 0.420103, antiderivative size = 439, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3887, 472, 579, 583, 522, 203}

$$\frac{58077\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{40960a^5d}-\frac{41693\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{49152a^4d}+\frac{8925\cot(c+dx)\sqrt{a\sec(c+dx)}}{32768a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
(74461*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(
32768*Sqrt[2]*a^(5/2)*d) + (8925*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(32
768*a^3*d) - (41693*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(49152*a^4*d
) + (58077*Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2))/(40960*a^5*d) - (9467
*Cos[c + d*x]*Cot[c + d*x]^5*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(5/2))
/(8192*a^5*d) - (2473*Cos[c + d*x]^2*Cot[c + d*x]^5*Sec[(c + d*x)/2]^4*(a +
 a*Sec[c + d*x])^(5/2))/(12288*a^5*d) - (155*Cos[c + d*x]^3*Cot[c + d*x]^5*
```

```
Sec[(c + d*x)/2]^6*(a + a*Sec[c + d*x])^(5/2))/(3072*a^5*d) - (7*Cos[c + d*
x]^4*Cot[c + d*x]^5*Sec[(c + d*x)/2]^8*(a + a*Sec[c + d*x])^(5/2))/(512*a^5
*d) - (Cos[c + d*x]^5*Cot[c + d*x]^5*Sec[(c + d*x)/2]^10*(a + a*Sec[c + d*x
])^(5/2))/(320*a^5*d)
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^6\left(1+ax^2\right)\left(2+ax^2\right)^6}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^5 d}$$

$$= -\frac{\cos^5(c+dx)\cot^5(c+dx)\sec^{10}\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{320a^5 d} - \frac{\text{Subst}\left(\int \frac{5a-15a^2x^{2}\ldots}{x^6\left(1+ax^2\right)\left(2+\ldots\right.}\ldots\right)}{1\ldots}$$

$$= -\frac{7\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{512a^5 d} - \frac{\cos^5(c+dx)\cot^5(c\ldots}{\ldots}$$

$$= -\frac{155\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{3072a^5 d} - \frac{7\cos^4(c+dx)\cot\ldots}{\ldots}$$

$$= -\frac{2473\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{12288a^5 d} - \frac{155\cos^3(c+dx)\ldots}{\ldots}$$

$$= -\frac{9467\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{8192a^5 d} - \frac{2473\cos^2(c+dx)\ldots}{\ldots}$$

$$= \frac{58077\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{40960a^5 d} - \frac{9467\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\ldots}{8192a^5 d}$$

$$= -\frac{41693\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{49152a^4 d} + \frac{58077\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{40960a^5 d} - \frac{94\ldots}{\ldots}$$

$$= \frac{8925\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{32768a^3 d} - \frac{41693\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{49152a^4 d} + \frac{58077\,\text{co}\ldots}{\ldots}$$

$$= \frac{8925\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{32768a^3 d} - \frac{41693\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{49152a^4 d} + \frac{58077\,\text{co}\ldots}{\ldots}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{74461\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{32768\sqrt{2}a^{5/2}d} + \frac{8925\cot(c+dx)\sqrt{a+a\sec(c\ldots}}{32768a^3 d}$$

Mathematica [C] time = 23.7097, size = 5698, normalized size = 12.98

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.444, size = 1412, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/983040/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos
(d*x+c))^5*(-983040*cos(d*x+c)^7*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)/cos(d*x+c))-1116915*cos(d*x+c)^7*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+
c)-1)/sin(d*x+c))-2949120*cos(d*x+c)^6*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)/cos(d*x+c))-3350745*cos(d*x+c)^6*sin(d*x+c)*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+co
s(d*x+c)-1)/sin(d*x+c))-983040*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x
+c)^5*sin(d*x+c)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)/cos(d*x+c))+1278126*cos(d*x+c)^8-1116915*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*cos(d*x+c)^5*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+4915200*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*2^(1/2)*cos(d*x+c)^4*sin(d*x+c)*arctanh(1/2*2^(1/2)*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+1363110*cos(d*x+c)
^7+5584575*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*ln(
-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c)
)+4915200*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos
(d*x+c))-1972170*cos(d*x+c)^6+5584575*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)+cos(d*x+c)-1)/sin(d*x+c))-983040*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))-2720050*cos(d*x+c)^5-1116915*cos(d*x+c)^
2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-2949120*2^(1/2)*cos(
d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+810890*cos(d*x+
c)^4-3350745*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(
```

```
-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c)
)-983040*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+16738
42*cos(d*x+c)^3-1116915*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)+30610*cos(d*x+c)^2-267750*cos(d*x+c))/sin(d*x+c)^15
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 9.6942, size = 613, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/983040*((2*(4*(6*(8*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a^3*sgn(tan(1/2*d*x
+ 1/2*c)^2 - 1)) - 91*sqrt(2)/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/
2*d*x + 1/2*c)^2 + 3043*sqrt(2)/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(
1/2*d*x + 1/2*c)^2 - 47185*sqrt(2)/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*t
an(1/2*d*x + 1/2*c)^2 + 349965*sqrt(2)/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
))*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c) - 1024*sqrt(2)*
(345*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
8 - 1230*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
a))^6*a + 1760*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^4*a^2 - 1150*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a))^2*a^3 + 299*a^4)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^5*sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c
)^2 - 1)))/d
```

3.207 $\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{9/2}f}+\frac{91\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{32\sqrt{2}a^{9/2}f}+\frac{27\tan(e+fx)}{32a^3f(a\sec(e+fx)+a)^{3/2}}+\frac{11\tan(e+fx)}{24a^2f(a\sec(e+fx)+a)^{5/2}}+\frac{}{3a}$$

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(9/2)*f) +
(91*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(32*
Sqrt[2]*a^(9/2)*f) + Tan[e + f*x]/(3*a*f*(a + a*Sec[e + f*x])^(7/2)) + (11*
Tan[e + f*x])/(24*a^2*f*(a + a*Sec[e + f*x])^(5/2)) + (27*Tan[e + f*x])/(32
*a^3*f*(a + a*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.1795, antiderivative size = 227, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3887, 471, 527, 522, 203}

$$\frac{27\sin(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{64a^4f\sqrt{a\sec(e+fx)+a}}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{9/2}f}+\frac{91\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{32\sqrt{2}a^{9/2}f}+\frac{\sin(e+fx)\cos^2(e+fx)\sec^6(}{24a^4f\sqrt{a\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + a*Sec[e + f*x])^(9/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(9/2)*f) +
(91*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(32*
Sqrt[2]*a^(9/2)*f) + (27*Sec[(e + f*x)/2]^2*Sin[e + f*x])/(64*a^4*f*Sqrt[a
+ a*Sec[e + f*x]]) + (11*Cos[e + f*x]*Sec[(e + f*x)/2]^4*Sin[e + f*x])/(96*
a^4*f*Sqrt[a + a*Sec[e + f*x]]) + (Cos[e + f*x]^2*Sec[(e + f*x)/2]^6*Sin[e
+ f*x])/(24*a^4*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)^4}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^3 f}$$

$$= \frac{\cos^2(e+fx)\sec^6\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{24a^4 f\sqrt{a+a\sec(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1-5ax^2}{(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{3a^4 f}$$

$$= \frac{11\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{96a^4 f\sqrt{a+a\sec(e+fx)}} + \frac{\cos^2(e+fx)\sec^6\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{24a^4 f\sqrt{a+a\sec(e+fx)}} +$$

$$= \frac{27\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{64a^4 f\sqrt{a+a\sec(e+fx)}} + \frac{11\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{96a^4 f\sqrt{a+a\sec(e+fx)}} + \frac{\cos^2(e+}{24}$$

$$= \frac{27\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{64a^4 f\sqrt{a+a\sec(e+fx)}} + \frac{11\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{96a^4 f\sqrt{a+a\sec(e+fx)}} + \frac{\cos^2(e+}{24}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{9/2} f} + \frac{91\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{32\sqrt{2}a^{9/2} f} + \frac{27\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{64a^4 f\sqrt{a+a\sec(e+fx)}} -$$

Mathematica [C] time = 23.6379, size = 5594, normalized size = 31.6

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^2/(a + a*Sec[e + f*x])^(9/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.212, size = 724, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+a*sec(f*x+e))^(9/2),x)
```

```
[Out] -1/192/f/a^5*(192*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^4*sin(f*x
+e)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*
x+e)/cos(f*x+e))+273*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^4*sin(
f*x+e)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/
sin(f*x+e))+384*2^(1/2)*cos(f*x+e)^3*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+
e)/cos(f*x+e))+546*cos(f*x+e)^3*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(
1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/si
n(f*x+e))-384*2^(1/2)*cos(f*x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/c
os(f*x+e))-314*cos(f*x+e)^5-546*cos(f*x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f
*x+e)-1)/sin(f*x+e))-192*2^(1/2)*sin(f*x+e)*arctanh(1/2*2^(1/2)*(-2*cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+
e)))^(1/2)+216*cos(f*x+e)^4-273*sin(f*x+e)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+
e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)+348*cos(f*x+e)^3-88*cos(f*x+e)^2-162*cos(f*x+e))*(1/cos(f*x+e)*a*
(1+cos(f*x+e)))^(1/2)/sin(f*x+e)^3/(1+cos(f*x+e))^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sec(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 3.59174, size = 1802, normalized size = 10.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(273*sqrt(2)*(cos(f*x + e)^4 + 4*cos(f*x + e)^3 + 6*cos(f*x + e)^2
+ 4*cos(f*x + e) + 1)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*co
```

```
s(f*x + e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 384*(cos(f*x + e)^
4 + 4*cos(f*x + e)^3 + 6*cos(f*x + e)^2 + 4*cos(f*x + e) + 1)*sqrt(-a)*log(
(2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 4*(157*
cos(f*x + e)^3 + 206*cos(f*x + e)^2 + 81*cos(f*x + e))*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sin(f*x + e))/(a^5*f*cos(f*x + e)^4 + 4*a^5*f*cos(f*x +
 e)^3 + 6*a^5*f*cos(f*x + e)^2 + 4*a^5*f*cos(f*x + e) + a^5*f), -1/192*(273
*sqrt(2)*(cos(f*x + e)^4 + 4*cos(f*x + e)^3 + 6*cos(f*x + e)^2 + 4*cos(f*x
+ e) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)/(sqrt(a)*sin(f*x + e))) - 384*(cos(f*x + e)^4 + 4*cos(f*x + e)^3
 + 6*cos(f*x + e)^2 + 4*cos(f*x + e) + 1)*sqrt(a)*arctan(sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(157*cos(f*x
 + e)^3 + 206*cos(f*x + e)^2 + 81*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*sin(f*x + e))/(a^5*f*cos(f*x + e)^4 + 4*a^5*f*cos(f*x + e)^3 +
 6*a^5*f*cos(f*x + e)^2 + 4*a^5*f*cos(f*x + e) + a^5*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+a*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sec(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.208 $\int (a + a\sec(c+dx))^n (e\tan(c+dx))^m\,dx$

Optimal. Leaf size=125

$$\frac{2^{m+n+1}(a\sec(c+dx)+a)^n(e\tan(c+dx))^{m+1}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+n+1}F_1\left(\frac{m+1}{2};m+n,1;\frac{m+3}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)}$$

```
[Out] (2^(1 + m + n)*AppellF1[(1 + m)/2, m + n, 1, (3 + m)/2, -((a - a*Sec[c + d*
x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 +
 Sec[c + d*x])^(-1))^(1 + m + n)*(a + a*Sec[c + d*x])^n*(e*Tan[c + d*x])^(1
 + m))/(d*e*(1 + m))
```

Rubi [A] time = 0.0884521, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {3889}

$$\frac{2^{m+n+1}(a\sec(c+dx)+a)^n(e\tan(c+dx))^{m+1}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+n+1}F_1\left(\frac{m+1}{2};m+n,1;\frac{m+3}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*(e*Tan[c + d*x])^m,x]
```


```
[Out] (2^(1 + m + n)*AppellF1[(1 + m)/2, m + n, 1, (3 + m)/2, -((a - a*Sec[c + d*
x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 +
 Sec[c + d*x])^(-1))^(1 + m + n)*(a + a*Sec[c + d*x])^n*(e*Tan[c + d*x])^(1
 + m))/(d*e*(1 + m))
```


Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a + a\sec(c+dx))^n (e\tan(c+dx))^m\,dx = \frac{2^{1+m+n} F_1\left(\frac{1+m}{2}; m+n, 1; \frac{3+m}{2}; -\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}, \frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{1+m+n}}{de(1+m)}$$

Mathematica [F] time = 1.25867, size = 0, normalized size = 0.

$$\int (a + a\sec(c+dx))^n (e\tan(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*(e*Tan[c + d*x])^m,x]
```

```
[Out] Integrate[(a + a*Sec[c + d*x])^n*(e*Tan[c + d*x])^m, x]
```

Maple [F] time = 0.735, size = 0, normalized size = 0.

$$\int (a + a\sec(dx+c))^n (e\tan(dx+c))^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*(e*tan(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))^n*(e*tan(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c) + a)^n (e\tan(dx+c))^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sec(dx+c)+a)^n(e\tan(dx+c))^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*(e*tan(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n(e\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

3.209 $\int (a + a\sec(c + dx))^3 (e\tan(c + dx))^m \, dx$

Optimal. Leaf size=243

$$\frac{a^3(e\tan(c+dx))^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(c+dx)\right)}{de(m+1)}+\frac{a^3\sec^3(c+dx)\cos^2(c+dx)^{\frac{m+4}{2}}(e\tan(c+dx)$$

```
[Out] (3*a^3*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)) + (a^3*Hypergeometric2F1[1,
(1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 +
m)) + (3*a^3*(Cos[c + d*x]^2)^((2 + m)/2)*Hypergeometric2F1[(1 + m)/2, (2 +
 m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x])^(1 + m))/(d
*e*(1 + m)) + (a^3*(Cos[c + d*x]^2)^((4 + m)/2)*Hypergeometric2F1[(1 + m)/2
, (4 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]^3*(e*Tan[c + d*x])^(1
+ m))/(d*e*(1 + m))
```

Rubi [A] time = 0.22916, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3886, 3476, 364, 2617, 2607, 32}

$$\frac{a^3(e\tan(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(c+dx)\right)}{de(m+1)}+\frac{a^3\sec^3(c+dx)\cos^2(c+dx)^{\frac{m+4}{2}}(e\tan(c+dx))^{m+1}\,{}_2F_1\left(\frac{m+1}{2},\frac{m+}{2}\right.}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*(e*Tan[c + d*x])^m,x]
```

```
[Out] (3*a^3*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)) + (a^3*Hypergeometric2F1[1,
(1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 +
m)) + (3*a^3*(Cos[c + d*x]^2)^((2 + m)/2)*Hypergeometric2F1[(1 + m)/2, (2 +
 m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x])^(1 + m))/(d
*e*(1 + m)) + (a^3*(Cos[c + d*x]^2)^((4 + m)/2)*Hypergeometric2F1[(1 + m)/2
, (4 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]^3*(e*Tan[c + d*x])^(1
+ m))/(d*e*(1 + m))
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a + a\sec(c+dx))^3(e\tan(c+dx))^m\,dx = \int \left(a^3(e\tan(c+dx))^m + 3a^3\sec(c+dx)(e\tan(c+dx))^m + 3a^3\sec^2(c+dx)(\right.$$
$$= a^3\int (e\tan(c+dx))^m\,dx + a^3\int \sec^3(c+dx)(e\tan(c+dx))^m\,dx + \left(3a^3\right)\int \mathrm{s}$$
$$= \frac{3a^3\cos^2(c+dx)^{\frac{2+m}{2}}\,{}_2F_1\left(\frac{1+m}{2},\frac{2+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sec(c+dx)(e\tan(c+}{de(1+m)}$$
$$= \frac{3a^3(e\tan(c+dx))^{1+m}}{de(1+m)} + \frac{a^3\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx)\right)(e\tan(c+dx))}{de(1+m)}$$

Mathematica [F] time = 2.57961, size = 0, normalized size = 0.

$$\int (a + a\sec(c+dx))^3(e\tan(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*(e*Tan[c + d*x])^m,x]
```

```
[Out] Integrate[(a + a*Sec[c + d*x])^3*(e*Tan[c + d*x])^m, x]
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (a + a\sec(dx+c))^3\,(e\tan(dx+c))^m\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(e*tan(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))^3*(e*tan(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^3\,(e\tan(dx+c))^m\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*(e*tan(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^3 \sec\left(dx+c\right)^3 + 3\,a^3 \sec\left(dx+c\right)^2 + 3\,a^3 \sec\left(dx+c\right) + a^3\right)\left(e\tan\left(dx+c\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)*(e*tan(d*x + c))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int (e\tan(c+dx))^m\, dx + \int 3\,(e\tan(c+dx))^m \sec(c+dx)\, dx + \int 3\,(e\tan(c+dx))^m \sec^2(c+dx)\, dx + \int (e\tan(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(e*tan(d*x+c))**m,x)
```

```
[Out] a**3*(Integral((e*tan(c + d*x))**m, x) + Integral(3*(e*tan(c + d*x))**m*sec
(c + d*x), x) + Integral(3*(e*tan(c + d*x))**m*sec(c + d*x)**2, x) + Integr
al((e*tan(c + d*x))**m*sec(c + d*x)**3, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^3 (e\tan(dx+c))^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(e*tan(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*(e*tan(d*x + c))^m, x)
```

3.210 $\int (a + a\sec(c + dx))^2 (e\tan(c + dx))^m \, dx$

Optimal. Leaf size=161

$$\frac{a^2(e\tan(c+dx))^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(c+dx)\right)}{de(m+1)}+\frac{2a^2\sec(c+dx)\cos^2(c+dx)^{\frac{m+2}{2}}(e\tan(c+dx)$$

```
[Out] (a^2*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)) + (a^2*Hypergeometric2F1[1, (1
 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)
) + (2*a^2*(Cos[c + d*x]^2)^((2 + m)/2)*Hypergeometric2F1[(1 + m)/2, (2 + m
)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x])^(1 + m))/(d*e
*(1 + m))
```

Rubi [A] time = 0.169426, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3886, 3476, 364, 2617, 2607, 32}

$$\frac{a^2(e\tan(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(c+dx)\right)}{de(m+1)}+\frac{2a^2\sec(c+dx)\cos^2(c+dx)^{\frac{m+2}{2}}(e\tan(c+dx))^{m+1}\,{}_2F_1\left(\frac{m+1}{2},\frac{m+}{2}\right.}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^m,x]
```


```
[Out] (a^2*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)) + (a^2*Hypergeometric2F1[1, (1
 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)
) + (2*a^2*(Cos[c + d*x]^2)^((2 + m)/2)*Hypergeometric2F1[(1 + m)/2, (2 + m
)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x])^(1 + m))/(d*e
*(1 + m))
```


Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2(e\tan(c+dx))^m\,dx = \int \left(a^2(e\tan(c+dx))^m + 2a^2\sec(c+dx)(e\tan(c+dx))^m + a^2\sec^2(c+dx)(\right.$$

$$= a^2\int (e\tan(c+dx))^m\,dx + a^2\int \sec^2(c+dx)(e\tan(c+dx))^m\,dx + \left(2a^2\right)\int$$

$$= \frac{2a^2\cos^2(c+dx)^{\frac{2+m}{2}}\,{}_2F_1\left(\frac{1+m}{2},\frac{2+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sec(c+dx)(e\tan(c+}{de(1+m)}$$

$$= \frac{a^2(e\tan(c+dx))^{1+m}}{de(1+m)} + \frac{a^2\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx)\right)(e\tan(c+dx))}{de(1+m)}$$

Mathematica [F] time = 0.999945, size = 0, normalized size = 0.

$$\int (a + a\sec(c + dx))^2 (e\tan(c + dx))^m \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^m,x]
```

```
[Out] Integrate[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^m, x]
```

Maple [F] time = 0.532, size = 0, normalized size = 0.

$$\int (a + a\sec(dx + c))^2 (e\tan(dx + c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx + c) + a)^2 (e\tan(dx + c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*(e*tan(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^2 \sec(dx + c)^2 + 2\,a^2 \sec(dx + c) + a^2\right)(e\tan(dx + c))^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*(e*tan(d*x + c))^m
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int (e\tan(c+dx))^m\,dx+\int 2\,(e\tan(c+dx))^m\sec(c+dx)\,dx+\int (e\tan(c+dx))^m\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*tan(d*x+c))**m,x)
```

```
[Out] a**2*(Integral((e*tan(c + d*x))**m, x) + Integral(2*(e*tan(c + d*x))**m*sec
(c + d*x), x) + Integral((e*tan(c + d*x))**m*sec(c + d*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^2\,(e\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*(e*tan(d*x + c))^m, x)
```

3.211 $\int (a + a\sec(c + dx))(e\tan(c + dx))^m\, dx$

Optimal. Leaf size=129

$$\frac{a(e\tan(c+dx))^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(c+dx)\right)}{de(m+1)}+\frac{a\sec(c+dx)\cos^2(c+dx)^{\frac{m+2}{2}}(e\tan(c+dx))^{m}}{}$$

```
[Out] (a*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d
*x])^(1 + m))/(d*e*(1 + m)) + (a*(Cos[c + d*x]^2)^((2 + m)/2)*Hypergeometri
c2F1[(1 + m)/2, (2 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c
 + d*x])^(1 + m))/(d*e*(1 + m))
```

Rubi [A] time = 0.0824419, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3884, 3476, 364, 2617}

$$\frac{a(e\tan(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(c+dx)\right)}{de(m+1)}+\frac{a\sec(c+dx)\cos^2(c+dx)^{\frac{m+2}{2}}(e\tan(c+dx))^{m+1}\,{}_2F_1\left(\frac{m+1}{2},\frac{m+2}{2};\right.}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^m,x]
```

```
[Out] (a*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d
*x])^(1 + m))/(d*e*(1 + m)) + (a*(Cos[c + d*x]^2)^((2 + m)/2)*Hypergeometri
c2F1[(1 + m)/2, (2 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c
 + d*x])^(1 + m))/(d*e*(1 + m))
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (a + a\sec(c+dx))(e\tan(c+dx))^m\,dx = a\int (e\tan(c+dx))^m\,dx + a\int \sec(c+dx)(e\tan(c+dx))^m\,dx$$

$$= \frac{a\cos^2(c+dx)^{\frac{2+m}{2}}\,{}_2F_1\left(\frac{1+m}{2},\frac{2+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sec(c+dx)(e\tan(c+dx$$

$$de(1+m)$$

$$= \frac{a\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx)\right)(e\tan(c+dx))^{1+m}}{de(1+m)} + \frac{a\cos^2(c+dx)^{\frac{2+m}{2}}\,2}{}$$

Mathematica [A] time = 0.702312, size = 105, normalized size = 0.81

$$\frac{a(e\tan(c+dx))^m\left(\frac{\tan(c+dx)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(c+dx)\right)}{m+1} + \csc(c+dx)\left(-\tan^2(c+dx)\right)^{\frac{1-m}{2}}\text{Hypergeometric2}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^m,x]
```

```
[Out] (a*(e*Tan[c + d*x])^m*((Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c +
 d*x]^2]*Tan[c + d*x])/(1 + m) + Csc[c + d*x]*Hypergeometric2F1[1/2, (1 - m
)/2, 3/2, Sec[c + d*x]^2]*(-Tan[c + d*x]^2)^((1 - m)/2)))/d
```

Maple [F] time = 0.68, size = 0, normalized size = 0.

$$\int (a + a\sec(dx + c))(e\tan(dx + c))^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*tan(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))*(e*tan(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx + c) + a)(e\tan(dx + c))^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sec(dx + c) + a)(e\tan(dx + c))^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int (e\tan(c + dx))^m\ dx + \int (e\tan(c + dx))^m \sec(c + dx)\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))**m,x)
```

```
[Out] a*(Integral((e*tan(c + d*x))**m, x) + Integral((e*tan(c + d*x))**m*sec(c +
d*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec (dx+c)+a)\,(e\tan (dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

3.212 $\int \frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=130

$$\frac{e(e\tan(c+dx))^{m-1}\text{Hypergeometric2F1}\left(1,\frac{m-1}{2},\frac{m+1}{2},-\tan^2(c+dx)\right)}{ad(1-m)}-\frac{e\sec(c+dx)\cos^2(c+dx)^{m/2}(e\tan(c+dx))^{m-1}}{ad}$$

```
[Out] (e*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c +
d*x])^(-1 + m))/(a*d*(1 - m)) - (e*(Cos[c + d*x]^2)^(m/2)*Hypergeometric2F1
[(-1 + m)/2, m/2, (1 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x])^
(-1 + m))/(a*d*(1 - m))
```

Rubi [A] time = 0.159216, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3888, 3884, 3476, 364, 2617}

$$\frac{e(e\tan(c+dx))^{m-1}\,{}_2F_1\left(1,\frac{m-1}{2};\frac{m+1}{2};-\tan^2(c+dx)\right)}{ad(1-m)}-\frac{e\sec(c+dx)\cos^2(c+dx)^{m/2}(e\tan(c+dx))^{m-1}\,{}_2F_1\left(\frac{m-1}{2},\frac{m}{2};\frac{m+1}{2};\right.}{ad(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x]),x]
```

```
[Out] (e*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c +
d*x])^(-1 + m))/(a*d*(1 - m)) - (e*(Cos[c + d*x]^2)^(m/2)*Hypergeometric2F1
[(-1 + m)/2, m/2, (1 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x])^
(-1 + m))/(a*d*(1 - m))
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
```

```
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx &= \frac{e^2\int(-a+a\sec(c+dx))(e\tan(c+dx))^{-2+m}\,dx}{a^2}\\ &= -\frac{e^2\int(e\tan(c+dx))^{-2+m}\,dx}{a}+\frac{e^2\int\sec(c+dx)(e\tan(c+dx))^{-2+m}\,dx}{a}\\ &= -\frac{e\cos^2(c+dx)^{m/2}\,{}_2F_1\left(\frac{1}{2}(-1+m),\frac{m}{2};\frac{1+m}{2};\sin^2(c+dx)\right)\sec(c+dx)(e\tan(c+dx))^{-1+m}}{ad(1-m)} - \frac{e^3\ldots}{\ldots}\\ &= \frac{e\,{}_2F_1\left(1,\frac{1}{2}(-1+m);\frac{1+m}{2};-\tan^2(c+dx)\right)(e\tan(c+dx))^{-1+m}}{ad(1-m)} - \frac{e\cos^2(c+dx)^{m/2}\,{}_2F_1\left(\frac{1}{2}(-1+\ldots\right.}{\ldots}\end{aligned}$$

Mathematica [F] time = 0.472587, size = 0, normalized size = 0.

$$\int \frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x]),x]
```

```
[Out] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x]), x]
```

Maple [F] time = 0.684, size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^m}{a+a\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c)),x)
```

```
[Out] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^m}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e\tan(dx+c))^m}{a\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((e*tan(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{(e\tan(c+dx))^m}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**m/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral((e*tan(c + d*x))**m/(sec(c + d*x) + 1), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^m}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

3.213 $\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=169

$$-\frac{e^3(e\tan(c+dx))^{m-3}\text{Hypergeometric2F1}\left(1,\frac{m-3}{2},\frac{m-1}{2},-\tan^2(c+dx)\right)}{a^2d(3-m)}+\frac{2e^3\sec(c+dx)\cos^2(c+dx)^{\frac{m-2}{2}}(e\tan(c+d$$

```
[Out] -((e^3*(e*Tan[c + d*x])^(-3 + m))/(a^2*d*(3 - m))) - (e^3*Hypergeometric2F1
[1, (-3 + m)/2, (-1 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d*x])^(-3 + m))/(a^
2*d*(3 - m)) + (2*e^3*(Cos[c + d*x]^2)^((-2 + m)/2)*Hypergeometric2F1[(-3 +
 m)/2, (-2 + m)/2, (-1 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x]
)^(-3 + m))/(a^2*d*(3 - m))
```

Rubi [A] time = 0.272374, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3888, 3886, 3476, 364, 2617, 2607, 32}

$$-\frac{e^3(e\tan(c+dx))^{m-3}\,{}_2F_1\left(1,\frac{m-3}{2};\frac{m-1}{2};-\tan^2(c+dx)\right)}{a^2d(3-m)}+\frac{2e^3\sec(c+dx)\cos^2(c+dx)^{\frac{m-2}{2}}(e\tan(c+dx))^{m-3}\,{}_2F_1\left(\frac{m-3}{2},\frac{m-}{2}\right.}{a^2d(3-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] -((e^3*(e*Tan[c + d*x])^(-3 + m))/(a^2*d*(3 - m))) - (e^3*Hypergeometric2F1
[1, (-3 + m)/2, (-1 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d*x])^(-3 + m))/(a^
2*d*(3 - m)) + (2*e^3*(Cos[c + d*x]^2)^((-2 + m)/2)*Hypergeometric2F1[(-3 +
 m)/2, (-2 + m)/2, (-1 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x]
)^(-3 + m))/(a^2*d*(3 - m))
```


Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
```

```
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \frac{e^4\int(-a+a\sec(c+dx))^2(e\tan(c+dx))^{-4+m}\,dx}{a^4}$$
$$= \frac{e^4\int\left(a^2(e\tan(c+dx))^{-4+m}-2a^2\sec(c+dx)(e\tan(c+dx))^{-4+m}+a^2\sec^2(c+dx)(e\tan(c+\right.}{a^4}$$
$$= \frac{e^4\int(e\tan(c+dx))^{-4+m}\,dx}{a^2}+\frac{e^4\int\sec^2(c+dx)(e\tan(c+dx))^{-4+m}\,dx}{a^2}-\frac{(2e^4)\int\sec(c+dx)}{}$$
$$= \frac{2e^3\cos^2(c+dx)^{\frac{1}{2}(-2+m)}\,{}_2F_1\left(\frac{1}{2}(-3+m),\frac{1}{2}(-2+m);\frac{1}{2}(-1+m);\sin^2(c+dx)\right)\sec(c+dx)(e\,\mathrm{t}}{a^2d(3-m)}$$
$$= -\frac{e^3(e\tan(c+dx))^{-3+m}}{a^2d(3-m)}-\frac{e^3\,{}_2F_1\left(1,\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);-\tan^2(c+dx)\right)(e\tan(c+dx))^{-3}}{a^2d(3-m)}$$

Mathematica [F] time = 0.858509, size = 0, normalized size = 0.

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^2, x]
```

Maple [F] time = 0.359, size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^m}{(a+a\sec(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^2,x)
```

```
[Out] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e\tan(dx+c))^m}{a^2\sec(dx+c)^2+2\,a^2\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((e*tan(d*x + c))^m/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{(e\tan(c+dx))^m}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**m/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral((e*tan(c + d*x))**m/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**
2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^2, x)
```

3.214 $\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=252

$$\frac{e^5(e\tan(c+dx))^{m-5}\text{Hypergeometric2F1}\left(1,\frac{m-5}{2},\frac{m-3}{2},-\tan^2(c+dx)\right)}{a^3d(5-m)}-\frac{e^5\sec^3(c+dx)\cos^2(c+dx)^{\frac{m-2}{2}}(e\tan(c+dx$$

```
[Out] (3*e^5*(e*Tan[c + d*x])^(-5 + m))/(a^3*d*(5 - m)) + (e^5*Hypergeometric2F1[
1, (-5 + m)/2, (-3 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d*x])^(-5 + m))/(a^3
*d*(5 - m)) - (3*e^5*(Cos[c + d*x]^2)^((-4 + m)/2)*Hypergeometric2F1[(-5 +
m)/2, (-4 + m)/2, (-3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x])
^(-5 + m))/(a^3*d*(5 - m)) - (e^5*(Cos[c + d*x]^2)^((-2 + m)/2)*Hypergeomet
ric2F1[(-5 + m)/2, (-2 + m)/2, (-3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]^3*(
e*Tan[c + d*x])^(-5 + m))/(a^3*d*(5 - m))
```

Rubi [A] time = 0.341305, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3888, 3886, 3476, 364, 2617, 2607, 32}

$$\frac{e^5(e\tan(c+dx))^{m-5}\,{}_2F_1\left(1,\frac{m-5}{2};\frac{m-3}{2};-\tan^2(c+dx)\right)}{a^3d(5-m)}-\frac{e^5\sec^3(c+dx)\cos^2(c+dx)^{\frac{m-2}{2}}(e\tan(c+dx))^{m-5}\,{}_2F_1\left(\frac{m-5}{2},\frac{m-}{2}\right.}{a^3d(5-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3*e^5*(e*Tan[c + d*x])^(-5 + m))/(a^3*d*(5 - m)) + (e^5*Hypergeometric2F1[
1, (-5 + m)/2, (-3 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d*x])^(-5 + m))/(a^3
*d*(5 - m)) - (3*e^5*(Cos[c + d*x]^2)^((-4 + m)/2)*Hypergeometric2F1[(-5 +
m)/2, (-4 + m)/2, (-3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x])
^(-5 + m))/(a^3*d*(5 - m)) - (e^5*(Cos[c + d*x]^2)^((-2 + m)/2)*Hypergeomet
ric2F1[(-5 + m)/2, (-2 + m)/2, (-3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]^3*(
e*Tan[c + d*x])^(-5 + m))/(a^3*d*(5 - m))
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
```

```
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = \frac{e^6\int(-a+a\sec(c+dx))^3(e\tan(c+dx))^{-6+m}\,dx}{a^6}$$
$$= \frac{e^6\int\left(-a^3(e\tan(c+dx))^{-6+m}+3a^3\sec(c+dx)(e\tan(c+dx))^{-6+m}-3a^3\sec^2(c+dx)(e\tan\right.}{a^6}$$
$$= -\frac{e^6\int(e\tan(c+dx))^{-6+m}\,dx}{a^3}+\frac{e^6\int\sec^3(c+dx)(e\tan(c+dx))^{-6+m}\,dx}{a^3}+\frac{\left(3e^6\right)\int\sec(c+}{}$$
$$= -\frac{3e^5\cos^2(c+dx)^{\frac{1}{2}(-4+m)}\,{}_2F_1\left(\frac{1}{2}(-5+m),\frac{1}{2}(-4+m);\frac{1}{2}(-3+m);\sin^2(c+dx)\right)\sec(c+dx)}{a^3d(5-m)}$$
$$= \frac{3e^5(e\tan(c+dx))^{-5+m}}{a^3d(5-m)}+\frac{e^5\,{}_2F_1\left(1,\frac{1}{2}(-5+m);\frac{1}{2}(-3+m);-\tan^2(c+dx)\right)(e\tan(c+dx))}{a^3d(5-m)}$$

Mathematica [F] time = 1.47004, size = 0, normalized size = 0.

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^3, x]
```

Maple [F] time = 0.451, size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^m}{(a+a\sec(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^3,x)
```

```
[Out] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \tan (dx + c))^m}{(a \sec (dx + c) + a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e \tan (dx + c))^m}{a^3 \sec (dx + c)^3 + 3 a^3 \sec (dx + c)^2 + 3 a^3 \sec (dx + c) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((e*tan(d*x + c))^m/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*
a^3*sec(d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{(e \tan (c+dx))^m}{\sec^3 (c+dx)+3 \sec^2 (c+dx)+3 \sec (c+dx)+1} \, dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**m/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral((e*tan(c + d*x))**m/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
 + d*x) + 1), x)/a**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^3, x)
```

3.215 $\int (a+a\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$

Optimal. Leaf size=131

$$\frac{2^{m+\frac{5}{2}}(a\sec(c+dx)+a)^{3/2}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+\frac{5}{2}}(e\tan(c+dx))^{m+1}F_1\left(\frac{m+1}{2};m+\frac{3}{2},1;\frac{m+3}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)}$$

```
[Out] (2^(5/2 + m)*AppellF1[(1 + m)/2, 3/2 + m, 1, (3 + m)/2, -((a - a*Sec[c + d*
x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 +
 Sec[c + d*x])^(-1))^(5/2 + m)*(a + a*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^
(1 + m))/(d*e*(1 + m))
```

Rubi [A] time = 0.0973926, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {3889}

$$\frac{2^{m+\frac{5}{2}}(a\sec(c+dx)+a)^{3/2}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+\frac{5}{2}}(e\tan(c+dx))^{m+1}F_1\left(\frac{m+1}{2};m+\frac{3}{2},1;\frac{m+3}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m,x]
```

```
[Out] (2^(5/2 + m)*AppellF1[(1 + m)/2, 3/2 + m, 1, (3 + m)/2, -((a - a*Sec[c + d*
x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 +
 Sec[c + d*x])^(-1))^(5/2 + m)*(a + a*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^
(1 + m))/(d*e*(1 + m))
```

Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx = \frac{2^{\frac{5}{2}+m}F_1\left(\frac{1+m}{2};\frac{3}{2}+m,1;\frac{3+m}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{5}{2}+m}(\cdots}{de(1+m)}$$

Mathematica [F] time = 3.44609, size = 0, normalized size = 0.

$$\int (a+a\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m,x]
```

```
[Out] Integrate[(a + a*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m, x]
```

Maple [F] time = 0.255, size = 0, normalized size = 0.

$$\int (a+a\sec(dx+c))^{\frac{3}{2}}(e\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{3}{2}}(e\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sec(dx+c)+a)^{\frac{3}{2}}(e\tan(dx+c))^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(e*tan(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{3}{2}}(e\tan(dx+c))^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

3.216 $\int \sqrt{a+a\sec(c+dx)}(e\tan(c+dx))^m\,dx$

Optimal. Leaf size=131

$$\frac{2^{m+\frac{3}{2}}\sqrt{a\sec(c+dx)+a}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+\frac{3}{2}}(e\tan(c+dx))^{m+1}F_1\left(\frac{m+1}{2};m+\frac{1}{2},1;\frac{m+3}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)}$$

```
[Out] (2^(3/2 + m)*AppellF1[(1 + m)/2, 1/2 + m, 1, (3 + m)/2, -((a - a*Sec[c + d*
x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 +
 Sec[c + d*x])^(-1))^(3/2 + m)*Sqrt[a + a*Sec[c + d*x]]*(e*Tan[c + d*x])^(1
 + m))/(d*e*(1 + m))
```

Rubi [A] time = 0.0855779, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {3889}

$$\frac{2^{m+\frac{3}{2}}\sqrt{a\sec(c+dx)+a}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+\frac{3}{2}}(e\tan(c+dx))^{m+1}F_1\left(\frac{m+1}{2};m+\frac{1}{2},1;\frac{m+3}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*(e*Tan[c + d*x])^m,x]
```


```
[Out] (2^(3/2 + m)*AppellF1[(1 + m)/2, 1/2 + m, 1, (3 + m)/2, -((a - a*Sec[c + d*
x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 +
 Sec[c + d*x])^(-1))^(3/2 + m)*Sqrt[a + a*Sec[c + d*x]]*(e*Tan[c + d*x])^(1
 + m))/(d*e*(1 + m))
```


Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}(e\tan(c+dx))^m\,dx = \frac{2^{\frac{3}{2}+m}F_1\left(\frac{1+m}{2};\frac{1}{2}+m,1;\frac{3+m}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{3}{2}+m}\sqrt{a-}}{de(1+m)}$$

Mathematica [F] time = 7.3573, size = 0, normalized size = 0.

$$\int \sqrt{a+a\sec(c+dx)}(e\tan(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*(e*Tan[c + d*x])^m,x]
```

```
[Out] Integrate[Sqrt[a + a*Sec[c + d*x]]*(e*Tan[c + d*x])^m, x]
```

Maple [F] time = 0.279, size = 0, normalized size = 0.

$$\int \sqrt{a+a\sec(dx+c)}\,(e\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec(dx+c)+a}\,(e\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a\sec\left(dx+c\right)+a}\left(e\tan\left(dx+c\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(e\tan\left(c+dx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*(e*tan(d*x+c))**m,x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(e*tan(c + d*x))**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sec\left(dx+c\right)+a}\left(e\tan\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

3.217 $\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=131

$$\frac{2^{m+\frac{1}{2}}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+\frac{1}{2}}(e\tan(c+dx))^{m+1}F_1\left(\frac{m+1}{2};m-\frac{1}{2},1;\frac{m+3}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (2^(1/2 + m)*AppellF1[(1 + m)/2, -1/2 + m, 1, (3 + m)/2, -((a - a*Sec[c + d
*x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1
+ Sec[c + d*x])^(-1))^(1/2 + m)*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt
[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.0884013, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {3889}

$$\frac{2^{m+\frac{1}{2}}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+\frac{1}{2}}(e\tan(c+dx))^{m+1}F_1\left(\frac{m+1}{2};m-\frac{1}{2},1;\frac{m+3}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^m/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2^(1/2 + m)*AppellF1[(1 + m)/2, -1/2 + m, 1, (3 + m)/2, -((a - a*Sec[c + d
*x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1
+ Sec[c + d*x])^(-1))^(1/2 + m)*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt
[a + a*Sec[c + d*x]])
```

Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1+m}{2};-\frac{1}{2}+m,1;\frac{3+m}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{1}{2}+m}(e\tan(c+dx))^{1+}}{de(1+m)\sqrt{a+a\sec(c+dx)}}$$

Mathematica [F] time = 2.21428, size = 0, normalized size = 0.

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^m/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] Integrate[(e*Tan[c + d*x])^m/Sqrt[a + a*Sec[c + d*x]], x]
```

Maple [F] time = 0.283, size = 0, normalized size = 0.

$$\int (e\tan(dx+c))^m \frac{1}{\sqrt{a+a\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^m}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*tan(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e\tan(dx+c))^m}{\sqrt{a\sec(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*tan(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a(\sec(c+dx)+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**m/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*tan(c + d*x))**m/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^m}{\sqrt{a\sec(dx+c)+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

3.218 $\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=131

$$\frac{2^{m-\frac{1}{2}}\left(\frac{1}{\sec(c+dx)+1}\right)^{m-\frac{1}{2}}(e\tan(c+dx))^{m+1}F_1\left(\frac{m+1}{2};m-\frac{3}{2},1;\frac{m+3}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (2^(-1/2 + m)*AppellF1[(1 + m)/2, -3/2 + m, 1, (3 + m)/2, -((a - a*Sec[c +
d*x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1
 + Sec[c + d*x])^(-1))^(-1/2 + m)*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)*(a
 + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.104877, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {3889}

$$\frac{2^{m-\frac{1}{2}}\left(\frac{1}{\sec(c+dx)+1}\right)^{m-\frac{1}{2}}(e\tan(c+dx))^{m+1}F_1\left(\frac{m+1}{2};m-\frac{3}{2},1;\frac{m+3}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2^(-1/2 + m)*AppellF1[(1 + m)/2, -3/2 + m, 1, (3 + m)/2, -((a - a*Sec[c +
d*x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1
 + Sec[c + d*x])^(-1))^(-1/2 + m)*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)*(a
 + a*Sec[c + d*x])^(3/2))
```

Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2^{-\frac{1}{2}+m}F_1\left(\frac{1+m}{2}; -\frac{3}{2}+m, 1; \frac{3+m}{2}; -\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}, \frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{-\frac{1}{2}+m}(e\tan(c+dx)}{de(1+m)(a+a\sec(c+dx))^{3/2}}$$

Mathematica [F] time = 9.73783, size = 0, normalized size = 0.

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^(3/2), x]
```

Maple [F] time = 0.251, size = 0, normalized size = 0.

$$\int (e\tan(dx+c))^m (a+a\sec(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\sec(dx+c)+a}\,(e\tan(dx+c))^m}{a^2\sec(dx+c)^2+2\,a^2\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*tan(d*x + c))^m/(a^2*sec(d*x + c)^2 +
2*a^2*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(e\tan(c+dx))^m}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**m/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((e*tan(c + d*x))**m/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^(3/2), x)
```

3.219 $\int (a+a\sec(c+dx))^n \tan^7(c+dx)\,dx$

Optimal. Leaf size=123

$$\frac{(a\sec(c+dx)+a)^{n+4}\text{Hypergeometric2F1}(1,n+4,n+5,\sec(c+dx)+1)}{a^4d(n+4)}+\frac{7(a\sec(c+dx)+a)^{n+4}}{a^4d(n+4)}-\frac{5(a\sec(c+dx)}{a^5d(n+}$$

```
[Out] (7*(a + a*Sec[c + d*x])^(4 + n))/(a^4*d*(4 + n)) + (Hypergeometric2F1[1, 4
+ n, 5 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(4 + n))/(a^4*d*(4 + n))
 - (5*(a + a*Sec[c + d*x])^(5 + n))/(a^5*d*(5 + n)) + (a + a*Sec[c + d*x])^
(6 + n)/(a^6*d*(6 + n))
```

Rubi [A] time = 0.100469, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3880, 88, 65}

$$\frac{(a\sec(c+dx)+a)^{n+4}\,{}_2F_1(1,n+4;n+5;\sec(c+dx)+1)}{a^4d(n+4)}+\frac{7(a\sec(c+dx)+a)^{n+4}}{a^4d(n+4)}-\frac{5(a\sec(c+dx)+a)^{n+5}}{a^5d(n+5)}+\frac{(a\sec(c}{a^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^7,x]
```

```
[Out] (7*(a + a*Sec[c + d*x])^(4 + n))/(a^4*d*(4 + n)) + (Hypergeometric2F1[1, 4
+ n, 5 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(4 + n))/(a^4*d*(4 + n))
 - (5*(a + a*Sec[c + d*x])^(5 + n))/(a^5*d*(5 + n)) + (a + a*Sec[c + d*x])^
(6 + n)/(a^6*d*(6 + n))
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^n\tan^7(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)^3(a+ax)^{3+n}}{x}\,dx,x,\sec(c+dx)\right)}{a^6d}$$

$$= \frac{\text{Subst}\left(\int \left(7a^3(a+ax)^{3+n} - \frac{a^3(a+ax)^{3+n}}{x} - 5a^2(a+ax)^{4+n} + a(a+ax)^{5+n}\right)dx, x \ldots}{a^6d}$$

$$= \frac{7(a+a\sec(c+dx))^{4+n}}{a^4d(4+n)} - \frac{5(a+a\sec(c+dx))^{5+n}}{a^5d(5+n)} + \frac{(a+a\sec(c+dx))^{6+n}}{a^6d(6+n)} - \ldots$$

$$= \frac{7(a+a\sec(c+dx))^{4+n}}{a^4d(4+n)} + \frac{{}_2F_1(1,4+n;5+n;1+\sec(c+dx))(a+a\sec(c+d\ldots}{a^4d(4+n)}$$

Mathematica [A] time = 0.41777, size = 87, normalized size = 0.71

$$\frac{(\sec(c+dx)+1)^4(a(\sec(c+dx)+1))^n\left(\frac{\text{Hypergeometric2F1}(1,n+4,n+5,\sec(c+dx)+1)}{n+4}+\frac{(\sec(c+dx)+1)^2}{n+6}-\frac{5(\sec(c+dx)+1)}{n+5}+\frac{7}{n+4}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^7,x]
```

```
[Out] ((1 + Sec[c + d*x])^4*(a*(1 + Sec[c + d*x]))^n*(7/(4 + n) + Hypergeometric2
F1[1, 4 + n, 5 + n, 1 + Sec[c + d*x]]/(4 + n) - (5*(1 + Sec[c + d*x]))/(5 +
 n) + (1 + Sec[c + d*x])^2/(6 + n)))/d
```

Maple [F] time = 0.439, size = 0, normalized size = 0.

$$\int (a+a\sec(dx+c))^n (\tan(dx+c))^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^7,x)
```

```
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^7,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sec(dx+c)+a)^n\tan(dx+c)^7, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^7, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**7,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n \tan(dx+c)^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^7, x)
```

3.220 $\int (a + a\sec(c + dx))^n \tan^5(c + dx)\,dx$

Optimal. Leaf size=97

$$-\frac{(a\sec(c+dx)+a)^{n+3}\text{Hypergeometric2F1}(1,n+3,n+4,\sec(c+dx)+1)}{a^3d(n+3)}-\frac{3(a\sec(c+dx)+a)^{n+3}}{a^3d(n+3)}+\frac{(a\sec(c+dx)}{a^4d(n+}$$

```
[Out] (-3*(a + a*Sec[c + d*x])^(3 + n))/(a^3*d*(3 + n)) - (Hypergeometric2F1[1, 3
 + n, 4 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3 + n))/(a^3*d*(3 + n)
) + (a + a*Sec[c + d*x])^(4 + n)/(a^4*d*(4 + n))
```

Rubi [A] time = 0.0817055, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3880, 88, 65}

$$-\frac{(a\sec(c+dx)+a)^{n+3}\,{}_2F_1(1,n+3;n+4;\sec(c+dx)+1)}{a^3d(n+3)}-\frac{3(a\sec(c+dx)+a)^{n+3}}{a^3d(n+3)}+\frac{(a\sec(c+dx)+a)^{n+4}}{a^4d(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^5,x]
```

```
[Out] (-3*(a + a*Sec[c + d*x])^(3 + n))/(a^3*d*(3 + n)) - (Hypergeometric2F1[1, 3
 + n, 4 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3 + n))/(a^3*d*(3 + n)
) + (a + a*Sec[c + d*x])^(4 + n)/(a^4*d*(4 + n))
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^n\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)^2(a+ax)^{2+n}}{x}\,dx, x, \sec(c+dx)\right)}{a^4d}$$
$$= \frac{\text{Subst}\left(\int \left(-3a^2(a+ax)^{2+n}+\frac{a^2(a+ax)^{2+n}}{x}+a(a+ax)^{3+n}\right)dx, x, \sec(c+dx)\right)}{a^4d}$$
$$= -\frac{3(a+a\sec(c+dx))^{3+n}}{a^3d(3+n)}+\frac{(a+a\sec(c+dx))^{4+n}}{a^4d(4+n)}+\frac{\text{Subst}\left(\int \frac{(a+ax)^{2+n}}{x}\,dx, x, \text{s}\right.}{a^2d}$$
$$= -\frac{3(a+a\sec(c+dx))^{3+n}}{a^3d(3+n)}-\frac{{}_2F_1(1,3+n;4+n;1+\sec(c+dx))(a+a\sec(c+}{a^3d(3+n)}$$

Mathematica [A] time = 0.144581, size = 72, normalized size = 0.74

$$\frac{(\sec(c+dx)+1)^3(a(\sec(c+dx)+1))^n(-(n+4)\text{Hypergeometric2F1}(1,n+3,n+4,\sec(c+dx)+1)+(n+3)\sec(c}{d(n+3)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^5,x]
```

```
[Out] ((1 + Sec[c + d*x])^3*(a*(1 + Sec[c + d*x]))^n*(-9 - 2*n - (4 + n)*Hypergeo
metric2F1[1, 3 + n, 4 + n, 1 + Sec[c + d*x]] + (3 + n)*Sec[c + d*x]))/(d*(3
 + n)*(4 + n))
```

Maple [F] time = 0.309, size = 0, normalized size = 0.

$$\int (a+a\sec(dx+c))^n(\tan(dx+c))^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^5,x)
```

```
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n \tan(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((a\sec(dx+c)+a)^n \tan(dx+c)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n \tan(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

3.221 $\int (a+a\sec(c+dx))^n \tan^3(c+dx)\,dx$

Optimal. Leaf size=69

$$\frac{(a\sec(c+dx)+a)^{n+2}\text{Hypergeometric2F1}(1,n+2,n+3,\sec(c+dx)+1)}{a^2d(n+2)}+\frac{(a\sec(c+dx)+a)^{n+2}}{a^2d(n+2)}$$

```
[Out] (a + a*Sec[c + d*x])^(2 + n)/(a^2*d*(2 + n)) + (Hypergeometric2F1[1, 2 + n,
 3 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2 + n))/(a^2*d*(2 + n))
```

Rubi [A] time = 0.0645631, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3880, 80, 65}

$$\frac{(a\sec(c+dx)+a)^{n+2}\,{}_2F_1(1,n+2;n+3;\sec(c+dx)+1)}{a^2d(n+2)}+\frac{(a\sec(c+dx)+a)^{n+2}}{a^2d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^3,x]
```

```
[Out] (a + a*Sec[c + d*x])^(2 + n)/(a^2*d*(2 + n)) + (Hypergeometric2F1[1, 2 + n,
 3 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2 + n))/(a^2*d*(2 + n))
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
```

```
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^n\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int\frac{(-a+ax)(a+ax)^{1+n}}{x}\,dx,x,\sec(c+dx)\right)}{a^2d}$$
$$= \frac{(a+a\sec(c+dx))^{2+n}}{a^2d(2+n)} - \frac{\text{Subst}\left(\int\frac{(a+ax)^{1+n}}{x}\,dx,x,\sec(c+dx)\right)}{ad}$$
$$= \frac{(a+a\sec(c+dx))^{2+n}}{a^2d(2+n)} + \frac{{}_2F_1(1,2+n;3+n;1+\sec(c+dx))(a+a\sec(c+dx$$

Mathematica [A] time = 0.0478846, size = 49, normalized size = 0.71

$$\frac{(\sec(c+dx)+1)^2(a(\sec(c+dx)+1))^n(\text{Hypergeometric2F1}(1,n+2,n+3,\sec(c+dx)+1)+1)}{d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^3,x]
```

```
[Out] ((1 + Hypergeometric2F1[1, 2 + n, 3 + n, 1 + Sec[c + d*x]])*(1 + Sec[c + d*
x])^2*(a*(1 + Sec[c + d*x]))^n)/(d*(2 + n))
```

Maple [F] time = 0.244, size = 0, normalized size = 0.

$$\int (a+a\sec(dx+c))^n(\tan(dx+c))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^3,x)
```

```
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n \tan(dx+c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sec(dx+c)+a)^n \tan(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a(\sec(c+dx)+1))^n \tan^3(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**3,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n \tan(dx+c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

3.222 $\int (a+a\sec(c+dx))^n\tan(c+dx)\,dx$

Optimal. Leaf size=43

$$-\frac{(a\sec(c+dx)+a)^{n+1}\text{Hypergeometric2F1}(1,n+1,n+2,\sec(c+dx)+1)}{ad(n+1)}$$

```
[Out] -((Hypergeometric2F1[1, 1 + n, 2 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x]
)^(1 + n))/(a*d*(1 + n)))
```

Rubi [A] time = 0.0413094, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3880, 65}

$$-\frac{(a\sec(c+dx)+a)^{n+1}\,{}_2F_1(1,n+1;n+2;\sec(c+dx)+1)}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x],x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + n, 2 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x]
)^(1 + n))/(a*d*(1 + n)))
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^n\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+ax)^n}{x}\,dx, x, \sec(c+dx)\right)}{d}$$
$$= -\frac{{}_2F_1(1,1+n;2+n;1+\sec(c+dx))(a+a\sec(c+dx))^{1+n}}{ad(1+n)}$$

Mathematica [A] time = 0.0421604, size = 43, normalized size = 1.

$$-\frac{(a(\sec(c+dx)+1))^{n+1}\text{Hypergeometric2F1}(1,n+1,n+2,\sec(c+dx)+1)}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x],x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + n, 2 + n, 1 + Sec[c + d*x]]*(a*(1 + Sec[c + d*x
]))^(1 + n))/(a*d*(1 + n)))
```

Maple [F] time = 0.317, size = 0, normalized size = 0.

$$\int (a+a\sec(dx+c))^n\tan(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c),x)
```

```
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n\tan(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sec\left(dx+c\right)+a\right)^{n}\tan\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{n}\tan\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sec\left(dx+c\right)+a\right)^{n}\tan\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

3.223 $\int \cot(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=74

$$\frac{(a\sec(c+dx)+a)^n\text{Hypergeometric2F1}(1,n,n+1,\sec(c+dx)+1)}{dn}-\frac{(a\sec(c+dx)+a)^n\text{Hypergeometric2F1}\left(1,\right.}{2dn}$$

```
[Out] -(Hypergeometric2F1[1, n, 1 + n, (1 + Sec[c + d*x])/2]*(a + a*Sec[c + d*x])
^n)/(2*d*n) + (Hypergeometric2F1[1, n, 1 + n, 1 + Sec[c + d*x]]*(a + a*Sec[
c + d*x])^n)/(d*n)
```

Rubi [A] time = 0.0608846, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3880, 86, 65, 68}

$$\frac{(a\sec(c+dx)+a)^n\,{}_2F_1(1,n;n+1;\sec(c+dx)+1)}{dn}-\frac{(a\sec(c+dx)+a)^n\,{}_2F_1\left(1,n;n+1;\frac{1}{2}(\sec(c+dx)+1)\right)}{2dn}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -(Hypergeometric2F1[1, n, 1 + n, (1 + Sec[c + d*x])/2]*(a + a*Sec[c + d*x])
^n)/(2*d*n) + (Hypergeometric2F1[1, n, 1 + n, 1 + Sec[c + d*x]]*(a + a*Sec[
c + d*x])^n)/(d*n)
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \cot(c+dx)(a+a\sec(c+dx))^n\,dx = \frac{a^2\,\text{Subst}\left(\int \frac{(a+ax)^{-1+n}}{x(-a+ax)}\,dx, x, \sec(c+dx)\right)}{d}$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{(a+ax)^{-1+n}}{x}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a^2\,\text{Subst}\left(\int \frac{(a+ax)^{-1+n}}{-a+ax}\,dx, x, \sec(c+\right.}{d}$$
$$= -\frac{{}_2F_1\left(1,n;1+n;\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^n}{2dn} + \frac{{}_2F_1(1,n;1+n;1+\text{s}}{}$$

Mathematica [A] time = 0.0413445, size = 57, normalized size = 0.77

$$-\frac{(a(\sec(c+dx)+1))^n\left(\text{Hypergeometric2F1}\left(1,n,n+1,\frac{1}{2}(\sec(c+dx)+1)\right)-2\text{Hypergeometric2F1}(1,n,n+1,\sec(\right.}{2dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -((Hypergeometric2F1[1, n, 1 + n, (1 + Sec[c + d*x])/2] - 2*Hypergeometric2
F1[1, n, 1 + n, 1 + Sec[c + d*x]])*(a*(1 + Sec[c + d*x]))^n)/(2*d*n)
```

Maple [F] time = 0.33, size = 0, normalized size = 0.

$$\int \cot(dx+c)(a+a\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c))^n,x)
```

```
[Out] int(cot(d*x+c)*(a+a*sec(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n\cot(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sec(dx+c)+a)^n\cot(dx+c)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a(\sec(c+dx)+1))^n\cot(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))**n,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n*cot(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n\cot(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

3.224 $\int \cot^3(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=127

$$-\frac{a(4-n)(a\sec(c+dx)+a)^{n-1}\text{Hypergeometric2F1}\left(1,n-1,n,\frac{1}{2}(\sec(c+dx)+1)\right)}{4d(1-n)}+\frac{a(a\sec(c+dx)+a)^{n-1}\text{Hyper}}{}$$

```
[Out] -(a*(4 - n)*Hypergeometric2F1[1, -1 + n, n, (1 + Sec[c + d*x])/2]*(a + a*Se
c[c + d*x])^(-1 + n))/(4*d*(1 - n)) + (a*Hypergeometric2F1[1, -1 + n, n, 1
+ Sec[c + d*x]]*(a + a*Sec[c + d*x])^(-1 + n))/(d*(1 - n)) + (a*(a + a*Sec[
c + d*x])^(-1 + n))/(2*d*(1 - Sec[c + d*x]))
```

Rubi [A] time = 0.109211, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3880, 103, 156, 65, 68}

$$-\frac{a(4-n)(a\sec(c+dx)+a)^{n-1}\,{}_2F_1\left(1,n-1;n;\frac{1}{2}(\sec(c+dx)+1)\right)}{4d(1-n)}+\frac{a(a\sec(c+dx)+a)^{n-1}\,{}_2F_1(1,n-1;n;\sec(c+dx)}{d(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -(a*(4 - n)*Hypergeometric2F1[1, -1 + n, n, (1 + Sec[c + d*x])/2]*(a + a*Se
c[c + d*x])^(-1 + n))/(4*d*(1 - n)) + (a*Hypergeometric2F1[1, -1 + n, n, 1
+ Sec[c + d*x]]*(a + a*Sec[c + d*x])^(-1 + n))/(d*(1 - n)) + (a*(a + a*Sec[
c + d*x])^(-1 + n))/(2*d*(1 - Sec[c + d*x]))
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^n\,dx = \frac{a^4\,\text{Subst}\left(\int \frac{(a+ax)^{-2+n}}{x(-a+ax)^2}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))} - \frac{a\,\text{Subst}\left(\int \frac{(a+ax)^{-2+n}\left(2a^2+a^2(2-n)x\right)}{x(-a+ax)}\,dx, x, \sec(c+dx)\right)}{2d}$$

$$= \frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))} + \frac{a^2\,\text{Subst}\left(\int \frac{(a+ax)^{-2+n}}{x}\,dx, x, \sec(c+dx)\right)}{d} - \frac{\left(a^3(4-n\right.}{}$$

$$= -\frac{a(4-n)\,{}_2F_1\left(1,-1+n;n;\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^{-1+n}}{4d(1-n)} + \frac{a\,{}_2F_1(1}{}$$

Mathematica [A] time = 0.216217, size = 96, normalized size = 0.76

$$-\frac{a(a(\sec(c+dx)+1))^{n-1}\left((n-4)(\sec(c+dx)-1)\text{Hypergeometric2F1}\left(1,n-1,n,\frac{1}{2}(\sec(c+dx)+1)\right)+4(\sec(c+d\right.}{4d(n-1)(\sec(c+dx)-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -(a*(-2 + 2*n + (-4 + n)*Hypergeometric2F1[1, -1 + n, n, (1 + Sec[c + d*x])
/2]*(-1 + Sec[c + d*x]) + 4*Hypergeometric2F1[1, -1 + n, n, 1 + Sec[c + d*x
]]*(-1 + Sec[c + d*x]))*(a*(1 + Sec[c + d*x]))^(-1 + n))/(4*d*(-1 + n)*(-1
+ Sec[c + d*x]))
```

Maple [F] time = 0.23, size = 0, normalized size = 0.

$$\int (\cot(dx+c))^{3}(a+a\sec(dx+c))^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^n,x)
```

```
[Out] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{n}\cot(dx+c)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sec(dx+c)+a)^n\cot(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n\cot(dx+c)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

3.225 $\int (a+a\sec(c+dx))^n \tan^4(c+dx)\,dx$

Optimal. Leaf size=106

$$\frac{2^{n+5}\tan^5(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+5}(a\sec(c+dx)+a)^n F_1\left(\frac{5}{2};n+4,1;\frac{7}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{5d}$$

```
[Out] (2^(5 + n)*AppellF1[5/2, 4 + n, 1, 7/2, -((a - a*Sec[c + d*x])/(a + a*Sec[c
 + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*x])^(
-1))^(5 + n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0553183, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3889}

$$\frac{2^{n+5}\tan^5(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+5}(a\sec(c+dx)+a)^n F_1\left(\frac{5}{2};n+4,1;\frac{7}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^4,x]
```



```
[Out] (2^(5 + n)*AppellF1[5/2, 4 + n, 1, 7/2, -((a - a*Sec[c + d*x])/(a + a*Sec[c
 + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*x])^(
-1))^(5 + n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x]^5)/(5*d)
```


Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^n \tan^4(c+dx)\,dx = \frac{2^{5+n}F_1\left(\frac{5}{2};4+n,1;\frac{7}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{5+n}(a+a\sec(c+}{5d}$$

Mathematica [F] time = 1.15102, size = 0, normalized size = 0.

$$\int (a + a\sec(c + dx))^n \tan^4(c + dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^4,x]
```

```
[Out] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^4, x]
```

Maple [F] time = 0.272, size = 0, normalized size = 0.

$$\int (a + a\sec(dx + c))^n (\tan(dx + c))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^4,x)
```

```
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx + c) + a)^n \tan(dx + c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sec(dx + c) + a)^n \tan(dx + c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n\tan(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

3.226 $\int (a+a\sec(c+dx))^n \tan^2(c+dx)\,dx$

Optimal. Leaf size=106

$$\frac{2^{n+3}\tan^3(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+3}(a\sec(c+dx)+a)^n F_1\left(\frac{3}{2};n+2,1;\frac{5}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{3d}$$

```
[Out] (2^(3 + n)*AppellF1[3/2, 2 + n, 1, 5/2, -((a - a*Sec[c + d*x])/(a + a*Sec[c
 + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*x])^(
-1))^(3 + n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0553637, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3889}

$$\frac{2^{n+3}\tan^3(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+3}(a\sec(c+dx)+a)^n F_1\left(\frac{3}{2};n+2,1;\frac{5}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^2,x]
```

```
[Out] (2^(3 + n)*AppellF1[3/2, 2 + n, 1, 5/2, -((a - a*Sec[c + d*x])/(a + a*Sec[c
 + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*x])^(
-1))^(3 + n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x]^3)/(3*d)
```

Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^n \tan^2(c+dx)\,dx = \frac{2^{3+n}F_1\left(\frac{3}{2};2+n,1;\frac{5}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{3+n}(a+a\sec(c+}{3d}$$

Mathematica [B] time = 14.9112, size = 2419, normalized size = 22.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^2,x]
```

```
[Out] (2^(3 + n)*Cos[(c + d*x)/2]*Sec[c + d*x]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x]
)^n*(a*(1 + Sec[c + d*x]))^n*Sin[(c + d*x)/2]^3*(-((Hypergeometric2F1[1/2,
1 + n, 3/2, Tan[(c + d*x)/2]^2] - 2*Hypergeometric2F1[1/2, 2 + n, 3/2, Tan[
(c + d*x)/2]^2])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n) + (3*AppellF1[1/2, n,
 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(-3*A
ppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(Appel
lF1[3/2, n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - n*AppellF1[3
/2, 1 + n, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/
2]^2)))/(d*(2^n*Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(-((
Hypergeometric2F1[1/2, 1 + n, 3/2, Tan[(c + d*x)/2]^2] - 2*Hypergeometric2F
1[1/2, 2 + n, 3/2, Tan[(c + d*x)/2]^2])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n
) + (3*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Co
s[(c + d*x)/2]^2)/(-3*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c
+ d*x)/2]^2] + 2*(AppellF1[3/2, n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2] - n*AppellF1[3/2, 1 + n, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2])*Tan[(c + d*x)/2]^2)) + 2^(1 + n)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])
^n*Tan[(c + d*x)/2]*(-(n*(Hypergeometric2F1[1/2, 1 + n, 3/2, Tan[(c + d*x)/
2]^2] - 2*Hypergeometric2F1[1/2, 2 + n, 3/2, Tan[(c + d*x)/2]^2])*(Cos[c +
d*x]*Sec[(c + d*x)/2]^2)^(-1 + n)*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos
[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])) - (3*AppellF1[1/2, n, 1, 3/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]*Sin[(c + d*x)/
2])/(-3*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] +
 2*(AppellF1[3/2, n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - n*A
ppellF1[3/2, 1 + n, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(
c + d*x)/2]^2) + (3*Cos[(c + d*x)/2]^2*(-(AppellF1[3/2, n, 2, 5/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (
n*AppellF1[3/2, 1 + n, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3))/(-3*AppellF1[1/2, n, 1, 3/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(AppellF1[3/2, n, 2, 5/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2] - n*AppellF1[3/2, 1 + n, 1, 5/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) - (3*AppellF1[1/2, n, 1, 3
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2*(2*(AppellF
1[3/2, n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - n*AppellF1[3/2
, 1 + n, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2] - 3*(-(AppellF1[3/2, n, 2, 5/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (n*AppellF1[3/2,
```

```
 1 + n, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2
*Tan[(c + d*x)/2])/3) + 2*Tan[(c + d*x)/2]^2*((-6*AppellF1[5/2, n, 3, 7/2,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2
])/5 + (3*n*AppellF1[5/2, 1 + n, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 - n*((-3*AppellF1[5/2, 1 + n,
 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2])/5 + (3*(1 + n)*AppellF1[5/2, 2 + n, 1, 7/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5))))/(-3*Appell
F1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(AppellF1[3
/2, n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - n*AppellF1[3/2, 1
 + n, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)
^2 - (Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(-(Csc[(c + d*x)/2]*Sec[(c + d*x)/
2]*(-Hypergeometric2F1[1/2, 2 + n, 3/2, Tan[(c + d*x)/2]^2] + (1 - Tan[(c +
 d*x)/2]^2)^(-2 - n))) + (Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(-Hypergeometri
c2F1[1/2, 1 + n, 3/2, Tan[(c + d*x)/2]^2] + (1 - Tan[(c + d*x)/2]^2)^(-1 -
n)))/2)) + 2^(1 + n)*n*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n)*Tan[(c +
d*x)/2]*(-((Hypergeometric2F1[1/2, 1 + n, 3/2, Tan[(c + d*x)/2]^2] - 2*Hype
rgeometric2F1[1/2, 2 + n, 3/2, Tan[(c + d*x)/2]^2])*(Cos[c + d*x]*Sec[(c +
d*x)/2]^2)^n) + (3*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d
*x)/2]^2]*Cos[(c + d*x)/2]^2)/(-3*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2] + 2*(AppellF1[3/2, n, 2, 5/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2] - n*AppellF1[3/2, 1 + n, 1, 5/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*
Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x])))
```

Maple [F] time = 0.21, size = 0, normalized size = 0.

$$\int (a + a\sec(dx + c))^n (\tan(dx + c))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^2,x)
```

```
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx + c) + a)^n \tan(dx + c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sec(dx+c)+a)^n\tan(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a(\sec(c+dx)+1))^n\tan^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**2,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n\tan(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

3.227 $\int \cot^2(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=102

$$-\frac{2^{n-1}\cot(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n-1}(a\sec(c+dx)+a)^n F_1\left(-\frac{1}{2};n-2,1;\frac{1}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{d}$$

```
[Out] -((2^(-1 + n)*AppellF1[-1/2, -2 + n, 1, 1/2, -((a - a*Sec[c + d*x])/(a + a*
Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*Cot[c + d*x]*((1
 + Sec[c + d*x])^(-1))^(-1 + n)*(a + a*Sec[c + d*x])^n)/d)
```

Rubi [A] time = 0.0553295, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3889}

$$-\frac{2^{n-1}\cot(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n-1}(a\sec(c+dx)+a)^n F_1\left(-\frac{1}{2};n-2,1;\frac{1}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -((2^(-1 + n)*AppellF1[-1/2, -2 + n, 1, 1/2, -((a - a*Sec[c + d*x])/(a + a*
Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*Cot[c + d*x]*((1
 + Sec[c + d*x])^(-1))^(-1 + n)*(a + a*Sec[c + d*x])^n)/d)
```

Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{2^{-1+n}F_1\left(-\frac{1}{2};-2+n,1;\frac{1}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\cot(c+dx)\left(\frac{1}{1+\sec(c+dx)}\right)^{-}}{d}$$

Mathematica [B] time = 15.3672, size = 2492, normalized size = 24.43

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] (2^(-3 + n)*Cos[c + d*x]^2*Csc[(c + d*x)/2]^3*Sec[(c + d*x)/2]*(Cos[(c + d*
x)/2]^2*Sec[c + d*x])^n*(a*(1 + Sec[c + d*x]))^n*((12*AppellF1[1/2, n, 1, 3
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sin[(c + d*x)/2]^2)/(-3*Appell
F1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(AppellF1[3
/2, n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - n*AppellF1[3/2, 1
 + n, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)
 + (Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(-Hypergeometric2F1[-1/2, n, 1/2, Ta
n[(c + d*x)/2]^2] + Hypergeometric2F1[1/2, n, 3/2, Tan[(c + d*x)/2]^2]*Tan[
(c + d*x)/2]^2)))/(d*(-(2^(-2 + n)*Csc[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*S
ec[c + d*x])^n*((12*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2]*Sin[(c + d*x)/2]^2)/(-3*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2
]^2, -Tan[(c + d*x)/2]^2] + 2*(AppellF1[3/2, n, 2, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] - n*AppellF1[3/2, 1 + n, 1, 5/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) + (Cos[c + d*x]*Sec[(c + d*x)/2]^
2)^n*(-Hypergeometric2F1[-1/2, n, 1/2, Tan[(c + d*x)/2]^2] + Hypergeometric
2F1[1/2, n, 3/2, Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2))) + 2^(-1 + n)*Cot
[(c + d*x)/2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*((12*AppellF1[1/2, n, 1,
3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]*Sin[(c + d*x
)/2])/(-3*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
 + 2*(AppellF1[3/2, n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - n
*AppellF1[3/2, 1 + n, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan
[(c + d*x)/2]^2) + (12*Sin[(c + d*x)/2]^2*(-(AppellF1[3/2, n, 2, 5/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3
+ (n*AppellF1[3/2, 1 + n, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*
Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3))/(-3*AppellF1[1/2, n, 1, 3/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(AppellF1[3/2, n, 2, 5/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2] - n*AppellF1[3/2, 1 + n, 1, 5/2, Tan[(c + d
*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) + n*(Cos[c + d*x]*Sec[(
c + d*x)/2]^2)^(-1 + n)*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*
Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-Hypergeometric2F1[-1/2, n, 1/2, Tan[
(c + d*x)/2]^2] + Hypergeometric2F1[1/2, n, 3/2, Tan[(c + d*x)/2]^2]*Tan[(c
 + d*x)/2]^2) - (12*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2]*Sin[(c + d*x)/2]^2*(2*(AppellF1[3/2, n, 2, 5/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2] - n*AppellF1[3/2, 1 + n, 1, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] - 3*(-(AppellF
1[3/2, n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]
```

```
^2*Tan[(c + d*x)/2])/3 + (n*AppellF1[3/2, 1 + n, 1, 5/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3) + 2*Tan[(c +
 d*x)/2]^2*((-6*AppellF1[5/2, n, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (3*n*AppellF1[5/2, 1 + n, 2
, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c +
 d*x)/2])/5 - n*((-3*AppellF1[5/2, 1 + n, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (3*(1 + n)*AppellF
1[5/2, 2 + n, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x
)/2]^2*Tan[(c + d*x)/2])/5))))/(-3*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2
]^2, -Tan[(c + d*x)/2]^2] + 2*(AppellF1[3/2, n, 2, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] - n*AppellF1[3/2, 1 + n, 1, 5/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)^2 + (Cos[c + d*x]*Sec[(c + d*x)/2
]^2)^n*(Hypergeometric2F1[1/2, n, 3/2, Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2] - (Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Hypergeometric2F1
[-1/2, n, 1/2, Tan[(c + d*x)/2]^2] - (1 - Tan[(c + d*x)/2]^2)^(-n)))/2 + (S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]*(-Hypergeometric2F1[1/2, n, 3/2, Tan[(c
+ d*x)/2]^2] + (1 - Tan[(c + d*x)/2]^2)^(-n)))/2)) + 2^(-1 + n)*n*Cot[(c +
d*x)/2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n)*((12*AppellF1[1/2, n, 1,
 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sin[(c + d*x)/2]^2)/(-3*Appe
llF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(AppellF1
[3/2, n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - n*AppellF1[3/2,
 1 + n, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^
2) + (Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(-Hypergeometric2F1[-1/2, n, 1/2,
Tan[(c + d*x)/2]^2] + Hypergeometric2F1[1/2, n, 3/2, Tan[(c + d*x)/2]^2]*Ta
n[(c + d*x)/2]^2))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos
[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))
```

Maple [F] time = 0.252, size = 0, normalized size = 0.

$$\int (\cot(dx+c))^2 (a + a\sec(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^n,x)

[Out] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c) + a)^n \cot(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sec(dx+c)+a)^n\cot(dx+c)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n\cot(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

3.228 $\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=106

$$-\frac{2^{n-3}\cot^3(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n-3}(a\sec(c+dx)+a)^nF_1\left(-\frac{3}{2};n-4,1;-\frac{1}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{3d}$$

```
[Out] -(2^(-3 + n)*AppellF1[-3/2, -4 + n, 1, -1/2, -((a - a*Sec[c + d*x])/(a + a*
Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*Cot[c + d*x]^3*(
(1 + Sec[c + d*x])^(-1))^(-3 + n)*(a + a*Sec[c + d*x])^n)/(3*d)
```

Rubi [A] time = 0.0555987, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3889}

$$-\frac{2^{n-3}\cot^3(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n-3}(a\sec(c+dx)+a)^nF_1\left(-\frac{3}{2};n-4,1;-\frac{1}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -(2^(-3 + n)*AppellF1[-3/2, -4 + n, 1, -1/2, -((a - a*Sec[c + d*x])/(a + a*
Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*Cot[c + d*x]^3*(
(1 + Sec[c + d*x])^(-1))^(-3 + n)*(a + a*Sec[c + d*x])^n)/(3*d)
```

Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])]])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx=-\frac{2^{-3+n}F_1\left(-\frac{3}{2};-4+n,1;-\frac{1}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\cot^3(c+dx)\left(\frac{1}{1+\sec(c+dx)}\right.}{3d}$$

Mathematica [F] time = 1.53106, size = 0, normalized size = 0.

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^n, x]
```

Maple [F] time = 0.303, size = 0, normalized size = 0.

$$\int (\cot(dx+c))^4 (a+a\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^n,x)
```

```
[Out] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n \cot(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sec(dx+c)+a)^n \cot(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="fricas")

[Out] integral((a*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**n,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sec (dx + c) + a)^n \cot (dx + c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="giac")

[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)

3.229 $\int (a+a\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=114

$$\frac{2^{n+\frac{7}{2}}\tan^{\frac{5}{2}}(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+\frac{5}{2}}(a\sec(c+dx)+a)^n F_1\left(\frac{5}{4};n+\frac{3}{2},1;\frac{9}{4};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{5d}$$

```
[Out] (2^(7/2 + n)*AppellF1[5/4, 3/2 + n, 1, 9/4, -((a - a*Sec[c + d*x])/(a + a*S
ec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*x
])^(-1))^(5/2 + n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x]^(5/2))/(5*d)
```

Rubi [A] time = 0.0640971, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {3889}

$$\frac{2^{n+\frac{7}{2}}\tan^{\frac{5}{2}}(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+\frac{5}{2}}(a\sec(c+dx)+a)^n F_1\left(\frac{5}{4};n+\frac{3}{2},1;\frac{9}{4};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^(3/2),x]
```


```
[Out] (2^(7/2 + n)*AppellF1[5/4, 3/2 + n, 1, 9/4, -((a - a*Sec[c + d*x])/(a + a*S
ec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*x
])^(-1))^(5/2 + n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x]^(5/2))/(5*d)
```


Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a + a\sec(c + dx))^n \tan^{\frac{3}{2}}(c + dx)\, dx = \frac{2^{\frac{7}{2}+n} F_1\left(\frac{5}{4}; \frac{3}{2} + n, 1; \frac{9}{4}; -\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}, \frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{5}{2}+n} (a + a\sec(c +}{5d}$$

Mathematica [B] time = 18.5428, size = 2072, normalized size = 18.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^(3/2),x]
```

```
[Out] (2^(1 + n)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(a*(1 + Sec[c + d*x]))^n*(-1
 + Tan[(c + d*x)/2])^(-1/2 - n)*(-2*AppellF1[1/4, 1/2 + n, 1, 5/4, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(1/2 + n
)*(-1 + Tan[(c + d*x)/2])^(1/2 + n) + (AppellF1[1/2, 1/2 + n, 3/2 + n, 3/2,
 Tan[(c + d*x)/2], -Tan[(c + d*x)/2]] + AppellF1[1/2, 3/2 + n, 1/2 + n, 3/2
, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]])*(1 - Tan[(c + d*x)/2])^(1/2 + n)*(-
1 + Tan[(c + d*x)/2]^2)^(1/2 + n))*Tan[c + d*x]^2)/(d*((2^n*Sec[c + d*x]^2*
(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(-1 + Tan[(c + d*x)/2])^(-1/2 - n)*(-2*
AppellF1[1/4, 1/2 + n, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Co
s[c + d*x]*Sec[(c + d*x)/2]^2)^(1/2 + n)*(-1 + Tan[(c + d*x)/2])^(1/2 + n)
+ (AppellF1[1/2, 1/2 + n, 3/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]
] + AppellF1[1/2, 3/2 + n, 1/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2
]])*(1 - Tan[(c + d*x)/2])^(1/2 + n)*(-1 + Tan[(c + d*x)/2]^2)^(1/2 + n)))/
Sqrt[Tan[c + d*x]] + 2^n*(-1/2 - n)*Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*
Sec[c + d*x])^n*(-1 + Tan[(c + d*x)/2])^(-3/2 - n)*(-2*AppellF1[1/4, 1/2 +
n, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c +
d*x)/2]^2)^(1/2 + n)*(-1 + Tan[(c + d*x)/2])^(1/2 + n) + (AppellF1[1/2, 1/2
 + n, 3/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]] + AppellF1[1/2, 3/
2 + n, 1/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]])*(1 - Tan[(c + d*
x)/2])^(1/2 + n)*(-1 + Tan[(c + d*x)/2]^2)^(1/2 + n))*Sqrt[Tan[c + d*x]] +
2^(1 + n)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(-1 + Tan[(c + d*x)/2])^(-1/2
 - n)*(-((1/2 + n)*AppellF1[1/4, 1/2 + n, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(1/2 +
 n)*(-1 + Tan[(c + d*x)/2])^(-1/2 + n)) - 2*(Cos[c + d*x]*Sec[(c + d*x)/2]^
2)^(1/2 + n)*(-1 + Tan[(c + d*x)/2])^(1/2 + n)*(-(AppellF1[5/4, 1/2 + n, 2,
 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2])/5 + ((1/2 + n)*AppellF1[5/4, 3/2 + n, 1, 9/4, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5) - 2*(1/2 + n)*
AppellF1[1/4, 1/2 + n, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Co
```

```
s[c + d*x]*Sec[(c + d*x)/2]^2)^(-1/2 + n)*(-1 + Tan[(c + d*x)/2])^(1/2 + n)
*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[
(c + d*x)/2]) + (1/2 + n)*(AppellF1[1/2, 1/2 + n, 3/2 + n, 3/2, Tan[(c + d*
x)/2], -Tan[(c + d*x)/2]] + AppellF1[1/2, 3/2 + n, 1/2 + n, 3/2, Tan[(c + d
*x)/2], -Tan[(c + d*x)/2]])*Sec[(c + d*x)/2]^2*(1 - Tan[(c + d*x)/2])^(1/2
+ n)*Tan[(c + d*x)/2]*(-1 + Tan[(c + d*x)/2]^2)^(-1/2 + n) - ((1/2 + n)*(Ap
pellF1[1/2, 1/2 + n, 3/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]] + A
ppellF1[1/2, 3/2 + n, 1/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]])*S
ec[(c + d*x)/2]^2*(1 - Tan[(c + d*x)/2])^(-1/2 + n)*(-1 + Tan[(c + d*x)/2]^
2)^(1/2 + n))/2 + (-((3/2 + n)*AppellF1[3/2, 1/2 + n, 5/2 + n, 5/2, Tan[(c
+ d*x)/2], -Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/6 + ((3/2 + n)*AppellF1[3
/2, 5/2 + n, 1/2 + n, 5/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]]*Sec[(c + d*
x)/2]^2)/6)*(1 - Tan[(c + d*x)/2])^(1/2 + n)*(-1 + Tan[(c + d*x)/2]^2)^(1/2
 + n))*Sqrt[Tan[c + d*x]] + 2^(1 + n)*n*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(
-1 + n)*(-1 + Tan[(c + d*x)/2])^(-1/2 - n)*(-2*AppellF1[1/4, 1/2 + n, 1, 5/
4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^
2)^(1/2 + n)*(-1 + Tan[(c + d*x)/2])^(1/2 + n) + (AppellF1[1/2, 1/2 + n, 3/
2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]] + AppellF1[1/2, 3/2 + n, 1
/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]])*(1 - Tan[(c + d*x)/2])^(
1/2 + n)*(-1 + Tan[(c + d*x)/2]^2)^(1/2 + n))*Sqrt[Tan[c + d*x]]*(-(Cos[(c
+ d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*
Tan[c + d*x]))
```

Maple [F] time = 0.282, size = 0, normalized size = 0.

$$\int (a + a \sec (dx + c))^n (\tan (dx + c))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^(3/2),x)

[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sec (dx + c) + a)^n \tan (dx + c)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sec(dx+c)+a)^n\tan(dx+c)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n\tan(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

3.230 $\int (a+a\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx$

Optimal. Leaf size=114

$$\frac{2^{n+\frac{5}{2}}\tan^{\frac{3}{2}}(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+\frac{3}{2}}(a\sec(c+dx)+a)^nF_1\left(\frac{3}{4};n+\frac{1}{2},1;\frac{7}{4};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{3d}$$

```
[Out] (2^(5/2 + n)*AppellF1[3/4, 1/2 + n, 1, 7/4, -((a - a*Sec[c + d*x])/(a + a*S
ec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*x
])^(-1))^(3/2 + n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x]^(3/2))/(3*d)
```

Rubi [A] time = 0.0583088, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {3889}

$$\frac{2^{n+\frac{5}{2}}\tan^{\frac{3}{2}}(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+\frac{3}{2}}(a\sec(c+dx)+a)^nF_1\left(\frac{3}{4};n+\frac{1}{2},1;\frac{7}{4};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]],x]
```

```
[Out] (2^(5/2 + n)*AppellF1[3/4, 1/2 + n, 1, 7/4, -((a - a*Sec[c + d*x])/(a + a*S
ec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*x
])^(-1))^(3/2 + n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x]^(3/2))/(3*d)
```

Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a + a\sec(c + dx))^n \sqrt{\tan(c + dx)}\, dx = \frac{2^{\frac{5}{2}+n} F_1\left(\frac{3}{4}; \frac{1}{2} + n, 1; \frac{7}{4}; -\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}, \frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{3}{2}+n} (a + a\sec(c +}{3d}$$

Mathematica [B] time = 2.08551, size = 238, normalized size = 2.09

$$\frac{56\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\tan(c+dx)}(a(\sec(c+dx)+1))^n}{d\left(6(\cos(c+dx)-1)\left(2F_1\left(\frac{7}{4}; n+\frac{1}{2}, 2; \frac{11}{4}; \tan^2\left(\frac{1}{2}(c+dx)\right), -\tan^2\left(\frac{1}{2}(c+dx)\right)\right) - (2n+1)F_1\left(\frac{7}{4}; n+\frac{3}{2}, 1; \frac{11}{4}; \tan^2\left(\frac{1}{2}(c+\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]],x]
```

```
[Out] (56*AppellF1[3/4, 1/2 + n, 1, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
*Cos[(c + d*x)/2]^3*(a*(1 + Sec[c + d*x]))^n*Sin[(c + d*x)/2]*Sqrt[Tan[c +
d*x]])/(d*(6*(2*AppellF1[7/4, 1/2 + n, 2, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2] - (1 + 2*n)*AppellF1[7/4, 3/2 + n, 1, 11/4, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + 21*AppellF1[3/4, 1/2 + n, 1,
 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x])))
```

Maple [F] time = 0.28, size = 0, normalized size = 0.

$$\int (a + a\sec(dx + c))^n \sqrt{\tan(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^(1/2),x)
```

```
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx + c) + a)^n \sqrt{\tan(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sec\left(dx+c\right)+a\right)^{n}\sqrt{\tan\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{n}\sqrt{\tan\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**(1/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n*sqrt(tan(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sec\left(dx+c\right)+a\right)^{n}\sqrt{\tan\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

3.231 $\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$

Optimal. Leaf size=111

$$\frac{2^{n+\frac{3}{2}}\sqrt{\tan(c+dx)}\left(\frac{1}{\sec(c+dx)+1}\right)^{n+\frac{1}{2}}(a\sec(c+dx)+a)^n F_1\left(\frac{1}{4};n-\frac{1}{2},1;\frac{5}{4};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{d}$$

```
[Out] (2^(3/2 + n)*AppellF1[1/4, -1/2 + n, 1, 5/4, -((a - a*Sec[c + d*x])/(a + a*
Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*
x])^(-1))^(1/2 + n)*(a + a*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]])/d
```

Rubi [A] time = 0.06267, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {3889}

$$\frac{2^{n+\frac{3}{2}}\sqrt{\tan(c+dx)}\left(\frac{1}{\sec(c+dx)+1}\right)^{n+\frac{1}{2}}(a\sec(c+dx)+a)^n F_1\left(\frac{1}{4};n-\frac{1}{2},1;\frac{5}{4};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]],x]
```

```
[Out] (2^(3/2 + n)*AppellF1[1/4, -1/2 + n, 1, 5/4, -((a - a*Sec[c + d*x])/(a + a*
Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*
x])^(-1))^(1/2 + n)*(a + a*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]])/d
```

Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx = \frac{2^{\frac{3}{2}+n}F_1\left(\frac{1}{4};-\frac{1}{2}+n,1;\frac{5}{4};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{1}{2}+n}(a+a\sec(c+dx))^n\sqrt{\mathrm{t}}}{d}$$

Mathematica [B] time = 1.37678, size = 229, normalized size = 2.06

$$\frac{10\cos(c+dx)(\cos(c+dx)+1)\sqrt{\tan(c+dx)}(a(\sec(c+dx)+1))^nF_1}{d\left(2(\cos(c+dx)-1)\left(2F_1\left(\frac{5}{4};n-\frac{1}{2},2;\frac{9}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+(1-2n)F_1\left(\frac{5}{4};n+\frac{1}{2},1;\frac{9}{4};\tan^2\left(\frac{1}{2}(c+\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]],x]
```

```
[Out] (10*AppellF1[1/4, -1/2 + n, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
]*Cos[c + d*x]*(1 + Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^n*Sqrt[Tan[c + d*x
]])/(d*(2*(2*AppellF1[5/4, -1/2 + n, 2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2] + (1 - 2*n)*AppellF1[5/4, 1/2 + n, 1, 9/4, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + 5*AppellF1[1/4, -1/2 + n, 1, 5/4
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x])))
```

Maple [F] time = 0.288, size = 0, normalized size = 0.

$$\int (a+a\sec(dx+c))^n\frac{1}{\sqrt{\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n/tan(d*x+c)^(1/2),x)
```

```
[Out] int((a+a*sec(d*x+c))^n/tan(d*x+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^n}{\sqrt{\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(a\sec(dx+c)+a)^n}{\sqrt{\tan(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a(\sec(c+dx)+1))^n}{\sqrt{\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n/tan(d*x+c)**(1/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n/sqrt(tan(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a\sec(dx+c)+a)^n}{\sqrt{\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

3.232 $\int \frac{(a+a\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=112

$$-\frac{2^{n+\frac{1}{2}}\left(\frac{1}{\sec(c+dx)+1}\right)^{n-\frac{1}{2}}(a\sec(c+dx)+a)^n F_1\left(-\frac{1}{4};n-\frac{3}{2},1;\frac{3}{4};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{d\sqrt{\tan(c+dx)}}$$

```
[Out] -((2^(1/2 + n)*AppellF1[-1/4, -3/2 + n, 1, 3/4, -((a - a*Sec[c + d*x])/(a +
 a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c +
 d*x])^(-1))^(-1/2 + n)*(a + a*Sec[c + d*x])^n)/(d*Sqrt[Tan[c + d*x]]))
```

Rubi [A] time = 0.0673982, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {3889}

$$-\frac{2^{n+\frac{1}{2}}\left(\frac{1}{\sec(c+dx)+1}\right)^{n-\frac{1}{2}}(a\sec(c+dx)+a)^n F_1\left(-\frac{1}{4};n-\frac{3}{2},1;\frac{3}{4};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n/Tan[c + d*x]^(3/2),x]
```

```
[Out] -((2^(1/2 + n)*AppellF1[-1/4, -3/2 + n, 1, 3/4, -((a - a*Sec[c + d*x])/(a +
 a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c +
 d*x])^(-1))^(-1/2 + n)*(a + a*Sec[c + d*x])^n)/(d*Sqrt[Tan[c + d*x]]))
```

Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2^{\frac{1}{2}+n}F_1\left(-\frac{1}{4};-\frac{3}{2}+n,1;\frac{3}{4};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{-\frac{1}{2}+n}(a+a\sec(c+dx))^n}{d\sqrt{\tan(c+dx)}}$$

Mathematica [B] time = 15.55, size = 2164, normalized size = 19.32

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n/Tan[c + d*x]^(3/2),x]
```

```
[Out] -(2^(1/2 + n)*Cot[c + d*x]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(Cos[(c +
d*x)/2]^2*Sec[c + d*x])^n*(a*(1 + Sec[c + d*x]))^n*(21*Hypergeometric2F1[-1
/4, -1/2 + n, 3/4, Tan[(c + d*x)/2]^2] + 7*AppellF1[3/4, -1/2 + n, 1, 7/4,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 + 7*Hypergeomet
ric2F1[3/4, -1/2 + n, 7/4, Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 - 3*Appel
lF1[7/4, -1/2 + n, 1, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c
 + d*x)/2]^4))/(21*d*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*((2^(-1/2 + n)*(
Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*Sec[c + d*x]^2*(Cos[(c + d*x)/2]^2*Sec[c
 + d*x])^n*(21*Hypergeometric2F1[-1/4, -1/2 + n, 3/4, Tan[(c + d*x)/2]^2] +
 7*AppellF1[3/4, -1/2 + n, 1, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
*Tan[(c + d*x)/2]^2 + 7*Hypergeometric2F1[3/4, -1/2 + n, 7/4, Tan[(c + d*x)
/2]^2]*Tan[(c + d*x)/2]^2 - 3*AppellF1[7/4, -1/2 + n, 1, 11/4, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^4))/(21*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Tan[c + d*x]^(3/2)) + (2^(-1/2 + n)*(Cos[c + d*x]*Sec[(c +
 d*x)/2]^2)^n*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*((Cos[c + d*x]*Sin[c + d*
x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x]))*(21*Hypergeomet
ric2F1[-1/4, -1/2 + n, 3/4, Tan[(c + d*x)/2]^2] + 7*AppellF1[3/4, -1/2 + n,
 1, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 + 7*Hy
pergeometric2F1[3/4, -1/2 + n, 7/4, Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2
- 3*AppellF1[7/4, -1/2 + n, 1, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Tan[(c + d*x)/2]^4))/(21*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*Sqrt[Ta
n[c + d*x]]) - (2^(1/2 + n)*n*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(Cos[(c +
 d*x)/2]^2*Sec[c + d*x])^(1 + n)*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[
c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(21*Hypergeometric2F1[-1/4, -
1/2 + n, 3/4, Tan[(c + d*x)/2]^2] + 7*AppellF1[3/4, -1/2 + n, 1, 7/4, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 + 7*Hypergeometric2F
1[3/4, -1/2 + n, 7/4, Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 - 3*AppellF1[7
/4, -1/2 + n, 1, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*
x)/2]^4))/(21*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Tan[c + d*x]]) - (
```

```
2^(1/2 + n)*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(Cos[(c + d*x)/2]^2*Sec[c +
 d*x])^n*(7*AppellF1[3/4, -1/2 + n, 1, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d
*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 7*Hypergeometric2F1[3/4, -1
/2 + n, 7/4, Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] - 6*Ap
pellF1[7/4, -1/2 + n, 1, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2]^3 + 7*Tan[(c + d*x)/2]^2*((-3*AppellF1[7/4
, -1/2 + n, 2, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2])/7 + (3*(-1/2 + n)*AppellF1[7/4, 1/2 + n, 1, 11/4, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]
)/7) - 3*Tan[(c + d*x)/2]^4*((-7*AppellF1[11/4, -1/2 + n, 2, 15/4, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/11 +
(7*(-1/2 + n)*AppellF1[11/4, 1/2 + n, 1, 15/4, Tan[(c + d*x)/2]^2, -Tan[(c
+ d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/11) + (21*Csc[(c + d*x)/2
]*Sec[(c + d*x)/2]*(Hypergeometric2F1[-1/4, -1/2 + n, 3/4, Tan[(c + d*x)/2]
^2] - (1 - Tan[(c + d*x)/2]^2)^(1/2 - n)))/4 + (21*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2]*(-Hypergeometric2F1[3/4, -1/2 + n, 7/4, Tan[(c + d*x)/2]^2] + (
1 - Tan[(c + d*x)/2]^2)^(1/2 - n)))/4))/(21*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[Tan[c + d*x]]) - (2^(1/2 + n)*n*(Cos[c + d*x]*Sec[(c + d*x)/2]^
2)^n*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n)*(21*Hypergeometric2F1[-1/4,
 -1/2 + n, 3/4, Tan[(c + d*x)/2]^2] + 7*AppellF1[3/4, -1/2 + n, 1, 7/4, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 + 7*Hypergeometric
2F1[3/4, -1/2 + n, 7/4, Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 - 3*AppellF1
[7/4, -1/2 + n, 1, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c +
d*x)/2]^4)*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*
x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(21*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]
)]*Sqrt[Tan[c + d*x]])))
```

Maple [F] time = 0.256, size = 0, normalized size = 0.

$$\int (a + a \sec (dx + c))^n (\tan (dx + c))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n/tan(d*x+c)^(3/2),x)
```

```
[Out] int((a+a*sec(d*x+c))^n/tan(d*x+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^n}{\tan(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(a\sec(dx+c)+a)^n}{\tan(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n/tan(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^n}{\tan(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

3.233 $\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=320

$$-\frac{a\sqrt{\sin(2c+2dx)}\tan^2(c+dx)\sec(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)(e\cot(c+dx))^{5/2}}{3d}+\frac{a\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\text{t}}{\sqrt{2}\text{d}}$$

```
[Out] (-2*(e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])*Tan[c + d*x])/(3*d) - (a*(e
*Cot[c + d*x])^(5/2)*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c
 + 2*d*x]]*Tan[c + d*x]^2)/(3*d) + (a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
]*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))/(Sqrt[2]*d) - (a*ArcTan[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))/(Sqrt[
2]*d) + (a*(e*Cot[c + d*x])^(5/2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[
c + d*x]]*Tan[c + d*x]^(5/2))/(2*Sqrt[2]*d) - (a*(e*Cot[c + d*x])^(5/2)*Log
[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(5/2))/(2*Sqrt
[2]*d)
```

Rubi [A] time = 0.263523, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {3900, 3882, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$\frac{a\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}{\sqrt{2}d}-\frac{a\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx}{\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x]),x]
```


```
[Out] (-2*(e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])*Tan[c + d*x])/(3*d) - (a*(e
*Cot[c + d*x])^(5/2)*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c
 + 2*d*x]]*Tan[c + d*x]^2)/(3*d) + (a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
]*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))/(Sqrt[2]*d) - (a*ArcTan[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))/(Sqrt[
2]*d) + (a*(e*Cot[c + d*x])^(5/2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[
c + d*x]]*Tan[c + d*x]^(5/2))/(2*Sqrt[2]*d) - (a*(e*Cot[c + d*x])^(5/2)*Log
[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(5/2))/(2*Sqrt
[2]*d)
```


Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \left((e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\int \frac{a+a\sec(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

$$= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} + \frac{1}{3}\Big(2(e\cot(c+dx))^{5/2}$$

$$= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} - \frac{1}{3}\Big(a(e\cot(c+dx))^{5/2}$$

$$= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} - \frac{\Big(a(e\cot(c+dx))^{5/2}\,\mathrm{si}}{3}$$

$$= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} - \frac{1}{3}\Big(a(e\cot(c+dx))^{5/2}$$

$$= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} - \frac{a(e\cot(c+dx))^{5/2}F\Big(}{}$$

$$= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} - \frac{a(e\cot(c+dx))^{5/2}F\Big(}{}$$

$$= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} - \frac{a(e\cot(c+dx))^{5/2}F\Big(}{}$$

$$= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} - \frac{a(e\cot(c+dx))^{5/2}F\Big(}{}$$

Mathematica [C] time = 3.67991, size = 185, normalized size = 0.58

$$-\frac{a\sec(c+dx)(e\cot(c+dx))^{5/2}\left(\sqrt{\cot(c+dx)}\left(-3\sqrt{\sin(2(c+dx))}\sin^{-1}(\cos(c+dx)-\sin(c+dx))+4(\cos(c+dx)+1\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] -(a*(e*Cot[c + d*x])^(5/2)*Sec[c + d*x]*(Sqrt[Cot[c + d*x]]*(4*(1 + Cos[c +
 d*x])*Cot[c + d*x] - 3*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[Sin[2*(c +
```

```
 d*x)]] + 3*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[
Sin[2*(c + d*x)]]) + 2*(-1)^(1/4)*Sqrt[Csc[c + d*x]^2]*EllipticF[I*ArcSinh[
(-1)^(1/4)*Sqrt[Cot[c + d*x]]], -1]*Sin[2*(c + d*x)]))/(6*d*Cot[c + d*x]^(5
/2))
```

Maple [C] time = 0.304, size = 658, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/6*a/d*2^(1/2)*(-1+cos(d*x+c))*(3*I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c
)-sin(d*x+c))/sin(d*x+c))^(1/2)-3*I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)
-sin(d*x+c))/sin(d*x+c))^(1/2)+3*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/si
n(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-si
n(d*x+c))/sin(d*x+c))^(1/2)+3*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d
*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d
*x+c))/sin(d*x+c))^(1/2)-4*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c
))^(1/2),1/2*2^(1/2))*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x
+c))^(1/2)+2*cos(d*x+c)*2^(1/2))*(cos(d*x+c)+1)^2*(e*cos(d*x+c)/sin(d*x+c))
^(5/2)/cos(d*x+c)^3/sin(d*x+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(5/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cot(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a), x)
```

3.234 $\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=346

$$\frac{a\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}{\sqrt{2}d}-\frac{a\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx}{\sqrt{2}d}$$

```
[Out] (-2*(e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])*Tan[c + d*x])/d - (2*a*(e*C
ot[c + d*x])^(3/2)*EllipticE[c - Pi/4 + d*x, 2]*Sin[c + d*x]*Tan[c + d*x])/
(d*Sqrt[Sin[2*c + 2*d*x]]) + (a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*(e*C
ot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))/(Sqrt[2]*d) - (a*ArcTan[1 + Sqrt[2]*
Sqrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))/(Sqrt[2]*d)
- (a*(e*Cot[c + d*x])^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*
x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*d) + (a*(e*Cot[c + d*x])^(3/2)*Log[1 + S
qrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*d)
 + (2*a*(e*Cot[c + d*x])^(3/2)*Sin[c + d*x]*Tan[c + d*x]^2)/d
```

Rubi [A] time = 0.279405, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 15, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {3900, 3882, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$\frac{a\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}{\sqrt{2}d}-\frac{a\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx}{\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*(e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])*Tan[c + d*x])/d - (2*a*(e*C
ot[c + d*x])^(3/2)*EllipticE[c - Pi/4 + d*x, 2]*Sin[c + d*x]*Tan[c + d*x])/
(d*Sqrt[Sin[2*c + 2*d*x]]) + (a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*(e*C
ot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))/(Sqrt[2]*d) - (a*ArcTan[1 + Sqrt[2]*
Sqrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))/(Sqrt[2]*d)
- (a*(e*Cot[c + d*x])^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*
x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*d) + (a*(e*Cot[c + d*x])^(3/2)*Log[1 + S
qrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*d)
 + (2*a*(e*Cot[c + d*x])^(3/2)*Sin[c + d*x]*Tan[c + d*x]^2)/d
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int \frac{a+a\sec(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} + \left(2(e\cot(c+dx))^{3/2}\,\text{ta}\right.$$

$$= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} - \left(a(e\cot(c+dx))^{3/2}\,\text{ta}\right.$$

$$= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} + \frac{2a(e\cot(c+dx))^{3/2}\,\text{si}}{}$$

$$= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} + \frac{2a(e\cot(c+dx))^{3/2}\,\text{si}}{}$$

$$= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} + \frac{2a(e\cot(c+dx))^{3/2}\,\text{si}}{}$$

$$= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} - \frac{2a(e\cot(c+dx))^{3/2}E\,(}{}$$

$$= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} - \frac{2a(e\cot(c+dx))^{3/2}E\,(}{}$$

$$= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} - \frac{2a(e\cot(c+dx))^{3/2}E\,(}{}$$

Mathematica [C] time = 1.14172, size = 191, normalized size = 0.55

$$\frac{ae(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{e\cot(c+dx)}\left(8\cot^2(c+dx)\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\cot^2(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*e*(1 + Cos[c + d*x])*Sqrt[e*Cot[c + d*x]]*Sec[(c + d*x)/2]^2*Sec[c + d*x
]*(8*Cot[c + d*x]^2*Hypergeometric2F1[3/4, 3/2, 7/4, -Cot[c + d*x]^2] + 3*S
qrt[Csc[c + d*x]^2]*(-4*Cos[c + d*x] - 4*Cos[c + d*x]^2 + ArcSin[Cos[c + d*
x] - Sin[c + d*x]]*Sqrt[Sin[2*(c + d*x)]] + Log[Cos[c + d*x] + Sin[c + d*x]
 + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]])))/(12*d*Sqrt[Csc[c + d*x
]^2])
```

Maple [C] time = 0.364, size = 1390, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] -1/2*a/d*2^(1/2)*(I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Elli
pticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*
cos(d*x+c)-I*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/
2))+I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-cos(d*x+c)*((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*((1-cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
),1/2+1/2*I,1/2*2^(1/2))-4*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^
(1/2))+2*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Elli
pticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-((1-cos(d*x
```

```
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
,1/2+1/2*I,1/2*2^(1/2))-4*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1
+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+2*((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+4*cos(d*x+c)*2^(1/2))*sin(d*x+c)*(
e*cos(d*x+c)/sin(d*x+c))^(3/2)/cos(d*x+c)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(3/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cot(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(3/2)*(a*sec(d*x + c) + a), x)
```

3.235 $\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=274

$$\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)\sqrt{e\cot(c+dx)}}{d}-\frac{a\sqrt{\tan(c+dx)}\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e}}{\sqrt{2}d}$$

```
[Out] (a*Sqrt[e*Cot[c + d*x]]*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[
2*c + 2*d*x]])/d - (a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[e*Cot[c +
 d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[2]*d) + (a*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c +
 d*x]]]*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[2]*d) - (a*Sqrt[e*Co
t[c + d*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[Tan[c +
 d*x]])/(2*Sqrt[2]*d) + (a*Sqrt[e*Cot[c + d*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]]*Sqrt[Tan[c + d*x]])/(2*Sqrt[2]*d)
```

Rubi [A] time = 0.201291, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3900, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$-\frac{a\sqrt{\tan(c+dx)}\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}}{\sqrt{2}d}+\frac{a\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\sqrt{\tan(c+dx)}\sqrt{e\cot(c+}}{\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x]),x]
```


```
[Out] (a*Sqrt[e*Cot[c + d*x]]*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[
2*c + 2*d*x]])/d - (a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[e*Cot[c +
 d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[2]*d) + (a*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c +
 d*x]]]*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[2]*d) - (a*Sqrt[e*Co
t[c + d*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[Tan[c +
 d*x]])/(2*Sqrt[2]*d) + (a*Sqrt[e*Cot[c + d*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]]*Sqrt[Tan[c + d*x]])/(2*Sqrt[2]*d)
```


Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&& !IntegerQ[m]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))\,dx = \left(\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{a+a\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx$$

$$= \left(a\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{1}{\sqrt{\tan(c+dx)}}\,dx + \left(a\sqrt{e\cot(c+dx)}\sqrt{\tan(c\right.$$

$$= \frac{\left(a\sqrt{e\cot(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(a\sqrt{e\cot(c+dx)}\sqrt{\mathrm{ta}\right.}{}$$

$$= \left(a\sqrt{e\cot(c+dx)}\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx + \frac{\left(2a\sqrt{e\,\mathrm{c}\right.}{}$$

$$= \frac{a\sqrt{e\cot(c+dx)}F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d} + \frac{\left(a\sqrt{e\cot(c+d\right.}{}$$

$$= \frac{a\sqrt{e\cot(c+dx)}F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d} + \frac{\left(a\sqrt{e\cot(c+d\right.}{}$$

$$= \frac{a\sqrt{e\cot(c+dx)}F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d} - \frac{a\sqrt{e\cot(c+dx}}{}$$

$$= \frac{a\sqrt{e\cot(c+dx)}F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d} - \frac{a\tan^{-1}\left(1-\sqrt{2}\right.}{}$$

<u>Mathematica</u> [C] time = 1.74716, size = 169, normalized size = 0.62

$$\frac{a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{e\cot(c+dx)}\left(\sqrt{\sin(2(c+dx))}\sqrt{\csc^2(c+dx)}\left(\log\left(\sin(c+dx)+\sqrt{\sin(2\right.\right.\right.}{4d\sqrt{\mathrm{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sqrt[e*Cot[c + d*x]]*Sec[(c + d*x)/2]^2*Sec[c + d*x]*
(4*(-1)^(1/4)*Sqrt[Cot[c + d*x]]*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Cot[c
+ d*x]]], -1] + Sqrt[Csc[c + d*x]^2]*(-ArcSin[Cos[c + d*x] - Sin[c + d*x]]
+ Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]])*Sqrt[Sin[2*(c
+ d*x)]]))/(4*d*Sqrt[Csc[c + d*x]^2])
```

Maple [C] time = 0.261, size = 289, normalized size = 1.1

$$-\frac{a\sqrt{2}\left(-1+\cos \left(dx+c\right)\right)\left(\cos \left(dx+c\right)+1\right)^{2}}{2\,d\left(\sin \left(dx+c\right)\right)^{2}\cos \left(dx+c\right)}\sqrt{\frac{e\cos \left(dx+c\right)}{\sin \left(dx+c\right)}}\sqrt{-\frac{-1+\cos \left(dx+c\right)-\sin \left(dx+c\right)}{\sin \left(dx+c\right)}}\sqrt{\frac{-1+\cos \left(dx+c\right)+}{\sin \left(dx+c\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*cot(d*x+c))^(1/2),x)
```

```
[Out] -1/2*a/d*2^(1/2)*(e*cos(d*x+c)/sin(d*x+c))^(1/2)*(-1+cos(d*x+c))*(-(-1+cos(
d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(I*EllipticPi((-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi((-(-1+cos(
d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+EllipticPi((-(-
1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+EllipticP
i((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)))/si
n(d*x+c)^2/cos(d*x+c)*(cos(d*x+c)+1)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \sqrt{e\cot(c+dx)}\,dx + \int \sqrt{e\cot(c+dx)}\sec(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*cot(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(sqrt(e*cot(c + d*x)), x) + Integral(sqrt(e*cot(c + d*x))*sec(c
+ d*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\cot(dx+c)}(a\sec(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a), x)
```

3.236 $\int \frac{a+a\sec(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$

Optimal. Leaf size=299

$$\frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}-\frac{a\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{a\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{a\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}\right.}{2\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

```
[Out] (2*a*Sin[c + d*x])/(d*Sqrt[e*Cot[c + d*x]]) - (2*a*Cos[c + d*x]*EllipticE[c
 - Pi/4 + d*x, 2])/(d*Sqrt[e*Cot[c + d*x]]*Sqrt[Sin[2*c + 2*d*x]]) - (a*Arc
Tan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d*Sqrt[e*Cot[c + d*x]]*Sqrt[T
an[c + d*x]]) + (a*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d*Sqrt[
e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) + (a*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
 + Tan[c + d*x]])/(2*Sqrt[2]*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) - (
a*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d*Sqrt[e*C
ot[c + d*x]]*Sqrt[Tan[c + d*x]])
```

Rubi [A] time = 0.233319, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {3900, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$\frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}-\frac{a\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{a\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{a\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}\right.}{2\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (2*a*Sin[c + d*x])/(d*Sqrt[e*Cot[c + d*x]]) - (2*a*Cos[c + d*x]*EllipticE[c
 - Pi/4 + d*x, 2])/(d*Sqrt[e*Cot[c + d*x]]*Sqrt[Sin[2*c + 2*d*x]]) - (a*Arc
Tan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d*Sqrt[e*Cot[c + d*x]]*Sqrt[T
an[c + d*x]]) + (a*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d*Sqrt[
e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) + (a*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
 + Tan[c + d*x]])/(2*Sqrt[2]*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) - (
a*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d*Sqrt[e*C
ot[c + d*x]]*Sqrt[Tan[c + d*x]])
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
```

```
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx = \frac{\int (a+a\sec(c+dx))\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$
$$= \frac{a\int \sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{a\int \sec(c+dx)\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$
$$= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{(2a)\int \cos(c+dx)\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$
$$= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{\left(2a\sqrt{\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\sin(c+dx)}} + \frac{(2a)\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,\right.}{d\sqrt{e\cot(c+dx)}\sqrt{}}$$
$$= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{(2a\cos(c+dx))\int \sqrt{\sin(2c+2dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} - \frac{a\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx}\right.}{d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$
$$= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} + \frac{a\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right.}{2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$
$$= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} + \frac{a\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx\right.}{2\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$
$$= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} - \frac{a\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{a\,\text{ta}}{\sqrt{2}d}$$

Mathematica [C] time = 1.5776, size = 189, normalized size = 0.63

$$\frac{a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(8\cot^3(c+dx)\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\cot^2(c+dx)\right)-3\cot(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sec[c + d*x]*(8*Cot[c + d*x]^3*Hyp
ergeometric2F1[3/4, 3/2, 7/4, -Cot[c + d*x]^2] - 3*Cot[c + d*x]*Sqrt[Csc[c
+ d*x]^2]*(-2 + 2*Cos[2*(c + d*x)] + ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sq
rt[Sin[2*(c + d*x)]] + Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*
x)]]]*Sqrt[Sin[2*(c + d*x)]])))/(12*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Csc[c + d*x
]^2])
```

Maple [C] time = 0.313, size = 1429, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x)
```

```
[Out] -1/2*a/d*2^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*(-I*cos(d*x+c)*(-(-1+co
s(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+I*cos(d*x+c)*(-(-1+cos(d
*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d
*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+(-(-1+cos(d*x+c)-sin(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c
))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)+(-(-1+cos(d*x+c)-sin(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1
/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)-4*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*
x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),
1/2*2^(1/2))*cos(d*x+c)+2*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*c
os(d*x+c)-I*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi
((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+I*(-
(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x
+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+(-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/
sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2
),1/2+1/2*I,1/2*2^(1/2))-4*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+
2*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos(
d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+2*cos(d*x+c)*2^(1/2)-2*2^
```

```
(1/2))/sin(d*x+c)^5/(e*cos(d*x+c)/sin(d*x+c))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{1}{\sqrt{e\cot(c+dx)}}\,dx + \int \frac{\sec(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(1/sqrt(e*cot(c + d*x)), x) + Integral(sec(c + d*x)/sqrt(e*cot(c
 + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sec(dx+c)+a}{\sqrt{e\cot(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(e*cot(d*x + c)), x)
```

3.237 $\int \frac{a+a\sec(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=320

$$-\frac{a\sqrt{\sin(2c+2dx)}\cot(c+dx)\csc(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3d(e\cot(c+dx))^{3/2}}+\frac{a\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{a\tan^{-1}\left(\sqrt{\right.}{\sqrt{2}d\tan^{\frac{3}{2}}(c}$$

```
[Out] (2*Cot[c + d*x]*(3*a + a*Sec[c + d*x]))/(3*d*(e*Cot[c + d*x])^(3/2)) - (a*C
ot[c + d*x]*Csc[c + d*x]*EllipticF[c - Pi/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x]
])/(3*d*(e*Cot[c + d*x])^(3/2)) + (a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]
)/(Sqrt[2]*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) - (a*ArcTan[1 + Sqr
t[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3
/2)) + (a*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d*
(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) - (a*Log[1 + Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(
3/2))
```

Rubi [A] time = 0.251212, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {3900, 3881, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$\frac{a\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{a\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}+\frac{2\cot(c+dx)(a\sec(c+dx)+3a)}{3d(e\cot(c+dx))^{3/2}}+\frac{a\log\left(\text{ta}\right.}{2\sqrt{2}a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/(e*Cot[c + d*x])^(3/2),x]
```


```
[Out] (2*Cot[c + d*x]*(3*a + a*Sec[c + d*x]))/(3*d*(e*Cot[c + d*x])^(3/2)) - (a*C
ot[c + d*x]*Csc[c + d*x]*EllipticF[c - Pi/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x]
])/(3*d*(e*Cot[c + d*x])^(3/2)) + (a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]
)/(Sqrt[2]*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) - (a*ArcTan[1 + Sqr
t[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3
/2)) + (a*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d*
(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) - (a*Log[1 + Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(
3/2))
```


Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \frac{\int (a+a\sec(c+dx))\tan^{\frac{3}{2}}(c+dx)\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{2\int \frac{\frac{3a}{2}+\frac{1}{2}a\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{3(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{a\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{3(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} - \frac{a\int \frac{1}{\sqrt{\tan(c+dx)}}\,d}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}}$$

$$= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{\left(a\cos^{\frac{3}{2}}(c+dx)\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{3(e\cot(c+dx))^{3/2}\sin^{\frac{3}{2}}(c+dx)} - \frac{a\,\mathrm{Subst}\left(\int \frac{\ }{\sqrt{}}\right.}{d(e\cot(c}$$

$$= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{\left(a\cot(c+dx)\csc(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}}{3(e\cot(c+dx))^{3/2}}$$

$$= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{a\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(2c+2dx}}{3d(e\cot(c+dx))^{3/2}}$$

$$= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{a\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(2c+2dx}}{3d(e\cot(c+dx))^{3/2}}$$

$$= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{a\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(2c+2dx}}{3d(e\cot(c+dx))^{3/2}}$$

$$= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{a\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(2c+2dx}}{3d(e\cot(c+dx))^{3/2}}$$

Mathematica [C] time = 2.41471, size = 224, normalized size = 0.7

$$\frac{a(\cos(c+dx)+1)\cos(2(c+dx))\csc(c+dx)\sqrt{\csc^2(c+dx)}\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\csc^2(c+dx)}\left(12\cos(c+dx)+3\sqrt{\sin(2(}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Cos[2*(c + d*x)]*Csc[c + d*x]*Sqrt[Csc[c + d*x]^2]*Se
c[(c + d*x)/2]^2*(-4*(-1)^(1/4)*Cot[c + d*x]^(3/2)*EllipticF[I*ArcSinh[(-1)
^(1/4)*Sqrt[Cot[c + d*x]]], -1] + Sqrt[Csc[c + d*x]^2]*(4 + 12*Cos[c + d*x]
 + 3*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Cot[c + d*x]*Sqrt[Sin[2*(c + d*x)]
] - 3*Cot[c + d*x]*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]
]*Sqrt[Sin[2*(c + d*x)]])))/(12*d*(e*Cot[c + d*x])^(3/2)*(-1 + Cot[c + d*x]
^2))
```

Maple [C] time = 0.258, size = 688, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*cot(d*x+c))^(3/2),x)
```

```
[Out] 1/6*a/d*2^(1/2)*(-1+cos(d*x+c))*(3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-3*I*EllipticPi(((1-cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+3*sin(d*x+c)*cos(d*x+c)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-4*sin(d*x+c)*cos(d*x+c)*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+3*sin(d*x+c)*cos(d*x+c)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+6*cos(d*x+c)^2*2^(1/2)-4*cos(
d*x+c)*2^(1/2)-2*2^(1/2))*(cos(d*x+c)+1)^2/(e*cos(d*x+c)/sin(d*x+c))^(3/2)/
sin(d*x+c)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{1}{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx+\int \frac{\sec\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))**(3/2),x)
```

```
[Out] a*(Integral((e*cot(c + d*x))**(-3/2), x) + Integral(sec(c + d*x)/(e*cot(c +
 d*x))**(3/2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sec\left(dx+c\right)+a}{\left(e\cot\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/(e*cot(d*x + c))^(3/2), x)
```

3.238 $\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=357

$$-\frac{2a^2\sqrt{\sin(2c+2dx)}\tan^2(c+dx)\sec(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)(e\cot(c+dx))^{5/2}}{3d}+\frac{a^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+a}\right.}{}$$

```
[Out] (-4*a^2*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x])/(3*d) - (4*a^2*(e*Cot[c + d*x]
)^(5/2)*Sec[c + d*x]*Tan[c + d*x])/(3*d) - (2*a^2*(e*Cot[c + d*x])^(5/2)*El
lipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]]*Tan[c + d*x]
^2)/(3*d) + (a^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(5
/2)*Tan[c + d*x]^(5/2))/(Sqrt[2]*d) - (a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c +
d*x]]]*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))/(Sqrt[2]*d) + (a^2*(e*Cot
[c + d*x])^(5/2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c +
 d*x]^(5/2))/(2*Sqrt[2]*d) - (a^2*(e*Cot[c + d*x])^(5/2)*Log[1 + Sqrt[2]*Sq
rt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(5/2))/(2*Sqrt[2]*d)
```

Rubi [A] time = 0.335922, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.68, Rules used = {3900, 3886, 3474, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2609, 2614, 2573, 2641, 2607, 30}

$$\frac{a^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}{\sqrt{2}d}-\frac{a^2\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\tan^{\frac{5}{2}}(c+dx)(e\cot(c-}{\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (-4*a^2*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x])/(3*d) - (4*a^2*(e*Cot[c + d*x]
)^(5/2)*Sec[c + d*x]*Tan[c + d*x])/(3*d) - (2*a^2*(e*Cot[c + d*x])^(5/2)*El
lipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]]*Tan[c + d*x]
^2)/(3*d) + (a^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(5
/2)*Tan[c + d*x]^(5/2))/(Sqrt[2]*d) - (a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c +
d*x]]]*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))/(Sqrt[2]*d) + (a^2*(e*Cot
[c + d*x])^(5/2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c +
 d*x]^(5/2))/(2*Sqrt[2]*d) - (a^2*(e*Cot[c + d*x])^(5/2)*Log[1 + Sqrt[2]*Sq
rt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(5/2))/(2*Sqrt[2]*d)
```


Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
```

```
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \left((e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\int \frac{(a+a\sec(c+dx))^2}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \left((e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\int\left(\frac{a^2}{\tan^{\frac{5}{2}}(c+dx)} + \frac{2a^2\sec(c+dx)}{\tan^{\frac{5}{2}}(c+dx)} + \frac{a^2}{\mathrm{t}}\right.$$

$$= \left(a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)}\,dx + \left(a^2(e\cot(c+dx)\right.$$

$$= -\frac{2a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} - \frac{4a^2(e\cot(c+dx))^{5/2}\sec(c+dx)\tan(}{3d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} - \frac{4a^2(e\cot(c+dx))^{5/2}\sec(c+dx)\tan(}{3d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} - \frac{4a^2(e\cot(c+dx))^{5/2}\sec(c+dx)\tan(}{3d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} - \frac{4a^2(e\cot(c+dx))^{5/2}\sec(c+dx)\tan(}{3d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} - \frac{4a^2(e\cot(c+dx))^{5/2}\sec(c+dx)\tan(}{3d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} - \frac{4a^2(e\cot(c+dx))^{5/2}\sec(c+dx)\tan(}{3d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} - \frac{4a^2(e\cot(c+dx))^{5/2}\sec(c+dx)\tan(}{3d}$$

Mathematica [C] time = 2.21904, size = 93, normalized size = 0.26

$$-\frac{2a^2e\cos^4\left(\frac{1}{2}(c+dx)\right)(e\cot(c+dx))^{3/2}\sec^4\left(\frac{1}{2}\cot^{-1}(\cot(c+dx))\right)\left(2\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-\tan^2(c+dx)\right)\right.}{3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*a^2*e*Cos[(c + d*x)/2]^4*(e*Cot[c + d*x])^(3/2)*(2 + 2*Hypergeometric2F
1[-3/4, 1/2, 1/4, -Tan[c + d*x]^2] - Hypergeometric2F1[3/4, 1, 7/4, -Cot[c
```

```
+ d*x]^2])*Sec[ArcCot[Cot[c + d*x]]/2]^4)/(3*d)
```

Maple [C] time = 0.27, size = 650, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6*a^2/d*2^(1/2)*(-1+cos(d*x+c))*(3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*sin(d*x+c)-3*I*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
),1/2+1/2*I,1/2*2^(1/2))+3*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1
/2*I,1/2*2^(1/2))+3*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/
2*2^(1/2))-2*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*
EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+4*cos(d
*x+c)*2^(1/2))*(e*cos(d*x+c)/sin(d*x+c))^(5/2)*(cos(d*x+c)+1)^2/sin(d*x+c)/
cos(d*x+c)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(5/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cot(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2, x)
```

3.239 $\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=343

$$-\frac{4a^2\sin(c+dx)(e\cot(c+dx))^{3/2}}{d}+\frac{a^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}{\sqrt{2}d}-\frac{a^2\tan^{-1}\left(\sqrt{2}\sqrt{\tan\right.}}{}$$

```
[Out] (-4*a^2*(e*Cot[c + d*x])^(3/2)*Sin[c + d*x])/d - (4*a^2*(e*Cot[c + d*x])^(3
/2)*Tan[c + d*x])/d - (4*a^2*(e*Cot[c + d*x])^(3/2)*EllipticE[c - Pi/4 + d*
x, 2]*Sin[c + d*x]*Tan[c + d*x])/(d*Sqrt[Sin[2*c + 2*d*x]]) + (a^2*ArcTan[1
 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))/(
Sqrt[2]*d) - (a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(
3/2)*Tan[c + d*x]^(3/2))/(Sqrt[2]*d) - (a^2*(e*Cot[c + d*x])^(3/2)*Log[1 -
Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*d
) + (a^2*(e*Cot[c + d*x])^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
+ d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*d)
```

Rubi [A] time = 0.329255, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.68, Rules used = {3900, 3886, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2608, 2615, 2572, 2639, 2607, 30}

$$-\frac{4a^2\sin(c+dx)(e\cot(c+dx))^{3/2}}{d}+\frac{a^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}{\sqrt{2}d}-\frac{a^2\tan^{-1}\left(\sqrt{2}\sqrt{\tan\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (-4*a^2*(e*Cot[c + d*x])^(3/2)*Sin[c + d*x])/d - (4*a^2*(e*Cot[c + d*x])^(3
/2)*Tan[c + d*x])/d - (4*a^2*(e*Cot[c + d*x])^(3/2)*EllipticE[c - Pi/4 + d*
x, 2]*Sin[c + d*x]*Tan[c + d*x])/(d*Sqrt[Sin[2*c + 2*d*x]]) + (a^2*ArcTan[1
 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))/(
Sqrt[2]*d) - (a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(
3/2)*Tan[c + d*x]^(3/2))/(Sqrt[2]*d) - (a^2*(e*Cot[c + d*x])^(3/2)*Log[1 -
Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*d
) + (a^2*(e*Cot[c + d*x])^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
+ d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*d)
```


Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2608

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(n + 1)), x] - Dist[(a^2*(m - 2))/(b^2*(n + 1)), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
```

```
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = \left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int \frac{(a+a\sec(c+dx))^2}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int \left(\frac{a^2}{\tan^{\frac{3}{2}}(c+dx)} + \frac{2a^2\sec(c+dx)}{\tan^{\frac{3}{2}}(c+dx)} + \frac{a^2\,\mathrm{se}}{\mathrm{tar}}\right.$$

$$= \left(a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)}\,dx + \left(a^2(e\cot(c+dx))^{3}\right.$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{2a^2(e\cot(c+dx))^{3/2}\tan(c+dx)}{d} - \left(a^2(\right.$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{4a^2(e\cot(c+dx))^{3/2}\tan(c+dx)}{d} - \frac{(4a}{}$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{4a^2(e\cot(c+dx))^{3/2}\tan(c+dx)}{d} - \frac{(4a}{}$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{4a^2(e\cot(c+dx))^{3/2}\tan(c+dx)}{d} - \frac{4a^2}{}$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{4a^2(e\cot(c+dx))^{3/2}\tan(c+dx)}{d} - \frac{4a^2}{}$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{4a^2(e\cot(c+dx))^{3/2}\tan(c+dx)}{d} - \frac{4a^2}{}$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{4a^2(e\cot(c+dx))^{3/2}\tan(c+dx)}{d} - \frac{4a^2}{}$$

Mathematica [C] time = 5.06208, size = 220, normalized size = 0.64

$$-\frac{a^2\cos^4\left(\frac{1}{2}(c+dx)\right)(e\cot(c+dx))^{3/2}\sec^4\left(\frac{1}{2}\cot^{-1}(\cot(c+dx))\right)\left(16\sqrt{\cot(c+dx)}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\tan\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*Cos[(c + d*x)/2]^4*(e*Cot[c + d*x])^(3/2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[
2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] +
```

```
16*Sqrt[Cot[c + d*x]] + 16*Sqrt[Cot[c + d*x]]*Hypergeometric2F1[-1/4, 1/2,
3/4, -Tan[c + d*x]^2] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c
+ d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])*Sec[
ArcCot[Cot[c + d*x]]/2]^4)/(4*d*Cot[c + d*x]^(3/2))
```

Maple [C] time = 0.28, size = 1392, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/2*a^2/d*2^(1/2)*(I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*El
lipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)*cos(d*x+c)-I*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1
+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(
1/2))-cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-cos
(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-8*cos(d*x+c)
*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+4*cos(d*x+c)*((1-cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2),1/2*2^(1/2))+I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*
2^(1/2))-I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((
1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2),1/2+1/2*I,1/2*2^(1/2))-8*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((
```

```
-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+4*(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+8*cos(d*x+c)*2^(1/2))*sin(d*x+c)
*(e*cos(d*x+c)/sin(d*x+c))^(3/2)/cos(d*x+c)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(3/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cot(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(3/2)*(a*sec(d*x + c) + a)^2, x)
```

3.240 $\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=311

$$\frac{2a^2\sqrt{\sin(2c+2dx)}\sec(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)\sqrt{e\cot(c+dx)}}{d}-\frac{a^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{\tan(c+dx)}\sqrt{}}{\sqrt{2}d}$$

```
[Out] (2*a^2*Sqrt[e*Cot[c + d*x]]*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[
Sin[2*c + 2*d*x]])/d - (a^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[e*C
ot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[2]*d) + (a^2*ArcTan[1 + Sqrt[2]*Sqrt
[Tan[c + d*x]]]*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[2]*d) - (a^2
*Sqrt[e*Cot[c + d*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sq
rt[Tan[c + d*x]])/(2*Sqrt[2]*d) + (a^2*Sqrt[e*Cot[c + d*x]]*Log[1 + Sqrt[2]
*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[Tan[c + d*x]])/(2*Sqrt[2]*d) + (2*
a^2*Sqrt[e*Cot[c + d*x]]*Tan[c + d*x])/d
```

Rubi [A] time = 0.287216, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3900, 3886, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641, 2607, 30}

$$-\frac{a^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}{\sqrt{2}d}+\frac{a^2\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\sqrt{\tan(c+dx)}\sqrt{e\cot(c+}}{\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (2*a^2*Sqrt[e*Cot[c + d*x]]*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[
Sin[2*c + 2*d*x]])/d - (a^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[e*C
ot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[2]*d) + (a^2*ArcTan[1 + Sqrt[2]*Sqrt
[Tan[c + d*x]]]*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[2]*d) - (a^2
*Sqrt[e*Cot[c + d*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sq
rt[Tan[c + d*x]])/(2*Sqrt[2]*d) + (a^2*Sqrt[e*Cot[c + d*x]]*Log[1 + Sqrt[2]
*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[Tan[c + d*x]])/(2*Sqrt[2]*d) + (2*
a^2*Sqrt[e*Cot[c + d*x]]*Tan[c + d*x])/d
```


Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
```

```
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2\,dx = \left(\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\tan(c+dx)}}\,dx$$

$$= \left(\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \left(\frac{a^2}{\sqrt{\tan(c+dx)}} + \frac{2a^2\sec(c+dx)}{\sqrt{\tan(c+dx)}} + \frac{a^2\sec}{\sqrt{\tan}}\right.$$

$$= \left(a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{1}{\sqrt{\tan(c+dx)}}\,dx + \left(a^2\sqrt{e\cot(c+dx)}\sqrt{t}\right.$$

$$= \frac{\left(2a^2\sqrt{e\cot(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(a^2\sqrt{e\cot(c+a}\right.}{}$$

$$= \frac{2a^2\sqrt{e\cot(c+dx)}\tan(c+dx)}{d} + \left(2a^2\sqrt{e\cot(c+dx)}\sec(c+dx)\sqrt{\sin(2c+2}\right.$$

$$= \frac{2a^2\sqrt{e\cot(c+dx)}F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d} + \frac{2a^2\sqrt{e\cot}}{}$$

$$= \frac{2a^2\sqrt{e\cot(c+dx)}F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d} + \frac{2a^2\sqrt{e\cot}}{}$$

$$= \frac{2a^2\sqrt{e\cot(c+dx)}F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d} - \frac{a^2\sqrt{e\cot(}}{}$$

$$= \frac{2a^2\sqrt{e\cot(c+dx)}F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d} - \frac{a^2\tan^{-1}\left(\right.}{}$$

Mathematica [C] time = 1.74705, size = 118, normalized size = 0.38

$$\frac{a^2e(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}\cot^{-1}(\cot(c+dx))\right)\left(6\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(c+dx)\right)-2\cot^2(c+dx)\text{Hy}\right.}{6d\sqrt{e\cot(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*e*(1 + Cos[c + d*x])^2*(3*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x
]^2] + 6*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c + d*x]^2] - 2*Cot[c + d*x]
```

```
^2*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])*Sec[ArcCot[Cot[c + d*x]
]/2]^4)/(6*d*Sqrt[e*Cot[c + d*x]])
```

Maple [C] time = 0.289, size = 653, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*cot(d*x+c))^(1/2),x)
```

```
[Out] -1/2*a^2/d*2^(1/2)*(cos(d*x+c)+1)^2*(e*cos(d*x+c)/sin(d*x+c))^(1/2)*(-1+cos
(d*x+c))*(I*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)-I*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2
+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+sin(
d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+sin(d*x+c)*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+2*sin(d*x+c)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-2*cos(d*x+c)*2^(1/2)+2*2^(1/2))/sin(d*x+
c)^3/cos(d*x+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int\sqrt{e\cot(c+dx)}\,dx+\int 2\sqrt{e\cot(c+dx)}\sec(c+dx)\,dx+\int\sqrt{e\cot(c+dx)}\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*cot(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(sqrt(e*cot(c + d*x)), x) + Integral(2*sqrt(e*cot(c + d*x))*s
ec(c + d*x), x) + Integral(sqrt(e*cot(c + d*x))*sec(c + d*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{e\cot(dx+c)}(a\sec(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a)^2, x)
```

3.241 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx$

Optimal. Leaf size=339

$$\frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}-\frac{a^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{a^2\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{2a^2\tan(c+dx)}{3d\sqrt{e\cot(c+dx)}}+\frac{a^2\log\left(\mathrm{ta}\right.}{2\sqrt{2}}$$

```
[Out] (4*a^2*Sin[c + d*x])/(d*Sqrt[e*Cot[c + d*x]]) - (4*a^2*Cos[c + d*x]*Ellipti
cE[c - Pi/4 + d*x, 2])/(d*Sqrt[e*Cot[c + d*x]]*Sqrt[Sin[2*c + 2*d*x]]) - (a
^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d*Sqrt[e*Cot[c + d*x]]*
Sqrt[Tan[c + d*x]]) + (a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]
*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) + (a^2*Log[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c +
d*x]]) - (a^2*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2
]*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) + (2*a^2*Tan[c + d*x])/(3*d*Sq
rt[e*Cot[c + d*x]])
```

Rubi [A] time = 0.318367, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {3900, 3886, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639, 2607, 30}

$$\frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}-\frac{a^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{a^2\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{2a^2\tan(c+dx)}{3d\sqrt{e\cot(c+dx)}}+\frac{a^2\log\left(\mathrm{ta}\right.}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (4*a^2*Sin[c + d*x])/(d*Sqrt[e*Cot[c + d*x]]) - (4*a^2*Cos[c + d*x]*Ellipti
cE[c - Pi/4 + d*x, 2])/(d*Sqrt[e*Cot[c + d*x]]*Sqrt[Sin[2*c + 2*d*x]]) - (a
^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d*Sqrt[e*Cot[c + d*x]]*
Sqrt[Tan[c + d*x]]) + (a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]
*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) + (a^2*Log[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c +
d*x]]) - (a^2*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2
]*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) + (2*a^2*Tan[c + d*x])/(3*d*Sq
rt[e*Cot[c + d*x]])
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \frac{\int (a+a\sec(c+dx))^2\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$

$$= \frac{\int \left(a^2\sqrt{\tan(c+dx)}+2a^2\sec(c+dx)\sqrt{\tan(c+dx)}+a^2\sec^2(c+dx)\sqrt{\tan(c+dx)}\right)\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$

$$= \frac{a^2\int \sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}+\frac{a^2\int \sec^2(c+dx)\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}+\frac{\left(2a^2\right)\int \sec(c+dx)\sqrt{}}{\sqrt{e\cot(c+dx)}\sqrt{\mathrm{t}}}$$

$$= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}-\frac{\left(4a^2\right)\int \cos(c+dx)\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}+\frac{a^2\,\mathrm{Subst}\left(\int \sqrt{x}\,dx,x,\tan(c+d\right.}{d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx}}$$

$$= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}+\frac{2a^2\tan(c+dx)}{3d\sqrt{e\cot(c+dx)}}-\frac{\left(4a^2\sqrt{\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}+\frac{2a^2\tan(c+dx)}{3d\sqrt{e\cot(c+dx)}}-\frac{\left(4a^2\cos(c+dx)\right)\int \sqrt{\sin(2c+2dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}}-\frac{a^2\,\mathrm{Subst}}{d\sqrt{e}}$$

$$= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}-\frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}}+\frac{2a^2\tan(c+dx)}{3d\sqrt{e\cot(c+dx)}}+\frac{a^2\,\mathrm{Subst}\left(\int \frac{}{1-}\right.}{2d\sqrt{e\cot}}$$

$$= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}-\frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}}+\frac{a^2\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(\right.}{2\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+}}$$

$$= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}-\frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}}-\frac{a^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}+$$

Mathematica [C] time = 10.8075, size = 221, normalized size = 0.65

$$\frac{a^2 \sin\left(\frac{1}{2}(c+dx)\right)\cos^5\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sec^4\left(\frac{1}{2}\cot^{-1}(\cot(c+dx))\right)\left(8\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\tan^2(c+d\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (a^2*Cos[(c + d*x)/2]^5*(4*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2]
 + 8*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] + 3*Sqrt[2]*Cot[c +
d*x]^(3/2)*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]
*Sqrt[Cot[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] -
 Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))*Sec[c + d*x]*Sec[ArcC
ot[Cot[c + d*x]]/2]^4*Sin[(c + d*x)/2])/(3*d*Sqrt[e*Cot[c + d*x]])
```

Maple [C] time = 0.356, size = 1504, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x)
```

```
[Out] 1/6*a^2/d*2^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*(3*I*EllipticPi((-(-1+
cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1
+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-3*I*EllipticPi((-(-1
+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-
1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-3*EllipticPi((-(-1+
cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1
+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+3*I*EllipticPi((-(-1
+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)-3*EllipticPi((-(-1+co
s(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+c
os(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-3*I*cos(d*x+c)*(-(-1+c
os(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
```

```
+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+24*EllipticE((-(-1+cos(
d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-si
n(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-12*EllipticF((-(-1+cos(d*x+c)-sin(
d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/s
in(d*x+c))^(1/2)*cos(d*x+c)^2-3*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1
/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2
*I,1/2*2^(1/2))*cos(d*x+c)-3*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,
1/2*2^(1/2))*cos(d*x+c)+24*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*
cos(d*x+c)-12*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Elliptic
F((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)-14
*cos(d*x+c)^2*2^(1/2)+12*cos(d*x+c)*2^(1/2)+2*2^(1/2))/sin(d*x+c)^5/cos(d*x
+c)/(e*cos(d*x+c)/sin(d*x+c))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int \frac{1}{\sqrt{e\cot(c+dx)}}\,dx+\int \frac{2\sec(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx+\int \frac{\sec^2(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*cot(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(1/sqrt(e*cot(c + d*x)), x) + Integral(2*sec(c + d*x)/sqrt(e*
cot(c + d*x)), x) + Integral(sec(c + d*x)**2/sqrt(e*cot(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^2}{\sqrt{e\cot(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(e*cot(d*x + c)), x)
```

3.242 $\int \frac{(a+a\sec(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=375

$$-\frac{2a^2\sqrt{\sin(2c+2dx)}\cot(c+dx)\csc(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3d(e\cot(c+dx))^{3/2}}+\frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}}+\frac{a^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(\right.}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)(e\cot(}$$

```
[Out] (2*a^2*Cot[c + d*x])/(d*(e*Cot[c + d*x])^(3/2)) + (4*a^2*Csc[c + d*x])/(3*d
*(e*Cot[c + d*x])^(3/2)) - (2*a^2*Cot[c + d*x]*Csc[c + d*x]*EllipticF[c - P
i/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x]])/(3*d*(e*Cot[c + d*x])^(3/2)) + (a^2*A
rcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d*(e*Cot[c + d*x])^(3/2)*Ta
n[c + d*x]^(3/2)) - (a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d
*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) + (a^2*Log[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x
]^(3/2)) - (a^2*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt
[2]*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) + (2*a^2*Tan[c + d*x])/(5*
d*(e*Cot[c + d*x])^(3/2))
```

Rubi [A] time = 0.328424, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.68, Rules used = {3900, 3886, 3473, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2611, 2614, 2573, 2641, 2607, 30}

$$\frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}}+\frac{a^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{a^2\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}+\frac{2a^2\tan(c+dx)}{5d(e\cot(c+dx))^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Cot[c + d*x])^(3/2),x]
```


```
[Out] (2*a^2*Cot[c + d*x])/(d*(e*Cot[c + d*x])^(3/2)) + (4*a^2*Csc[c + d*x])/(3*d
*(e*Cot[c + d*x])^(3/2)) - (2*a^2*Cot[c + d*x]*Csc[c + d*x]*EllipticF[c - P
i/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x]])/(3*d*(e*Cot[c + d*x])^(3/2)) + (a^2*A
rcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d*(e*Cot[c + d*x])^(3/2)*Ta
n[c + d*x]^(3/2)) - (a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d
*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) + (a^2*Log[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x
]^(3/2)) - (a^2*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt
[2]*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) + (2*a^2*Tan[c + d*x])/(5*
```

```
d*(e*Cot[c + d*x])^(3/2))
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \frac{\int (a+a\sec(c+dx))^2\tan^{\frac{3}{2}}(c+dx)\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\int \left(a^2\tan^{\frac{3}{2}}(c+dx)+2a^2\sec(c+dx)\tan^{\frac{3}{2}}(c+dx)+a^2\sec^2(c+dx)\tan^{\frac{3}{2}}(c+dx)\right)dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{a^2\int \tan^{\frac{3}{2}}(c+dx)\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} + \frac{a^2\int \sec^2(c+dx)\tan^{\frac{3}{2}}(c+dx)\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} + \frac{\left(2a^2\right)\int \sec(c+dx)}{(e\cot(c+dx))^{3/2}}$$

$$= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} - \frac{\left(2a^2\right)\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{3(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} - \frac{a^2\int}{(e\cot(c+a}$$

$$= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} + \frac{2a^2\tan(c+dx)}{5d(e\cot(c+dx))^{3/2}} - \frac{\left(2a^2\cos^{\frac{3}{2}}(c+dx)\right)\int \frac{}{\sqrt{}}}{3(e\cot(c+dx))^{3/2}}$$

$$= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} + \frac{2a^2\tan(c+dx)}{5d(e\cot(c+dx))^{3/2}} - \frac{\left(2a^2\cot(c+dx)\csc(c+\right.}{3(}$$

$$= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} - \frac{2a^2\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{}}{3d(e\cot(c+dx))^{3/2}}$$

$$= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} - \frac{2a^2\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{}}{3d(e\cot(c+dx))^{3/2}}$$

$$= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} - \frac{2a^2\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{}}{3d(e\cot(c+dx))^{3/2}}$$

$$= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} - \frac{2a^2\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{}}{3d(e\cot(c+dx))^{3/2}}$$

Mathematica [C] time = 5.92444, size = 127, normalized size = 0.34

$$\frac{a^2\sin^2(c+dx)(\sec(c+dx)+1)^2\sec^4\left(\frac{1}{2}\cot^{-1}(\cot(c+dx))\right)\left(\text{Hypergeometric2F1}\left(-\frac{5}{4},1,-\frac{1}{4},-\cot^2(c+dx)\right)+2\left(\text{H}\right.\right.}{10de\sqrt{e\cot(c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (a^2*(Hypergeometric2F1[-5/4, 1, -1/4, -Cot[c + d*x]^2] + 2*(5*Cot[c + d*x]
^2*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2] + Hypergeometric2F1[1/2
, 5/4, 9/4, -Tan[c + d*x]^2]))*(1 + Sec[c + d*x])^2*Sec[ArcCot[Cot[c + d*x]
]/2]^4*Sin[c + d*x]^2)/(10*d*e*Sqrt[e*Cot[c + d*x]])
```

Maple [C] time = 0.309, size = 721, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(3/2),x)
```

```
[Out] 1/30*a^2/d*2^(1/2)*(-1+cos(d*x+c))*(-15*I*sin(d*x+c)*EllipticPi(((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+15*I*sin(d*x+c)*EllipticPi(((
1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+15*sin(d*x+c)*Ellipt
icPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+15*sin(d*x+c)
*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1
/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-10*sin
(d*x+c)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+24*cos(d*x
+c)^3*2^(1/2)-4*cos(d*x+c)^2*2^(1/2)-14*cos(d*x+c)*2^(1/2)-6*2^(1/2))*(cos(
d*x+c)+1)^2/(e*cos(d*x+c)/sin(d*x+c))^(3/2)/cos(d*x+c)/sin(d*x+c)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*cot(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^2}{(e\cot(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/(e*cot(d*x + c))^(3/2), x)
```

3.243 $\int \frac{(e\cot(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=405

$$\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}{\sqrt{2}ad}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}{\sqrt{2}ad}$$

```
[Out] (2*Cot[c + d*x]*(e*Cot[c + d*x])^(3/2)*(1 - Sec[c + d*x]))/(5*a*d) - (2*(e*
Cot[c + d*x])^(3/2)*(5 - 3*Sec[c + d*x])*Tan[c + d*x])/(5*a*d) + (6*(e*Cot[
c + d*x])^(3/2)*EllipticE[c - Pi/4 + d*x, 2]*Sin[c + d*x]*Tan[c + d*x])/(5*
a*d*Sqrt[Sin[2*c + 2*d*x]]) + (ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*(e*Co
t[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))/(Sqrt[2]*a*d) - (ArcTan[1 + Sqrt[2]*S
qrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))/(Sqrt[2]*a*d)
 - ((e*Cot[c + d*x])^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*a*d) + ((e*Cot[c + d*x])^(3/2)*Log[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*a*d
) - (6*(e*Cot[c + d*x])^(3/2)*Sin[c + d*x]*Tan[c + d*x]^2)/(5*a*d)
```

Rubi [A] time = 0.426719, antiderivative size = 405, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {3900, 3888, 3882, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}{\sqrt{2}ad}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}{\sqrt{2}ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```


```
[Out] (2*Cot[c + d*x]*(e*Cot[c + d*x])^(3/2)*(1 - Sec[c + d*x]))/(5*a*d) - (2*(e*
Cot[c + d*x])^(3/2)*(5 - 3*Sec[c + d*x])*Tan[c + d*x])/(5*a*d) + (6*(e*Cot[
c + d*x])^(3/2)*EllipticE[c - Pi/4 + d*x, 2]*Sin[c + d*x]*Tan[c + d*x])/(5*
a*d*Sqrt[Sin[2*c + 2*d*x]]) + (ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*(e*Co
t[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))/(Sqrt[2]*a*d) - (ArcTan[1 + Sqrt[2]*S
qrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))/(Sqrt[2]*a*d)
 - ((e*Cot[c + d*x])^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*a*d) + ((e*Cot[c + d*x])^(3/2)*Log[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*a*d
```

```
) - (6*(e*Cot[c + d*x])^(3/2)*Sin[c + d*x]*Tan[c + d*x]^2)/(5*a*d)
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
```

```
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int \frac{1}{(a+a\sec(c+dx))\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{\left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int \frac{-a+a\sec(c+dx)}{\tan^{\frac{7}{2}}(c+dx)}\,dx}{a^2}$$

$$= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} + \frac{\left(2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int \frac{\frac{5a}{2}-\frac{3}{2}a\sec}{\tan^{\frac{3}{2}}(c}}{5a^2}$$

$$= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))\tan(c+}{5ad}$$

$$= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))\tan(c+}{5ad}$$

$$= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))\tan(c+}{5ad}$$

$$= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))\tan(c+}{5ad}$$

$$= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))\tan(c+}{5ad}$$

$$= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))\tan(c+}{5ad}$$

$$= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))\tan(c+}{5ad}$$

$$= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))\tan(c+}{5ad}$$

Mathematica [C] time = 4.00642, size = 316, normalized size = 0.78

$$-\frac{e\sin^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\sqrt{\sec^2(c+dx)}+1\right)\sqrt{e\cot(c+dx)}\left(-40\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\tan^2(c+dx)\right)+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(e*Sqrt[e*Cot[c + d*x]]*(30*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]
*Cot[c + d*x]^(3/2) - 30*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]*Cot
[c + d*x]^(3/2) + 120*Cot[c + d*x]^2 - 24*Cot[c + d*x]^4 + 24*Cot[c + d*x]^
4*Hypergeometric2F1[-5/4, -1/2, -1/4, -Tan[c + d*x]^2] - 120*Cot[c + d*x]^2
*Hypergeometric2F1[-1/2, -1/4, 3/4, -Tan[c + d*x]^2] - 40*Hypergeometric2F1
[1/2, 3/4, 7/4, -Tan[c + d*x]^2] + 15*Sqrt[2]*Cot[c + d*x]^(3/2)*Log[1 - Sq
rt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 15*Sqrt[2]*Cot[c + d*x]^(3/2)*Lo
g[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])*Sec[c + d*x]*(1 + Sqrt[Se
c[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(30*a*d)
```

Maple [C] time = 0.297, size = 2113, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/10/a/d*2^(1/2)*(-1+cos(d*x+c))*(5*I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+
1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+10*I*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2),1/2+1/2*I,1/2*2^(1/2))+5*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2
*2^(1/2))*cos(d*x+c)^2+5*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1
/2))*cos(d*x+c)^2-12*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*Ell
ipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)
^2+6*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2-10*I*((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)-5*I*((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+10*cos(d*x+c)*((1-co
```

```
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+10*cos(d*x+c)*((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-24*cos(d*x+c)*((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2),1/2*2^(1/2))+12*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*
2^(1/2))-5*I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+5*I*((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+5*((1-cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+5*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2
*I,1/2*2^(1/2))-12*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellip
ticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+6*((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2),1/2*2^(1/2))-6*cos(d*x+c)^2*2^(1/2)-4*cos(d*x+c)*2^(1/
2))*(e*cos(d*x+c)/sin(d*x+c))^(3/2)/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \cot(dx + c))^{\frac{3}{2}}}{a \sec(dx + c) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

3.244 $\int \frac{\sqrt{e\cot(c+dx)}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=325

$$-\frac{\sqrt{\sin(2c+2dx)}\sec(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)\sqrt{e\cot(c+dx)}}{3ad}-\frac{\sqrt{\tan(c+dx)}\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\,\text{co}}}{\sqrt{2}ad}$$

```
[Out] (2*Cot[c + d*x]*Sqrt[e*Cot[c + d*x]]*(1 - Sec[c + d*x]))/(3*a*d) - (Sqrt[e*
Cot[c + d*x]]*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*
x]])/(3*a*d) - (ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[e*Cot[c + d*x]]
*Sqrt[Tan[c + d*x]])/(Sqrt[2]*a*d) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
]*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[2]*a*d) - (Sqrt[e*Cot[c +
d*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[Tan[c + d*x]]
)/(2*Sqrt[2]*a*d) + (Sqrt[e*Cot[c + d*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]
] + Tan[c + d*x]]*Sqrt[Tan[c + d*x]])/(2*Sqrt[2]*a*d)
```

Rubi [A] time = 0.324659, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3900, 3888, 3882, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$-\frac{\sqrt{\tan(c+dx)}\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}}{\sqrt{2}ad}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}{\sqrt{2}ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cot[c + d*x]]/(a + a*Sec[c + d*x]),x]
```


```
[Out] (2*Cot[c + d*x]*Sqrt[e*Cot[c + d*x]]*(1 - Sec[c + d*x]))/(3*a*d) - (Sqrt[e*
Cot[c + d*x]]*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*
x]])/(3*a*d) - (ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[e*Cot[c + d*x]]
*Sqrt[Tan[c + d*x]])/(Sqrt[2]*a*d) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
]*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[2]*a*d) - (Sqrt[e*Cot[c +
d*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[Tan[c + d*x]]
)/(2*Sqrt[2]*a*d) + (Sqrt[e*Cot[c + d*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]
] + Tan[c + d*x]]*Sqrt[Tan[c + d*x]])/(2*Sqrt[2]*a*d)
```


Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
```

```
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
```

```
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\sec(c+dx)}\,dx = \left(\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{1}{(a+a\sec(c+dx))\sqrt{\tan(c+dx)}}\,dx$$

$$= \frac{\left(\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{-a+a\sec(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}(1-\sec(c+dx))}{3ad} + \frac{\left(2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{\frac{3a}{2}-\frac{1}{2}a\sec(c+}{\sqrt{\tan(c+dx}}}{3a^2}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}(1-\sec(c+dx))}{3ad} - \frac{\left(\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{3a}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}(1-\sec(c+dx))}{3ad} - \frac{\left(\sqrt{e\cot(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin}}}{3a\sqrt{\cos(c+dx)}}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}(1-\sec(c+dx))}{3ad} - \frac{\left(\sqrt{e\cot(c+dx)}\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)}{3a}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}(1-\sec(c+dx))}{3ad} - \frac{\sqrt{e\cot(c+dx)}F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{}}{3ad}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}(1-\sec(c+dx))}{3ad} - \frac{\sqrt{e\cot(c+dx)}F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{}}{3ad}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}(1-\sec(c+dx))}{3ad} - \frac{\sqrt{e\cot(c+dx)}F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{}}{3ad}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}(1-\sec(c+dx))}{3ad} - \frac{\sqrt{e\cot(c+dx)}F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{}}{3ad}$$

Mathematica [C] time = 1.58069, size = 135, normalized size = 0.42

$$-\frac{4\sin^2\left(\frac{1}{2}(c+dx)\right)\csc(c+dx)\left(\sqrt{\sec^2(c+dx)}+1\right)\sqrt{e\cot(c+dx)}\left(3\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(c+dx)\right)+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-4*Sqrt[e*Cot[c + d*x]]*Csc[c + d*x]*(Cot[c + d*x]^2*Hypergeometric2F1[-3/
4, -1/2, 1/4, -Tan[c + d*x]^2] + 3*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c
+ d*x]^2] + Cot[c + d*x]^2*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*
x]^2]))*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(3*a*d)
```

Maple [C] time = 0.301, size = 1269, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/6/a/d*2^(1/2)*(e*cos(d*x+c)/sin(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*
x+c))^2*(3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/
2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*sin(d*x+c)-3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),
1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*sin(d*x+c)+3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)*sin(d*x+c)+3*sin(d*x+c)*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2
-1/2*I,1/2*2^(1/2))-3*I*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*
I,1/2*2^(1/2))+3*sin(d*x+c)*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+
1/2*I,1/2*2^(1/2))-8*sin(d*x+c)*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1
/2*2^(1/2))+3*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1
```

```
/2))+3*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*Ellipt
icPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-8*
sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+2*cos(d*x+c)^2*2^(1/
2)-2*cos(d*x+c)*2^(1/2))/sin(d*x+c)^5/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\cot(dx+c)}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cot(d*x + c))/(a*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{e\cot(c+dx)}}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(e*cot(c + d*x))/(sec(c + d*x) + 1), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\cot(dx+c)}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cot(d*x + c))/(a*sec(d*x + c) + a), x)
```

3.245 $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=347

$$\frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}ad\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{2\cot(c+dx)(1-\sec(c+}{ad\sqrt{e\cot(c+dx)}}$$

```
[Out] (2*Cot[c + d*x]*(1 - Sec[c + d*x]))/(a*d*Sqrt[e*Cot[c + d*x]]) + (2*Sin[c +
 d*x])/(a*d*Sqrt[e*Cot[c + d*x]]) - (2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*
x, 2])/(a*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Sin[2*c + 2*d*x]]) - ArcTan[1 - Sqrt[
2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]
) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a*d*Sqrt[e*Cot[c + d*x]
]*Sqrt[Tan[c + d*x]]) + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/
(2*Sqrt[2]*a*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) - Log[1 + Sqrt[2]*S
qrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*Sqrt[e*Cot[c + d*x]]*Sqrt[
Tan[c + d*x]])
```

Rubi [A] time = 0.356534, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {3900, 3888, 3882, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$\frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}ad\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{2\cot(c+dx)(1-\sec(c+}{ad\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] (2*Cot[c + d*x]*(1 - Sec[c + d*x]))/(a*d*Sqrt[e*Cot[c + d*x]]) + (2*Sin[c +
 d*x])/(a*d*Sqrt[e*Cot[c + d*x]]) - (2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*
x, 2])/(a*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Sin[2*c + 2*d*x]]) - ArcTan[1 - Sqrt[
2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]
) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a*d*Sqrt[e*Cot[c + d*x]
]*Sqrt[Tan[c + d*x]]) + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/
(2*Sqrt[2]*a*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) - Log[1 + Sqrt[2]*S
qrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*Sqrt[e*Cot[c + d*x]]*Sqrt[
Tan[c + d*x]])
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\sqrt{\tan(c+dx)}}{a+a\sec(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$

$$= \frac{\int \frac{-a+a\sec(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$

$$= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\int \left(\frac{a}{2}+\frac{1}{2}a\sec(c+dx)\right)\sqrt{\tan(c+dx)}\,dx}{a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$

$$= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{\int \sqrt{\tan(c+dx)}\,dx}{a\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{\int \sec(c+}{a\sqrt{e\cot(c}$$

$$= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}} - \frac{2\int \cos(c+dx)\sqrt{\tan(c}{a\sqrt{e\cot(c+dx)}\sqrt{\tan}}$$

$$= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\right)\int \sqrt{\cos}}{a\sqrt{e\cot(c+}}$$

$$= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}} - \frac{(2\cos(c+dx))\int \sqrt{\sin(}}{a\sqrt{e\cot(c+dx)}\sqrt{\sin}}$$

$$= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}} - \frac{2\cos(c+dx)E\left(c-\frac{\pi}{4}\right.}{ad\sqrt{e\cot(c+dx)}\sqrt{\sin(}}$$

$$= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}} - \frac{2\cos(c+dx)E\left(c-\frac{\pi}{4}\right.}{ad\sqrt{e\cot(c+dx)}\sqrt{\sin(}}$$

$$= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}} - \frac{2\cos(c+dx)E\left(c-\frac{\pi}{4}\right.}{ad\sqrt{e\cot(c+dx)}\sqrt{\sin(}}$$

<u>Mathematica</u> [C] time = 3.49984, size = 249, normalized size = 0.72

$$-\frac{\sin^2\left(\frac{1}{2}(c+dx)\right)\csc(c+dx)\left(\sqrt{\sec^2(c+dx)}+1\right)\left(8\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\tan^2(c+dx)\right)+24\cot^2(c+dx)\text{F}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] -(Csc[c + d*x]*(24*Cot[c + d*x]^2*Hypergeometric2F1[-1/2, -1/4, 3/4, -Tan[c
 + d*x]^2] + 8*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] - 3*Cot[c
```

```
+ d*x]^(3/2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*
ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 8*Sqrt[Cot[c + d*x]] + Sqrt[2]*Log
[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*S
qrt[Cot[c + d*x]] + Cot[c + d*x]]))*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x
)/2]^2)/(6*a*d*Sqrt[e*Cot[c + d*x]])
```

Maple [C] time = 0.303, size = 359, normalized size = 1.

$$-\frac{\sqrt{2}\left(\cos\left(dx+c\right)+1\right)^{2}\left(-1+\cos\left(dx+c\right)\right)}{2\,ad\left(\sin\left(dx+c\right)\right)^{3}}\sqrt{\frac{-1+\cos\left(dx+c\right)}{\sin\left(dx+c\right)}}\sqrt{\frac{-1+\cos\left(dx+c\right)+\sin\left(dx+c\right)}{\sin\left(dx+c\right)}}\sqrt{-\frac{-1+\cos\left(dx+c\right)-}{\sin\left(dx+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x)
```

```
[Out] -1/2/a/d*2^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*(cos(
d*x+c)+1)^2*(I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/
2-1/2*I,1/2*2^(1/2))-I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^
(1/2),1/2+1/2*I,1/2*2^(1/2))+4*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d
*x+c))^(1/2),1/2*2^(1/2))-2*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+
c))^(1/2),1/2*2^(1/2))-EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^
(1/2),1/2-1/2*I,1/2*2^(1/2))-EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*
x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)))*(-1+cos(d*x+c))/sin(d*x+c)^3/(e*cos(d*x
+c)/sin(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\cot\left(dx+c\right)}\left(a\sec\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sqrt{e\cot{(c+dx)}}\sec{(c+dx)}+\sqrt{e\cot{(c+dx)}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*cot(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*cot(c + d*x))*sec(c + d*x) + sqrt(e*cot(c + d*x))), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\cot{(dx+c)}}(a\sec{(dx+c)}+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a)), x)
```

3.246 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=290

$$\frac{\sqrt{\sin(2c+2dx)}\cot(c+dx)\csc(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{ad(e\cot(c+dx))^{3/2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{\tan^{-1}\left(\sqrt{2}\right.}{\sqrt{2}ad\tan^{\frac{3}{2}}(c\right.}$$

```
[Out] (Cot[c + d*x]*Csc[c + d*x]*EllipticF[c - Pi/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*
x]])/(a*d*(e*Cot[c + d*x])^(3/2)) + ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/
(Sqrt[2]*a*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) - ArcTan[1 + Sqrt[2
]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2
)) + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*C
ot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
 + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))
```

Rubi [A] time = 0.289011, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.56, Rules used = {3900, 3888, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}ad\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}+\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}ad\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])),x]
```


```
[Out] (Cot[c + d*x]*Csc[c + d*x]*EllipticF[c - Pi/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*
x]])/(a*d*(e*Cot[c + d*x])^(3/2)) + ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/
(Sqrt[2]*a*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) - ArcTan[1 + Sqrt[2
]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2
)) + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*C
ot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
 + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))
```


Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

```
&& !IntegerQ[m]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\tan^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\int \frac{-a+a\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{\int \frac{1}{\sqrt{\tan(c+dx)}}\,dx}{a(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} + \frac{\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{a(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\cos^{\frac{3}{2}}(c+dx)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{a(e\cot(c+dx))^{3/2}\sin^{\frac{3}{2}}(c+dx)} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx, x, \tan(c+d\right.}{ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx}$$

$$= \frac{\left(\cot(c+dx)\csc(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{a(e\cot(c+dx))^{3/2}} - \frac{2\,\text{Subst}\left(\int \frac{}{1+}\right.}{ad(e\cot(c+}$$

$$= \frac{\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(2c+2dx)}}{ad(e\cot(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\right.}{ad(e\cot(c+d}$$

$$= \frac{\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(2c+2dx)}}{ad(e\cot(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}}\right.}{2ad(e\cot(c}$$

$$= \frac{\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(2c+2dx)}}{ad(e\cot(c+dx))^{3/2}} + \frac{\log\left(1-\sqrt{2}\sqrt{t}\right.}{2\sqrt{2}ad(e\cot(}$$

$$= \frac{\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(2c+2dx)}}{ad(e\cot(c+dx))^{3/2}} + \frac{\tan^{-1}\left(1-\right.}{\sqrt{2}ad(e\cot(c+}$$

Mathematica [C] time = 8.21183, size = 112, normalized size = 0.39

$$\frac{4\sin^2\left(\frac{1}{2}(c+dx)\right)\cot^2(c+dx)\csc(c+dx)\left(\sqrt{\sec^2(c+dx)}+1\right)\left(3\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(c+dx)\right)+\cot\right.}{3ad(e\cot(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (4*Cot[c + d*x]^2*Csc[c + d*x]*(3*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c +
 d*x]^2] + Cot[c + d*x]^2*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])*
```

```
(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(3*a*d*(e*Cot[c + d*x])^(3/2
))
```

Maple [C] time = 0.259, size = 323, normalized size = 1.1

$$-\frac{\sqrt{2}\,(\cos(dx+c)+1)^2\,(-1+\cos(dx+c))\cos(dx+c)}{2\,ad\,(\sin(dx+c))^4}\left(i\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\mathrm{E}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/2/a/d*2^(1/2)*(cos(d*x+c)+1)^2*(I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-I*EllipticPi(((1-cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-EllipticPi(((1-cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+4*EllipticF(((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-EllipticPi(((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)))*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*(-1+cos(d*x+c))*cos(d*x+c)/(e*cos(d*x+c)/sin(d*x
+c))^(3/2)/sin(d*x+c)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cot(dx+c))^{\frac{3}{2}}\,(a\sec(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*cot(d*x + c))^(3/2)*(a*sec(d*x + c) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cot(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*cot(d*x + c))^(3/2)*(a*sec(d*x + c) + a)), x)
```

3.247 $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=325

$$\frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}ad\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}-\frac{\log\left(\tan(c+a\right.}{2\sqrt{2}ad\tan^{\frac{5}{2}}}$$

```
[Out] (2*Cos[c + d*x]*Cot[c + d*x])/(a*d*(e*Cot[c + d*x])^(5/2)) - (2*Cos[c + d*x
]*Cot[c + d*x]^2*EllipticE[c - Pi/4 + d*x, 2])/(a*d*(e*Cot[c + d*x])^(5/2)*
Sqrt[Sin[2*c + 2*d*x]]) + ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a
*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2)) - ArcTan[1 + Sqrt[2]*Sqrt[Tan
[c + d*x]]]/(Sqrt[2]*a*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2)) - Log[1
 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c + d*x
])^(5/2)*Tan[c + d*x]^(5/2)) + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))
```

Rubi [A] time = 0.323062, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3900, 3888, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$\frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}ad\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}-\frac{\log\left(\tan(c+a\right.}{2\sqrt{2}ad\tan^{\frac{5}{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])),x]
```


```
[Out] (2*Cos[c + d*x]*Cot[c + d*x])/(a*d*(e*Cot[c + d*x])^(5/2)) - (2*Cos[c + d*x
]*Cot[c + d*x]^2*EllipticE[c - Pi/4 + d*x, 2])/(a*d*(e*Cot[c + d*x])^(5/2)*
Sqrt[Sin[2*c + 2*d*x]]) + ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a
*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2)) - ArcTan[1 + Sqrt[2]*Sqrt[Tan
[c + d*x]]]/(Sqrt[2]*a*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2)) - Log[1
 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c + d*x
])^(5/2)*Tan[c + d*x]^(5/2)) + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))
```


Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
```

```
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
```

```
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\tan^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

$$= \frac{\int(-a+a\sec(c+dx))\sqrt{\tan(c+dx)}\,dx}{a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

$$= -\frac{\int\sqrt{\tan(c+dx)}\,dx}{a(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} + \frac{\int\sec(c+dx)\sqrt{\tan(c+dx)}\,dx}{a(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}} - \frac{2\int\cos(c+dx)\sqrt{\tan(c+dx)}\,dx}{a(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} - \frac{\text{Subst}\left(\int \frac{}{1}\right.}{ad(e\cot(c}$$

$$= \frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}} - \frac{\left(2\cos^{\frac{5}{2}}(c+dx)\right)\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}{a(e\cot(c+dx))^{5/2}\sin^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}} - \frac{\left(2\cos(c+dx)\cot^2(c+dx)\right)\int\sqrt{\sin(2c+2dx)}}{a(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+2dx)}}$$

$$= \frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}} - \frac{2\cos(c+dx)\cot^2(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{ad(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+2dx)}} -$$

$$= \frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}} - \frac{2\cos(c+dx)\cot^2(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{ad(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+2dx)}} -$$

$$= \frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}} - \frac{2\cos(c+dx)\cot^2(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{ad(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+2dx)}} +$$

Mathematica [C] time = 56.8628, size = 194, normalized size = 0.6

$$-\frac{\sin^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\sqrt{\sec^2(c+dx)}+1\right)\sqrt{e\cot(c+dx)}\left(3\sqrt{2}\cot^{\frac{3}{2}}(c+dx)\left(\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+d}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] -(Sqrt[e*Cot[c + d*x]]*(-8*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2
] + 3*Sqrt[2]*Cot[c + d*x]^(3/2)*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]
- 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[c + d
*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))
*Sec[c + d*x]*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(6*a*d*e^3)
```

Maple [C] time = 0.287, size = 1419, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/2/a/d*2^(1/2)*(-1+cos(d*x+c))^2*(I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1
/2*I,1/2*2^(1/2))*cos(d*x+c)-I*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1
/2+1/2*I,1/2*2^(1/2))-cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,
1/2*2^(1/2))-cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/
2))-4*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+2*cos(d*x+c)*(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+I*((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
),1/2-1/2*I,1/2*2^(1/2))-I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^
(1/2))-((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
```

```
+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-4*((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),
1/2*2^(1/2))+2*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF
(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+2*cos(d*x+c)*2^(
1/2)-2*2^(1/2))*cos(d*x+c)^2*(cos(d*x+c)+1)^2/sin(d*x+c)^7/(e*cos(d*x+c)/si
n(d*x+c))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cot(dx+c))^{\frac{5}{2}}\,(a\sec(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cot(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a)), x)
```

3.248 $\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=335

$$-\frac{\sqrt{\sin(2c+2dx)}\cot^3(c+dx)\csc(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3ad(e\cot(c+dx))^{7/2}}-\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}}-\frac{\tan^{-1}\left(1-\right.}{\sqrt{2}ad\tan^{\frac{7}{2}}(c+}$$

```
[Out] (-2*Cot[c + d*x]^3*(3 - Sec[c + d*x]))/(3*a*d*(e*Cot[c + d*x])^(7/2)) - (Co
t[c + d*x]^3*Csc[c + d*x]*EllipticF[c - Pi/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x
]])/(3*a*d*(e*Cot[c + d*x])^(7/2)) - ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]
/(Sqrt[2]*a*d*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2)) + ArcTan[1 + Sqrt[
2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a*d*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/
2)) - Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*
Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2)) + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]
] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2))
```

Rubi [A] time = 0.337375, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3900, 3888, 3881, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$-\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}ad\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}-\frac{\log\left(\right.}{2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(7/2)*(a + a*Sec[c + d*x])),x]
```


```
[Out] (-2*Cot[c + d*x]^3*(3 - Sec[c + d*x]))/(3*a*d*(e*Cot[c + d*x])^(7/2)) - (Co
t[c + d*x]^3*Csc[c + d*x]*EllipticF[c - Pi/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x
]])/(3*a*d*(e*Cot[c + d*x])^(7/2)) - ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]
/(Sqrt[2]*a*d*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2)) + ArcTan[1 + Sqrt[
2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a*d*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/
2)) - Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*
Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2)) + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]
] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2))
```


Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
```

```
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
```

```
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\tan^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

$$= \frac{\int(-a+a\sec(c+dx))\tan^{\frac{3}{2}}(c+dx)\,dx}{a^2(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{2\int \frac{-\frac{3a}{2}+\frac{1}{2}a\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{3a^2(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{3a(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} + \frac{\ldots}{a(e\,c\ldots}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\cos^{\frac{7}{2}}(c+dx)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{3a(e\cot(c+dx))^{7/2}\sin^{\frac{7}{2}}(c+dx)} + \ldots$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\left(\cot^3(c+dx)\csc(c+dx)\sqrt{\sin(2c+2dx)}\ldots\right.}{3a(e\cot(c+dx))^{7/2}\ldots}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\cot^3(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\ldots\right.}{3ad(e\cot(c+dx))^{7/2}\ldots}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\cot^3(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\ldots\right.}{3ad(e\cot(c+dx))^{7/2}\ldots}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\cot^3(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\ldots\right.}{3ad(e\cot(c+dx))^{7/2}\ldots}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\cot^3(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\ldots\right.}{3ad(e\cot(c+dx))^{7/2}\ldots}$$

Mathematica [C] time = 15.1426, size = 130, normalized size = 0.39

$$-\frac{4\sin^2\left(\frac{1}{2}(c+dx)\right)\csc(c+dx)\left(\sqrt{\sec^2(c+dx)}+1\right)\sqrt{e\cot(c+dx)}\left(-3\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\tan^2(c+dx)\right)\right.}{3a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cot[c + d*x])^(7/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (-4*Sqrt[e*Cot[c + d*x]]*Csc[c + d*x]*(3 - 3*Hypergeometric2F1[-1/2, 1/4, 5
/4, -Tan[c + d*x]^2] + 3*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c + d*x]^2]
+ Cot[c + d*x]^2*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])*(1 + Sqrt
[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(3*a*d*e^4)
```

Maple [C] time = 0.28, size = 698, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/6/a/d*2^(1/2)*(-1+cos(d*x+c))*(3*I*sin(d*x+c)*cos(d*x+c)*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*I*sin(d*x+c)*cos(d*x+c)*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*sin(d*x+c)*cos(d*x+c)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+8*sin(d*x+c)*cos(d*x+c)*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-3*sin(d*x+c)*cos(d*x+c)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-6*cos(d*x+c)^2*2^(1/2)+8*cos(
d*x+c)*2^(1/2)-2*2^(1/2))*cos(d*x+c)^2*(cos(d*x+c)+1)^2/(e*cos(d*x+c)/sin(d
*x+c))^(7/2)/sin(d*x+c)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cot(dx+c))^{\frac{7}{2}}(a\sec(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*cot(d*x + c))^(7/2)*(a*sec(d*x + c) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cot(dx+c))^{\frac{7}{2}}(a\sec(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*cot(d*x + c))^(7/2)*(a*sec(d*x + c) + a)), x)
```

3.249 $\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=371

$$-\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}}-\frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}+\frac{\tan^{-1}\left(\sqrt{2}\right.}{\sqrt{2}ad\tan^{\frac{9}{2}}(c+}$$

```
[Out] (-6*Cos[c + d*x]*Cot[c + d*x]^3)/(5*a*d*(e*Cot[c + d*x])^(9/2)) - (2*Cot[c
+ d*x]^3*(5 - 3*Sec[c + d*x]))/(15*a*d*(e*Cot[c + d*x])^(9/2)) + (6*Cos[c +
 d*x]*Cot[c + d*x]^4*EllipticE[c - Pi/4 + d*x, 2])/(5*a*d*(e*Cot[c + d*x])^
(9/2)*Sqrt[Sin[2*c + 2*d*x]]) - ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqr
t[2]*a*d*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2)) + ArcTan[1 + Sqrt[2]*Sq
rt[Tan[c + d*x]]]/(Sqrt[2]*a*d*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2)) +
 Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c
 + d*x])^(9/2)*Tan[c + d*x]^(9/2)) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + T
an[c + d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2))
```

Rubi [A] time = 0.367597, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {3900, 3888, 3881, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$-\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}}-\frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}+\frac{\tan^{-1}\left(\sqrt{2}\right.}{\sqrt{2}ad\tan^{\frac{9}{2}}(c+}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(9/2)*(a + a*Sec[c + d*x])),x]
```


```
[Out] (-6*Cos[c + d*x]*Cot[c + d*x]^3)/(5*a*d*(e*Cot[c + d*x])^(9/2)) - (2*Cot[c
+ d*x]^3*(5 - 3*Sec[c + d*x]))/(15*a*d*(e*Cot[c + d*x])^(9/2)) + (6*Cos[c +
 d*x]*Cot[c + d*x]^4*EllipticE[c - Pi/4 + d*x, 2])/(5*a*d*(e*Cot[c + d*x])^
(9/2)*Sqrt[Sin[2*c + 2*d*x]]) - ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqr
t[2]*a*d*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2)) + ArcTan[1 + Sqrt[2]*Sq
rt[Tan[c + d*x]]]/(Sqrt[2]*a*d*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2)) +
 Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c
 + d*x])^(9/2)*Tan[c + d*x]^(9/2)) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + T
an[c + d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2))
```


Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\tan^{\frac{9}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}$$

$$= \frac{\int(-a+a\sec(c+dx))\tan^{\frac{5}{2}}(c+dx)\,dx}{a^2(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}$$

$$= -\frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} - \frac{2\int\left(-\frac{5a}{2}+\frac{3}{2}a\sec(c+dx)\right)\sqrt{\tan(c+a}}{5a^2(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx}$$

$$= -\frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} - \frac{3\int\sec(c+dx)\sqrt{\tan(c+dx)}\,dx}{5a(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} + \frac{}{a(}$$

$$= -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} - \frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} + \frac{6\int\cos(}{5a(e\cot(}$$

$$= -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} - \frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} + \frac{\left(6\cos^{\frac{9}{2}}(c\right.}{5}$$

$$= -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} - \frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} + \frac{\left(6\cos(c\right.}{5a}$$

$$= -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} - \frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} + \frac{6\cos(c+}{5ad(e}$$

$$= -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} - \frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} + \frac{6\cos(c+}{5ad(e}$$

$$= -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} - \frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} + \frac{6\cos(c+}{5ad(e}$$

Mathematica [C] time = 19.2914, size = 261, normalized size = 0.7

$$\frac{\sin^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\sqrt{\sec^2(c+dx)}+1\right)\sqrt{e\cot(c+dx)}\left(8\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\tan^2(c+dx)\right)-8\text{H}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cot[c + d*x])^(9/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Sqrt[e*Cot[c + d*x]]*(-8 + 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]
]*Cot[c + d*x]^(3/2) - 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]*Cot
[c + d*x]^(3/2) + 8*Hypergeometric2F1[-1/2, 3/4, 7/4, -Tan[c + d*x]^2] - 8*
Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] + 3*Sqrt[2]*Cot[c + d*x]^
(3/2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 3*Sqrt[2]*Cot[c
+ d*x]^(3/2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])*Sec[c + d*
x]*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(6*a*d*e^5)
```

Maple [C] time = 0.28, size = 1505, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/30/a/d*2^(1/2)*(-1+cos(d*x+c))^2*(-15*I*EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^
(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+15*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2)*cos(d*x+c)^2-15*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*cos(d*x+c)^3+15*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*cos(d*x+c)^3-15*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*cos(d*x+c)^3-15*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*cos(d*x+c)^3+18*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x
+c)^3-36*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2)
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3-15*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(
```

```
d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-15*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+18*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2-36*((-1+cos(d*x+c))/sin(d*x+c))^
(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
),1/2*2^(1/2))*cos(d*x+c)^2+28*cos(d*x+c)^3*2^(1/2)-24*cos(d*x+c)^2*2^(1/2)
-10*cos(d*x+c)*2^(1/2)+6*2^(1/2))*cos(d*x+c)^2*(cos(d*x+c)+1)^2/(e*cos(d*x+
c)/sin(d*x+c))^(9/2)/sin(d*x+c)^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cot(dx+c))^{\frac{9}{2}}(a\sec(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*cot(d*x + c))^(9/2)*(a*sec(d*x + c) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(9/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cot(dx+c))^{\frac{9}{2}}(a\sec(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*cot(d*x + c))^(9/2)*(a*sec(d*x + c) + a)), x)
```

3.250 $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=413

$$-\frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}}+\frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}}-\frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}}+\frac{4\cot^2(c+dx)\csc(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}}-\frac{\tan^{-1}\left(1-\sqrt{2}\right.}{\sqrt{2}a^2d\sqrt{\tan(c+}}$$

```
[Out] (2*Cot[c + d*x])/(a^2*d*Sqrt[e*Cot[c + d*x]]) - (12*Cos[c + d*x]*Cot[c + d*
x])/(5*a^2*d*Sqrt[e*Cot[c + d*x]]) - (4*Cot[c + d*x]^3)/(5*a^2*d*Sqrt[e*Cot
[c + d*x]]) + (4*Cot[c + d*x]^2*Csc[c + d*x])/(5*a^2*d*Sqrt[e*Cot[c + d*x]]
) - (12*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2])/(5*a^2*d*Sqrt[e*Cot[c +
d*x]]*Sqrt[Sin[2*c + 2*d*x]]) - ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqr
t[2]*a^2*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) + ArcTan[1 + Sqrt[2]*Sq
rt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) +
 Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*Sqrt[e
*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + T
an[c + d*x]]/(2*Sqrt[2]*a^2*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]])
```

Rubi [A] time = 0.427095, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 19, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.76, Rules used = {3900, 3888, 3886, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2609, 2608, 2615, 2572, 2639, 2607, 30}

$$-\frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}}+\frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}}-\frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}}+\frac{4\cot^2(c+dx)\csc(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}}-\frac{\tan^{-1}\left(1-\sqrt{2}\right.}{\sqrt{2}a^2d\sqrt{\tan(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```


```
[Out] (2*Cot[c + d*x])/(a^2*d*Sqrt[e*Cot[c + d*x]]) - (12*Cos[c + d*x]*Cot[c + d*
x])/(5*a^2*d*Sqrt[e*Cot[c + d*x]]) - (4*Cot[c + d*x]^3)/(5*a^2*d*Sqrt[e*Cot
[c + d*x]]) + (4*Cot[c + d*x]^2*Csc[c + d*x])/(5*a^2*d*Sqrt[e*Cot[c + d*x]]
) - (12*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2])/(5*a^2*d*Sqrt[e*Cot[c +
d*x]]*Sqrt[Sin[2*c + 2*d*x]]) - ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqr
t[2]*a^2*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) + ArcTan[1 + Sqrt[2]*Sq
rt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) +
 Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*Sqrt[e
*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + T
an[c + d*x]]/(2*Sqrt[2]*a^2*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]])
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2608

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(n + 1)), x] - Dist[(a^2*(m - 2))/(b^2*(n + 1)), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$

$$= \frac{\int \frac{(-a+a\sec(c+dx))^2}{\tan^{\frac{7}{2}}(c+dx)}\,dx}{a^4\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$

$$= \frac{\int \left(\frac{a^2}{\tan^{\frac{7}{2}}(c+dx)} - \frac{2a^2\sec(c+dx)}{\tan^{\frac{7}{2}}(c+dx)} + \frac{a^2\sec^2(c+dx)}{\tan^{\frac{7}{2}}(c+dx)}\right)dx}{a^4\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$

$$= \frac{\int \frac{1}{\tan^{\frac{7}{2}}(c+dx)}\,dx}{a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{\int \frac{\sec^2(c+dx)}{\tan^{\frac{7}{2}}(c+dx)}\,dx}{a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} - \frac{2 \int}{a^2\sqrt{e\cot(}}$$

$$= -\frac{2\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(}}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} + \frac{4\,\mathrm{co}}{5}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} + \frac{4\,\mathrm{co}}{5}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} + \frac{4\,\mathrm{co}}{5}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} + \frac{4\,\mathrm{co}}{5}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} + \frac{4\,\mathrm{co}}{5}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} + \frac{4\,\mathrm{co}}{5}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} + \frac{4\,\mathrm{co}}{5}$$

Mathematica [F] time = 12.9766, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])^2), x]
```

Maple [C] time = 0.315, size = 2117, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x)
```

```
[Out] -1/10/a^2/d*2^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^3*(-5*I*cos(d*x+c)^2*(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-10*I*cos(d*x+c)*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+10*I*((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)+5*I*cos(d*x+c)^2*((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-5*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2
+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+24*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1
+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2)*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1
/2))*cos(d*x+c)^2-12*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*Ell
ipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)
^2+5*I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*I*((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
```

```
^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-10*cos(d*x+c)*((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-10*cos(d*x+c)*((1-cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+48*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2),1/2*2^(1/2))-24*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-
5*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+24*((1-cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/
2*2^(1/2))-12*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+2*cos(d*x+c)^2*2^
(1/2)-2*cos(d*x+c)*2^(1/2))/sin(d*x+c)^7/(e*cos(d*x+c)/sin(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e \cot (dx+c)}(a \sec (dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**2/(e*cot(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e \cot (dx+c)}(a \sec (dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a)^2), x)
```

3.251 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=359

$$\frac{2\sqrt{\sin(2c+2dx)}\cot(c+dx)\csc(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3a^2d(e\cot(c+dx))^{3/2}}-\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}}+\frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}}$$

```
[Out] (-4*Cot[c + d*x]^3)/(3*a^2*d*(e*Cot[c + d*x])^(3/2)) + (4*Cot[c + d*x]^2*Cs
c[c + d*x])/(3*a^2*d*(e*Cot[c + d*x])^(3/2)) + (2*Cot[c + d*x]*Csc[c + d*x]
*EllipticF[c - Pi/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x]])/(3*a^2*d*(e*Cot[c + d
*x])^(3/2)) + ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*(e*Cot[
c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
]/(Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) + Log[1 - Sqrt[
2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(3/
2)*Tan[c + d*x]^(3/2)) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]
/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))
```

Rubi [A] time = 0.388679, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 18, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.72, Rules used = {3900, 3888, 3886, 3474, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2609, 2614, 2573, 2641, 2607, 30}

$$-\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}}+\frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+}\right.}{\sqrt{2}a^2d\tan^{\frac{3}{2}}(c+dx)(e\,\text{co}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2),x]
```


```
[Out] (-4*Cot[c + d*x]^3)/(3*a^2*d*(e*Cot[c + d*x])^(3/2)) + (4*Cot[c + d*x]^2*Cs
c[c + d*x])/(3*a^2*d*(e*Cot[c + d*x])^(3/2)) + (2*Cot[c + d*x]*Csc[c + d*x]
*EllipticF[c - Pi/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x]])/(3*a^2*d*(e*Cot[c + d
*x])^(3/2)) + ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*(e*Cot[
c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
]/(Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) + Log[1 - Sqrt[
2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(3/
2)*Tan[c + d*x]^(3/2)) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]
/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))
```


Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
```

```
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\int \frac{(-a+a\sec(c+dx))^2}{\tan^{\frac{5}{2}}(c+dx)}\,dx}{a^4(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\int \left(\frac{a^2}{\tan^{\frac{5}{2}}(c+dx)} - \frac{2a^2\sec(c+dx)}{\tan^{\frac{5}{2}}(c+dx)} + \frac{a^2\sec^2(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}\right)dx}{a^4(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)}\,dx}{a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} + \frac{\int \frac{\sec^2(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}\,dx}{a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} - \cdots$$

$$= -\frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{2\int \frac{\sec(c+\cdots}{\sqrt{\tan(c+\cdots}}}{3a^2(e\cot(c+dx))^{3/\cdots}}$$

$$= -\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{\left(2\cos^{\frac{3}{2}}(c+dx)\right)\int \cdots}{3a^2(e\cot(c+d\cdots}$$

$$= -\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{\left(2\cot(c+dx)\csc(\cdots\right.}{\cdots}$$

$$= -\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{2\cot(c+dx)\csc(c\cdots}{3\cdots}$$

$$= -\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{2\cot(c+dx)\csc(c\cdots}{3\cdots}$$

$$= -\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{2\cot(c+dx)\csc(c\cdots}{3\cdots}$$

$$= -\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{2\cot(c+dx)\csc(c\cdots}{3\cdots}$$

Mathematica [F] time = 9.4704, size = 0, normalized size = 0.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2), x]
```

Maple [C] time = 0.277, size = 1267, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/a^2/d*2^(1/2)*(-1+cos(d*x+c))^2*(3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-3*I*EllipticPi(((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+3*sin(d*x+c)*cos(d*x+c)*(
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*sin(d*x+c)*cos(d*x+
c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-10*sin(d*x+c)*cos
(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+3*I*EllipticPi(((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-3*I*sin(d*x+c)*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*sin(d*x+c)*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2),1/2+1/2*I,1/2*2^(1/2))-10*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
```

```
)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/
2*2^(1/2))+4*cos(d*x+c)^2*2^(1/2)-4*cos(d*x+c)*2^(1/2))*cos(d*x+c)*(cos(d*x
+c)+1)^2/(e*cos(d*x+c)/sin(d*x+c))^(3/2)/sin(d*x+c)^7
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(3/2)/(a+a*sec(d*x+c))**2,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\cot\left(dx+c\right)\right)^{\frac{3}{2}}\left(a\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cot(d*x + c))^(3/2)*(a*sec(d*x + c) + a)^2), x)
```

3.252 $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=355

$$-\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}}+\frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+d}\right.}{\sqrt{2}a^2d\tan^{\frac{5}{2}}(c+dx)(e\cot}$$

```
[Out] (-4*Cot[c + d*x]^3)/(a^2*d*(e*Cot[c + d*x])^(5/2)) + (4*Cos[c + d*x]*Cot[c
+ d*x]^3)/(a^2*d*(e*Cot[c + d*x])^(5/2)) + (4*Cos[c + d*x]*Cot[c + d*x]^2*E
llipticE[c - Pi/4 + d*x, 2])/(a^2*d*(e*Cot[c + d*x])^(5/2)*Sqrt[Sin[2*c + 2
*d*x]]) + ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*(e*Cot[c +
d*x])^(5/2)*Tan[c + d*x]^(5/2)) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(S
qrt[2]*a^2*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2)) - Log[1 - Sqrt[2]*S
qrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(5/2)*T
an[c + d*x]^(5/2)) + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*
Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))
```

Rubi [A] time = 0.417798, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 18, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.72, Rules used = {3900, 3888, 3886, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2608, 2615, 2572, 2639, 2607, 30}

$$-\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}}+\frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+d}\right.}{\sqrt{2}a^2d\tan^{\frac{5}{2}}(c+dx)(e\cot}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2),x]
```


```
[Out] (-4*Cot[c + d*x]^3)/(a^2*d*(e*Cot[c + d*x])^(5/2)) + (4*Cos[c + d*x]*Cot[c
+ d*x]^3)/(a^2*d*(e*Cot[c + d*x])^(5/2)) + (4*Cos[c + d*x]*Cot[c + d*x]^2*E
llipticE[c - Pi/4 + d*x, 2])/(a^2*d*(e*Cot[c + d*x])^(5/2)*Sqrt[Sin[2*c + 2
*d*x]]) + ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*(e*Cot[c +
d*x])^(5/2)*Tan[c + d*x]^(5/2)) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(S
qrt[2]*a^2*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2)) - Log[1 - Sqrt[2]*S
qrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(5/2)*T
an[c + d*x]^(5/2)) + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*
Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))
```


Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2608

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(n + 1)), x] - Dist[(a^2*(m - 2))/(b^2*(n + 1)), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
```

```
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

$$= \frac{\int \frac{(-a+a\sec(c+dx))^2}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{a^4(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

$$= \frac{\int \left(\frac{a^2}{\tan^{\frac{3}{2}}(c+dx)} - \frac{2a^2\sec(c+dx)}{\tan^{\frac{3}{2}}(c+dx)} + \frac{a^2\sec^2(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\right)dx}{a^4(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

$$= \frac{\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} + \frac{\int \frac{\sec^2(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} - \frac{}{a^2(}$$

$$= -\frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} - \frac{\int\sqrt{\tan(c+d}}{a^2(e\cot(c+dx))^{5/2}\,\mathrm{t}}$$

$$= -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{\left(4\cos^{\frac{5}{2}}(c+dx)\right)\int\sqrt{}}{a^2(e\cot(c+}$$

$$= -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{\left(4\cos(c+dx)\cot^2(\right.}{a^2(e\cot(c+}$$

$$= -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^2(c}{a^2d(e\cot(c+dx)}$$

$$= -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^2(c}{a^2d(e\cot(c+dx)}$$

$$= -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^2(c}{a^2d(e\cot(c+dx)}$$

$$= -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^2(c}{a^2d(e\cot(c+dx)}$$

<u>Mathematica</u> [F] time = 4.93531, size = 0, normalized size = 0.

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2), x]
```

Maple [C] time = 0.28, size = 367, normalized size = 1.

$$\frac{\sqrt{2}\,(\cos(dx+c)+1)^2\,(-1+\cos(dx+c))\,(\cos(dx+c))^2}{2\,da^2\,(\sin(dx+c))^5}\left(i\text{EllipticPi}\left(\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/2/a^2/d*2^(1/2)*(cos(d*x+c)+1)^2*(I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c
))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi((-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+8*EllipticE((-(-1+cos(d
*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-4*EllipticF((-(-1+cos(d*x+
c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-EllipticPi((-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-EllipticPi((-(-1+cos(d*
x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)))*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(
d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*(-1+cos(d*x+c))*cos(d*x+c)^2/(e*cos(d*
x+c)/sin(d*x+c))^(5/2)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cot(dx+c))^{\frac{5}{2}}\,(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cot(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2), x)
```

3.253 $\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=321

$$-\frac{2\sqrt{\sin(2c+2dx)}\cot^3(c+dx)\csc(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{a^2d(e\cot(c+dx))^{7/2}}+\frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(}\right.}{\sqrt{2}a^2d\tan^{\frac{7}{2}}(c+dx)(e\,co}$$

```
[Out] (2*Cot[c + d*x]^3)/(a^2*d*(e*Cot[c + d*x])^(7/2)) - (2*Cot[c + d*x]^3*Csc[c
 + d*x]*EllipticF[c - Pi/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x]])/(a^2*d*(e*Cot[
c + d*x])^(7/2)) - ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*(e
*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2)) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c +
d*x]]]/(Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2)) - Log[1 -
Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x]
)^(7/2)*Tan[c + d*x]^(7/2)) + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2))
```

Rubi [A] time = 0.372385, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {3900, 3888, 3886, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641, 2607, 30}

$$\frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}a^2d\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}-\frac{\log\left(\tan(c+d\right.}{2\sqrt{2}a^2d\tan^{\frac{7}{2}}(}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (2*Cot[c + d*x]^3)/(a^2*d*(e*Cot[c + d*x])^(7/2)) - (2*Cot[c + d*x]^3*Csc[c
 + d*x]*EllipticF[c - Pi/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x]])/(a^2*d*(e*Cot[
c + d*x])^(7/2)) - ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*(e
*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2)) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c +
d*x]]]/(Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2)) - Log[1 -
Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x]
)^(7/2)*Tan[c + d*x]^(7/2)) + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2))
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
```

```
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

$$= \frac{\int \frac{(-a+a\sec(c+dx))^2}{\sqrt{\tan(c+dx)}}\,dx}{a^4(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

$$= \frac{\int \left(\frac{a^2}{\sqrt{\tan(c+dx)}} - \frac{2a^2\sec(c+dx)}{\sqrt{\tan(c+dx)}} + \frac{a^2\sec^2(c+dx)}{\sqrt{\tan(c+dx)}}\right)dx}{a^4(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

$$= \frac{\int \frac{1}{\sqrt{\tan(c+dx)}}\,dx}{a^2(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} + \frac{\int \frac{\sec^2(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{a^2(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} - \frac{}{a}$$

$$= -\frac{\left(2\cos^{\frac{7}{2}}(c+dx)\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{a^2(e\cot(c+dx))^{7/2}\sin^{\frac{7}{2}}(c+dx)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, \tan(c+\right.}{a^2d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c}$$

$$= \frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}} - \frac{\left(2\cot^3(c+dx)\csc(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{}{\sqrt{\sin}}}{a^2(e\cot(c+dx))^{7/2}}$$

$$= \frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}} - \frac{2\cot^3(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(}}{a^2d(e\cot(c+dx))^{7/2}}$$

$$= \frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}} - \frac{2\cot^3(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(}}{a^2d(e\cot(c+dx))^{7/2}}$$

$$= \frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}} - \frac{2\cot^3(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(}}{a^2d(e\cot(c+dx))^{7/2}}$$

$$= \frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}} - \frac{2\cot^3(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(}}{a^2d(e\cot(c+dx))^{7/2}}$$

Mathematica [F] time = 4.75572, size = 0, normalized size = 0.

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((e*Cot[c + d*x])^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] Integrate[1/((e*Cot[c + d*x])^(7/2)*(a + a*Sec[c + d*x])^2), x]
```

Maple [C] time = 0.3, size = 653, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/2/a^2/d*2^(1/2)*(I*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)-I*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)+sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+sin
(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-6*sin(d*x+c)
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-2*cos(d*x+c)*2^(1/2)+2*2^(1/2)
)*(-1+cos(d*x+c))*cos(d*x+c)^3*(cos(d*x+c)+1)^2/sin(d*x+c)^7/(e*cos(d*x+c)/
sin(d*x+c))^(7/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(7/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cot(dx+c))^{\frac{7}{2}}(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cot(d*x + c))^(7/2)*(a*sec(d*x + c) + a)^2), x)
```

3.254 $\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=357

$$\frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}}-\frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+d}\right.}{\sqrt{2}a^2d\tan^{\frac{9}{2}}(c+dx)(e\cot(}$$

```
[Out] (2*Cot[c + d*x]^3)/(3*a^2*d*(e*Cot[c + d*x])^(9/2)) - (4*Cos[c + d*x]*Cot[c
 + d*x]^3)/(a^2*d*(e*Cot[c + d*x])^(9/2)) + (4*Cos[c + d*x]*Cot[c + d*x]^4*
EllipticE[c - Pi/4 + d*x, 2])/(a^2*d*(e*Cot[c + d*x])^(9/2)*Sqrt[Sin[2*c +
2*d*x]]) - ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*(e*Cot[c +
 d*x])^(9/2)*Tan[c + d*x]^(9/2)) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(
Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2)) + Log[1 - Sqrt[2]*
Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(9/2)*
Tan[c + d*x]^(9/2)) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2
*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2))
```

Rubi [A] time = 0.393955, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.68, Rules used = {3900, 3888, 3886, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639, 2607, 30}

$$\frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}}-\frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+d}\right.}{\sqrt{2}a^2d\tan^{\frac{9}{2}}(c+dx)(e\cot(}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(9/2)*(a + a*Sec[c + d*x])^2),x]
```


```
[Out] (2*Cot[c + d*x]^3)/(3*a^2*d*(e*Cot[c + d*x])^(9/2)) - (4*Cos[c + d*x]*Cot[c
 + d*x]^3)/(a^2*d*(e*Cot[c + d*x])^(9/2)) + (4*Cos[c + d*x]*Cot[c + d*x]^4*
EllipticE[c - Pi/4 + d*x, 2])/(a^2*d*(e*Cot[c + d*x])^(9/2)*Sqrt[Sin[2*c +
2*d*x]]) - ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*(e*Cot[c +
 d*x])^(9/2)*Tan[c + d*x]^(9/2)) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(
Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2)) + Log[1 - Sqrt[2]*
Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(9/2)*
Tan[c + d*x]^(9/2)) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2
*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2))
```


Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}$$
$$= \frac{\int (-a+a\sec(c+dx))^2\sqrt{\tan(c+dx)}\,dx}{a^4(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}$$
$$= \frac{\int \left(a^2\sqrt{\tan(c+dx)} - 2a^2\sec(c+dx)\sqrt{\tan(c+dx)} + a^2\sec^2(c+dx)\sqrt{\tan(\right.}{a^4(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}$$
$$= \frac{\int \sqrt{\tan(c+dx)}\,dx}{a^2(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} + \frac{\int \sec^2(c+dx)\sqrt{\tan(c+dx)}\,dx}{a^2(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} - \frac{2\int}{a^2(}$$
$$= -\frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}} + \frac{4\int \cos(c+dx)\sqrt{\tan(c+dx)}\,dx}{a^2(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} + \frac{\text{Subst}}{a^2d(e\text{co}}$$
$$= \frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}} - \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}} + \frac{\left(4\cos^{\frac{9}{2}}(c+dx)\right)\int\sqrt{}}{a^2(e\cot(c+}$$
$$= \frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}} - \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}} + \frac{(4\cos(c+dx)\cot^4(c}{a^2(e\cot(c+d}$$
$$= \frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}} - \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}} + \frac{4\cos(c+dx)\cot^4(c}{a^2d(e\cot(c+dx)}$$
$$= \frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}} - \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}} + \frac{4\cos(c+dx)\cot^4(c}{a^2d(e\cot(c+dx)}$$
$$= \frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}} - \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}} + \frac{4\cos(c+dx)\cot^4(c}{a^2d(e\cot(c+dx)}$$

Mathematica [F] time = 52.0152, size = 0, normalized size = 0.

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((e*Cot[c + d*x])^(9/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] Integrate[1/((e*Cot[c + d*x])^(9/2)*(a + a*Sec[c + d*x])^2), x]
```

Maple [C] time = 0.268, size = 1480, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/a^2/d*2^(1/2)*(-1+cos(d*x+c))^2*(3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*cos(d*x+c)^2+3*I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*
I,1/2*2^(1/2))*cos(d*x+c)-3*I*cos(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)-3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*E
llipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2
))*cos(d*x+c)^2-3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*Ellipt
icPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*co
s(d*x+c)^2-24*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2+12*(
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2-3*cos(d*x+c)*((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*cos(d*x+c)*((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-24*cos(d*x+c)*((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*
```

```
x+c))^(1/2),1/2*2^(1/2))+12*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2
^(1/2))+10*cos(d*x+c)^2*2^(1/2)-12*cos(d*x+c)*2^(1/2)+2*2^(1/2))*cos(d*x+c)
^3*(cos(d*x+c)+1)^2/(e*cos(d*x+c)/sin(d*x+c))^(9/2)/sin(d*x+c)^9
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(9/2)/(a+a*sec(d*x+c))**2,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cot(dx+c))^{\frac{9}{2}}(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cot(d*x + c))^(9/2)*(a*sec(d*x + c) + a)^2), x)
```

3.255 $\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=389

$$\frac{2\sqrt{\sin(2c+2dx)}\cot^5(c+dx)\csc(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3a^2d(e\cot(c+dx))^{11/2}}+\frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}}+\frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}}-\frac{4}{\ }$$

```
[Out] (2*Cot[c + d*x]^3)/(5*a^2*d*(e*Cot[c + d*x])^(11/2)) + (2*Cot[c + d*x]^5)/(
a^2*d*(e*Cot[c + d*x])^(11/2)) - (4*Cot[c + d*x]^4*Csc[c + d*x])/(3*a^2*d*(
e*Cot[c + d*x])^(11/2)) + (2*Cot[c + d*x]^5*Csc[c + d*x]*EllipticF[c - Pi/4
 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x]])/(3*a^2*d*(e*Cot[c + d*x])^(11/2)) + ArcT
an[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(11/2)*T
an[c + d*x]^(11/2)) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d
*(e*Cot[c + d*x])^(11/2)*Tan[c + d*x]^(11/2)) + Log[1 - Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(11/2)*Tan[c + d*
x]^(11/2)) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*
a^2*d*(e*Cot[c + d*x])^(11/2)*Tan[c + d*x]^(11/2))
```

Rubi [A] time = 0.48042, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 18, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.72, Rules used = {3900, 3888, 3886, 3473, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2611, 2614, 2573, 2641, 2607, 30}

$$\frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}}+\frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}}-\frac{4\cot^4(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{11/2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\tan^{\frac{11}{2}}(c+dx)(e\cot(c+dx))^{11/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(11/2)*(a + a*Sec[c + d*x])^2),x]
```


```
[Out] (2*Cot[c + d*x]^3)/(5*a^2*d*(e*Cot[c + d*x])^(11/2)) + (2*Cot[c + d*x]^5)/(
a^2*d*(e*Cot[c + d*x])^(11/2)) - (4*Cot[c + d*x]^4*Csc[c + d*x])/(3*a^2*d*(
e*Cot[c + d*x])^(11/2)) + (2*Cot[c + d*x]^5*Csc[c + d*x]*EllipticF[c - Pi/4
 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x]])/(3*a^2*d*(e*Cot[c + d*x])^(11/2)) + ArcT
an[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(11/2)*T
an[c + d*x]^(11/2)) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d
*(e*Cot[c + d*x])^(11/2)*Tan[c + d*x]^(11/2)) + Log[1 - Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(11/2)*Tan[c + d*
x]^(11/2)) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*
a^2*d*(e*Cot[c + d*x])^(11/2)*Tan[c + d*x]^(11/2))
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}$$

$$= \frac{\int(-a+a\sec(c+dx))^2\tan^{\frac{3}{2}}(c+dx)\,dx}{a^4(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}$$

$$= \frac{\int\Big(a^2\tan^{\frac{3}{2}}(c+dx)-2a^2\sec(c+dx)\tan^{\frac{3}{2}}(c+dx)+a^2\sec^2(c+dx)\tan^{\frac{3}{2}}(}{a^4(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}$$

$$= \frac{\int\tan^{\frac{3}{2}}(c+dx)\,dx}{a^2(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)} + \frac{\int\sec^2(c+dx)\tan^{\frac{3}{2}}(c+dx)\,dx}{a^2(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)} -$$

$$= \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{11/2}} + \frac{2\int\frac{\sec(c+d}{\sqrt{\tan(c+}}}{3a^2(e\cot(c+dx))^{11/}}$$

$$= \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}} + \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+dx)\csc(c+d}{3a^2d(e\cot(c+dx))^{11/2}}$$

$$= \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}} + \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+dx)\csc(c+d}{3a^2d(e\cot(c+dx))^{11/2}}$$

$$= \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}} + \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+dx)\csc(c+d}{3a^2d(e\cot(c+dx))^{11/2}}$$

$$= \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}} + \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+dx)\csc(c+d}{3a^2d(e\cot(c+dx))^{11/2}}$$

$$= \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}} + \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+dx)\csc(c+d}{3a^2d(e\cot(c+dx))^{11/2}}$$

$$= \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}} + \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+dx)\csc(c+d}{3a^2d(e\cot(c+dx))^{11/2}}$$

Mathematica [F] time = 13.1504, size = 0, normalized size = 0.

$$\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((e*Cot[c + d*x])^(11/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] Integrate[1/((e*Cot[c + d*x])^(11/2)*(a + a*Sec[c + d*x])^2), x]
```

Maple [C] time = 0.274, size = 721, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/30/a^2/d*2^(1/2)*(-1+cos(d*x+c))*(15*I*sin(d*x+c)*EllipticPi(((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-15*I*sin(d*x+c)*EllipticPi(((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+15*sin(d*x+c)*Ellipti
cPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+15*sin(d*x+c)*
EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/
2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-50*sin(
d*x+c)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+24*cos(d*x+
c)^3*2^(1/2)-44*cos(d*x+c)^2*2^(1/2)+26*cos(d*x+c)*2^(1/2)-6*2^(1/2))*cos(d
*x+c)^3*(cos(d*x+c)+1)^2/(e*cos(d*x+c)/sin(d*x+c))^(11/2)/sin(d*x+c)^9
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(11/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cot(dx+c))^{\frac{11}{2}}(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cot(d*x + c))^(11/2)*(a*sec(d*x + c) + a)^2), x)
```

3.256 $\int (a+b\sec(c+dx))\tan^7(c+dx)\,dx$

Optimal. Leaf size=111

$$\frac{\tan^6(c+dx)(7a+6b\sec(c+dx))}{42d}-\frac{\tan^4(c+dx)(35a+24b\sec(c+dx))}{140d}+\frac{\tan^2(c+dx)(35a+16b\sec(c+dx))}{70d}+\frac{a\,\mathrm{lo}}{}$$

```
[Out] (a*Log[Cos[c + d*x]])/d - (16*b*Sec[c + d*x])/(35*d) + ((35*a + 16*b*Sec[c
+ d*x])*Tan[c + d*x]^2)/(70*d) - ((35*a + 24*b*Sec[c + d*x])*Tan[c + d*x]^4
)/(140*d) + ((7*a + 6*b*Sec[c + d*x])*Tan[c + d*x]^6)/(42*d)
```

Rubi [A] time = 0.156044, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3881, 3884, 3475, 2606, 8}

$$\frac{\tan^6(c+dx)(7a+6b\sec(c+dx))}{42d}-\frac{\tan^4(c+dx)(35a+24b\sec(c+dx))}{140d}+\frac{\tan^2(c+dx)(35a+16b\sec(c+dx))}{70d}+\frac{a\,\mathrm{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^7,x]
```


```
[Out] (a*Log[Cos[c + d*x]])/d - (16*b*Sec[c + d*x])/(35*d) + ((35*a + 16*b*Sec[c
+ d*x])*Tan[c + d*x]^2)/(70*d) - ((35*a + 24*b*Sec[c + d*x])*Tan[c + d*x]^4
)/(140*d) + ((7*a + 6*b*Sec[c + d*x])*Tan[c + d*x]^6)/(42*d)
```


Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))\tan^7(c+dx)\,dx &= \frac{(7a+6b\sec(c+dx))\tan^6(c+dx)}{42d}-\frac{1}{7}\int (7a+6b\sec(c+dx))\tan^5(c+dx)\,dx\\ &= -\frac{(35a+24b\sec(c+dx))\tan^4(c+dx)}{140d}+\frac{(7a+6b\sec(c+dx))\tan^6(c+dx)}{42d}+\frac{1}{35}\\ &= \frac{(35a+16b\sec(c+dx))\tan^2(c+dx)}{70d}-\frac{(35a+24b\sec(c+dx))\tan^4(c+dx)}{140d}+\frac{(}{}\\ &= \frac{(35a+16b\sec(c+dx))\tan^2(c+dx)}{70d}-\frac{(35a+24b\sec(c+dx))\tan^4(c+dx)}{140d}+\frac{(}{}\\ &= \frac{a\log(\cos(c+dx))}{d}+\frac{(35a+16b\sec(c+dx))\tan^2(c+dx)}{70d}-\frac{(35a+24b\sec(c+}{140}\\ &= \frac{a\log(\cos(c+dx))}{d}-\frac{16b\sec(c+dx)}{35d}+\frac{(35a+16b\sec(c+dx))\tan^2(c+dx)}{70d}-\frac{(}{}\end{aligned}$$

Mathematica [A] time = 0.447335, size = 106, normalized size = 0.95

$$\frac{a\left(2\tan^6(c+dx)-3\tan^4(c+dx)+6\tan^2(c+dx)+12\log(\cos(c+dx))\right)}{12d}+\frac{b\sec^7(c+dx)}{7d}-\frac{3b\sec^5(c+dx)}{5d}+\frac{b\sec^3(c}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^7,x]
```

```
[Out] -((b*Sec[c + d*x])/d) + (b*Sec[c + d*x]^3)/d - (3*b*Sec[c + d*x]^5)/(5*d) +
 (b*Sec[c + d*x]^7)/(7*d) + (a*(12*Log[Cos[c + d*x]] + 6*Tan[c + d*x]^2 - 3
```

```
*Tan[c + d*x]^4 + 2*Tan[c + d*x]^6))/(12*d)
```

Maple [B] time = 0.045, size = 216, normalized size = 2.

$$\frac{a(\tan(dx+c))^6}{6d}-\frac{a(\tan(dx+c))^4}{4d}+\frac{(\tan(dx+c))^2 a}{2d}+\frac{a\ln(\cos(dx+c))}{d}+\frac{b(\sin(dx+c))^8}{7d(\cos(dx+c))^7}-\frac{b(\sin(dx+c))^8}{35d(\cos(dx+c))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c)^7,x)
```

```
[Out] 1/6/d*a*tan(d*x+c)^6-1/4/d*a*tan(d*x+c)^4+1/2/d*a*tan(d*x+c)^2+a*ln(cos(d*x
+c))/d+1/7/d*b*sin(d*x+c)^8/cos(d*x+c)^7-1/35/d*b*sin(d*x+c)^8/cos(d*x+c)^5
+1/35/d*b*sin(d*x+c)^8/cos(d*x+c)^3-1/7/d*b*sin(d*x+c)^8/cos(d*x+c)-16/35/d
*b*cos(d*x+c)-1/7/d*b*cos(d*x+c)*sin(d*x+c)^6-6/35/d*b*cos(d*x+c)*sin(d*x+c
)^4-8/35/d*b*cos(d*x+c)*sin(d*x+c)^2
```

Maxima [A] time = 1.03822, size = 127, normalized size = 1.14

$$\frac{420\,a\log(\cos(dx+c))-\frac{420\,b\cos(dx+c)^6-630\,a\cos(dx+c)^5-420\,b\cos(dx+c)^4+315\,a\cos(dx+c)^3+252\,b\cos(dx+c)^2-70\,a\cos(dx+c)-60\,b}{\cos(dx+c)^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/420*(420*a*log(cos(d*x + c)) - (420*b*cos(d*x + c)^6 - 630*a*cos(d*x + c)
^5 - 420*b*cos(d*x + c)^4 + 315*a*cos(d*x + c)^3 + 252*b*cos(d*x + c)^2 - 7
0*a*cos(d*x + c) - 60*b)/cos(d*x + c)^7)/d
```

Fricas [A] time = 1.49646, size = 284, normalized size = 2.56

$$\frac{420\,a\cos(dx+c)^7\log(-\cos(dx+c))-420\,b\cos(dx+c)^6+630\,a\cos(dx+c)^5+420\,b\cos(dx+c)^4-315\,a\cos(dx-}{420\,d\cos(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/420*(420*a*cos(d*x + c)^7*log(-cos(d*x + c)) - 420*b*cos(d*x + c)^6 + 630
*a*cos(d*x + c)^5 + 420*b*cos(d*x + c)^4 - 315*a*cos(d*x + c)^3 - 252*b*cos
(d*x + c)^2 + 70*a*cos(d*x + c) + 60*b)/(d*cos(d*x + c)^7)
```

Sympy [A] time = 19.3387, size = 148, normalized size = 1.33

$$\begin{cases} -\frac{a\log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{a\tan^6(c+dx)}{6d} - \frac{a\tan^4(c+dx)}{4d} + \frac{a\tan^2(c+dx)}{2d} + \frac{b\tan^6(c+dx)\sec(c+dx)}{7d} - \frac{6b\tan^4(c+dx)\sec(c+dx)}{35d} + \frac{8b\tan^2(c+dx)\sec \\ x\,(a+b\sec(c))\tan^7(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)**7,x)
```

```
[Out] Piecewise((-a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**6/(6*d) - a*
tan(c + d*x)**4/(4*d) + a*tan(c + d*x)**2/(2*d) + b*tan(c + d*x)**6*sec(c +
 d*x)/(7*d) - 6*b*tan(c + d*x)**4*sec(c + d*x)/(35*d) + 8*b*tan(c + d*x)**2
*sec(c + d*x)/(35*d) - 16*b*sec(c + d*x)/(35*d), Ne(d, 0)), (x*(a + b*sec(c
))*tan(c)**7, True))
```

Giac [B] time = 8.00617, size = 428, normalized size = 3.86

$$-\frac{420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{1089\,a+384\,b+\frac{8463\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2688\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{28749\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] -1/420*(420*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*a*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (1089*a + 384*b + 84
63*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2688*b*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) + 28749*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 8064*b
*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 56035*a*(cos(d*x + c) - 1)^3/(
cos(d*x + c) + 1)^3 + 13440*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 5
```

```
6035*a*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 28749*a*(cos(d*x + c) -
1)^5/(cos(d*x + c) + 1)^5 + 8463*a*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^
6 + 1089*a*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/((cos(d*x + c) - 1)/(
cos(d*x + c) + 1) + 1)^7)/d
```

3.257 $\int (a+b\sec(c+dx))\tan^5(c+dx)\,dx$

Optimal. Leaf size=84

$$\frac{\tan^4(c+dx)(5a+4b\sec(c+dx))}{20d}-\frac{\tan^2(c+dx)(15a+8b\sec(c+dx))}{30d}-\frac{a\log(\cos(c+dx))}{d}+\frac{8b\sec(c+dx)}{15d}$$

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (8*b*Sec[c + d*x])/(15*d) - ((15*a + 8*b*Sec[c
 + d*x])*Tan[c + d*x]^2)/(30*d) + ((5*a + 4*b*Sec[c + d*x])*Tan[c + d*x]^4)
/(20*d)
```

Rubi [A] time = 0.0935648, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3881, 3884, 3475, 2606, 8}

$$\frac{\tan^4(c+dx)(5a+4b\sec(c+dx))}{20d}-\frac{\tan^2(c+dx)(15a+8b\sec(c+dx))}{30d}-\frac{a\log(\cos(c+dx))}{d}+\frac{8b\sec(c+dx)}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^5,x]
```



```
[Out] -((a*Log[Cos[c + d*x]])/d) + (8*b*Sec[c + d*x])/(15*d) - ((15*a + 8*b*Sec[c
 + d*x])*Tan[c + d*x]^2)/(30*d) + ((5*a + 4*b*Sec[c + d*x])*Tan[c + d*x]^4)
/(20*d)
```


Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))\tan^5(c+dx)\,dx = \frac{(5a+4b\sec(c+dx))\tan^4(c+dx)}{20d} - \frac{1}{5}\int (5a+4b\sec(c+dx))\tan^3(c+dx)\,dx$$

$$= -\frac{(15a+8b\sec(c+dx))\tan^2(c+dx)}{30d} + \frac{(5a+4b\sec(c+dx))\tan^4(c+dx)}{20d} + \frac{1}{15}$$

$$= -\frac{(15a+8b\sec(c+dx))\tan^2(c+dx)}{30d} + \frac{(5a+4b\sec(c+dx))\tan^4(c+dx)}{20d} + a$$

$$= -\frac{a\log(\cos(c+dx))}{d} - \frac{(15a+8b\sec(c+dx))\tan^2(c+dx)}{30d} + \frac{(5a+4b\sec(c+d}{20}$$

$$= -\frac{a\log(\cos(c+dx))}{d} + \frac{8b\sec(c+dx)}{15d} - \frac{(15a+8b\sec(c+dx))\tan^2(c+dx)}{30d} +$$

Mathematica [A] time = 0.189151, size = 82, normalized size = 0.98

$$-\frac{a\left(-\tan^4(c+dx)+2\tan^2(c+dx)+4\log(\cos(c+dx))\right)}{4d} + \frac{b\sec^5(c+dx)}{5d} - \frac{2b\sec^3(c+dx)}{3d} + \frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] (b*Sec[c + d*x])/d - (2*b*Sec[c + d*x]^3)/(3*d) + (b*Sec[c + d*x]^5)/(5*d)
- (a*(4*Log[Cos[c + d*x]] + 2*Tan[c + d*x]^2 - Tan[c + d*x]^4))/(4*d)
```

Maple [B] time = 0.04, size = 161, normalized size = 1.9

$$\frac{a (\tan (dx+c))^4}{4 d} - \frac{(\tan (dx+c))^2 a}{2 d} - \frac{a \ln (\cos (dx+c))}{d} + \frac{b (\sin (dx+c))^6}{5 d (\cos (dx+c))^5} - \frac{b (\sin (dx+c))^6}{15 d (\cos (dx+c))^3} + \frac{b (\sin (dx+c))^6}{5 d \cos (dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c)^5,x)
```

```
[Out] 1/4/d*a*tan(d*x+c)^4-1/2/d*a*tan(d*x+c)^2-a*ln(cos(d*x+c))/d+1/5/d*b*sin(d*
x+c)^6/cos(d*x+c)^5-1/15/d*b*sin(d*x+c)^6/cos(d*x+c)^3+1/5/d*b*sin(d*x+c)^6
/cos(d*x+c)+8/15/d*b*cos(d*x+c)+1/5/d*b*cos(d*x+c)*sin(d*x+c)^4+4/15/d*b*co
s(d*x+c)*sin(d*x+c)^2
```

Maxima [A] time = 0.988523, size = 97, normalized size = 1.15

$$-\frac{60 a \log (\cos (dx+c)) - \frac{60 b \cos(dx+c)^4 - 60 a \cos(dx+c)^3 - 40 b \cos(dx+c)^2 + 15 a \cos(dx+c) + 12 b}{\cos(dx+c)^5}}{60 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(60*a*log(cos(d*x + c)) - (60*b*cos(d*x + c)^4 - 60*a*cos(d*x + c)^3
- 40*b*cos(d*x + c)^2 + 15*a*cos(d*x + c) + 12*b)/cos(d*x + c)^5)/d
```

Fricas [A] time = 1.34255, size = 216, normalized size = 2.57

$$-\frac{60 a \cos (dx+c)^5 \log (-\cos (dx+c)) - 60 b \cos (dx+c)^4 + 60 a \cos (dx+c)^3 + 40 b \cos (dx+c)^2 - 15 a \cos (dx+c) -}{60 d \cos (dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/60*(60*a*cos(d*x + c)^5*log(-cos(d*x + c)) - 60*b*cos(d*x + c)^4 + 60*a*
cos(d*x + c)^3 + 40*b*cos(d*x + c)^2 - 15*a*cos(d*x + c) - 12*b)/(d*cos(d*x
```

```
+ c)^5)
```

Sympy [A] time = 5.30308, size = 112, normalized size = 1.33

$$\begin{cases} \frac{a\log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{a\tan^4(c+dx)}{4d} - \frac{a\tan^2(c+dx)}{2d} + \frac{b\tan^4(c+dx)\sec(c+dx)}{5d} - \frac{4b\tan^2(c+dx)\sec(c+dx)}{15d} + \frac{8b\sec(c+dx)}{15d} & \text{for } d\neq 0 \\ x\left(a+b\sec(c)\right)\tan^5(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)**5,x)
```

```
[Out] Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**4/(4*d) - a*t
an(c + d*x)**2/(2*d) + b*tan(c + d*x)**4*sec(c + d*x)/(5*d) - 4*b*tan(c + d
*x)**2*sec(c + d*x)/(15*d) + 8*b*sec(c + d*x)/(15*d), Ne(d, 0)), (x*(a + b*
sec(c))*tan(c)**5, True))
```

Giac [B] time = 3.54973, size = 335, normalized size = 3.99

$$\frac{60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{137\,a+64\,b+\frac{805\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{320\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{1970\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{640\,b(\cos\cdots}{(\cos\cdots}}{\left(\frac{\cos(dx+c)-\cdots}{\cos(dx+c)+\cdots}\right.}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (137*a + 64*b + 805*a*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + 320*b*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 1970*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 640*b*(cos(d*x
+ c) - 1)^2/(cos(d*x + c) + 1)^2 + 1970*a*(cos(d*x + c) - 1)^3/(cos(d*x + c
) + 1)^3 + 805*a*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*a*(cos(d*x
 + c) - 1)^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 1)^5)/d
```

3.258 $\int (a+b\sec(c+dx))\tan^3(c+dx)\,dx$

Optimal. Leaf size=55

$$\frac{\tan^2(c+dx)(3a+2b\sec(c+dx))}{6d}+\frac{a\log(\cos(c+dx))}{d}-\frac{2b\sec(c+dx)}{3d}$$

```
[Out] (a*Log[Cos[c + d*x]])/d - (2*b*Sec[c + d*x])/(3*d) + ((3*a + 2*b*Sec[c + d*
x])*Tan[c + d*x]^2)/(6*d)
```

Rubi [A] time = 0.0661531, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3881, 3884, 3475, 2606, 8}

$$\frac{\tan^2(c+dx)(3a+2b\sec(c+dx))}{6d}+\frac{a\log(\cos(c+dx))}{d}-\frac{2b\sec(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] (a*Log[Cos[c + d*x]])/d - (2*b*Sec[c + d*x])/(3*d) + ((3*a + 2*b*Sec[c + d*
x])*Tan[c + d*x]^2)/(6*d)
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))\tan^3(c+dx)\,dx &= \frac{(3a+2b\sec(c+dx))\tan^2(c+dx)}{6d}-\frac{1}{3}\int (3a+2b\sec(c+dx))\tan(c+dx)\,dx\\ &= \frac{(3a+2b\sec(c+dx))\tan^2(c+dx)}{6d}-a\int \tan(c+dx)\,dx-\frac{1}{3}(2b)\int \sec(c+dx)\\ &= \frac{a\log(\cos(c+dx))}{d}+\frac{(3a+2b\sec(c+dx))\tan^2(c+dx)}{6d}-\frac{(2b)\,\text{Subst}(\int 1\,dx,x}{3d}\\ &= \frac{a\log(\cos(c+dx))}{d}-\frac{2b\sec(c+dx)}{3d}+\frac{(3a+2b\sec(c+dx))\tan^2(c+dx)}{6d}\end{aligned}$$

Mathematica [A] time = 0.155668, size = 55, normalized size = 1.

$$\frac{a\left(\tan^2(c+dx)+2\log(\cos(c+dx))\right)}{2d}+\frac{b\sec^3(c+dx)}{3d}-\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] -((b*Sec[c + d*x])/d) + (b*Sec[c + d*x]^3)/(3*d) + (a*(2*Log[Cos[c + d*x]]
+ Tan[c + d*x]^2))/(2*d)
```

Maple [B] time = 0.037, size = 104, normalized size = 1.9

$$\frac{(\tan(dx+c))^2 a}{2d}+\frac{a\ln(\cos(dx+c))}{d}+\frac{b(\sin(dx+c))^4}{3d(\cos(dx+c))^3}-\frac{b(\sin(dx+c))^4}{3d\cos(dx+c)}-\frac{b\cos(dx+c)(\sin(dx+c))^2}{3d}-\frac{2b\cos}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c)^3,x)
```

```
[Out] 1/2/d*a*tan(d*x+c)^2+a*ln(cos(d*x+c))/d+1/3/d*b*sin(d*x+c)^4/cos(d*x+c)^3-1
/3/d*b*sin(d*x+c)^4/cos(d*x+c)-1/3/d*b*cos(d*x+c)*sin(d*x+c)^2-2/3/d*b*cos(
d*x+c)
```

Maxima [A] time = 0.982901, size = 68, normalized size = 1.24

$$\frac{6\,a\log\left(\cos\left(dx+c\right)\right)-\frac{6\,b\cos\left(dx+c\right)^{2}-3\,a\cos\left(dx+c\right)-2\,b}{\cos\left(dx+c\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(6*a*log(cos(d*x + c)) - (6*b*cos(d*x + c)^2 - 3*a*cos(d*x + c) - 2*b)/
cos(d*x + c)^3)/d
```

Fricas [A] time = 1.09481, size = 149, normalized size = 2.71

$$\frac{6\,a\cos\left(dx+c\right)^{3}\log\left(-\cos\left(dx+c\right)\right)-6\,b\cos\left(dx+c\right)^{2}+3\,a\cos\left(dx+c\right)+2\,b}{6\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*cos(d*x + c)^3*log(-cos(d*x + c)) - 6*b*cos(d*x + c)^2 + 3*a*cos(d
*x + c) + 2*b)/(d*cos(d*x + c)^3)
```

Sympy [A] time = 1.27028, size = 76, normalized size = 1.38

$$\begin{cases} -\frac{a\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{a\tan^{2}\left(c+dx\right)}{2d}+\frac{b\tan^{2}\left(c+dx\right)\sec\left(c+dx\right)}{3d}-\frac{2b\sec\left(c+dx\right)}{3d} & \text{for}\ d\neq 0 \\ x\left(a+b\sec\left(c\right)\right)\tan^{3}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)**3,x)
```

```
[Out] Piecewise((-a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**2/(2*d) + b*
tan(c + d*x)**2*sec(c + d*x)/(3*d) - 2*b*sec(c + d*x)/(3*d), Ne(d, 0)), (x*
(a + b*sec(c))*tan(c)**3, True))
```

<u>Giac</u> [B] time = 1.75327, size = 242, normalized size = 4.4

$$-\frac{6\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-6\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{11\,a+8\,b+\frac{45\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{24\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{45\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{11\,a(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/6*(6*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 6*a*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (11*a + 8*b + 45*a*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 24*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 + 45*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 11*a*(cos(d*x + c) - 1)
^3/(cos(d*x + c) + 1)^3)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^3)/d
```

3.259 $\int (a + b\sec(c + dx))\tan(c + dx)\,dx$

Optimal. Leaf size=25

$$\frac{b\sec(c + dx)}{d} - \frac{a\log(\cos(c + dx))}{d}$$

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (b*Sec[c + d*x])/d
```

Rubi [A] time = 0.0367905, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3884, 3475, 2606, 8}

$$\frac{b\sec(c + dx)}{d} - \frac{a\log(\cos(c + dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (b*Sec[c + d*x])/d
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))\tan(c+dx)\,dx &= a\int \tan(c+dx)\,dx + b\int \sec(c+dx)\tan(c+dx)\,dx\\ &= -\frac{a\log(\cos(c+dx))}{d} + \frac{b\,\text{Subst}(\int 1\,dx, x, \sec(c+dx))}{d}\\ &= -\frac{a\log(\cos(c+dx))}{d} + \frac{b\sec(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0139561, size = 25, normalized size = 1.

$$\frac{b\sec(c+dx)}{d} - \frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (b*Sec[c + d*x])/d
```

Maple [A] time = 0.016, size = 25, normalized size = 1.

$$\frac{a\ln(\sec(dx+c))}{d} + \frac{b\sec(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c),x)
```

```
[Out] 1/d*a*ln(sec(d*x+c))+b*sec(d*x+c)/d
```

Maxima [A] time = 0.992142, size = 35, normalized size = 1.4

$$-\frac{a\log(\cos(dx+c)) - \frac{b}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*log(cos(d*x + c)) - b/cos(d*x + c))/d
```

Fricas [A] time = 0.770841, size = 80, normalized size = 3.2

$$-\frac{a\cos(dx+c)\log(-\cos(dx+c))-b}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -(a*cos(d*x + c)*log(-cos(d*x + c)) - b)/(d*cos(d*x + c))
```

Sympy [A] time = 0.98261, size = 37, normalized size = 1.48

$$\begin{cases} \frac{a\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{b\sec(c+dx)}{d} & \text{for } d\neq 0 \\ x(a+b\sec(c))\tan(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c),x)
```

```
[Out] Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + b*sec(c + d*x)/d, Ne(d, 0)),
(x*(a + b*sec(c))*tan(c), True))
```

Giac [B] time = 1.20217, size = 144, normalized size = 5.76

$$\frac{a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{a+2b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c),x, algorithm="giac")
```

```
[Out] (a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - a*log(abs(-(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a + 2*b + a*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

3.260 $\int \cot(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=43

$$\frac{(a+b)\log(1-\cos(c+dx))}{2d}+\frac{(a-b)\log(\cos(c+dx)+1)}{2d}$$

```
[Out] ((a + b)*Log[1 - Cos[c + d*x]])/(2*d) + ((a - b)*Log[1 + Cos[c + d*x]])/(2*
d)
```

Rubi [A] time = 0.080858, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3883, 2668, 633, 31}

$$\frac{(a+b)\log(1-\cos(c+dx))}{2d}+\frac{(a-b)\log(\cos(c+dx)+1)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sec[c + d*x]),x]
```

```
[Out] ((a + b)*Log[1 - Cos[c + d*x]])/(2*d) + ((a - b)*Log[1 + Cos[c + d*x]])/(2*
d)
```

Rule 3883

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))/cot[(c_.) + (d_.)*(x_)], x_Symbo
l] :> Int[(b + a*Sin[c + d*x])/Cos[c + d*x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+b\sec(c+dx))\,dx &= \int (b+a\cos(c+dx))\csc(c+dx)\,dx\\ &= -\frac{a\,\text{Subst}\left(\int \frac{b+x}{a^2-x^2}\,dx, x, a\cos(c+dx)\right)}{d}\\ &= -\frac{(a-b)\,\text{Subst}\left(\int \frac{1}{-a-x}\,dx, x, a\cos(c+dx)\right)}{2d} - \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{a-x}\,dx, x, a\cos(c\right.}{2d}\\ &= \frac{(a+b)\log(1-\cos(c+dx))}{2d} + \frac{(a-b)\log(1+\cos(c+dx))}{2d}\end{aligned}$$

Mathematica [A] time = 0.0357285, size = 60, normalized size = 1.4

$$\frac{a(\log(\tan(c+dx))+\log(\cos(c+dx)))}{d}+\frac{b\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{b\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sec[c + d*x]),x]
```

```
[Out] -((b*Log[Cos[c/2 + (d*x)/2]])/d) + (b*Log[Sin[c/2 + (d*x)/2]])/d + (a*(Log[
Cos[c + d*x]] + Log[Tan[c + d*x]]))/d
```

Maple [A] time = 0.034, size = 35, normalized size = 0.8

$$\frac{a\ln(\sin(dx+c))}{d}+\frac{b\ln(\csc(dx+c)-\cot(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*a*ln(sin(d*x+c))+1/d*b*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 0.973253, size = 46, normalized size = 1.07

$$\frac{(a-b)\log\left(\cos\left(dx+c\right)+1\right)+(a+b)\log\left(\cos\left(dx+c\right)-1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*((a - b)*log(cos(d*x + c) + 1) + (a + b)*log(cos(d*x + c) - 1))/d
```

Fricas [A] time = 0.969679, size = 113, normalized size = 2.63

$$\frac{(a-b)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+(a+b)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((a - b)*log(1/2*cos(d*x + c) + 1/2) + (a + b)*log(-1/2*cos(d*x + c) +
1/2))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))\cot(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x), x)
```

Giac [A] time = 1.28501, size = 82, normalized size = 1.91

$$\frac{(a+b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-2\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((a + b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 2*a*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

3.261 $\int \cot^3(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=72

$$-\frac{(2a+b)\log(1-\cos(c+dx))}{4d}-\frac{(2a-b)\log(\cos(c+dx)+1)}{4d}-\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}$$

```
[Out] -((2*a + b)*Log[1 - Cos[c + d*x]])/(4*d) - ((2*a - b)*Log[1 + Cos[c + d*x]]
)/(4*d) - (Cot[c + d*x]^2*(a + b*Sec[c + d*x]))/(2*d)
```

Rubi [A] time = 0.108431, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3882, 3883, 2668, 633, 31}

$$-\frac{(2a+b)\log(1-\cos(c+dx))}{4d}-\frac{(2a-b)\log(\cos(c+dx)+1)}{4d}-\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

```
[Out] -((2*a + b)*Log[1 - Cos[c + d*x]])/(4*d) - ((2*a - b)*Log[1 + Cos[c + d*x]]
)/(4*d) - (Cot[c + d*x]^2*(a + b*Sec[c + d*x]))/(2*d)
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3883

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))/cot[(c_.) + (d_.)*(x_)], x_Symbo
l] :> Int[(b + a*Sin[c + d*x])/Cos[c + d*x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
```

```
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^3(c+dx)(a+b\sec(c+dx))\,dx &= -\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}+\frac{1}{2}\int \cot(c+dx)(-2a-b\sec(c+dx))\,dx\\ &= -\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}+\frac{1}{2}\int (-b-2a\cos(c+dx))\csc(c+dx)\,dx\\ &= -\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}+\frac{a\,\text{Subst}\left(\int \frac{-b+x}{4a^2-x^2}\,dx,x,-2a\cos(c+dx)\right)}{d}\\ &= -\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}+\frac{(2a-b)\,\text{Subst}\left(\int \frac{1}{2a-x}\,dx,x,-2a\cos(c+dx\right.}{4d}\\ &= -\frac{(2a+b)\log(1-\cos(c+dx))}{4d}-\frac{(2a-b)\log(1+\cos(c+dx))}{4d}-\frac{\cot^2(c+dx)(a}{}\end{aligned}$$

Mathematica [A] time = 1.35824, size = 114, normalized size = 1.58

$$-\frac{a\left(\cot^2(c+dx)+2\log(\tan(c+dx))+2\log(\cos(c+dx))\right)}{2d}-\frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{b\log\left(\sin\left(\frac{1}{2}(c\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

```
[Out] -(b*Csc[(c + d*x)/2]^2)/(8*d) + (b*Log[Cos[(c + d*x)/2]])/(2*d) - (b*Log[Si
n[(c + d*x)/2]])/(2*d) - (a*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[T
an[c + d*x]]))/(2*d) + (b*Sec[(c + d*x)/2]^2)/(8*d)
```

Maple [A] time = 0.047, size = 85, normalized size = 1.2

$$-\frac{(\cot(dx+c))^2 a}{2d}-\frac{a\ln(\sin(dx+c))}{d}-\frac{b(\cos(dx+c))^3}{2d(\sin(dx+c))^2}-\frac{b\cos(dx+c)}{2d}-\frac{b\ln(\csc(dx+c)-\cot(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sec(d*x+c)),x)
```

```
[Out] -1/2/d*a*cot(d*x+c)^2-1/d*a*ln(sin(d*x+c))-1/2/d*b/sin(d*x+c)^2*cos(d*x+c)^
3-1/2/d*b*cos(d*x+c)-1/2/d*b*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 0.979701, size = 84, normalized size = 1.17

$$-\frac{(2a-b)\log(\cos(dx+c)+1)+(2a+b)\log(\cos(dx+c)-1)-\frac{2(b\cos(dx+c)+a)}{\cos(dx+c)^2-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*((2*a - b)*log(cos(d*x + c) + 1) + (2*a + b)*log(cos(d*x + c) - 1) - 2
*(b*cos(d*x + c) + a)/(cos(d*x + c)^2 - 1))/d
```

Fricas [A] time = 0.72683, size = 254, normalized size = 3.53

$$\frac{2b\cos(dx+c)-\left((2a-b)\cos(dx+c)^2-2a+b\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-\left((2a+b)\cos(dx+c)^2-2a-b\right)\log\left(-\frac{1}{2}\right.}{4\left(d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*cos(d*x + c) - ((2*a - b)*cos(d*x + c)^2 - 2*a + b)*log(1/2*cos(d*
x + c) + 1/2) - ((2*a + b)*cos(d*x + c)^2 - 2*a - b)*log(-1/2*cos(d*x + c)
```

```
+ 1/2) + 2*a)/(d*cos(d*x + c)^2 - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sec(c + dx))\cot^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x)**3, x)
```

Giac [B] time = 1.27541, size = 230, normalized size = 3.19

$$-\frac{2(2a+b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 8a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - \frac{\left(a+b+\frac{4a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1} - \frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/8*(2*(2*a + b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 8*a*l
og(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a + b + 4*a*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*
(cos(d*x + c) + 1)/(cos(d*x + c) - 1) - a*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/d
```

3.262 $\int \cot^5(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=102

$$\frac{(8a+3b)\log(1-\cos(c+dx))}{16d}+\frac{(8a-3b)\log(\cos(c+dx)+1)}{16d}-\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}+\frac{\cot^2(c+dx)(4a+3b}{8d}$$

```
[Out] ((8*a + 3*b)*Log[1 - Cos[c + d*x]])/(16*d) + ((8*a - 3*b)*Log[1 + Cos[c + d
*x]])/(16*d) - (Cot[c + d*x]^4*(a + b*Sec[c + d*x]))/(4*d) + (Cot[c + d*x]^
2*(4*a + 3*b*Sec[c + d*x]))/(8*d)
```

Rubi [A] time = 0.131437, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3882, 3883, 2668, 633, 31}

$$\frac{(8a+3b)\log(1-\cos(c+dx))}{16d}+\frac{(8a-3b)\log(\cos(c+dx)+1)}{16d}-\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}+\frac{\cot^2(c+dx)(4a+3b}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

```
[Out] ((8*a + 3*b)*Log[1 - Cos[c + d*x]])/(16*d) + ((8*a - 3*b)*Log[1 + Cos[c + d
*x]])/(16*d) - (Cot[c + d*x]^4*(a + b*Sec[c + d*x]))/(4*d) + (Cot[c + d*x]^
2*(4*a + 3*b*Sec[c + d*x]))/(8*d)
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3883

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))/cot[(c_.) + (d_.)*(x_)], x_Symbo
l] :> Int[(b + a*Sin[c + d*x])/Cos[c + d*x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \cot^5(c+dx)(a+b\sec(c+dx))\,dx = -\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d} + \frac{1}{4}\int \cot^3(c+dx)(-4a-3b\sec(c+dx))\,dx$$
$$= -\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d} + \frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{8d} + \frac{1}{8}\int \text{co}$$
$$= -\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d} + \frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{8d} + \frac{1}{8}\int (3$$
$$= -\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d} + \frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{8d} - \frac{a\,\text{Subs}}{}$$
$$= -\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d} + \frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{8d} - \frac{(8a-3}{}$$
$$= \frac{(8a+3b)\log(1-\cos(c+dx))}{16d} + \frac{(8a-3b)\log(1+\cos(c+dx))}{16d} - \frac{\cot^4(c+dx)(}{}$$

Mathematica [A] time = 0.3295, size = 166, normalized size = 1.63

$$\frac{a\left(-\cot^4(c+dx)+2\cot^2(c+dx)+4\log(\tan(c+dx))+4\log(\cos(c+dx))\right)}{4d} - \frac{b\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{5b\csc^2\left(\frac{1}{2}(c+dx\right.}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

```
[Out] (5*b*Csc[(c + d*x)/2]^2)/(32*d) - (b*Csc[(c + d*x)/2]^4)/(64*d) - (3*b*Log[
Cos[(c + d*x)/2]])/(8*d) + (3*b*Log[Sin[(c + d*x)/2]])/(8*d) + (a*(2*Cot[c
+ d*x]^2 - Cot[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Tan[c + d*x]]))/(4*
d) - (5*b*Sec[(c + d*x)/2]^2)/(32*d) + (b*Sec[(c + d*x)/2]^4)/(64*d)
```

Maple [A] time = 0.044, size = 134, normalized size = 1.3

$$-\frac{a(\cot(dx+c))^4}{4d}+\frac{(\cot(dx+c))^2 a}{2d}+\frac{a\ln(\sin(dx+c))}{d}-\frac{b(\cos(dx+c))^5}{4d(\sin(dx+c))^4}+\frac{b(\cos(dx+c))^5}{8d(\sin(dx+c))^2}+\frac{b(\cos(dx+c))^3}{8d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+b*sec(d*x+c)),x)
```

```
[Out] -1/4/d*a*cot(d*x+c)^4+1/2/d*a*cot(d*x+c)^2+1/d*a*ln(sin(d*x+c))-1/4/d*b/sin
(d*x+c)^4*cos(d*x+c)^5+1/8/d*b/sin(d*x+c)^2*cos(d*x+c)^5+1/8/d*b*cos(d*x+c)
^3+3/8/d*b*cos(d*x+c)+3/8/d*b*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 0.981967, size = 134, normalized size = 1.31

$$\frac{(8a-3b)\log(\cos(dx+c)+1)+(8a+3b)\log(\cos(dx+c)-1)-\frac{2\left(5b\cos(dx+c)^3+8a\cos(dx+c)^2-3b\cos(dx+c)-6a\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*((8*a - 3*b)*log(cos(d*x + c) + 1) + (8*a + 3*b)*log(cos(d*x + c) - 1)
 - 2*(5*b*cos(d*x + c)^3 + 8*a*cos(d*x + c)^2 - 3*b*cos(d*x + c) - 6*a)/(co
s(d*x + c)^4 - 2*cos(d*x + c)^2 + 1))/d
```

Fricas [A] time = 0.834091, size = 440, normalized size = 4.31

$$-\frac{10b\cos(dx+c)^3+16a\cos(dx+c)^2-6b\cos(dx+c)-\left((8a-3b)\cos(dx+c)^4-2(8a-3b)\cos(dx+c)^2+8a-3\right.}{16\left(d\cos(dx+c)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(10*b*cos(d*x + c)^3 + 16*a*cos(d*x + c)^2 - 6*b*cos(d*x + c) - ((8*a
 - 3*b)*cos(d*x + c)^4 - 2*(8*a - 3*b)*cos(d*x + c)^2 + 8*a - 3*b)*log(1/2*
cos(d*x + c) + 1/2) - ((8*a + 3*b)*cos(d*x + c)^4 - 2*(8*a + 3*b)*cos(d*x +
 c)^2 + 8*a + 3*b)*log(-1/2*cos(d*x + c) + 1/2) - 12*a)/(d*cos(d*x + c)^4 -
 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.40751, size = 359, normalized size = 3.52

$$\frac{4\,(8\,a+3\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-64\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a+b+\frac{12\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{8\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{48\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{18\,b(\cos(dx+c)-}{(\cos(dx+c)+1}\right.}{(\cos(dx+c)-1)^2}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/64*(4*(8*a + 3*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 64*
a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a + b + 12*a*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 48*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 18*b*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2 - 12*a
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*b*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + b*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2)/d
```

3.263 $\int \cot^7(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=130

$$-\frac{(16a+5b)\log(1-\cos(c+dx))}{32d}-\frac{(16a-5b)\log(\cos(c+dx)+1)}{32d}-\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}+\frac{\cot^4(c+dx)(6a+}{24}$$

```
[Out] -((16*a + 5*b)*Log[1 - Cos[c + d*x]])/(32*d) - ((16*a - 5*b)*Log[1 + Cos[c
+ d*x]])/(32*d) - (Cot[c + d*x]^6*(a + b*Sec[c + d*x]))/(6*d) + (Cot[c + d*
x]^4*(6*a + 5*b*Sec[c + d*x]))/(24*d) - (Cot[c + d*x]^2*(8*a + 5*b*Sec[c +
d*x]))/(16*d)
```

Rubi [A] time = 0.175971, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3882, 3883, 2668, 633, 31}

$$-\frac{(16a+5b)\log(1-\cos(c+dx))}{32d}-\frac{(16a-5b)\log(\cos(c+dx)+1)}{32d}-\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}+\frac{\cot^4(c+dx)(6a+}{24}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*(a + b*Sec[c + d*x]),x]
```



```
[Out] -((16*a + 5*b)*Log[1 - Cos[c + d*x]])/(32*d) - ((16*a - 5*b)*Log[1 + Cos[c
+ d*x]])/(32*d) - (Cot[c + d*x]^6*(a + b*Sec[c + d*x]))/(6*d) + (Cot[c + d*
x]^4*(6*a + 5*b*Sec[c + d*x]))/(24*d) - (Cot[c + d*x]^2*(8*a + 5*b*Sec[c +
d*x]))/(16*d)
```


Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3883

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))/cot[(c_.) + (d_.)*(x_)], x_Symbo
l] :> Int[(b + a*Sin[c + d*x])/Cos[c + d*x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \cot^7(c+dx)(a+b\sec(c+dx))\,dx = -\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d} + \frac{1}{6}\int \cot^5(c+dx)(-6a-5b\sec(c+dx))\,dx$$

$$= -\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d} + \frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{24d} + \frac{1}{24}\int \text{c}$$

$$= -\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d} + \frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{24d} - \frac{\cot^2(c}{}$$

$$= -\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d} + \frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{24d} - \frac{\cot^2(c}{}$$

$$= -\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d} + \frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{24d} - \frac{\cot^2(c}{}$$

$$= -\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d} + \frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{24d} - \frac{\cot^2(c}{}$$

$$= -\frac{(16a+5b)\log(1-\cos(c+dx))}{32d} - \frac{(16a-5b)\log(1+\cos(c+dx))}{32d} - \frac{\cot^6(c+}{}$$

Mathematica [A] time = 0.580025, size = 216, normalized size = 1.66

$$-\frac{a\left(2\cot^6(c+dx)-3\cot^4(c+dx)+6\cot^2(c+dx)+12\log(\tan(c+dx))+12\log(\cos(c+dx))\right)}{12d} - \frac{b\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-11*b*Csc[(c + d*x)/2]^2)/(64*d) + (b*Csc[(c + d*x)/2]^4)/(32*d) - (b*Csc[
(c + d*x)/2]^6)/(384*d) + (5*b*Log[Cos[(c + d*x)/2]])/(16*d) - (5*b*Log[Sin
[(c + d*x)/2]])/(16*d) - (a*(6*Cot[c + d*x]^2 - 3*Cot[c + d*x]^4 + 2*Cot[c
+ d*x]^6 + 12*Log[Cos[c + d*x]] + 12*Log[Tan[c + d*x]]))/(12*d) + (11*b*Sec
[(c + d*x)/2]^2)/(64*d) - (b*Sec[(c + d*x)/2]^4)/(32*d) + (b*Sec[(c + d*x)/
2]^6)/(384*d)
```

Maple [A] time = 0.046, size = 185, normalized size = 1.4

$$-\frac{a\left(\cot\left(dx+c\right)\right)^{6}}{6d}+\frac{a\left(\cot\left(dx+c\right)\right)^{4}}{4d}-\frac{\left(\cot\left(dx+c\right)\right)^{2}a}{2d}-\frac{a\ln\left(\sin\left(dx+c\right)\right)}{d}-\frac{b\left(\cos\left(dx+c\right)\right)^{7}}{6d\left(\sin\left(dx+c\right)\right)^{6}}+\frac{b\left(\cos\left(dx+c\right)\right)^{7}}{24d\left(\sin\left(dx+c\right)\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7*(a+b*sec(d*x+c)),x)
```

```
[Out] -1/6/d*a*cot(d*x+c)^6+1/4/d*a*cot(d*x+c)^4-1/2/d*a*cot(d*x+c)^2-1/d*a*ln(si
n(d*x+c))-1/6/d*b/sin(d*x+c)^6*cos(d*x+c)^7+1/24/d*b/sin(d*x+c)^4*cos(d*x+c
)^7-1/16/d*b/sin(d*x+c)^2*cos(d*x+c)^7-1/16/d*b*cos(d*x+c)^5-5/48/d*b*cos(d
*x+c)^3-5/16/d*b*cos(d*x+c)-5/16/d*b*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 0.994773, size = 180, normalized size = 1.38

$$-\frac{3\left(16a-5b\right)\log\left(\cos\left(dx+c\right)+1\right)+3\left(16a+5b\right)\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(33b\cos\left(dx+c\right)^{5}+72a\cos\left(dx+c\right)^{4}-40b\cos\left(dx+c\right)^{3}-108a\cdots\right.}{\cos\left(dx+c\right)^{6}-3\cos\left(dx+c\right)^{4}+3\cos\left(\cdots\right.}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(3*(16*a - 5*b)*log(cos(d*x + c) + 1) + 3*(16*a + 5*b)*log(cos(d*x +
c) - 1) - 2*(33*b*cos(d*x + c)^5 + 72*a*cos(d*x + c)^4 - 40*b*cos(d*x + c)^
3 - 108*a*cos(d*x + c)^2 + 15*b*cos(d*x + c) + 44*a)/(cos(d*x + c)^6 - 3*co
s(d*x + c)^4 + 3*cos(d*x + c)^2 - 1))/d
```

Fricas [A] time = 0.823228, size = 630, normalized size = 4.85

$$\frac{66\,b\cos\left(dx+c\right)^{5}+144\,a\cos\left(dx+c\right)^{4}-80\,b\cos\left(dx+c\right)^{3}-216\,a\cos\left(dx+c\right)^{2}+30\,b\cos\left(dx+c\right)-3\,\Big((16\,a-5\,b)\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(66*b*cos(d*x + c)^5 + 144*a*cos(d*x + c)^4 - 80*b*cos(d*x + c)^3 - 21
6*a*cos(d*x + c)^2 + 30*b*cos(d*x + c) - 3*((16*a - 5*b)*cos(d*x + c)^6 - 3
*(16*a - 5*b)*cos(d*x + c)^4 + 3*(16*a - 5*b)*cos(d*x + c)^2 - 16*a + 5*b)*
log(1/2*cos(d*x + c) + 1/2) - 3*((16*a + 5*b)*cos(d*x + c)^6 - 3*(16*a + 5*
b)*cos(d*x + c)^4 + 3*(16*a + 5*b)*cos(d*x + c)^2 - 16*a - 5*b)*log(-1/2*co
s(d*x + c) + 1/2) + 88*a)/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(
d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33109, size = 483, normalized size = 3.72

$$-\frac{12\,(16\,a+5\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-384\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a+b+\frac{12\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{87\,a(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{45\,b(\cos}{(\cos(}\right.}{(\cos(dx}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/384*(12*(16*a + 5*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) -
 384*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a + b + 12*a
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*b*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) + 87*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 45*b*(cos(d*x +
c) - 1)^2/(cos(d*x + c) + 1)^2 + 352*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) +
 1)^3 + 110*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)
^3/(cos(d*x + c) - 1)^3 - 87*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 45*b
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2 + 9*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - a*(cos(d*x +
 c) - 1)^3/(cos(d*x + c) + 1)^3 + b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)
^3)/d
```

3.264 $\int (a+b\sec(c+dx))\tan^6(c+dx)\,dx$

Optimal. Leaf size=102

$$\frac{\tan^5(c+dx)(6a+5b\sec(c+dx))}{30d}-\frac{\tan^3(c+dx)(8a+5b\sec(c+dx))}{24d}+\frac{\tan(c+dx)(16a+5b\sec(c+dx))}{16d}-ax-\frac{5b}{}$$

```
[Out] -(a*x) - (5*b*ArcTanh[Sin[c + d*x]])/(16*d) + ((16*a + 5*b*Sec[c + d*x])*Ta
n[c + d*x])/(16*d) - ((8*a + 5*b*Sec[c + d*x])*Tan[c + d*x]^3)/(24*d) + ((6
*a + 5*b*Sec[c + d*x])*Tan[c + d*x]^5)/(30*d)
```

Rubi [A] time = 0.0953595, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3881, 3770}

$$\frac{\tan^5(c+dx)(6a+5b\sec(c+dx))}{30d}-\frac{\tan^3(c+dx)(8a+5b\sec(c+dx))}{24d}+\frac{\tan(c+dx)(16a+5b\sec(c+dx))}{16d}-ax-\frac{5b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^6,x]
```


```
[Out] -(a*x) - (5*b*ArcTanh[Sin[c + d*x]])/(16*d) + ((16*a + 5*b*Sec[c + d*x])*Ta
n[c + d*x])/(16*d) - ((8*a + 5*b*Sec[c + d*x])*Tan[c + d*x]^3)/(24*d) + ((6
*a + 5*b*Sec[c + d*x])*Tan[c + d*x]^5)/(30*d)
```


Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))\tan^6(c+dx)\,dx = \frac{(6a+5b\sec(c+dx))\tan^5(c+dx)}{30d} - \frac{1}{6}\int (6a+5b\sec(c+dx))\tan^4(c+dx)\,dx$$
$$= -\frac{(8a+5b\sec(c+dx))\tan^3(c+dx)}{24d} + \frac{(6a+5b\sec(c+dx))\tan^5(c+dx)}{30d} + \frac{1}{24}$$
$$= \frac{(16a+5b\sec(c+dx))\tan(c+dx)}{16d} - \frac{(8a+5b\sec(c+dx))\tan^3(c+dx)}{24d} + (6a+$$
$$= -ax + \frac{(16a+5b\sec(c+dx))\tan(c+dx)}{16d} - \frac{(8a+5b\sec(c+dx))\tan^3(c+dx)}{24d} +$$
$$= -ax - \frac{5b\tanh^{-1}(\sin(c+dx))}{16d} + \frac{(16a+5b\sec(c+dx))\tan(c+dx)}{16d} - (8a+5b\,\mathrm{se}$$

Mathematica [A] time = 1.02845, size = 103, normalized size = 1.01

$$\frac{\frac{1}{8}\tan(c+dx)\sec^5(c+dx)(1168a\cos(c+dx)+568a\cos(3(c+dx))+184a\cos(5(c+dx))+140b\cos(2(c+dx))+165b\,\mathrm{c}}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^6,x]
```

```
[Out] (-240*a*ArcTan[Tan[c + d*x]] - 75*b*ArcTanh[Sin[c + d*x]] + ((295*b + 1168*
a*Cos[c + d*x] + 140*b*Cos[2*(c + d*x)] + 568*a*Cos[3*(c + d*x)] + 165*b*Co
s[4*(c + d*x)] + 184*a*Cos[5*(c + d*x)])*Sec[c + d*x]^5*Tan[c + d*x])/8)/(2
40*d)
```

Maple [A] time = 0.042, size = 178, normalized size = 1.8

$$\frac{(\tan(dx+c))^5 a}{5d} - \frac{a(\tan(dx+c))^3}{3d} + \frac{a\tan(dx+c)}{d} - ax - \frac{ac}{d} + \frac{b(\sin(dx+c))^7}{6d(\cos(dx+c))^6} - \frac{b(\sin(dx+c))^7}{24d(\cos(dx+c))^4} + \frac{b(\sin(dx}{16d(\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c)^6,x)
```

```
[Out] 1/5/d*tan(d*x+c)^5*a-1/3/d*a*tan(d*x+c)^3+1/d*a*tan(d*x+c)-a*x-1/d*a*c+1/6/
d*b*sin(d*x+c)^7/cos(d*x+c)^6-1/24/d*b*sin(d*x+c)^7/cos(d*x+c)^4+1/16/d*b*s
in(d*x+c)^7/cos(d*x+c)^2+1/16/d*sin(d*x+c)^5*b+5/48/d*b*sin(d*x+c)^3+5/16/d
```

```
*sin(d*x+c)*b-5/16/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.46158, size = 181, normalized size = 1.77

$$\frac{32\left(3\tan\left(dx+c\right)^{5}-5\tan\left(dx+c\right)^{3}-15\,dx-15\,c+15\tan\left(dx+c\right)\right)a-5\,b\left(\frac{2\left(33\sin(dx+c)^{5}-40\sin(dx+c)^{3}+15\sin(dx+c)\right)}{\sin(dx+c)^{6}-3\sin(dx+c)^{4}+3\sin(dx+c)^{2}-1}+15\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x
 + c))*a - 5*b*(2*(33*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 15*sin(d*x + c))
/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) + 15*log(sin(d*
x + c) + 1) - 15*log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 0.943063, size = 377, normalized size = 3.7

$$-\frac{480\,adx\cos\left(dx+c\right)^{6}+75\,b\cos\left(dx+c\right)^{6}\log\left(\sin\left(dx+c\right)+1\right)-75\,b\cos\left(dx+c\right)^{6}\log\left(-\sin\left(dx+c\right)+1\right)-2\left(368\,a\cos\right.}{480\,d\cos\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/480*(480*a*d*x*cos(d*x + c)^6 + 75*b*cos(d*x + c)^6*log(sin(d*x + c) + 1
) - 75*b*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 2*(368*a*cos(d*x + c)^5 +
165*b*cos(d*x + c)^4 - 176*a*cos(d*x + c)^3 - 130*b*cos(d*x + c)^2 + 48*a*c
os(d*x + c) + 40*b)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sec\left(c+dx\right)\right)\tan^{6}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)**6,x)
```

```
[Out] Integral((a + b*sec(c + d*x))*tan(c + d*x)**6, x)
```

Giac [B] time = 5.0167, size = 308, normalized size = 3.02

$$-\frac{240\,(dx+c)a+75\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-75\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(240\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-75\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] -1/240*(240*(d*x + c)*a + 75*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 75*b*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(240*a*tan(1/2*d*x + 1/2*c)^11 - 75*b*
tan(1/2*d*x + 1/2*c)^11 - 1520*a*tan(1/2*d*x + 1/2*c)^9 + 425*b*tan(1/2*d*x
 + 1/2*c)^9 + 4128*a*tan(1/2*d*x + 1/2*c)^7 - 990*b*tan(1/2*d*x + 1/2*c)^7
- 4128*a*tan(1/2*d*x + 1/2*c)^5 - 990*b*tan(1/2*d*x + 1/2*c)^5 + 1520*a*tan
(1/2*d*x + 1/2*c)^3 + 425*b*tan(1/2*d*x + 1/2*c)^3 - 240*a*tan(1/2*d*x + 1/
2*c) - 75*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.265 $\int (a + b\sec(c + dx))\tan^4(c + dx)\,dx$

Optimal. Leaf size=73

$$\frac{\tan^3(c+dx)(4a+3b\sec(c+dx))}{12d} - \frac{\tan(c+dx)(8a+3b\sec(c+dx))}{8d} + ax + \frac{3b\tanh^{-1}(\sin(c+dx))}{8d}$$

```
[Out] a*x + (3*b*ArcTanh[Sin[c + d*x]])/(8*d) - ((8*a + 3*b*Sec[c + d*x])*Tan[c +
 d*x])/(8*d) + ((4*a + 3*b*Sec[c + d*x])*Tan[c + d*x]^3)/(12*d)
```

Rubi [A] time = 0.065274, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3881, 3770}

$$\frac{\tan^3(c+dx)(4a+3b\sec(c+dx))}{12d} - \frac{\tan(c+dx)(8a+3b\sec(c+dx))}{8d} + ax + \frac{3b\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] a*x + (3*b*ArcTanh[Sin[c + d*x]])/(8*d) - ((8*a + 3*b*Sec[c + d*x])*Tan[c +
 d*x])/(8*d) + ((4*a + 3*b*Sec[c + d*x])*Tan[c + d*x]^3)/(12*d)
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))\tan^4(c+dx)\,dx = \frac{(4a+3b\sec(c+dx))\tan^3(c+dx)}{12d} - \frac{1}{4}\int (4a+3b\sec(c+dx))\tan^2(c+dx)\,dx$$
$$= -\frac{(8a+3b\sec(c+dx))\tan(c+dx)}{8d} + \frac{(4a+3b\sec(c+dx))\tan^3(c+dx)}{12d} + \frac{1}{8}\int ($$
$$= ax - \frac{(8a+3b\sec(c+dx))\tan(c+dx)}{8d} + \frac{(4a+3b\sec(c+dx))\tan^3(c+dx)}{12d} + \frac{1}{8}$$
$$= ax + \frac{3b\tanh^{-1}(\sin(c+dx))}{8d} - \frac{(8a+3b\sec(c+dx))\tan(c+dx)}{8d} + \frac{(4a+3b\sec($$

Mathematica [A] time = 0.577929, size = 79, normalized size = 1.08

$$\frac{\tan(c+dx)\sec^3(c+dx)(-(32a\cos(c+dx)+16a\cos(3(c+dx))+15b\cos(2(c+dx))+3b))+48a\tan^{-1}(\tan(c+dx))+}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (48*a*ArcTan[Tan[c + d*x]] + 18*b*ArcTanh[Sin[c + d*x]] - (3*b + 32*a*Cos[c
 + d*x] + 15*b*Cos[2*(c + d*x)] + 16*a*Cos[3*(c + d*x)])*Sec[c + d*x]^3*Tan
[c + d*x])/(48*d)
```

Maple [A] time = 0.04, size = 127, normalized size = 1.7

$$\frac{a(\tan(dx+c))^3}{3d} - \frac{a\tan(dx+c)}{d} + ax + \frac{ac}{d} + \frac{b(\sin(dx+c))^5}{4d(\cos(dx+c))^4} - \frac{b(\sin(dx+c))^5}{8d(\cos(dx+c))^2} - \frac{b(\sin(dx+c))^3}{8d} - \frac{3\sin(dx+c}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c)^4,x)
```

```
[Out] 1/3/d*a*tan(d*x+c)^3-1/d*a*tan(d*x+c)+a*x+1/d*a*c+1/4/d*b*sin(d*x+c)^5/cos(
d*x+c)^4-1/8/d*b*sin(d*x+c)^5/cos(d*x+c)^2-1/8/d*b*sin(d*x+c)^3-3/8/d*sin(d
*x+c)*b+3/8/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.47086, size = 138, normalized size = 1.89

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a+3\,b\left(\frac{2\left(5\,\sin(dx+c)^{3}-3\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(d\right.\right.\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a + 3*b*(2*(5*sin(
d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) + 3*lo
g(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 0.923648, size = 302, normalized size = 4.14

$$\frac{48\,adx\cos\left(dx+c\right)^{4}+9\,b\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-9\,b\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)-2\left(32\,a\cos\left(dx\right.\right.}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/48*(48*a*d*x*cos(d*x + c)^4 + 9*b*cos(d*x + c)^4*log(sin(d*x + c) + 1) -
9*b*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 2*(32*a*cos(d*x + c)^3 + 15*b*c
os(d*x + c)^2 - 8*a*cos(d*x + c) - 6*b)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec\left(c+dx\right)\right)\tan^{4}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))*tan(c + d*x)**4, x)
```

Giac [B] time = 2.32488, size = 232, normalized size = 3.18

$$\frac{24\,(dx+c)a+9\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-9\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-104\,a\tan\cdots\right.}{\cdots}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/24*(24*(d*x + c)*a + 9*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*b*log(abs
(tan(1/2*d*x + 1/2*c) - 1)) + 2*(24*a*tan(1/2*d*x + 1/2*c)^7 - 9*b*tan(1/2*
d*x + 1/2*c)^7 - 104*a*tan(1/2*d*x + 1/2*c)^5 + 33*b*tan(1/2*d*x + 1/2*c)^5
 + 104*a*tan(1/2*d*x + 1/2*c)^3 + 33*b*tan(1/2*d*x + 1/2*c)^3 - 24*a*tan(1/
2*d*x + 1/2*c) - 9*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/
d
```

3.266 $\int (a+b\sec(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=45

$$\frac{\tan(c+dx)(2a+b\sec(c+dx))}{2d}-ax-\frac{b\tanh^{-1}(\sin(c+dx))}{2d}$$

```
[Out] -(a*x) - (b*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*a + b*Sec[c + d*x])*Tan[c +
d*x])/(2*d)
```

Rubi [A] time = 0.0354112, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3881, 3770}

$$\frac{\tan(c+dx)(2a+b\sec(c+dx))}{2d}-ax-\frac{b\tanh^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*x) - (b*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*a + b*Sec[c + d*x])*Tan[c +
d*x])/(2*d)
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))\tan^2(c+dx)\,dx = \frac{(2a+b\sec(c+dx))\tan(c+dx)}{2d} - \frac{1}{2}\int (2a+b\sec(c+dx))\,dx$$
$$= -ax + \frac{(2a+b\sec(c+dx))\tan(c+dx)}{2d} - \frac{1}{2}b\int \sec(c+dx)\,dx$$
$$= -ax - \frac{b\tanh^{-1}(\sin(c+dx))}{2d} + \frac{(2a+b\sec(c+dx))\tan(c+dx)}{2d}$$

Mathematica [A] time = 0.0244875, size = 60, normalized size = 1.33

$$-\frac{a\tan^{-1}(\tan(c+dx))}{d} + \frac{a\tan(c+dx)}{d} - \frac{b\tanh^{-1}(\sin(c+dx))}{2d} + \frac{b\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -((a*ArcTan[Tan[c + d*x]])/d) - (b*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c
+ d*x])/d + (b*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.03, size = 78, normalized size = 1.7

$$-ax + \frac{a\tan(dx+c)}{d} - \frac{ac}{d} + \frac{b(\sin(dx+c))^3}{2d(\cos(dx+c))^2} + \frac{\sin(dx+c)\,b}{2d} - \frac{b\ln(\sec(dx+c)+\tan(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c)^2,x)
```

```
[Out] -a*x+1/d*a*tan(d*x+c)-1/d*a*c+1/2/d*b*sin(d*x+c)^3/cos(d*x+c)^2+1/2/d*sin(d
*x+c)*b-1/2/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.49497, size = 88, normalized size = 1.96

$$-\frac{4(dx+c-\tan(dx+c))a + b\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} + \log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*(d*x + c - tan(d*x + c))*a + b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1)
 + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)))/d
```

Fricas [B] time = 0.903781, size = 234, normalized size = 5.2

$$-\frac{4\,adx\cos\left(dx+c\right)^{2}+b\cos\left(dx+c\right)^{2}\log\left(\sin\left(dx+c\right)+1\right)-b\cos\left(dx+c\right)^{2}\log\left(-\sin\left(dx+c\right)+1\right)-2\left(2\,a\cos\left(dx+c\right)\right.}{4\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*d*x*cos(d*x + c)^2 + b*cos(d*x + c)^2*log(sin(d*x + c) + 1) - b*c
os(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(2*a*cos(d*x + c) + b)*sin(d*x + c
))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec\left(c+dx\right)\right)\tan^{2}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))*tan(c + d*x)**2, x)
```

Giac [B] time = 1.51081, size = 155, normalized size = 3.44

$$-\frac{2\left(dx+c\right)a+b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,a\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)*a + b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b*log(abs(tan(
1/2*d*x + 1/2*c) - 1)) + 2*(2*a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/
2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
1/2*c)^2 - 1)^2)/d
```

3.267 $\int \cot^2(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=26

$$-\frac{\cot(c+dx)(a+b\sec(c+dx))}{d}-ax$$

```
[Out] -(a*x) - (Cot[c + d*x]*(a + b*Sec[c + d*x]))/d
```

Rubi [A] time = 0.0264657, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3882, 8}

$$-\frac{\cot(c+dx)(a+b\sec(c+dx))}{d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

```
[Out] -(a*x) - (Cot[c + d*x]*(a + b*Sec[c + d*x]))/d
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)(a+b\sec(c+dx))\,dx &= -\frac{\cot(c+dx)(a+b\sec(c+dx))}{d}-\int a\,dx\\ &= -ax-\frac{\cot(c+dx)(a+b\sec(c+dx))}{d}\end{aligned}$$

Mathematica [C] time = 0.0203991, size = 43, normalized size = 1.65

$$-\frac{a\cot(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)}{d}-\frac{b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

```
[Out] -((b*Csc[c + d*x])/d) - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Ta
n[c + d*x]^2])/d
```

Maple [A] time = 0.035, size = 35, normalized size = 1.4

$$\frac{1}{d}\left(a\left(-\cot\left(dx+c\right)-dx-c\right)-\frac{b}{\sin\left(dx+c\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*(a*(-cot(d*x+c)-d*x-c)-b/sin(d*x+c))
```

Maxima [A] time = 1.47307, size = 42, normalized size = 1.62

$$-\frac{\left(dx+c+\frac{1}{\tan(dx+c)}\right)a+\frac{b}{\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -((d*x + c + 1/tan(d*x + c))*a + b/sin(d*x + c))/d
```

Fricas [A] time = 0.807459, size = 82, normalized size = 3.15

$$-\frac{adx\sin\left(dx+c\right)+a\cos\left(dx+c\right)+b}{d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(a*d*x*sin(d*x + c) + a*cos(d*x + c) + b)/(d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))\cot^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x)**2, x)
```

Giac [A] time = 1.15339, size = 70, normalized size = 2.69

$$-\frac{2\,(dx+c)a-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{a+b}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)*a - a*tan(1/2*d*x + 1/2*c) + b*tan(1/2*d*x + 1/2*c) + (a
+ b)/tan(1/2*d*x + 1/2*c))/d
```

3.268 $\int \cot^4(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=55

$$-\frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d}+\frac{\cot(c+dx)(3a+2b\sec(c+dx))}{3d}+ax$$

```
[Out] a*x - (Cot[c + d*x]^3*(a + b*Sec[c + d*x]))/(3*d) + (Cot[c + d*x]*(3*a + 2*
b*Sec[c + d*x]))/(3*d)
```

Rubi [A] time = 0.0517472, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3882, 8}

$$-\frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d}+\frac{\cot(c+dx)(3a+2b\sec(c+dx))}{3d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

```
[Out] a*x - (Cot[c + d*x]^3*(a + b*Sec[c + d*x]))/(3*d) + (Cot[c + d*x]*(3*a + 2*
b*Sec[c + d*x]))/(3*d)
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sec(c+dx))\,dx = -\frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d} + \frac{1}{3}\int \cot^2(c+dx)(-3a-2b\sec(c+dx))\,dx$$
$$= -\frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d} + \frac{\cot(c+dx)(3a+2b\sec(c+dx))}{3d} + \frac{1}{3}\int 3a$$
$$= ax - \frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d} + \frac{\cot(c+dx)(3a+2b\sec(c+dx))}{3d}$$

Mathematica [C] time = 0.0294127, size = 62, normalized size = 1.13

$$-\frac{a\cot^3(c+dx)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)}{3d} - \frac{b\csc^3(c+dx)}{3d} + \frac{b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/(3*d) - (a*Cot[c + d*x]^3*Hypergeom
etric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d)
```

Maple [A] time = 0.042, size = 86, normalized size = 1.6

$$\frac{1}{d}\left(a\left(-\frac{(\cot(dx+c))^3}{3} + \cot(dx+c) + dx + c\right) + b\left(-\frac{(\cos(dx+c))^4}{3(\sin(dx+c))^3} + \frac{(\cos(dx+c))^4}{3\sin(dx+c)} + \frac{\left(2+(\cos(dx+c))^2\right)\sin(dx}{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+b*(-1/3/sin(d*x+c)^3*cos(d*x+c)
^4+1/3/sin(d*x+c)*cos(d*x+c)^4+1/3*(2+cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 1.45948, size = 80, normalized size = 1.45

$$\frac{\left(3\,dx + 3\,c + \frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a + \frac{\left(3\,\sin(dx+c)^2-1\right)b}{\sin(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*((3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a + (3*sin(d*x +
 c)^2 - 1)*b/sin(d*x + c)^3)/d
```

Fricas [A] time = 0.783235, size = 213, normalized size = 3.87

$$\frac{4\,a\cos\left(dx+c\right)^{3}+3\,b\cos\left(dx+c\right)^{2}-3\,a\cos\left(dx+c\right)+3\left(adx\cos\left(dx+c\right)^{2}-adx\right)\sin\left(dx+c\right)-2\,b}{3\left(d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(4*a*cos(d*x + c)^3 + 3*b*cos(d*x + c)^2 - 3*a*cos(d*x + c) + 3*(a*d*x*
cos(d*x + c)^2 - a*d*x)*sin(d*x + c) - 2*b)/((d*cos(d*x + c)^2 - d)*sin(d*x
 + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec\left(c+dx\right)\right)\cot^{4}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x)**4, x)
```

Giac [B] time = 1.31355, size = 151, normalized size = 2.75

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+24\left(dx+c\right)a-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+}{\tan\left(\frac{1}{2}\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 + 24*(d*x + c)*a
- 15*a*tan(1/2*d*x + 1/2*c) + 9*b*tan(1/2*d*x + 1/2*c) + (15*a*tan(1/2*d*x
+ 1/2*c)^2 + 9*b*tan(1/2*d*x + 1/2*c)^2 - a - b)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.269 $\int \cot^6(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=84

$$-\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}+\frac{\cot^3(c+dx)(5a+4b\sec(c+dx))}{15d}-\frac{\cot(c+dx)(15a+8b\sec(c+dx))}{15d}-ax$$

```
[Out] -(a*x) - (Cot[c + d*x]^5*(a + b*Sec[c + d*x]))/(5*d) + (Cot[c + d*x]^3*(5*a
 + 4*b*Sec[c + d*x]))/(15*d) - (Cot[c + d*x]*(15*a + 8*b*Sec[c + d*x]))/(15
*d)
```

Rubi [A] time = 0.0811312, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3882, 8}

$$-\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}+\frac{\cot^3(c+dx)(5a+4b\sec(c+dx))}{15d}-\frac{\cot(c+dx)(15a+8b\sec(c+dx))}{15d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Sec[c + d*x]),x]
```


```
[Out] -(a*x) - (Cot[c + d*x]^5*(a + b*Sec[c + d*x]))/(5*d) + (Cot[c + d*x]^3*(5*a
 + 4*b*Sec[c + d*x]))/(15*d) - (Cot[c + d*x]*(15*a + 8*b*Sec[c + d*x]))/(15
*d)
```


Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^6(c+dx)(a+b\sec(c+dx))\,dx = -\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d} + \frac{1}{5}\int \cot^4(c+dx)(-5a-4b\sec(c+dx))\,dx$$
$$= -\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d} + \frac{\cot^3(c+dx)(5a+4b\sec(c+dx))}{15d} + \frac{1}{15}\int c$$
$$= -\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d} + \frac{\cot^3(c+dx)(5a+4b\sec(c+dx))}{15d} - \frac{\cot(c+}{}$$
$$= -ax - \frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d} + \frac{\cot^3(c+dx)(5a+4b\sec(c+dx))}{15d} - \frac{c}{}$$

Mathematica [C] time = 0.0367839, size = 79, normalized size = 0.94

$$-\frac{a\cot^5(c+dx)\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(c+dx)\right)}{5d} - \frac{b\csc^5(c+dx)}{5d} + \frac{2b\csc^3(c+dx)}{3d} - \frac{b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Sec[c + d*x]),x]
```

```
[Out] -((b*Csc[c + d*x])/d) + (2*b*Csc[c + d*x]^3)/(3*d) - (b*Csc[c + d*x]^5)/(5*
d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])/(
5*d)
```

Maple [A] time = 0.045, size = 129, normalized size = 1.5

$$\frac{1}{d}\left(a\left(-\frac{(\cot(dx+c))^5}{5} + \frac{(\cot(dx+c))^3}{3} - \cot(dx+c) - dx - c\right) + b\left(-\frac{(\cos(dx+c))^6}{5(\sin(dx+c))^5} + \frac{(\cos(dx+c))^6}{15(\sin(dx+c))^3} - \frac{(\cos(dx}{5\sin(}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+b*(-1/5/sin(d*
x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c)*cos(d*x+c
)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 1.46946, size = 107, normalized size = 1.27

$$-\frac{\left(15\,dx+15\,c+\frac{15\,\tan\left(dx+c\right)^{4}-5\,\tan\left(dx+c\right)^{2}+3}{\tan\left(dx+c\right)^{5}}\right)a+\frac{\left(15\,\sin\left(dx+c\right)^{4}-10\,\sin\left(dx+c\right)^{2}+3\right)b}{\sin\left(dx+c\right)^{5}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/15*((15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x
+ c)^5)*a + (15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 + 3)*b/sin(d*x + c)^5)/d
```

Fricas [A] time = 0.806891, size = 343, normalized size = 4.08

$$-\frac{23\,a\cos\left(dx+c\right)^{5}+15\,b\cos\left(dx+c\right)^{4}-35\,a\cos\left(dx+c\right)^{3}-20\,b\cos\left(dx+c\right)^{2}+15\,a\cos\left(dx+c\right)+15\,(adx\cos\left(dx+c\right)^{}}{15\,\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(23*a*cos(d*x + c)^5 + 15*b*cos(d*x + c)^4 - 35*a*cos(d*x + c)^3 - 20
*b*cos(d*x + c)^2 + 15*a*cos(d*x + c) + 15*(a*d*x*cos(d*x + c)^4 - 2*a*d*x*
cos(d*x + c)^2 + a*d*x)*sin(d*x + c) + 8*b)/((d*cos(d*x + c)^4 - 2*d*cos(d*
x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28717, size = 230, normalized size = 2.74

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-35\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+25\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-480\,(dx+c)a+330\,a\,\mathrm{ta}}{480}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/480*(3*a*tan(1/2*d*x + 1/2*c)^5 - 3*b*tan(1/2*d*x + 1/2*c)^5 - 35*a*tan(1
/2*d*x + 1/2*c)^3 + 25*b*tan(1/2*d*x + 1/2*c)^3 - 480*(d*x + c)*a + 330*a*t
an(1/2*d*x + 1/2*c) - 150*b*tan(1/2*d*x + 1/2*c) - (330*a*tan(1/2*d*x + 1/2
*c)^4 + 150*b*tan(1/2*d*x + 1/2*c)^4 - 35*a*tan(1/2*d*x + 1/2*c)^2 - 25*b*t
an(1/2*d*x + 1/2*c)^2 + 3*a + 3*b)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.270 $\int \cot^8(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=111

$$-\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d}+\frac{\cot^5(c+dx)(7a+6b\sec(c+dx))}{35d}-\frac{\cot^3(c+dx)(35a+24b\sec(c+dx))}{105d}+\frac{\cot(c+a}{}$$

```
[Out] a*x - (Cot[c + d*x]^7*(a + b*Sec[c + d*x]))/(7*d) + (Cot[c + d*x]^5*(7*a +
6*b*Sec[c + d*x]))/(35*d) + (Cot[c + d*x]*(35*a + 16*b*Sec[c + d*x]))/(35*d
) - (Cot[c + d*x]^3*(35*a + 24*b*Sec[c + d*x]))/(105*d)
```

Rubi [A] time = 0.111427, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3882, 8}

$$-\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d}+\frac{\cot^5(c+dx)(7a+6b\sec(c+dx))}{35d}-\frac{\cot^3(c+dx)(35a+24b\sec(c+dx))}{105d}+\frac{\cot(c+a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8*(a + b*Sec[c + d*x]),x]
```



```
[Out] a*x - (Cot[c + d*x]^7*(a + b*Sec[c + d*x]))/(7*d) + (Cot[c + d*x]^5*(7*a +
6*b*Sec[c + d*x]))/(35*d) + (Cot[c + d*x]*(35*a + 16*b*Sec[c + d*x]))/(35*d
) - (Cot[c + d*x]^3*(35*a + 24*b*Sec[c + d*x]))/(105*d)
```


Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^8(c+dx)(a+b\sec(c+dx))\,dx = -\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d} + \frac{1}{7}\int \cot^6(c+dx)(-7a-6b\sec(c+dx))\,dx$$
$$= -\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d} + \frac{\cot^5(c+dx)(7a+6b\sec(c+dx))}{35d} + \frac{1}{35}\int c$$
$$= -\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d} + \frac{\cot^5(c+dx)(7a+6b\sec(c+dx))}{35d} - \frac{\cot^3(c}{}$$
$$= -\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d} + \frac{\cot^5(c+dx)(7a+6b\sec(c+dx))}{35d} + \frac{\cot(c+}{}$$
$$= ax - \frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d} + \frac{\cot^5(c+dx)(7a+6b\sec(c+dx))}{35d} + \frac{\cot}{}$$

Mathematica [C] time = 0.0460286, size = 92, normalized size = 0.83

$$-\frac{a\cot^7(c+dx)\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},-\tan^2(c+dx)\right)}{7d} - \frac{b\csc^7(c+dx)}{7d} + \frac{3b\csc^5(c+dx)}{5d} - \frac{b\csc^3(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/d + (3*b*Csc[c + d*x]^5)/(5*d) - (b
*Csc[c + d*x]^7)/(7*d) - (a*Cot[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/2,
 -Tan[c + d*x]^2])/(7*d)
```

Maple [A] time = 0.051, size = 162, normalized size = 1.5

$$\frac{1}{d}\left(a\left(-\frac{(\cot(dx+c))^7}{7} + \frac{(\cot(dx+c))^5}{5} - \frac{(\cot(dx+c))^3}{3} + \cot(dx+c) + dx + c\right) + b\left(-\frac{(\cos(dx+c))^8}{7(\sin(dx+c))^7} + \frac{(\cos(dx}{35(\sin(d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^8*(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/7*cot(d*x+c)^7+1/5*cot(d*x+c)^5-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+
c)+b*(-1/7/sin(d*x+c)^7*cos(d*x+c)^8+1/35/sin(d*x+c)^5*cos(d*x+c)^8-1/35/si
n(d*x+c)^3*cos(d*x+c)^8+1/7/sin(d*x+c)*cos(d*x+c)^8+1/7*(16/5+cos(d*x+c)^6+
6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 1.48094, size = 135, normalized size = 1.22

$$\frac{\left(105\,dx+105\,c+\frac{105\,\tan(dx+c)^6-35\,\tan(dx+c)^4+21\,\tan(dx+c)^2-15}{\tan(dx+c)^7}\right)a+\frac{3\left(35\,\sin(dx+c)^6-35\,\sin(dx+c)^4+21\,\sin(dx+c)^2-5\right)b}{\sin(dx+c)^7}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/105*((105*d*x + 105*c + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan(
d*x + c)^2 - 15)/tan(d*x + c)^7)*a + 3*(35*sin(d*x + c)^6 - 35*sin(d*x + c)
^4 + 21*sin(d*x + c)^2 - 5)*b/sin(d*x + c)^7)/d
```

Fricas [A] time = 0.805925, size = 477, normalized size = 4.3

$$\frac{176\,a\cos\left(dx+c\right)^7+105\,b\cos\left(dx+c\right)^6-406\,a\cos\left(dx+c\right)^5-210\,b\cos\left(dx+c\right)^4+350\,a\cos\left(dx+c\right)^3+168\,b\cos\left(dx+\right.}{105\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/105*(176*a*cos(d*x + c)^7 + 105*b*cos(d*x + c)^6 - 406*a*cos(d*x + c)^5 -
 210*b*cos(d*x + c)^4 + 350*a*cos(d*x + c)^3 + 168*b*cos(d*x + c)^2 - 105*a
*cos(d*x + c) + 105*(a*d*x*cos(d*x + c)^6 - 3*a*d*x*cos(d*x + c)^4 + 3*a*d*
x*cos(d*x + c)^2 - a*d*x)*sin(d*x + c) - 48*b)/((d*cos(d*x + c)^6 - 3*d*cos
(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**8*(a+b*sec(d*x+c)),x)
```

[Out] Timed out

Giac [B] time = 1.47965, size = 304, normalized size = 2.74

$$\frac{15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-189\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+147\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+1295\,a\tan\left(\frac{1}{2}\,dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/13440*(15*a*tan(1/2*d*x + 1/2*c)^7 - 15*b*tan(1/2*d*x + 1/2*c)^7 - 189*a*
tan(1/2*d*x + 1/2*c)^5 + 147*b*tan(1/2*d*x + 1/2*c)^5 + 1295*a*tan(1/2*d*x
+ 1/2*c)^3 - 735*b*tan(1/2*d*x + 1/2*c)^3 + 13440*(d*x + c)*a - 9765*a*tan(
1/2*d*x + 1/2*c) + 3675*b*tan(1/2*d*x + 1/2*c) + (9765*a*tan(1/2*d*x + 1/2*
c)^6 + 3675*b*tan(1/2*d*x + 1/2*c)^6 - 1295*a*tan(1/2*d*x + 1/2*c)^4 - 735*
b*tan(1/2*d*x + 1/2*c)^4 + 189*a*tan(1/2*d*x + 1/2*c)^2 + 147*b*tan(1/2*d*x
 + 1/2*c)^2 - 15*a - 15*b)/tan(1/2*d*x + 1/2*c)^7)/d
```

3.271 $\int (a+b\sec (c+dx))^2 \tan ^9(c+dx) \, dx$

Optimal. Leaf size=185

$$\frac{a^2\sec^8(c+dx)}{8d}-\frac{2a^2\sec^6(c+dx)}{3d}+\frac{3a^2\sec^4(c+dx)}{2d}-\frac{2a^2\sec^2(c+dx)}{d}-\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec^9(c+dx)}{9d}-\frac{8a}{}$$

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a*b*Sec[c + d*x])/d - (2*a^2*Sec[c + d*x]
^2)/d - (8*a*b*Sec[c + d*x]^3)/(3*d) + (3*a^2*Sec[c + d*x]^4)/(2*d) + (12*a
*b*Sec[c + d*x]^5)/(5*d) - (2*a^2*Sec[c + d*x]^6)/(3*d) - (8*a*b*Sec[c + d*
x]^7)/(7*d) + (a^2*Sec[c + d*x]^8)/(8*d) + (2*a*b*Sec[c + d*x]^9)/(9*d) + (
b^2*Tan[c + d*x]^10)/(10*d)
```

Rubi [A] time = 0.131187, antiderivative size = 217, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 948}

$$\frac{(a^2-4b^2)\sec^8(c+dx)}{8d}-\frac{(2a^2-3b^2)\sec^6(c+dx)}{3d}+\frac{(3a^2-2b^2)\sec^4(c+dx)}{2d}-\frac{(4a^2-b^2)\sec^2(c+dx)}{2d}-\frac{a^2\log(\cos(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^9,x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a*b*Sec[c + d*x])/d - ((4*a^2 - b^2)*Sec[
c + d*x]^2)/(2*d) - (8*a*b*Sec[c + d*x]^3)/(3*d) + ((3*a^2 - 2*b^2)*Sec[c +
 d*x]^4)/(2*d) + (12*a*b*Sec[c + d*x]^5)/(5*d) - ((2*a^2 - 3*b^2)*Sec[c + d
*x]^6)/(3*d) - (8*a*b*Sec[c + d*x]^7)/(7*d) + ((a^2 - 4*b^2)*Sec[c + d*x]^8
)/(8*d) + (2*a*b*Sec[c + d*x]^9)/(9*d) + (b^2*Sec[c + d*x]^10)/(10*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
```

```
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\tan^9(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^2\left(b^2-x^2\right)^4}{x}\,dx, x, b\sec(c+dx)\right)}{b^8 d}$$
$$= \frac{\text{Subst}\left(\int \left(2ab^8+\frac{a^2b^8}{x}-b^6\left(4a^2-b^2\right)x-8ab^6x^2+2b^4\left(3a^2-2b^2\right)x^3+12ab^4\right.\right.}{}$$
$$= -\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}-\frac{\left(4a^2-b^2\right)\sec^2(c+dx)}{2d}-\frac{8ab\sec^3(}{3d}$$

Mathematica [A] time = 0.423763, size = 173, normalized size = 0.94

$$\frac{315\left(a^2-4b^2\right)\sec^8(c+dx)-840\left(2a^2-3b^2\right)\sec^6(c+dx)+1260\left(3a^2-2b^2\right)\sec^4(c+dx)-1260\left(4a^2-b^2\right)\sec^2(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^9,x]
```

```
[Out] (-2520*a^2*Log[Cos[c + d*x]] + 5040*a*b*Sec[c + d*x] - 1260*(4*a^2 - b^2)*S
ec[c + d*x]^2 - 6720*a*b*Sec[c + d*x]^3 + 1260*(3*a^2 - 2*b^2)*Sec[c + d*x]
^4 + 6048*a*b*Sec[c + d*x]^5 - 840*(2*a^2 - 3*b^2)*Sec[c + d*x]^6 - 2880*a*
b*Sec[c + d*x]^7 + 315*(a^2 - 4*b^2)*Sec[c + d*x]^8 + 560*a*b*Sec[c + d*x]^
9 + 252*b^2*Sec[c + d*x]^10)/(2520*d)
```

Maple [A] time = 0.056, size = 317, normalized size = 1.7

$$\frac{(\tan(dx+c))^8 a^2}{8d}-\frac{a^2(\tan(dx+c))^6}{6d}+\frac{a^2(\tan(dx+c))^4}{4d}-\frac{a^2(\tan(dx+c))^2}{2d}-\frac{a^2\ln(\cos(dx+c))}{d}+\frac{2ab(\sin(dx+}{9d(\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^9,x)
```

```
[Out] 1/8/d*tan(d*x+c)^8*a^2-1/6/d*a^2*tan(d*x+c)^6+1/4/d*a^2*tan(d*x+c)^4-1/2/d*
a^2*tan(d*x+c)^2-a^2*ln(cos(d*x+c))/d+2/9/d*a*b*sin(d*x+c)^10/cos(d*x+c)^9-
2/63/d*a*b*sin(d*x+c)^10/cos(d*x+c)^7+2/105/d*a*b*sin(d*x+c)^10/cos(d*x+c)^
5-2/63/d*a*b*sin(d*x+c)^10/cos(d*x+c)^3+2/9/d*a*b*sin(d*x+c)^10/cos(d*x+c)+
256/315/d*cos(d*x+c)*a*b+2/9/d*a*b*cos(d*x+c)*sin(d*x+c)^8+16/63/d*a*b*cos(
d*x+c)*sin(d*x+c)^6+32/105/d*a*b*cos(d*x+c)*sin(d*x+c)^4+128/315/d*a*b*cos(
d*x+c)*sin(d*x+c)^2+1/10/d*b^2*sin(d*x+c)^10/cos(d*x+c)^10
```

<u>Maxima</u> [A] time = 1.0182, size = 235, normalized size = 1.27

$$-\frac{2520\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{5040\,ab\cos(dx+c)^9-6720\,ab\cos(dx+c)^7-1260\left(4\,a^2-b^2\right)\cos(dx+c)^8+6048\,ab\cos(dx+c)^5+1260\left(3\,a^2-2\,b^2\right)\cos(dx+c)^6-2}{\cos(dx+c)^{10}}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -1/2520*(2520*a^2*log(cos(d*x + c)) - (5040*a*b*cos(d*x + c)^9 - 6720*a*b*c
os(d*x + c)^7 - 1260*(4*a^2 - b^2)*cos(d*x + c)^8 + 6048*a*b*cos(d*x + c)^5
 + 1260*(3*a^2 - 2*b^2)*cos(d*x + c)^6 - 2880*a*b*cos(d*x + c)^3 - 840*(2*a
^2 - 3*b^2)*cos(d*x + c)^4 + 560*a*b*cos(d*x + c) + 315*(a^2 - 4*b^2)*cos(d
*x + c)^2 + 252*b^2)/cos(d*x + c)^10)/d
```

<u>Fricas</u> [A] time = 0.93027, size = 483, normalized size = 2.61

$$-\frac{2520\,a^2\cos\left(dx+c\right)^{10}\log\left(-\cos\left(dx+c\right)\right)-5040\,ab\cos\left(dx+c\right)^9+6720\,ab\cos\left(dx+c\right)^7+1260\left(4\,a^2-b^2\right)\cos\left(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/2520*(2520*a^2*cos(d*x + c)^10*log(-cos(d*x + c)) - 5040*a*b*cos(d*x + c
)^9 + 6720*a*b*cos(d*x + c)^7 + 1260*(4*a^2 - b^2)*cos(d*x + c)^8 - 6048*a*
b*cos(d*x + c)^5 - 1260*(3*a^2 - 2*b^2)*cos(d*x + c)^6 + 2880*a*b*cos(d*x +
 c)^3 + 840*(2*a^2 - 3*b^2)*cos(d*x + c)^4 - 560*a*b*cos(d*x + c) - 315*(a^
2 - 4*b^2)*cos(d*x + c)^2 - 252*b^2)/(d*cos(d*x + c)^10)
```

Sympy [A] time = 90.7571, size = 314, normalized size = 1.7

$$\begin{cases} \frac{a^2\log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{a^2\tan^8(c+dx)}{8d} - \frac{a^2\tan^6(c+dx)}{6d} + \frac{a^2\tan^4(c+dx)}{4d} - \frac{a^2\tan^2(c+dx)}{2d} + \frac{2ab\tan^8(c+dx)\sec(c+dx)}{9d} - \frac{16ab\tan^6(c+dx)\sec(}{63d} \\ x\,(a+b\sec(c))^2\tan^9(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**9,x)
```

```
[Out] Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**8/(8*d)
 - a**2*tan(c + d*x)**6/(6*d) + a**2*tan(c + d*x)**4/(4*d) - a**2*tan(c + d
*x)**2/(2*d) + 2*a*b*tan(c + d*x)**8*sec(c + d*x)/(9*d) - 16*a*b*tan(c + d*
x)**6*sec(c + d*x)/(63*d) + 32*a*b*tan(c + d*x)**4*sec(c + d*x)/(105*d) - 1
28*a*b*tan(c + d*x)**2*sec(c + d*x)/(315*d) + 256*a*b*sec(c + d*x)/(315*d)
+ b**2*tan(c + d*x)**8*sec(c + d*x)**2/(10*d) - b**2*tan(c + d*x)**6*sec(c
+ d*x)**2/(10*d) + b**2*tan(c + d*x)**4*sec(c + d*x)**2/(10*d) - b**2*tan(c
 + d*x)**2*sec(c + d*x)**2/(10*d) + b**2*sec(c + d*x)**2/(10*d), Ne(d, 0)),
 (x*(a + b*sec(c))**2*tan(c)**9, True))
```

Giac [B] time = 13.7938, size = 660, normalized size = 3.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="giac")
```

```
[Out] 1/2520*(2520*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 252
0*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (7381*a^2 + 40
96*a*b + 78850*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 40960*a*b*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 382545*a^2*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2 + 184320*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 111420
0*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 491520*a*b*(cos(d*x + c)
- 1)^3/(cos(d*x + c) + 1)^3 + 2171610*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c
) + 1)^4 + 860160*a*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 2736972*a
^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 516096*a*b*(cos(d*x + c) - 1
)^5/(cos(d*x + c) + 1)^5 - 258048*b^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) +
1)^5 + 2171610*a^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 1114200*a^2*
(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 382545*a^2*(cos(d*x + c) - 1)^8
```

```
/(cos(d*x + c) + 1)^8 + 78850*a^2*(cos(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9
 + 7381*a^2*(cos(d*x + c) - 1)^10/(cos(d*x + c) + 1)^10)/((cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 1)^10)/d
```

3.272 $\int (a+b\sec(c+dx))^2\tan^7(c+dx)\,dx$

Optimal. Leaf size=149

$$\frac{a^2\sec^6(c+dx)}{6d}-\frac{3a^2\sec^4(c+dx)}{4d}+\frac{3a^2\sec^2(c+dx)}{2d}+\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec^7(c+dx)}{7d}-\frac{6ab\sec^5(c+dx)}{5d}+$$

```
[Out] (a^2*Log[Cos[c + d*x]])/d - (2*a*b*Sec[c + d*x])/d + (3*a^2*Sec[c + d*x]^2)
/(2*d) + (2*a*b*Sec[c + d*x]^3)/d - (3*a^2*Sec[c + d*x]^4)/(4*d) - (6*a*b*S
ec[c + d*x]^5)/(5*d) + (a^2*Sec[c + d*x]^6)/(6*d) + (2*a*b*Sec[c + d*x]^7)/
(7*d) + (b^2*Tan[c + d*x]^8)/(8*d)
```

Rubi [A] time = 0.110769, antiderivative size = 169, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 948}

$$\frac{(a^2-3b^2)\sec^6(c+dx)}{6d}-\frac{3(a^2-b^2)\sec^4(c+dx)}{4d}+\frac{(3a^2-b^2)\sec^2(c+dx)}{2d}+\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^7,x]
```

```
[Out] (a^2*Log[Cos[c + d*x]])/d - (2*a*b*Sec[c + d*x])/d + ((3*a^2 - b^2)*Sec[c +
 d*x]^2)/(2*d) + (2*a*b*Sec[c + d*x]^3)/d - (3*(a^2 - b^2)*Sec[c + d*x]^4)/
(4*d) - (6*a*b*Sec[c + d*x]^5)/(5*d) + ((a^2 - 3*b^2)*Sec[c + d*x]^6)/(6*d)
 + (2*a*b*Sec[c + d*x]^7)/(7*d) + (b^2*Sec[c + d*x]^8)/(8*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
```

```
& EqQ[d, 0]))
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\tan^7(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a+x)^2\left(b^2-x^2\right)^3}{x}\,dx, x, b\sec(c+dx)\right)}{b^6 d}$$
$$= -\frac{\text{Subst}\left(\int \left(2ab^6+\frac{a^2b^6}{x}-b^4\left(3a^2-b^2\right)x-6ab^4x^2+3b^2\left(a^2-b^2\right)x^3+6ab^2x^4\right.\right.}{b^6 d}$$
$$= \frac{a^2\log(\cos(c+dx))}{d}-\frac{2ab\sec(c+dx)}{d}+\frac{\left(3a^2-b^2\right)\sec^2(c+dx)}{2d}+\frac{2ab\sec^3(c+}{d}$$

Mathematica [A] time = 0.330653, size = 138, normalized size = 0.93

$$\frac{140\left(a^2-3b^2\right)\sec^6(c+dx)-630\left(a^2-b^2\right)\sec^4(c+dx)+420\left(3a^2-b^2\right)\sec^2(c+dx)+840a^2\log(\cos(c+dx))+240ab\,s}{840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^7,x]
```

```
[Out] (840*a^2*Log[Cos[c + d*x]] - 1680*a*b*Sec[c + d*x] + 420*(3*a^2 - b^2)*Sec[
c + d*x]^2 + 1680*a*b*Sec[c + d*x]^3 - 630*(a^2 - b^2)*Sec[c + d*x]^4 - 100
8*a*b*Sec[c + d*x]^5 + 140*(a^2 - 3*b^2)*Sec[c + d*x]^6 + 240*a*b*Sec[c + d
*x]^7 + 105*b^2*Sec[c + d*x]^8)/(840*d)
```

Maple [A] time = 0.052, size = 256, normalized size = 1.7

$$\frac{a^2(\tan(dx+c))^6}{6d}-\frac{a^2(\tan(dx+c))^4}{4d}+\frac{a^2(\tan(dx+c))^2}{2d}+\frac{a^2\ln(\cos(dx+c))}{d}+\frac{2ab(\sin(dx+c))^8}{7d(\cos(dx+c))^7}-\frac{2ab(\sin(dx+}{35d(\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^7,x)
```

```
[Out] 1/6/d*a^2*tan(d*x+c)^6-1/4/d*a^2*tan(d*x+c)^4+1/2/d*a^2*tan(d*x+c)^2+a^2*ln
(cos(d*x+c))/d+2/7/d*a*b*sin(d*x+c)^8/cos(d*x+c)^7-2/35/d*a*b*sin(d*x+c)^8/
```

```
cos(d*x+c)^5+2/35/d*a*b*sin(d*x+c)^8/cos(d*x+c)^3-2/7/d*a*b*sin(d*x+c)^8/co
s(d*x+c)-32/35/d*cos(d*x+c)*a*b-2/7/d*a*b*cos(d*x+c)*sin(d*x+c)^6-12/35/d*a
*b*cos(d*x+c)*sin(d*x+c)^4-16/35/d*a*b*cos(d*x+c)*sin(d*x+c)^2+1/8/d*b^2*si
n(d*x+c)^8/cos(d*x+c)^8
```

Maxima [A] time = 1.03587, size = 188, normalized size = 1.26

$$\frac{840\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{1680\,ab\cos(dx+c)^7-1680\,ab\cos(dx+c)^5-420\left(3\,a^2-b^2\right)\cos(dx+c)^6+1008\,ab\cos(dx+c)^3+630\left(a^2-b^2\right)\cos(dx+c)^4-240\,ab\cos}{\cos(dx+c)^8}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/840*(840*a^2*log(cos(d*x + c)) - (1680*a*b*cos(d*x + c)^7 - 1680*a*b*cos(
d*x + c)^5 - 420*(3*a^2 - b^2)*cos(d*x + c)^6 + 1008*a*b*cos(d*x + c)^3 + 6
30*(a^2 - b^2)*cos(d*x + c)^4 - 240*a*b*cos(d*x + c) - 140*(a^2 - 3*b^2)*co
s(d*x + c)^2 - 105*b^2)/cos(d*x + c)^8)/d
```

Fricas [A] time = 0.983282, size = 383, normalized size = 2.57

$$\frac{840\,a^2\cos\left(dx+c\right)^8\log\left(-\cos\left(dx+c\right)\right)-1680\,ab\cos\left(dx+c\right)^7+1680\,ab\cos\left(dx+c\right)^5+420\left(3\,a^2-b^2\right)\cos\left(dx+c\right)^6-}{840\,d\cos\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/840*(840*a^2*cos(d*x + c)^8*log(-cos(d*x + c)) - 1680*a*b*cos(d*x + c)^7
+ 1680*a*b*cos(d*x + c)^5 + 420*(3*a^2 - b^2)*cos(d*x + c)^6 - 1008*a*b*cos
(d*x + c)^3 - 630*(a^2 - b^2)*cos(d*x + c)^4 + 240*a*b*cos(d*x + c) + 140*(
a^2 - 3*b^2)*cos(d*x + c)^2 + 105*b^2)/(d*cos(d*x + c)^8)
```

Sympy [A] time = 25.2943, size = 252, normalized size = 1.69

$$\begin{cases} -\frac{a^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^2\tan^6\left(c+dx\right)}{6d}-\frac{a^2\tan^4\left(c+dx\right)}{4d}+\frac{a^2\tan^2\left(c+dx\right)}{2d}+\frac{2ab\tan^6\left(c+dx\right)\sec\left(c+dx\right)}{7d}-\frac{12ab\tan^4\left(c+dx\right)\sec\left(c+dx\right)}{35d}+\frac{16ab\,\mathrm{ta}}{} \\ x\left(a+b\sec\left(c\right)\right)^2\tan^7\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**7,x)
```

```
[Out] Piecewise((-a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**6/(6*d
) - a**2*tan(c + d*x)**4/(4*d) + a**2*tan(c + d*x)**2/(2*d) + 2*a*b*tan(c +
 d*x)**6*sec(c + d*x)/(7*d) - 12*a*b*tan(c + d*x)**4*sec(c + d*x)/(35*d) +
16*a*b*tan(c + d*x)**2*sec(c + d*x)/(35*d) - 32*a*b*sec(c + d*x)/(35*d) + b
**2*tan(c + d*x)**6*sec(c + d*x)**2/(8*d) - b**2*tan(c + d*x)**4*sec(c + d*
x)**2/(8*d) + b**2*tan(c + d*x)**2*sec(c + d*x)**2/(8*d) - b**2*sec(c + d*x
)**2/(8*d), Ne(d, 0)), (x*(a + b*sec(c))**2*tan(c)**7, True))
```

Giac [B] time = 7.90808, size = 560, normalized size = 3.76

$$-\frac{840\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-840\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{2283\,a^2+1536\,ab+\frac{19944\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{12288\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{77364\,a^2(\cos\ldots}{(\cos(d\ldots}}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] -1/840*(840*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 840*
a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (2283*a^2 + 1536
*a*b + 19944*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12288*a*b*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 77364*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2 + 43008*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 175448*a^2
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 86016*a*b*(cos(d*x + c) - 1)^3
/(cos(d*x + c) + 1)^3 + 231490*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^
4 + 53760*a*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 26880*b^2*(cos(d*
x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 175448*a^2*(cos(d*x + c) - 1)^5/(cos(d
*x + c) + 1)^5 + 77364*a^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 1994
4*a^2*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 2283*a^2*(cos(d*x + c) -
1)^8/(cos(d*x + c) + 1)^8)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^8)/d
```

3.273 $\int (a+b\sec(c+dx))^2\tan^5(c+dx)\,dx$

Optimal. Leaf size=115

$$\frac{a^2\sec^4(c+dx)}{4d}-\frac{a^2\sec^2(c+dx)}{d}-\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec^5(c+dx)}{5d}-\frac{4ab\sec^3(c+dx)}{3d}+\frac{2ab\sec(c+dx)}{d}+\frac{b^2\cdots}{}$$

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a*b*Sec[c + d*x])/d - (a^2*Sec[c + d*x]^2
)/d - (4*a*b*Sec[c + d*x]^3)/(3*d) + (a^2*Sec[c + d*x]^4)/(4*d) + (2*a*b*Se
c[c + d*x]^5)/(5*d) + (b^2*Tan[c + d*x]^6)/(6*d)
```

Rubi [A] time = 0.091351, antiderivative size = 131, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 948}

$$\frac{\left(a^2-2b^2\right)\sec^4(c+dx)}{4d}-\frac{\left(2a^2-b^2\right)\sec^2(c+dx)}{2d}-\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec^5(c+dx)}{5d}-\frac{4ab\sec^3(c+dx)}{3d}+\frac{2ab\,\mathrm{se}\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a*b*Sec[c + d*x])/d - ((2*a^2 - b^2)*Sec[
c + d*x]^2)/(2*d) - (4*a*b*Sec[c + d*x]^3)/(3*d) + ((a^2 - 2*b^2)*Sec[c + d
*x]^4)/(4*d) + (2*a*b*Sec[c + d*x]^5)/(5*d) + (b^2*Sec[c + d*x]^6)/(6*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^2}{x}\,dx, x, b\sec(c+dx)\right)}{b^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(2ab^4+\frac{a^2b^4}{x}-b^2\left(2a^2-b^2\right)x-4ab^2x^2+\left(a^2-2b^2\right)x^3+2ax^4+x^5\right) d\right.}{b^4 d}$$
$$= -\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}-\frac{\left(2a^2-b^2\right)\sec^2(c+dx)}{2d}-\frac{4ab\sec^3(c}{3d}$$

<u>Mathematica</u> [A] time = 0.257131, size = 105, normalized size = 0.91

$$\frac{15\left(a^2-2b^2\right)\sec^4(c+dx)+30\left(b^2-2a^2\right)\sec^2(c+dx)-60a^2\log(\cos(c+dx))+24ab\sec^5(c+dx)-80ab\sec^3(c+dx)+}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] (-60*a^2*Log[Cos[c + d*x]] + 120*a*b*Sec[c + d*x] + 30*(-2*a^2 + b^2)*Sec[c
 + d*x]^2 - 80*a*b*Sec[c + d*x]^3 + 15*(a^2 - 2*b^2)*Sec[c + d*x]^4 + 24*a*
b*Sec[c + d*x]^5 + 10*b^2*Sec[c + d*x]^6)/(60*d)
```

<u>Maple</u> [A] time = 0.051, size = 197, normalized size = 1.7

$$\frac{a^2(\tan(dx+c))^4}{4d}-\frac{a^2(\tan(dx+c))^2}{2d}-\frac{a^2\ln(\cos(dx+c))}{d}+\frac{2ab(\sin(dx+c))^6}{5d(\cos(dx+c))^5}-\frac{2ab(\sin(dx+c))^6}{15d(\cos(dx+c))^3}+\frac{2ab(\sin(dx}{5d\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^5,x)
```

```
[Out] 1/4/d*a^2*tan(d*x+c)^4-1/2/d*a^2*tan(d*x+c)^2-a^2*ln(cos(d*x+c))/d+2/5/d*a*
b*sin(d*x+c)^6/cos(d*x+c)^5-2/15/d*a*b*sin(d*x+c)^6/cos(d*x+c)^3+2/5/d*a*b*
sin(d*x+c)^6/cos(d*x+c)+16/15/d*cos(d*x+c)*a*b+2/5/d*a*b*cos(d*x+c)*sin(d*x
+c)^4+8/15/d*a*b*cos(d*x+c)*sin(d*x+c)^2+1/6/d*b^2*sin(d*x+c)^6/cos(d*x+c)^
6
```

Maxima [A] time = 0.98191, size = 146, normalized size = 1.27

$$-\frac{60\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{120\,ab\cos(dx+c)^5-80\,ab\cos(dx+c)^3-30\left(2\,a^2-b^2\right)\cos(dx+c)^4+24\,ab\cos(dx+c)+15\left(a^2-2\,b^2\right)\cos(dx+c)^2+10\,b^2}{\cos(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(60*a^2*log(cos(d*x + c)) - (120*a*b*cos(d*x + c)^5 - 80*a*b*cos(d*x
+ c)^3 - 30*(2*a^2 - b^2)*cos(d*x + c)^4 + 24*a*b*cos(d*x + c) + 15*(a^2 -
2*b^2)*cos(d*x + c)^2 + 10*b^2)/cos(d*x + c)^6)/d
```

Fricas [A] time = 0.804059, size = 293, normalized size = 2.55

$$-\frac{60\,a^2\cos\left(dx+c\right)^6\log\left(-\cos\left(dx+c\right)\right)-120\,ab\cos\left(dx+c\right)^5+80\,ab\cos\left(dx+c\right)^3+30\left(2\,a^2-b^2\right)\cos\left(dx+c\right)^4-24\,a}{60\,d\cos\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/60*(60*a^2*cos(d*x + c)^6*log(-cos(d*x + c)) - 120*a*b*cos(d*x + c)^5 +
80*a*b*cos(d*x + c)^3 + 30*(2*a^2 - b^2)*cos(d*x + c)^4 - 24*a*b*cos(d*x +
c) - 15*(a^2 - 2*b^2)*cos(d*x + c)^2 - 10*b^2)/(d*cos(d*x + c)^6)
```

Sympy [A] time = 9.82042, size = 189, normalized size = 1.64

$$\begin{cases}\frac{a^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^2\tan^4\left(c+dx\right)}{4d}-\frac{a^2\tan^2\left(c+dx\right)}{2d}+\frac{2ab\tan^4\left(c+dx\right)\sec\left(c+dx\right)}{5d}-\frac{8ab\tan^2\left(c+dx\right)\sec\left(c+dx\right)}{15d}+\frac{16ab\sec\left(c+dx\right)}{15d}+\frac{b^2\tan^4\left(\right.}{} \\ x\left(a+b\sec\left(c\right)\right)^2\tan^5\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**5,x)
```

```
[Out] Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**4/(4*d)
 - a**2*tan(c + d*x)**2/(2*d) + 2*a*b*tan(c + d*x)**4*sec(c + d*x)/(5*d) -
```

```
8*a*b*tan(c + d*x)**2*sec(c + d*x)/(15*d) + 16*a*b*sec(c + d*x)/(15*d) + b*
*2*tan(c + d*x)**4*sec(c + d*x)**2/(6*d) - b**2*tan(c + d*x)**2*sec(c + d*x
)**2/(6*d) + b**2*sec(c + d*x)**2/(6*d), Ne(d, 0)), (x*(a + b*sec(c))**2*ta
n(c)**5, True))
```

Giac [B] time = 3.56158, size = 460, normalized size = 4.

$$\frac{60\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-60\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{147\,a^2+128\,ab+\frac{1002\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{768\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2925\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^2*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (147*a^2 + 128*a*b +
 1002*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 768*a*b*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 2925*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
 1920*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 4140*a^2*(cos(d*x + c
) - 1)^3/(cos(d*x + c) + 1)^3 + 1280*a*b*(cos(d*x + c) - 1)^3/(cos(d*x + c)
 + 1)^3 - 640*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 2925*a^2*(cos
(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1002*a^2*(cos(d*x + c) - 1)^5/(cos(
d*x + c) + 1)^5 + 147*a^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6)/((cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^6)/d
```

3.274 $\int (a + b\sec(c + dx))^2 \tan^3(c + dx)\,dx$

Optimal. Leaf size=87

$$\frac{(a^2 - b^2)\sec^2(c + dx)}{2d} + \frac{a^2\log(\cos(c + dx))}{d} + \frac{2ab\sec^3(c + dx)}{3d} - \frac{2ab\sec(c + dx)}{d} + \frac{b^2\sec^4(c + dx)}{4d}$$

```
[Out] (a^2*Log[Cos[c + d*x]])/d - (2*a*b*Sec[c + d*x])/d + ((a^2 - b^2)*Sec[c + d
*x]^2)/(2*d) + (2*a*b*Sec[c + d*x]^3)/(3*d) + (b^2*Sec[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.069152, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 894}

$$\frac{(a^2 - b^2)\sec^2(c + dx)}{2d} + \frac{a^2\log(\cos(c + dx))}{d} + \frac{2ab\sec^3(c + dx)}{3d} - \frac{2ab\sec(c + dx)}{d} + \frac{b^2\sec^4(c + dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (a^2*Log[Cos[c + d*x]])/d - (2*a*b*Sec[c + d*x])/d + ((a^2 - b^2)*Sec[c + d
*x]^2)/(2*d) + (2*a*b*Sec[c + d*x]^3)/(3*d) + (b^2*Sec[c + d*x]^4)/(4*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\tan^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)}{x}\,dx, x, b\sec(c+dx)\right)}{b^2d}$$
$$= -\frac{\text{Subst}\left(\int \left(2ab^2+\frac{a^2b^2}{x}-\left(a^2-b^2\right)x-2ax^2-x^3\right)dx, x, b\sec(c+dx)\right)}{b^2d}$$
$$= \frac{a^2\log(\cos(c+dx))}{d}-\frac{2ab\sec(c+dx)}{d}+\frac{\left(a^2-b^2\right)\sec^2(c+dx)}{2d}+\frac{2ab\sec^3(c+d}{3d}$$

Mathematica [A] time = 0.435898, size = 74, normalized size = 0.85

$$\frac{6\left(a^2-b^2\right)\sec^2(c+dx)+12a^2\log(\cos(c+dx))+8ab\sec^3(c+dx)-24ab\sec(c+dx)+3b^2\sec^4(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (12*a^2*Log[Cos[c + d*x]] - 24*a*b*Sec[c + d*x] + 6*(a^2 - b^2)*Sec[c + d*x
]^2 + 8*a*b*Sec[c + d*x]^3 + 3*b^2*Sec[c + d*x]^4)/(12*d)
```

Maple [A] time = 0.048, size = 136, normalized size = 1.6

$$\frac{a^2(\tan(dx+c))^2}{2d}+\frac{a^2\ln(\cos(dx+c))}{d}+\frac{2ab(\sin(dx+c))^4}{3d(\cos(dx+c))^3}-\frac{2ab(\sin(dx+c))^4}{3d\cos(dx+c)}-\frac{2ab\cos(dx+c)(\sin(dx+c))^2}{3d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^3,x)
```

```
[Out] 1/2/d*a^2*tan(d*x+c)^2+a^2*ln(cos(d*x+c))/d+2/3/d*a*b*sin(d*x+c)^4/cos(d*x+
c)^3-2/3/d*a*b*sin(d*x+c)^4/cos(d*x+c)-2/3/d*a*b*cos(d*x+c)*sin(d*x+c)^2-4/
3/d*cos(d*x+c)*a*b+1/4/d*b^2*sin(d*x+c)^4/cos(d*x+c)^4
```

Maxima [A] time = 0.999052, size = 101, normalized size = 1.16

$$\frac{12a^2\log(\cos(dx+c))-\frac{24ab\cos(dx+c)^3-8ab\cos(dx+c)-6\left(a^2-b^2\right)\cos(dx+c)^2-3b^2}{\cos(dx+c)^4}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(12*a^2*log(cos(d*x + c)) - (24*a*b*cos(d*x + c)^3 - 8*a*b*cos(d*x + c
) - 6*(a^2 - b^2)*cos(d*x + c)^2 - 3*b^2)/cos(d*x + c)^4)/d
```

Fricas [A] time = 0.969567, size = 205, normalized size = 2.36

$$\frac{12\,a^2\cos\left(dx+c\right)^4\log\left(-\cos\left(dx+c\right)\right)-24\,ab\cos\left(dx+c\right)^3+8\,ab\cos\left(dx+c\right)+6\left(a^2-b^2\right)\cos\left(dx+c\right)^2+3\,b^2}{12\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(12*a^2*cos(d*x + c)^4*log(-cos(d*x + c)) - 24*a*b*cos(d*x + c)^3 + 8*
a*b*cos(d*x + c) + 6*(a^2 - b^2)*cos(d*x + c)^2 + 3*b^2)/(d*cos(d*x + c)^4)
```

Sympy [A] time = 4.02762, size = 126, normalized size = 1.45

$$\begin{cases} -\frac{a^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^2\tan^2\left(c+dx\right)}{2d}+\frac{2ab\tan^2\left(c+dx\right)\sec\left(c+dx\right)}{3d}-\frac{4ab\sec\left(c+dx\right)}{3d}+\frac{b^2\tan^2\left(c+dx\right)\sec^2\left(c+dx\right)}{4d}-\frac{b^2\sec^2\left(c+dx\right)}{4d} & \text{for } d \neq \\ x\left(a+b\sec\left(c\right)\right)^2\tan^3\left(c\right) & \text{otherv} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**3,x)
```

```
[Out] Piecewise((-a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**2/(2*d
) + 2*a*b*tan(c + d*x)**2*sec(c + d*x)/(3*d) - 4*a*b*sec(c + d*x)/(3*d) + b
**2*tan(c + d*x)**2*sec(c + d*x)**2/(4*d) - b**2*sec(c + d*x)**2/(4*d), Ne(
d, 0)), (x*(a + b*sec(c))**2*tan(c)**3, True))
```

<u>Giac</u> [B] time = 1.93279, size = 360, normalized size = 4.14

$$-\frac{12\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-12\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{25\,a^2+32\,ab+\frac{124\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{128\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{198\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+9\cdots}{\left(\frac{\cos(dx\cdots}{\cos(dx\cdots}\right.}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/12*(12*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 12*a^2
*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (25*a^2 + 32*a*b +
124*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 128*a*b*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1) + 198*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 96
*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 48*b^2*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2 + 124*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3
+ 25*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/((cos(d*x + c) - 1)/(co
s(d*x + c) + 1) + 1)^4)/d
```

3.275 $\int (a+b\sec(c+dx))^2\tan(c+dx)\,dx$

Optimal. Leaf size=47

$$-\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{b^2\sec^2(c+dx)}{2d}$$

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a*b*Sec[c + d*x])/d + (b^2*Sec[c + d*x]^2
)/(2*d)
```

Rubi [A] time = 0.0336619, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3885, 43}

$$-\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{b^2\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a*b*Sec[c + d*x])/d + (b^2*Sec[c + d*x]^2
)/(2*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^2}{x}\,dx, x, b\sec(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(2a+\frac{a^2}{x}+x\right)dx, x, b\sec(c+dx)\right)}{d}$$
$$= -\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{b^2\sec^2(c+dx)}{2d}$$

Mathematica [A] time = 0.0568751, size = 42, normalized size = 0.89

$$\frac{-2a^2\log(\cos(c+dx))+4ab\sec(c+dx)+b^2\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] (-2*a^2*Log[Cos[c + d*x]] + 4*a*b*Sec[c + d*x] + b^2*Sec[c + d*x]^2)/(2*d)
```

Maple [A] time = 0.015, size = 45, normalized size = 1.

$$\frac{b^2(\sec(dx+c))^2}{2d}+2\frac{ab\sec(dx+c)}{d}+\frac{a^2\ln(\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c),x)
```

```
[Out] 1/2*b^2*sec(d*x+c)^2/d+2*a*b*sec(d*x+c)/d+1/d*a^2*ln(sec(d*x+c))
```

Maxima [A] time = 0.974877, size = 57, normalized size = 1.21

$$-\frac{2a^2\log(\cos(dx+c))-\frac{4ab\cos(dx+c)+b^2}{\cos(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a^2*log(cos(d*x + c)) - (4*a*b*cos(d*x + c) + b^2)/cos(d*x + c)^2)/
d
```

Fricas [A] time = 0.844613, size = 127, normalized size = 2.7

$$-\frac{2\,a^2\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)-4\,ab\cos\left(dx+c\right)-b^2}{2\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*cos(d*x + c)^2*log(-cos(d*x + c)) - 4*a*b*cos(d*x + c) - b^2)/(
d*cos(d*x + c)^2)
```

Sympy [A] time = 0.682281, size = 60, normalized size = 1.28

$$\begin{cases}\frac{a^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{2ab\sec\left(c+dx\right)}{d}+\frac{b^2\sec^2\left(c+dx\right)}{2d} & \text{for}\ d\neq 0\\ x\left(a+b\sec\left(c\right)\right)^2\tan\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c),x)
```

```
[Out] Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + 2*a*b*sec(c + d*x)/d + b**
2*sec(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*sec(c))**2*tan(c), True))
```

Giac [B] time = 1.2018, size = 258, normalized size = 5.49

$$\frac{2\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-2\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{3\,a^2+8\,ab+\frac{6\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{8\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a^2(\cos(dx+c)-\cdots}{(\cos(dx+c)+1)\cdots}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2*a^2*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (3*a^2 + 8*a*b + 6*a^2*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*a*b*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) - 4*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^2*(cos(d*x + c
) - 1)^2/(cos(d*x + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^
2)/d
```

3.276 $\int \cot(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=61

$$\frac{a^2\log(\cos(c+dx))}{d}+\frac{(a+b)^2\log(1-\sec(c+dx))}{2d}+\frac{(a-b)^2\log(\sec(c+dx)+1)}{2d}$$

```
[Out] (a^2*Log[Cos[c + d*x]])/d + ((a + b)^2*Log[1 - Sec[c + d*x]])/(2*d) + ((a -
 b)^2*Log[1 + Sec[c + d*x]])/(2*d)
```

Rubi [A] time = 0.0996327, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3885, 1802}

$$\frac{a^2\log(\cos(c+dx))}{d}+\frac{(a+b)^2\log(1-\sec(c+dx))}{2d}+\frac{(a-b)^2\log(\sec(c+dx)+1)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Log[Cos[c + d*x]])/d + ((a + b)^2*Log[1 - Sec[c + d*x]])/(2*d) + ((a -
 b)^2*Log[1 + Sec[c + d*x]])/(2*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \cot(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{b^2\,\text{Subst}\left(\int \frac{(a+x)^2}{x(b^2-x^2)}\,dx, x, b\sec(c+dx)\right)}{d}$$
$$= -\frac{b^2\,\text{Subst}\left(\int \left(\frac{(a+b)^2}{2b^2(b-x)}+\frac{a^2}{b^2x}-\frac{(a-b)^2}{2b^2(b+x)}\right)dx, x, b\sec(c+dx)\right)}{d}$$
$$= \frac{a^2\log(\cos(c+dx))}{d}+\frac{(a+b)^2\log(1-\sec(c+dx))}{2d}+\frac{(a-b)^2\log(1+\sec(c+dx)}{2d}$$

Mathematica [A] time = 0.10629, size = 53, normalized size = 0.87

$$\frac{(a+b)^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+(a-b)^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-b^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((a - b)^2*Log[Cos[(c + d*x)/2]] - b^2*Log[Cos[c + d*x]] + (a + b)^2*Log[Si
n[(c + d*x)/2]])/d
```

Maple [A] time = 0.039, size = 53, normalized size = 0.9

$$\frac{b^2\ln(\tan(dx+c))}{d}+2\,\frac{ab\ln(\csc(dx+c)-\cot(dx+c))}{d}+\frac{a^2\ln(\sin(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*b^2*ln(tan(d*x+c))+2/d*a*b*ln(csc(d*x+c)-cot(d*x+c))+1/d*a^2*ln(sin(d*x
+c))
```

Maxima [A] time = 0.984351, size = 84, normalized size = 1.38

$$-\frac{2\,b^2\log(\cos(dx+c))-\left(a^2-2\,ab+b^2\right)\log(\cos(dx+c)+1)-\left(a^2+2\,ab+b^2\right)\log(\cos(dx+c)-1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^2*log(cos(d*x + c)) - (a^2 - 2*a*b + b^2)*log(cos(d*x + c) + 1) -
 (a^2 + 2*a*b + b^2)*log(cos(d*x + c) - 1))/d
```

Fricas [A] time = 0.873844, size = 184, normalized size = 3.02

$$-\frac{2\,b^{2}\log\left(-\cos\left(dx+c\right)\right)-\left(a^{2}-2\,ab+b^{2}\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(a^{2}+2\,ab+b^{2}\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*log(-cos(d*x + c)) - (a^2 - 2*a*b + b^2)*log(1/2*cos(d*x + c) +
 1/2) - (a^2 + 2*a*b + b^2)*log(-1/2*cos(d*x + c) + 1/2))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec\left(c+dx\right)\right)^{2}\cot\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cot(c + d*x), x)
```

Giac [A] time = 1.38499, size = 136, normalized size = 2.23

$$-\frac{2\,a^{2}\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)+2\,b^{2}\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\left(a^{2}+2\,ab+b^{2}\right)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) + 2*b^2*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a^2 + 2*a*b + b^2)*lo
g(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)))/d
```

3.277 $\int \cot^3(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=92

$$-\frac{\cot^2(c+dx)\left(a^2+2ab\sec(c+dx)+b^2\right)}{2d}-\frac{a^2\log(\cos(c+dx))}{d}-\frac{a(a+b)\log(1-\sec(c+dx))}{2d}-\frac{a(a-b)\log(\sec(c+dx)+1)}{2d}$$

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) - (a*(a + b)*Log[1 - Sec[c + d*x]])/(2*d) - (a
*(a - b)*Log[1 + Sec[c + d*x]])/(2*d) - (Cot[c + d*x]^2*(a^2 + b^2 + 2*a*b*
Sec[c + d*x]))/(2*d)
```

Rubi [A] time = 0.129499, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3885, 1805, 801}

$$-\frac{\cot^2(c+dx)\left(a^2+2ab\sec(c+dx)+b^2\right)}{2d}-\frac{a^2\log(\cos(c+dx))}{d}-\frac{a(a+b)\log(1-\sec(c+dx))}{2d}-\frac{a(a-b)\log(\sec(c+dx)+1)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) - (a*(a + b)*Log[1 - Sec[c + d*x]])/(2*d) - (a
*(a - b)*Log[1 + Sec[c + d*x]])/(2*d) - (Cot[c + d*x]^2*(a^2 + b^2 + 2*a*b*
Sec[c + d*x]))/(2*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
```

```
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{b^4\,\text{Subst}\left(\int \frac{(a+x)^2}{x\left(b^2-x^2\right)^2}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= -\frac{\cot^2(c+dx)\left(a^2+b^2+2ab\sec(c+dx)\right)}{2d} - \frac{b^2\,\text{Subst}\left(\int \frac{-2a^2-2ax}{x\left(b^2-x^2\right)}\,dx, x, b\sec(c+\right.}{2d}$$

$$= -\frac{\cot^2(c+dx)\left(a^2+b^2+2ab\sec(c+dx)\right)}{2d} - \frac{b^2\,\text{Subst}\left(\int \left(-\frac{a(a+b)}{b^2(b-x)} - \frac{2a^2}{b^2x} + \frac{a(a-b}{b^2(b+}\right.\right.}{2d}$$

$$= -\frac{a^2\log(\cos(c+dx))}{d} - \frac{a(a+b)\log(1-\sec(c+dx))}{2d} - \frac{a(a-b)\log(1+\sec(c+}{2d}$$

Mathematica [A] time = 0.458545, size = 82, normalized size = 0.89

$$-\frac{(a+b)^2\csc^2\left(\frac{1}{2}(c+dx)\right)+(a-b)^2\sec^2\left(\frac{1}{2}(c+dx)\right)+8a\left((a+b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+(a-b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((a + b)^2*Csc[(c + d*x)/2]^2 + 8*a*((a - b)*Log[Cos[(c + d*x)/2]] + (a +
b)*Log[Sin[(c + d*x)/2]]) + (a - b)^2*Sec[(c + d*x)/2]^2)/(8*d)
```

Maple [A] time = 0.052, size = 108, normalized size = 1.2

$$-\frac{a^2(\cot(dx+c))^2}{2d} - \frac{a^2\ln(\sin(dx+c))}{d} - \frac{ab(\cos(dx+c))^3}{d(\sin(dx+c))^2} - \frac{a\cos(dx+c)\,b}{d} - \frac{ab\ln(\csc(dx+c)-\cot(dx+c))}{d} - \frac{}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/2/d*a^2*cot(d*x+c)^2-1/d*a^2*ln(sin(d*x+c))-1/d*a*b/sin(d*x+c)^2*cos(d*x
+c)^3-1/d*cos(d*x+c)*a*b-1/d*a*b*ln(csc(d*x+c)-cot(d*x+c))-1/2/d*b^2/sin(d*
x+c)^2
```

Maxima [A] time = 0.987193, size = 97, normalized size = 1.05

$$-\frac{\left(a^2-ab\right)\log\left(\cos\left(dx+c\right)+1\right)+\left(a^2+ab\right)\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\,ab\cos(dx+c)+a^2+b^2}{\cos(dx+c)^2-1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*((a^2 - a*b)*log(cos(d*x + c) + 1) + (a^2 + a*b)*log(cos(d*x + c) - 1)
 - (2*a*b*cos(d*x + c) + a^2 + b^2)/(cos(d*x + c)^2 - 1))/d
```

Fricas [A] time = 0.790496, size = 275, normalized size = 2.99

$$\frac{2\,ab\cos\left(dx+c\right)+a^2+b^2-\left(\left(a^2-ab\right)\cos\left(dx+c\right)^2-a^2+ab\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(\left(a^2+ab\right)\cos\left(dx+c\right)^2-a^2-ab\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*cos(d*x + c) + a^2 + b^2 - ((a^2 - a*b)*cos(d*x + c)^2 - a^2 + a
*b)*log(1/2*cos(d*x + c) + 1/2) - ((a^2 + a*b)*cos(d*x + c)^2 - a^2 - a*b)*
log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec\left(c+dx\right)\right)^2\cot^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cot(c + d*x)**3, x)
```

Giac [B] time = 1.3528, size = 282, normalized size = 3.07

$$\frac{8\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-4\left(a^2+ab\right)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)+\frac{\left(a^2+2\,ab+b^2\right.}{}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) + a^2*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*(a^2 + a*b)*log(abs(-co
s(d*x + c) + 1)/abs(cos(d*x + c) + 1)) + (a^2 + 2*a*b + b^2 + 4*a^2*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 4*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/d
```

3.278 $\int \cot^5(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=126

$$-\frac{\cot^4(c+dx)\left(a^2+2ab\sec(c+dx)+b^2\right)}{4d}+\frac{a^2\log(\cos(c+dx))}{d}+\frac{a(4a+3b)\log(1-\sec(c+dx))}{8d}+\frac{a(4a-3b)\log(\sec}{8d}$$

```
[Out] (a^2*Log[Cos[c + d*x]])/d + (a*(4*a + 3*b)*Log[1 - Sec[c + d*x]])/(8*d) + (
a*(4*a - 3*b)*Log[1 + Sec[c + d*x]])/(8*d) + (a*Cot[c + d*x]^2*(2*a + 3*b*S
ec[c + d*x]))/(4*d) - (Cot[c + d*x]^4*(a^2 + b^2 + 2*a*b*Sec[c + d*x]))/(4*
d)
```

Rubi [A] time = 0.159482, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3885, 1805, 823, 801}

$$-\frac{\cot^4(c+dx)\left(a^2+2ab\sec(c+dx)+b^2\right)}{4d}+\frac{a^2\log(\cos(c+dx))}{d}+\frac{a(4a+3b)\log(1-\sec(c+dx))}{8d}+\frac{a(4a-3b)\log(\sec}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Sec[c + d*x])^2,x]
```


```
[Out] (a^2*Log[Cos[c + d*x]])/d + (a*(4*a + 3*b)*Log[1 - Sec[c + d*x]])/(8*d) + (
a*(4*a - 3*b)*Log[1 + Sec[c + d*x]])/(8*d) + (a*Cot[c + d*x]^2*(2*a + 3*b*S
ec[c + d*x]))/(4*d) - (Cot[c + d*x]^4*(a^2 + b^2 + 2*a*b*Sec[c + d*x]))/(4*
d)
```


Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
```

```
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \cot^5(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{b^6\,\text{Subst}\left(\int \frac{(a+x)^2}{x\left(b^2-x^2\right)^3}\,dx,x,b\sec(c+dx)\right)}{d}$$

$$= -\frac{\cot^4(c+dx)\left(a^2+b^2+2ab\sec(c+dx)\right)}{4d}+\frac{b^4\,\text{Subst}\left(\int \frac{-4a^2-6ax}{x\left(b^2-x^2\right)^2}\,dx,x,b\sec(c\right.}{4d}$$

$$= \frac{a\cot^2(c+dx)(2a+3b\sec(c+dx))}{4d}-\frac{\cot^4(c+dx)\left(a^2+b^2+2ab\sec(c+dx)\right)}{4d}$$

$$= \frac{a\cot^2(c+dx)(2a+3b\sec(c+dx))}{4d}-\frac{\cot^4(c+dx)\left(a^2+b^2+2ab\sec(c+dx)\right)}{4d}$$

$$= \frac{a^2\log(\cos(c+dx))}{d}+\frac{a(4a+3b)\log(1-\sec(c+dx))}{8d}+\frac{a(4a-3b)\log(1+\sec(}{8d}$$

Mathematica [A] time = 3.22511, size = 148, normalized size = 1.17

$$\frac{2\left(7a^2+10ab+3b^2\right)\csc^2\left(\frac{1}{2}(c+dx)\right)+2\left(7a^2-10ab+3b^2\right)\sec^2\left(\frac{1}{2}(c+dx)\right)-(a+b)^2\csc^4\left(\frac{1}{2}(c+dx)\right)-(a-b)^2\sec^4\left(\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(7*a^2 + 10*a*b + 3*b^2)*Csc[(c + d*x)/2]^2 - (a + b)^2*Csc[(c + d*x)/2]
^4 + 16*a*((4*a - 3*b)*Log[Cos[(c + d*x)/2]] + (4*a + 3*b)*Log[Sin[(c + d*x
)/2]]) + 2*(7*a^2 - 10*a*b + 3*b^2)*Sec[(c + d*x)/2]^2 - (a - b)^2*Sec[(c +
 d*x)/2]^4)/(64*d)
```

Maple [A] time = 0.053, size = 169, normalized size = 1.3

$$-\frac{a^2 (\cot (dx+c))^4}{4d}+\frac{a^2 (\cot (dx+c))^2}{2d}+\frac{a^2 \ln (\sin (dx+c))}{d}-\frac{ab (\cos (dx+c))^5}{2d (\sin (dx+c))^4}+\frac{ab (\cos (dx+c))^5}{4d (\sin (dx+c))^2}+\frac{ab (\cos (dx+}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/4/d*a^2*cot(d*x+c)^4+1/2/d*a^2*cot(d*x+c)^2+1/d*a^2*ln(sin(d*x+c))-1/2/d
*a*b/sin(d*x+c)^4*cos(d*x+c)^5+1/4/d*a*b/sin(d*x+c)^2*cos(d*x+c)^5+1/4/d*a*
b*cos(d*x+c)^3+3/4/d*cos(d*x+c)*a*b+3/4/d*a*b*ln(csc(d*x+c)-cot(d*x+c))-1/4
/d*b^2/sin(d*x+c)^4*cos(d*x+c)^4
```

Maxima [A] time = 1.06366, size = 165, normalized size = 1.31

$$\frac{\left(4a^2-3ab\right)\log (\cos (dx+c)+1)+\left(4a^2+3ab\right)\log (\cos (dx+c)-1)-\frac{2\left(5ab\cos(dx+c)^3-3ab\cos(dx+c)+2\left(2a^2+b^2\right)\cos(dx+c)^2-\right.}{\cos(dx+c)^4-2\cos(dx+c)^2+1}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*((4*a^2 - 3*a*b)*log(cos(d*x + c) + 1) + (4*a^2 + 3*a*b)*log(cos(d*x +
c) - 1) - 2*(5*a*b*cos(d*x + c)^3 - 3*a*b*cos(d*x + c) + 2*(2*a^2 + b^2)*co
s(d*x + c)^2 - 3*a^2 - b^2)/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1))/d
```

Fricas [A] time = 0.989525, size = 504, normalized size = 4.

$$-\frac{10\,ab\cos(dx+c)^3-6\,ab\cos(dx+c)+4\left(2\,a^2+b^2\right)\cos(dx+c)^2-6\,a^2-2\,b^2-\left(\left(4\,a^2-3\,ab\right)\cos(dx+c)^4-2\left(4\,a^2-\right.\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(10*a*b*cos(d*x + c)^3 - 6*a*b*cos(d*x + c) + 4*(2*a^2 + b^2)*cos(d*x
+ c)^2 - 6*a^2 - 2*b^2 - ((4*a^2 - 3*a*b)*cos(d*x + c)^4 - 2*(4*a^2 - 3*a*b
)*cos(d*x + c)^2 + 4*a^2 - 3*a*b)*log(1/2*cos(d*x + c) + 1/2) - ((4*a^2 + 3
*a*b)*cos(d*x + c)^4 - 2*(4*a^2 + 3*a*b)*cos(d*x + c)^2 + 4*a^2 + 3*a*b)*lo
g(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.45574, size = 486, normalized size = 3.86

$$-\frac{64\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)+\frac{12\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{16\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4\,b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{2\,ab(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/64*(64*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) + 12*a^2
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 16*a*b*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) + 4*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a^2*(cos(d*x + c
```

```
) - 1)^2/(cos(d*x + c) + 1)^2 - 2*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2 + b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 8*(4*a^2 + 3*a*b)*lo
g(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) + (a^2 + 2*a*b + b^2 + 12*a
^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 16*a*b*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 4*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 48*a^2*(cos(d*
x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 36*a*b*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2)/d
```

3.279 $\int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx$

Optimal. Leaf size=157

$$\frac{a^2\tan^5(c+dx)}{5d}-\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}-a^2x-\frac{5ab\tanh^{-1}(\sin(c+dx))}{8d}+\frac{ab\tan^5(c+dx)\sec(c+dx)}{3d}-\frac{5a}{}$$

```
[Out] -(a^2*x) - (5*a*b*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*Tan[c + d*x])/d + (5*
a*b*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (a^2*Tan[c + d*x]^3)/(3*d) - (5*a*b*
Sec[c + d*x]*Tan[c + d*x]^3)/(12*d) + (a^2*Tan[c + d*x]^5)/(5*d) + (a*b*Sec
[c + d*x]*Tan[c + d*x]^5)/(3*d) + (b^2*Tan[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.197791, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3886, 3473, 8, 2611, 3770, 2607, 30}

$$\frac{a^2\tan^5(c+dx)}{5d}-\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}-a^2x-\frac{5ab\tanh^{-1}(\sin(c+dx))}{8d}+\frac{ab\tan^5(c+dx)\sec(c+dx)}{3d}-\frac{5a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^6,x]
```

```
[Out] -(a^2*x) - (5*a*b*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*Tan[c + d*x])/d + (5*
a*b*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (a^2*Tan[c + d*x]^3)/(3*d) - (5*a*b*
Sec[c + d*x]*Tan[c + d*x]^3)/(12*d) + (a^2*Tan[c + d*x]^5)/(5*d) + (a*b*Sec
[c + d*x]*Tan[c + d*x]^5)/(3*d) + (b^2*Tan[c + d*x]^7)/(7*d)
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx = \int \left(a^2\tan^6(c+dx)+2ab\sec(c+dx)\tan^6(c+dx)+b^2\sec^2(c+dx)\tan^6(c+\right.$$

$$= a^2\int \tan^6(c+dx)\,dx+(2ab)\int \sec(c+dx)\tan^6(c+dx)\,dx+b^2\int \sec^2(c+d$$

$$= \frac{a^2\tan^5(c+dx)}{5d}+\frac{ab\sec(c+dx)\tan^5(c+dx)}{3d}-a^2\int \tan^4(c+dx)\,dx-\frac{1}{3}(5a$$

$$= -\frac{a^2\tan^3(c+dx)}{3d}-\frac{5ab\sec(c+dx)\tan^3(c+dx)}{12d}+\frac{a^2\tan^5(c+dx)}{5d}+\frac{ab\sec(}{}$$

$$= \frac{a^2\tan(c+dx)}{d}+\frac{5ab\sec(c+dx)\tan(c+dx)}{8d}-\frac{a^2\tan^3(c+dx)}{3d}-\frac{5ab\sec(c+}{}$$

$$= -a^2x-\frac{5ab\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\tan(c+dx)}{d}+\frac{5ab\sec(c+dx)\tan(c+d}{8d}$$

Mathematica [A] time = 1.35533, size = 293, normalized size = 1.87

$$\frac{-2100\sec^6(c+dx)\left(7a^2(c+dx)-\left(a^2+b^2\right)\tan(c+dx)\right)+\sec^7(c+dx)\left(-\left(-3444a^2\sin(3(c+dx))-1988a^2\sin(5(c+d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^6,x]
```

```
[Out] (16800*a*b*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]]) - Sec[c + d*x]^7*(8820*a^2*(c + d*x)*Cos[3*(c + d*x)]
 + 2940*a^2*(c + d*x)*Cos[5*(c + d*x)] + 420*a^2*c*Cos[7*(c + d*x)] + 420*a
^2*d*x*Cos[7*(c + d*x)] - 3444*a^2*Sin[3*(c + d*x)] + 1260*b^2*Sin[3*(c + d
*x)] - 980*a*b*Sin[4*(c + d*x)] - 1988*a^2*Sin[5*(c + d*x)] - 420*b^2*Sin[5
*(c + d*x)] - 1155*a*b*Sin[6*(c + d*x)] - 644*a^2*Sin[7*(c + d*x)] + 60*b^2
*Sin[7*(c + d*x)]) + 5950*a*b*Sec[c + d*x]^5*Tan[c + d*x] - 2100*Sec[c + d*
x]^6*(7*a^2*(c + d*x) - (a^2 + b^2)*Tan[c + d*x]))/(26880*d)
```

Maple [A] time = 0.051, size = 219, normalized size = 1.4

$$\frac{a^2(\tan(dx+c))^5}{5d}-\frac{a^2(\tan(dx+c))^3}{3d}+\frac{a^2\tan(dx+c)}{d}-a^2x-\frac{a^2c}{d}+\frac{ab(\sin(dx+c))^7}{3d(\cos(dx+c))^6}-\frac{ab(\sin(dx+c))^7}{12d(\cos(dx+c))^4}+\frac{ab(\mathrm{si}}{8d(\mathrm{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^6,x)
```

```
[Out] 1/5*a^2*tan(d*x+c)^5/d-1/3*a^2*tan(d*x+c)^3/d+a^2*tan(d*x+c)/d-a^2*x-1/d*a^
2*c+1/3/d*a*b*sin(d*x+c)^7/cos(d*x+c)^6-1/12/d*a*b*sin(d*x+c)^7/cos(d*x+c)^
4+1/8/d*a*b*sin(d*x+c)^7/cos(d*x+c)^2+1/8/d*a*b*sin(d*x+c)^5+5/24/d*a*b*sin
(d*x+c)^3+5/8/d*a*b*sin(d*x+c)-5/8/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/7/d*b^
2*sin(d*x+c)^7/cos(d*x+c)^7
```

Maxima [A] time = 1.46323, size = 203, normalized size = 1.29

$$\frac{240\,b^2\tan(dx+c)^7+112\left(3\tan(dx+c)^5-5\tan(dx+c)^3-15\,dx-15\,c+15\tan(dx+c)\right)a^2-35\,ab\left(\frac{2\left(33\sin(dx+c)^5-40\right.}{\sin(dx+c)^6-3\sin}\right.}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/1680*(240*b^2*tan(d*x + c)^7 + 112*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 -
 15*d*x - 15*c + 15*tan(d*x + c))*a^2 - 35*a*b*(2*(33*sin(d*x + c)^5 - 40*s
in(d*x + c)^3 + 15*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin
(d*x + c)^2 - 1) + 15*log(sin(d*x + c) + 1) - 15*log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 0.834162, size = 494, normalized size = 3.15

$$-\frac{1680\,a^2dx\cos(dx+c)^7 + 525\,ab\cos(dx+c)^7\log(\sin(dx+c)+1) - 525\,ab\cos(dx+c)^7\log(-\sin(dx+c)+1) - 2(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/1680*(1680*a^2*d*x*cos(d*x + c)^7 + 525*a*b*cos(d*x + c)^7*log(sin(d*x +
 c) + 1) - 525*a*b*cos(d*x + c)^7*log(-sin(d*x + c) + 1) - 2*(1155*a*b*cos(
d*x + c)^5 + 8*(161*a^2 - 15*b^2)*cos(d*x + c)^6 - 910*a*b*cos(d*x + c)^3 -
 8*(77*a^2 - 45*b^2)*cos(d*x + c)^4 + 280*a*b*cos(d*x + c) + 24*(7*a^2 - 15
*b^2)*cos(d*x + c)^2 + 120*b^2)*sin(d*x + c))/(d*cos(d*x + c)^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sec(c+dx))^2 \tan^6(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**6,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*tan(c + d*x)**6, x)
```

Giac [A] time = 5.02981, size = 381, normalized size = 2.43

$$-\frac{840\,(dx+c)a^2 + 525\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 525\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(840\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13} - 525\,ab}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] -1/840*(840*(d*x + c)*a^2 + 525*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 52
5*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(840*a^2*tan(1/2*d*x + 1/2*c)^
13 - 525*a*b*tan(1/2*d*x + 1/2*c)^13 - 6160*a^2*tan(1/2*d*x + 1/2*c)^11 + 3
500*a*b*tan(1/2*d*x + 1/2*c)^11 + 19768*a^2*tan(1/2*d*x + 1/2*c)^9 - 9905*a
*b*tan(1/2*d*x + 1/2*c)^9 - 28896*a^2*tan(1/2*d*x + 1/2*c)^7 + 7680*b^2*tan
(1/2*d*x + 1/2*c)^7 + 19768*a^2*tan(1/2*d*x + 1/2*c)^5 + 9905*a*b*tan(1/2*d
*x + 1/2*c)^5 - 6160*a^2*tan(1/2*d*x + 1/2*c)^3 - 3500*a*b*tan(1/2*d*x + 1/
2*c)^3 + 840*a^2*tan(1/2*d*x + 1/2*c) + 525*a*b*tan(1/2*d*x + 1/2*c))/(tan(
1/2*d*x + 1/2*c)^2 - 1)^7)/d
```

3.280 $\int (a+b\sec(c+dx))^2\tan^4(c+dx)\,dx$

Optimal. Leaf size=116

$$\frac{a^2\tan^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+a^2x+\frac{3ab\tanh^{-1}(\sin(c+dx))}{4d}+\frac{ab\tan^3(c+dx)\sec(c+dx)}{2d}-\frac{3ab\tan(c+dx)\mathrm{sec}}{4d}$$

```
[Out] a^2*x + (3*a*b*ArcTanh[Sin[c + d*x]])/(4*d) - (a^2*Tan[c + d*x])/d - (3*a*b
*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (a^2*Tan[c + d*x]^3)/(3*d) + (a*b*Sec[c
 + d*x]*Tan[c + d*x]^3)/(2*d) + (b^2*Tan[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.152166, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3886, 3473, 8, 2611, 3770, 2607, 30}

$$\frac{a^2\tan^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+a^2x+\frac{3ab\tanh^{-1}(\sin(c+dx))}{4d}+\frac{ab\tan^3(c+dx)\sec(c+dx)}{2d}-\frac{3ab\tan(c+dx)\mathrm{sec}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] a^2*x + (3*a*b*ArcTanh[Sin[c + d*x]])/(4*d) - (a^2*Tan[c + d*x])/d - (3*a*b
*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (a^2*Tan[c + d*x]^3)/(3*d) + (a*b*Sec[c
 + d*x]*Tan[c + d*x]^3)/(2*d) + (b^2*Tan[c + d*x]^5)/(5*d)
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\tan^4(c+dx)\,dx = \int \left(a^2\tan^4(c+dx)+2ab\sec(c+dx)\tan^4(c+dx)+b^2\sec^2(c+dx)\tan^4(c+d$$

$$= a^2\int \tan^4(c+dx)\,dx+(2ab)\int \sec(c+dx)\tan^4(c+dx)\,dx+b^2\int \sec^2(c+dx$$

$$= \frac{a^2\tan^3(c+dx)}{3d}+\frac{ab\sec(c+dx)\tan^3(c+dx)}{2d}-a^2\int \tan^2(c+dx)\,dx-\frac{1}{2}(3ab)$$

$$= -\frac{a^2\tan(c+dx)}{d}-\frac{3ab\sec(c+dx)\tan(c+dx)}{4d}+\frac{a^2\tan^3(c+dx)}{3d}+\frac{ab\sec(c+$$

$$= a^2x+\frac{3ab\tanh^{-1}(\sin(c+dx))}{4d}-\frac{a^2\tan(c+dx)}{d}-\frac{3ab\sec(c+dx)\tan(c+dx)}{4d}$$

Mathematica [B] time = 0.938164, size = 355, normalized size = 3.06

$$\frac{\sec^5(c+dx)\left(-80a^2\sin(c+dx)-160a^2\sin(3(c+dx))-80a^2\sin(5(c+dx))+60a^2c\cos(5(c+dx))+60a^2dx\cos(5(c+$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] (Sec[c + d*x]^5*(60*a^2*c*Cos[5*(c + d*x)] + 60*a^2*d*x*Cos[5*(c + d*x)] -
45*a*b*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 45*a*b*C
os[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 150*a*Cos[c + d*
x]*(4*a*(c + d*x) - 3*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*b*Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 75*a*Cos[3*(c + d*x)]*(4*a*(c + d*x
) - 3*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*b*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]]) - 80*a^2*Sin[c + d*x] + 120*b^2*Sin[c + d*x] - 60*a*b
*Sin[2*(c + d*x)] - 160*a^2*Sin[3*(c + d*x)] - 60*b^2*Sin[3*(c + d*x)] - 15
0*a*b*Sin[4*(c + d*x)] - 80*a^2*Sin[5*(c + d*x)] + 12*b^2*Sin[5*(c + d*x)])
)/(960*d)
```

Maple [A] time = 0.049, size = 164, normalized size = 1.4

$$\frac{a^2 (\tan(dx+c))^3}{3d} - \frac{a^2 \tan(dx+c)}{d} + a^2x + \frac{a^2c}{d} + \frac{ab (\sin(dx+c))^5}{2d (\cos(dx+c))^4} - \frac{ab (\sin(dx+c))^5}{4d (\cos(dx+c))^2} - \frac{ab (\sin(dx+c))^3}{4d} - \frac{3\,ab\,\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^4,x)
```

```
[Out] 1/3*a^2*tan(d*x+c)^3/d-a^2*tan(d*x+c)/d+a^2*x+1/d*a^2*c+1/2/d*a*b*sin(d*x+c
)^5/cos(d*x+c)^4-1/4/d*a*b*sin(d*x+c)^5/cos(d*x+c)^2-1/4/d*a*b*sin(d*x+c)^3
-3/4/d*a*b*sin(d*x+c)+3/4/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/5/d*b^2*sin(d*x
+c)^5/cos(d*x+c)^5
```

Maxima [A] time = 1.53085, size = 159, normalized size = 1.37

$$\frac{24\,b^2 \tan(dx+c)^5 + 40\left(\tan(dx+c)^3 + 3\,dx + 3\,c - 3\,\tan(dx+c)\right)a^2 + 15\,ab\left(\frac{2\left(5\,\sin(dx+c)^3 - 3\,\sin(dx+c)\right)}{\sin(dx+c)^4 - 2\,\sin(dx+c)^2 + 1} + 3\,\log(\sin(dx\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/120*(24*b^2*tan(d*x + c)^5 + 40*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x
 + c))*a^2 + 15*a*b*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4
- 2*sin(d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)
))/d
```

Fricas [A] time = 0.808717, size = 393, normalized size = 3.39

$$\frac{120\,a^2dx\cos(dx+c)^5 + 45\,ab\cos(dx+c)^5\log(\sin(dx+c)+1) - 45\,ab\cos(dx+c)^5\log(-\sin(dx+c)+1) - 2\left(75\,ab\,c\right.}{120\,d\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/120*(120*a^2*d*x*cos(d*x + c)^5 + 45*a*b*cos(d*x + c)^5*log(sin(d*x + c)
+ 1) - 45*a*b*cos(d*x + c)^5*log(-sin(d*x + c) + 1) - 2*(75*a*b*cos(d*x + c
)^3 + 4*(20*a^2 - 3*b^2)*cos(d*x + c)^4 - 30*a*b*cos(d*x + c) - 4*(5*a^2 -
6*b^2)*cos(d*x + c)^2 - 12*b^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sec(c+dx))^2\tan^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*tan(c + d*x)**4, x)
```

Giac [B] time = 2.33104, size = 297, normalized size = 2.56

$$60\,(dx+c)a^2 + 45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(60\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 45\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/60*(60*(d*x + c)*a^2 + 45*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*a*b
*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(60*a^2*tan(1/2*d*x + 1/2*c)^9 - 45
*a*b*tan(1/2*d*x + 1/2*c)^9 - 320*a^2*tan(1/2*d*x + 1/2*c)^7 + 210*a*b*tan(
1/2*d*x + 1/2*c)^7 + 520*a^2*tan(1/2*d*x + 1/2*c)^5 - 192*b^2*tan(1/2*d*x +
 1/2*c)^5 - 320*a^2*tan(1/2*d*x + 1/2*c)^3 - 210*a*b*tan(1/2*d*x + 1/2*c)^3
 + 60*a^2*tan(1/2*d*x + 1/2*c) + 45*a*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
+ 1/2*c)^2 - 1)^5)/d
```

3.281 $\int (a+b\sec(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=70

$$\frac{a^2\tan(c+dx)}{d}-a^2x-\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{d}+\frac{b^2\tan^3(c+dx)}{3d}$$

```
[Out] -(a^2*x) - (a*b*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d + (a*b*Sec[
c + d*x]*Tan[c + d*x])/d + (b^2*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.114441, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3886, 3473, 8, 2611, 3770, 2607, 30}

$$\frac{a^2\tan(c+dx)}{d}-a^2x-\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{d}+\frac{b^2\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^2,x]
```


```
[Out] -(a^2*x) - (a*b*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d + (a*b*Sec[
c + d*x]*Tan[c + d*x])/d + (b^2*Tan[c + d*x]^3)/(3*d)
```


Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^2\tan^2(c+dx)\,dx &= \int \left(a^2\tan^2(c+dx)+2ab\sec(c+dx)\tan^2(c+dx)+b^2\sec^2(c+dx)\tan^2(c+\right.\\ &= a^2\int \tan^2(c+dx)\,dx+(2ab)\int \sec(c+dx)\tan^2(c+dx)\,dx+b^2\int \sec^2(c+d\\ &= \frac{a^2\tan(c+dx)}{d}+\frac{ab\sec(c+dx)\tan(c+dx)}{d}-a^2\int 1\,dx-(ab)\int \sec(c+dx)\\ &= -a^2x-\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{ab\sec(c+dx)\tan(c+dx)}{d}\end{aligned}$$

Mathematica [B] time = 1.16121, size = 201, normalized size = 2.87

$$\frac{\sec^3(c+dx)\left(2\sin(c+dx)\left(\left(3a^2-b^2\right)\cos(2(c+dx))+3a^2+6ab\cos(c+dx)+b^2\right)-9a\cos(c+dx)\left(a(c+dx)-b\log\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (Sec[c + d*x]^3*(-9*a*Cos[c + d*x]*(a*(c + d*x) - b*Log[Cos[(c + d*x)/2] -
Sin[(c + d*x)/2]] + b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 3*a*Cos[3
*(c + d*x)]*(a*(c + d*x) - b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + b*L
og[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 2*(3*a^2 + b^2 + 6*a*b*Cos[c + d
*x] + (3*a^2 - b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(12*d)
```

Maple [A] time = 0.04, size = 109, normalized size = 1.6

$$-a^2x+\frac{a^2\tan(dx+c)}{d}-\frac{a^2c}{d}+\frac{ab(\sin(dx+c))^3}{d(\cos(dx+c))^2}+\frac{ab\sin(dx+c)}{d}-\frac{ab\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b^2(\sin(dx+}{3d(\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^2,x)
```

```
[Out] -a^2*x+a^2*tan(d*x+c)/d-1/d*a^2*c+1/d*a*b*sin(d*x+c)^3/cos(d*x+c)^2+1/d*a*b
*sin(d*x+c)-1/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/3/d*b^2*sin(d*x+c)^3/cos(d*
x+c)^3
```

Maxima [A] time = 1.48126, size = 111, normalized size = 1.59

$$\frac{2b^2\tan(dx+c)^3-6(dx+c-\tan(dx+c))a^2-3ab\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^2*tan(d*x + c)^3 - 6*(d*x + c - tan(d*x + c))*a^2 - 3*a*b*(2*sin(d
*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
)))/d
```

Fricas [A] time = 0.942503, size = 294, normalized size = 4.2

$$-\frac{6a^2dx\cos(dx+c)^3+3ab\cos(dx+c)^3\log(\sin(dx+c)+1)-3ab\cos(dx+c)^3\log(-\sin(dx+c)+1)-2\left(3ab\cos(\right.}{6d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*a^2*d*x*cos(d*x + c)^3 + 3*a*b*cos(d*x + c)^3*log(sin(d*x + c) + 1)
 - 3*a*b*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(3*a*b*cos(d*x + c) + (3
*a^2 - b^2)*cos(d*x + c)^2 + b^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sec(c + dx))^2 \tan^2(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*tan(c + d*x)**2, x)
```

Giac [B] time = 1.66288, size = 213, normalized size = 3.04

$$-\frac{3(dx+c)a^2 + 3ab\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 3ab\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) + \frac{2\left(3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 \cdots\right.}{}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)*a^2 + 3*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*a*b*
tan(1/2*d*x + 1/2*c)^5 - 6*a^2*tan(1/2*d*x + 1/2*c)^3 + 4*b^2*tan(1/2*d*x +
 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) + 3*a*b*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.282 $\int \cot^2(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=48

$$-\frac{a^2\cot(c+dx)}{d}+a^2(-x)-\frac{2ab\csc(c+dx)}{d}-\frac{b^2\cot(c+dx)}{d}$$

```
[Out] -(a^2*x) - (a^2*Cot[c + d*x])/d - (b^2*Cot[c + d*x])/d - (2*a*b*Csc[c + d*x
])/d
```

Rubi [A] time = 0.0746202, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3886, 3473, 8, 2606, 3767}

$$-\frac{a^2\cot(c+dx)}{d}+a^2(-x)-\frac{2ab\csc(c+dx)}{d}-\frac{b^2\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*x) - (a^2*Cot[c + d*x])/d - (b^2*Cot[c + d*x])/d - (2*a*b*Csc[c + d*x
])/d
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \cot^2(c+dx)(a+b\sec(c+dx))^2\,dx &= \int \left(a^2\cot^2(c+dx)+2ab\cot(c+dx)\csc(c+dx)+b^2\csc^2(c+dx)\right)dx \\
&= a^2\int \cot^2(c+dx)\,dx+(2ab)\int \cot(c+dx)\csc(c+dx)\,dx+b^2\int \csc^2(c+dx \\
&= -\frac{a^2\cot(c+dx)}{d}-a^2\int 1\,dx-\frac{(2ab)\,\mathrm{Subst}(\int 1\,dx,x,\csc(c+dx))}{d}-\frac{b^2\,\mathrm{Subst}(}{} \\
&= -a^2x-\frac{a^2\cot(c+dx)}{d}-\frac{b^2\cot(c+dx)}{d}-\frac{2ab\csc(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.366417, size = 39, normalized size = 0.81

$$-\frac{\left(a^2+b^2\right)\cot(c+dx)+a(a(c+dx)+2b\csc(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((a^2 + b^2)*Cot[c + d*x] + a*(a*(c + d*x) + 2*b*Csc[c + d*x]))/d)
```

Maple [A] time = 0.038, size = 49, normalized size = 1.

$$\frac{1}{d}\left(a^2\left(-\cot(dx+c)-dx-c\right)-2\,\frac{ab}{\sin(dx+c)}-b^2\cot(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-cot(d*x+c)-d*x-c)-2*a*b/sin(d*x+c)-b^2*cot(d*x+c))
```

Maxima [A] time = 1.46353, size = 63, normalized size = 1.31

$$-\frac{\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2+\frac{2\,ab}{\sin(dx+c)}+\frac{b^2}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((d*x + c + 1/tan(d*x + c))*a^2 + 2*a*b/sin(d*x + c) + b^2/tan(d*x + c))/d
```

Fricas [A] time = 0.852383, size = 104, normalized size = 2.17

$$-\frac{a^2dx\sin\left(dx+c\right)+2\,ab+\left(a^2+b^2\right)\cos\left(dx+c\right)}{d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*d*x*sin(d*x + c) + 2*a*b + (a^2 + b^2)*cos(d*x + c))/(d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec\left(c+dx\right)\right)^2\cot^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cot(c + d*x)**2, x)
```

Giac [A] time = 1.36982, size = 108, normalized size = 2.25

$$-\frac{2\,(dx+c)a^2-a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{a^2+2\,ab+b^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)*a^2 - a^2*tan(1/2*d*x + 1/2*c) + 2*a*b*tan(1/2*d*x + 1/2*
c) - b^2*tan(1/2*d*x + 1/2*c) + (a^2 + 2*a*b + b^2)/tan(1/2*d*x + 1/2*c))/d
```

3.283 $\int \cot^4(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=85

$$-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+a^2x-\frac{2ab\csc^3(c+dx)}{3d}+\frac{2ab\csc(c+dx)}{d}-\frac{b^2\cot^3(c+dx)}{3d}$$

```
[Out] a^2*x + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^3)/(3*d) - (b^2*Cot[c + d*
x]^3)/(3*d) + (2*a*b*Csc[c + d*x])/d - (2*a*b*Csc[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.115351, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3886, 3473, 8, 2606, 2607, 30}

$$-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+a^2x-\frac{2ab\csc^3(c+dx)}{3d}+\frac{2ab\csc(c+dx)}{d}-\frac{b^2\cot^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```


```
[Out] a^2*x + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^3)/(3*d) - (b^2*Cot[c + d*
x]^3)/(3*d) + (2*a*b*Csc[c + d*x])/d - (2*a*b*Csc[c + d*x]^3)/(3*d)
```


Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^2\,dx = \int \left(a^2\cot^4(c+dx)+2ab\cot^3(c+dx)\csc(c+dx)+b^2\cot^2(c+dx)\csc^2(c+d\right.$$

$$= a^2\int \cot^4(c+dx)\,dx+(2ab)\int \cot^3(c+dx)\csc(c+dx)\,dx+b^2\int \cot^2(c+d$$

$$= -\frac{a^2\cot^3(c+dx)}{3d}-a^2\int \cot^2(c+dx)\,dx-\frac{(2ab)\operatorname{Subst}\left(\int\left(-1+x^2\right)dx,x,\csc\right.}{d}$$

$$= \frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{b^2\cot^3(c+dx)}{3d}+\frac{2ab\csc(c+dx)}{d}-\frac{2ab\operatorname{cs}}{}$$

$$= a^2x+\frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{b^2\cot^3(c+dx)}{3d}+\frac{2ab\csc(c+dx)}{d}-$$

Mathematica [A] time = 0.460439, size = 122, normalized size = 1.44

$$-\frac{\csc^3(c+dx)\left(-9a^2c\sin(c+dx)-9a^2dx\sin(c+dx)+3a^2c\sin(3(c+dx))+3a^2dx\sin(3(c+dx))+4a^2\cos(3(c+dx))\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]^3*(-4*a*b + 3*b^2*Cos[c + d*x] + 12*a*b*Cos[2*(c + d*x)] + 4
*a^2*Cos[3*(c + d*x)] + b^2*Cos[3*(c + d*x)] - 9*a^2*c*Sin[c + d*x] - 9*a^2
```

```
*d*x*Sin[c + d*x] + 3*a^2*c*Sin[3*(c + d*x)] + 3*a^2*d*x*Sin[3*(c + d*x)]))
/(12*d)
```

Maple [A] time = 0.047, size = 111, normalized size = 1.3

$$\frac{1}{d}\left(a^2\left(-\frac{(\cot(dx+c))^3}{3}+\cot(dx+c)+dx+c\right)+2ab\left(-1/3\,\frac{(\cos(dx+c))^4}{(\sin(dx+c))^3}+1/3\,\frac{(\cos(dx+c))^4}{\sin(dx+c)}+1/3\,\left(2+(\cos(dx+c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+2*a*b*(-1/3/sin(d*x+c)^3*cos(
d*x+c)^4+1/3/sin(d*x+c)*cos(d*x+c)^4+1/3*(2+cos(d*x+c)^2)*sin(d*x+c))-1/3*b
^2/sin(d*x+c)^3*cos(d*x+c)^3)
```

Maxima [A] time = 1.47951, size = 103, normalized size = 1.21

$$\frac{\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2+\frac{2\left(3\,\sin(dx+c)^2-1\right)ab}{\sin(dx+c)^3}-\frac{b^2}{\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 + 2*(3*sin(d
*x + c)^2 - 1)*a*b/sin(d*x + c)^3 - b^2/tan(d*x + c)^3)/d
```

Fricas [A] time = 0.831908, size = 240, normalized size = 2.82

$$\frac{6\,ab\cos(dx+c)^2+\left(4\,a^2+b^2\right)\cos(dx+c)^3-3\,a^2\cos(dx+c)-4\,ab+3\left(a^2dx\cos(dx+c)^2-a^2dx\right)\sin(dx+c)}{3\left(d\cos(dx+c)^2-d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(6*a*b*cos(d*x + c)^2 + (4*a^2 + b^2)*cos(d*x + c)^3 - 3*a^2*cos(d*x +
c) - 4*a*b + 3*(a^2*d*x*cos(d*x + c)^2 - a^2*d*x)*sin(d*x + c))/((d*cos(d*x
 + c)^2 - d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sec(c + dx))^2 \cot^4(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cot(c + d*x)**4, x)
```

Giac [B] time = 1.39209, size = 238, normalized size = 2.8

$$\frac{a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - 2\,ab\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + b^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + 24\,(dx + c)a^2 - 15\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 18\,ab\tan}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 - 2*a*b*tan(1/2*d*x + 1/2*c)^3 + b^2*tan(1
/2*d*x + 1/2*c)^3 + 24*(d*x + c)*a^2 - 15*a^2*tan(1/2*d*x + 1/2*c) + 18*a*b
*tan(1/2*d*x + 1/2*c) - 3*b^2*tan(1/2*d*x + 1/2*c) + (15*a^2*tan(1/2*d*x +
1/2*c)^2 + 18*a*b*tan(1/2*d*x + 1/2*c)^2 + 3*b^2*tan(1/2*d*x + 1/2*c)^2 - a
^2 - 2*a*b - b^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.284 $\int \cot^6(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=122

$$-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-a^2x-\frac{2ab\csc^5(c+dx)}{5d}+\frac{4ab\csc^3(c+dx)}{3d}-\frac{2ab\csc(c+dx)}{d}-\frac{b^2}{}$$

```
[Out] -(a^2*x) - (a^2*Cot[c + d*x])/d + (a^2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c +
 d*x]^5)/(5*d) - (b^2*Cot[c + d*x]^5)/(5*d) - (2*a*b*Csc[c + d*x])/d + (4*a
*b*Csc[c + d*x]^3)/(3*d) - (2*a*b*Csc[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.134744, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3886, 3473, 8, 2606, 194, 2607, 30}

$$-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-a^2x-\frac{2ab\csc^5(c+dx)}{5d}+\frac{4ab\csc^3(c+dx)}{3d}-\frac{2ab\csc(c+dx)}{d}-\frac{b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Sec[c + d*x])^2,x]
```


```
[Out] -(a^2*x) - (a^2*Cot[c + d*x])/d + (a^2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c +
 d*x]^5)/(5*d) - (b^2*Cot[c + d*x]^5)/(5*d) - (2*a*b*Csc[c + d*x])/d + (4*a
*b*Csc[c + d*x]^3)/(3*d) - (2*a*b*Csc[c + d*x]^5)/(5*d)
```


Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \cot^6(c+dx)(a+b\sec(c+dx))^2\,dx = \int \left(a^2\cot^6(c+dx)+2ab\cot^5(c+dx)\csc(c+dx)+b^2\cot^4(c+dx)\csc^2(c+d\right.$$

$$= a^2\int \cot^6(c+dx)\,dx+(2ab)\int \cot^5(c+dx)\csc(c+dx)\,dx+b^2\int \cot^4(c+d$$

$$= -\frac{a^2\cot^5(c+dx)}{5d}-a^2\int \cot^4(c+dx)\,dx-\frac{(2ab)\,\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx,x,\text{cs}\right.}{d}$$

$$= \frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(c+dx)}{5d}-\frac{b^2\cot^5(c+dx)}{5d}+a^2\int \cot^2(c+dx)\,dx-$$

$$= -\frac{a^2\cot(c+dx)}{d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(c+dx)}{5d}-\frac{b^2\cot^5(c+dx)}{5d}-\frac{2ab\,c}{}$$

$$= -a^2x-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(c+dx)}{5d}-\frac{b^2\cot^5(c+dx)}{5d}-$$

Mathematica [A] time = 0.591659, size = 198, normalized size = 1.62

$$-\frac{\csc^5(c+dx)\left(10\left(5a^2+3b^2\right)\cos(c+dx)+150a^2c\sin(c+dx)+150a^2dx\sin(c+dx)-75a^2c\sin(3(c+dx))-75a^2dx\,\text{s}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]^5*(116*a*b + 10*(5*a^2 + 3*b^2)*Cos[c + d*x] - 80*a*b*Cos[2*
(c + d*x)] - 25*a^2*Cos[3*(c + d*x)] + 15*b^2*Cos[3*(c + d*x)] + 60*a*b*Cos
[4*(c + d*x)] + 23*a^2*Cos[5*(c + d*x)] + 3*b^2*Cos[5*(c + d*x)] + 150*a^2*
c*Sin[c + d*x] + 150*a^2*d*x*Sin[c + d*x] - 75*a^2*c*Sin[3*(c + d*x)] - 75*
a^2*d*x*Sin[3*(c + d*x)] + 15*a^2*c*Sin[5*(c + d*x)] + 15*a^2*d*x*Sin[5*(c
+ d*x)]))/(240*d)
```

Maple [A] time = 0.05, size = 154, normalized size = 1.3

$$\frac{1}{d}\left(a^2\left(-\frac{(\cot(dx+c))^5}{5}+\frac{(\cot(dx+c))^3}{3}-\cot(dx+c)-dx-c\right)+2\,ab\left(-1/5\,\frac{(\cos(dx+c))^6}{(\sin(dx+c))^5}+1/15\,\frac{(\cos(dx+c))^6}{(\sin(dx+c))^3}-1/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+2*a*b*(-1/5/
sin(d*x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c)*cos
(d*x+c)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-1/5*b^2/sin(d
*x+c)^5*cos(d*x+c)^5)
```

Maxima [A] time = 1.499, size = 130, normalized size = 1.07

$$-\frac{\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^2+\frac{2\left(15\,\sin(dx+c)^4-10\,\sin(dx+c)^2+3\right)ab}{\sin(dx+c)^5}+\frac{3\,b^2}{\tan(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/15*((15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x
+ c)^5)*a^2 + 2*(15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 + 3)*a*b/sin(d*x + c
)^5 + 3*b^2/tan(d*x + c)^5)/d
```

Fricas [A] time = 0.781459, size = 382, normalized size = 3.13

$$-\frac{30\,ab\cos\left(dx+c\right)^{4}+\left(23\,a^{2}+3\,b^{2}\right)\cos\left(dx+c\right)^{5}-35\,a^{2}\cos\left(dx+c\right)^{3}-40\,ab\cos\left(dx+c\right)^{2}+15\,a^{2}\cos\left(dx+c\right)+16\,a}{15\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(30*a*b*cos(d*x + c)^4 + (23*a^2 + 3*b^2)*cos(d*x + c)^5 - 35*a^2*cos
(d*x + c)^3 - 40*a*b*cos(d*x + c)^2 + 15*a^2*cos(d*x + c) + 16*a*b + 15*(a^
2*d*x*cos(d*x + c)^4 - 2*a^2*d*x*cos(d*x + c)^2 + a^2*d*x)*sin(d*x + c))/((
d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.4415, size = 369, normalized size = 3.02

$$3\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-6\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-35\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+50\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*a*b*tan(1/2*d*x + 1/2*c)^5 + 3*b^2*
tan(1/2*d*x + 1/2*c)^5 - 35*a^2*tan(1/2*d*x + 1/2*c)^3 + 50*a*b*tan(1/2*d*x
```

```
 + 1/2*c)^3 - 15*b^2*tan(1/2*d*x + 1/2*c)^3 - 480*(d*x + c)*a^2 + 330*a^2*t
an(1/2*d*x + 1/2*c) - 300*a*b*tan(1/2*d*x + 1/2*c) + 30*b^2*tan(1/2*d*x + 1
/2*c) - (330*a^2*tan(1/2*d*x + 1/2*c)^4 + 300*a*b*tan(1/2*d*x + 1/2*c)^4 +
30*b^2*tan(1/2*d*x + 1/2*c)^4 - 35*a^2*tan(1/2*d*x + 1/2*c)^2 - 50*a*b*tan(
1/2*d*x + 1/2*c)^2 - 15*b^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2 + 6*a*b + 3*b^2)
/tan(1/2*d*x + 1/2*c)^5)/d
```

3.285 $\int \cot^8(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=153

$$-\frac{a^2\cot^7(c+dx)}{7d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+a^2x-\frac{2ab\csc^7(c+dx)}{7d}+\frac{6ab\csc^5(c+dx)}{5d}-$$

```
[Out] a^2*x + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^3)/(3*d) + (a^2*Cot[c + d*
x]^5)/(5*d) - (a^2*Cot[c + d*x]^7)/(7*d) - (b^2*Cot[c + d*x]^7)/(7*d) + (2*
a*b*Csc[c + d*x])/d - (2*a*b*Csc[c + d*x]^3)/d + (6*a*b*Csc[c + d*x]^5)/(5*
d) - (2*a*b*Csc[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.147713, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3886, 3473, 8, 2606, 194, 2607, 30}

$$-\frac{a^2\cot^7(c+dx)}{7d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+a^2x-\frac{2ab\csc^7(c+dx)}{7d}+\frac{6ab\csc^5(c+dx)}{5d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8*(a + b*Sec[c + d*x])^2,x]
```



```
[Out] a^2*x + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^3)/(3*d) + (a^2*Cot[c + d*
x]^5)/(5*d) - (a^2*Cot[c + d*x]^7)/(7*d) - (b^2*Cot[c + d*x]^7)/(7*d) + (2*
a*b*Csc[c + d*x])/d - (2*a*b*Csc[c + d*x]^3)/d + (6*a*b*Csc[c + d*x]^5)/(5*
d) - (2*a*b*Csc[c + d*x]^7)/(7*d)
```


Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \cot^8(c+dx)(a+b\sec(c+dx))^2\,dx = \int \left(a^2\cot^8(c+dx)+2ab\cot^7(c+dx)\csc(c+dx)+b^2\cot^6(c+dx)\csc^2(c+d\right.$$
$$= a^2\int \cot^8(c+dx)\,dx+(2ab)\int \cot^7(c+dx)\csc(c+dx)\,dx+b^2\int \cot^6(c+dx$$
$$= -\frac{a^2\cot^7(c+dx)}{7d}-a^2\int \cot^6(c+dx)\,dx-\frac{(2ab)\,\mathrm{Subst}\left(\int\left(-1+x^2\right)^3\,dx,x,\mathrm{cs}\right.}{d}$$
$$= \frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^7(c+dx)}{7d}-\frac{b^2\cot^7(c+dx)}{7d}+a^2\int \cot^4(c+dx)\,dx-$$
$$= -\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^7(c+dx)}{7d}-\frac{b^2\cot^7(c+dx)}{7d}+\frac{2ab}{}$$
$$= \frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^7(c+dx)}{7d}-\frac{b^2\cot}{}$$
$$= a^2x+\frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^7(c+dx)}{7d}-$$

Mathematica [A] time = 0.814682, size = 257, normalized size = 1.68

$$-\frac{\csc^7(c+dx)\left(-3675a^2c\sin(c+dx)-3675a^2dx\sin(c+dx)+2205a^2c\sin(3(c+dx))+2205a^2dx\sin(3(c+dx))-735\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]^7*(-1272*a*b + 525*b^2*Cos[c + d*x] + 3612*a*b*Cos[2*(c + d*
x)] + 1176*a^2*Cos[3*(c + d*x)] + 315*b^2*Cos[3*(c + d*x)] - 840*a*b*Cos[4*
(c + d*x)] - 392*a^2*Cos[5*(c + d*x)] + 105*b^2*Cos[5*(c + d*x)] + 420*a*b*
Cos[6*(c + d*x)] + 176*a^2*Cos[7*(c + d*x)] + 15*b^2*Cos[7*(c + d*x)] - 367
5*a^2*c*Sin[c + d*x] - 3675*a^2*d*x*Sin[c + d*x] + 2205*a^2*c*Sin[3*(c + d*
x)] + 2205*a^2*d*x*Sin[3*(c + d*x)] - 735*a^2*c*Sin[5*(c + d*x)] - 735*a^2*
d*x*Sin[5*(c + d*x)] + 105*a^2*c*Sin[7*(c + d*x)] + 105*a^2*d*x*Sin[7*(c +
d*x)]))/(6720*d)
```

Maple [A] time = 0.055, size = 187, normalized size = 1.2

$$\frac{1}{d}\left(a^2\left(-\frac{(\cot(dx+c))^7}{7}+\frac{(\cot(dx+c))^5}{5}-\frac{(\cot(dx+c))^3}{3}+\cot(dx+c)+dx+c\right)+2\,ab\left(-1/7\,\frac{(\cos(dx+c))^8}{(\sin(dx+c))^7}+1/35\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^8*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/7*cot(d*x+c)^7+1/5*cot(d*x+c)^5-1/3*cot(d*x+c)^3+cot(d*x+c)+d*
x+c)+2*a*b*(-1/7/sin(d*x+c)^7*cos(d*x+c)^8+1/35/sin(d*x+c)^5*cos(d*x+c)^8-1
/35/sin(d*x+c)^3*cos(d*x+c)^8+1/7/sin(d*x+c)*cos(d*x+c)^8+1/7*(16/5+cos(d*x
+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-1/7*b^2/sin(d*x+c)^7*c
os(d*x+c)^7)
```

Maxima [A] time = 1.53239, size = 157, normalized size = 1.03

$$\frac{\left(105\,dx+105\,c+\frac{105\,\tan(dx+c)^6-35\,\tan(dx+c)^4+21\,\tan(dx+c)^2-15}{\tan(dx+c)^7}\right)a^2+\frac{6\left(35\,\sin(dx+c)^6-35\,\sin(dx+c)^4+21\,\sin(dx+c)^2-5\right)ab}{\sin(dx+c)^7}-\frac{15\,b^2}{\tan(dx+c)^7}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*((105*d*x + 105*c + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan(
d*x + c)^2 - 15)/tan(d*x + c)^7)*a^2 + 6*(35*sin(d*x + c)^6 - 35*sin(d*x +
c)^4 + 21*sin(d*x + c)^2 - 5)*a*b/sin(d*x + c)^7 - 15*b^2/tan(d*x + c)^7)/d
```

Fricas [A] time = 0.95232, size = 524, normalized size = 3.42

$$\frac{210\,ab\cos(dx+c)^6+\left(176\,a^2+15\,b^2\right)\cos(dx+c)^7-406\,a^2\cos(dx+c)^5-420\,ab\cos(dx+c)^4+350\,a^2\cos(dx+c)^3+\cdots}{105\left(d\cos(dx+c)^6-3\,d\cos\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(210*a*b*cos(d*x + c)^6 + (176*a^2 + 15*b^2)*cos(d*x + c)^7 - 406*a^2
*cos(d*x + c)^5 - 420*a*b*cos(d*x + c)^4 + 350*a^2*cos(d*x + c)^3 + 336*a*b
*cos(d*x + c)^2 - 105*a^2*cos(d*x + c) - 96*a*b + 105*(a^2*d*x*cos(d*x + c)
^6 - 3*a^2*d*x*cos(d*x + c)^4 + 3*a^2*d*x*cos(d*x + c)^2 - a^2*d*x)*sin(d*x
 + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*si
n(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**8*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29837, size = 494, normalized size = 3.23

$$\frac{15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-30\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+15\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-189\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+294\,ab\tan\left(\frac{1}{2}\,dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/13440*(15*a^2*tan(1/2*d*x + 1/2*c)^7 - 30*a*b*tan(1/2*d*x + 1/2*c)^7 + 15
*b^2*tan(1/2*d*x + 1/2*c)^7 - 189*a^2*tan(1/2*d*x + 1/2*c)^5 + 294*a*b*tan(
1/2*d*x + 1/2*c)^5 - 105*b^2*tan(1/2*d*x + 1/2*c)^5 + 1295*a^2*tan(1/2*d*x
+ 1/2*c)^3 - 1470*a*b*tan(1/2*d*x + 1/2*c)^3 + 315*b^2*tan(1/2*d*x + 1/2*c)
^3 + 13440*(d*x + c)*a^2 - 9765*a^2*tan(1/2*d*x + 1/2*c) + 7350*a*b*tan(1/2
*d*x + 1/2*c) - 525*b^2*tan(1/2*d*x + 1/2*c) + (9765*a^2*tan(1/2*d*x + 1/2*
c)^6 + 7350*a*b*tan(1/2*d*x + 1/2*c)^6 + 525*b^2*tan(1/2*d*x + 1/2*c)^6 - 1
295*a^2*tan(1/2*d*x + 1/2*c)^4 - 1470*a*b*tan(1/2*d*x + 1/2*c)^4 - 315*b^2*
tan(1/2*d*x + 1/2*c)^4 + 189*a^2*tan(1/2*d*x + 1/2*c)^2 + 294*a*b*tan(1/2*d
*x + 1/2*c)^2 + 105*b^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2 - 30*a*b - 15*b^2)/
tan(1/2*d*x + 1/2*c)^7)/d
```

3.286 $\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=250

$$\frac{(a^2-4b^2)\sec^5(c+dx)}{5b^3d}-\frac{a(a^2-4b^2)\sec^4(c+dx)}{4b^4d}+\frac{(-4a^2b^2+a^4+6b^4)\sec^3(c+dx)}{3b^5d}-\frac{a(-4a^2b^2+a^4+6b^4)\sec^2(c+$$

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - ((a^2 - b^2)^4*Log[a + b*Sec[c + d*x]])/(a*b^8
*d) + ((a^6 - 4*a^4*b^2 + 6*a^2*b^4 - 4*b^6)*Sec[c + d*x])/(b^7*d) - (a*(a^
4 - 4*a^2*b^2 + 6*b^4)*Sec[c + d*x]^2)/(2*b^6*d) + ((a^4 - 4*a^2*b^2 + 6*b^
4)*Sec[c + d*x]^3)/(3*b^5*d) - (a*(a^2 - 4*b^2)*Sec[c + d*x]^4)/(4*b^4*d) +
 ((a^2 - 4*b^2)*Sec[c + d*x]^5)/(5*b^3*d) - (a*Sec[c + d*x]^6)/(6*b^2*d) +
Sec[c + d*x]^7/(7*b*d)
```

Rubi [A] time = 0.196866, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 894}

$$\frac{(a^2-4b^2)\sec^5(c+dx)}{5b^3d}-\frac{a(a^2-4b^2)\sec^4(c+dx)}{4b^4d}+\frac{(-4a^2b^2+a^4+6b^4)\sec^3(c+dx)}{3b^5d}-\frac{a(-4a^2b^2+a^4+6b^4)\sec^2(c+$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^9/(a + b*Sec[c + d*x]),x]
```


```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - ((a^2 - b^2)^4*Log[a + b*Sec[c + d*x]])/(a*b^8
*d) + ((a^6 - 4*a^4*b^2 + 6*a^2*b^4 - 4*b^6)*Sec[c + d*x])/(b^7*d) - (a*(a^
4 - 4*a^2*b^2 + 6*b^4)*Sec[c + d*x]^2)/(2*b^6*d) + ((a^4 - 4*a^2*b^2 + 6*b^
4)*Sec[c + d*x]^3)/(3*b^5*d) - (a*(a^2 - 4*b^2)*Sec[c + d*x]^4)/(4*b^4*d) +
 ((a^2 - 4*b^2)*Sec[c + d*x]^5)/(5*b^3*d) - (a*Sec[c + d*x]^6)/(6*b^2*d) +
Sec[c + d*x]^7/(7*b*d)
```


Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^4}{x(a+x)}\,dx, x, b\sec(c+dx)\right)}{b^8 d}$$

$$= \frac{\text{Subst}\left(\int \left(a^6\left(1+\frac{-4a^4b^2+6a^2b^4-4b^6}{a^6}\right)+\frac{b^8}{ax}-a\left(a^4-4a^2b^2+6b^4\right)x+\left(a^4-4a^2b^2+6b^4\right)x^2-a\right.\right.}{b^8 d}$$

$$= -\frac{\log(\cos(c+dx))}{ad}-\frac{\left(a^2-b^2\right)^4\log(a+b\sec(c+dx))}{ab^8d}+\frac{\left(a^6-4a^4b^2+6a^2b^4-4b^6\right)\sec(c+}{b^7 d}$$

Mathematica [B] time = 6.22729, size = 520, normalized size = 2.08

$$\frac{\left(-4a^2b^2+a^4+6b^4\right)\sec^4(c+dx)(a\cos(c+dx)+b)}{3b^5d(a+b\sec(c+dx))}-\frac{a\left(-4a^2b^2+a^4+6b^4\right)\sec^3(c+dx)(a\cos(c+dx)+b)}{2b^6d(a+b\sec(c+dx))}-\frac{\left(2b^2-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^9/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((a^7 - 4*a^5*b^2 + 6*a^3*b^4 - 4*a*b^6)*(b + a*Cos[c + d*x])*Log[Cos[c + d
*x]]*Sec[c + d*x])/(b^8*d*(a + b*Sec[c + d*x])) + ((-a^8 + 4*a^6*b^2 - 6*a^
4*b^4 + 4*a^2*b^6 - b^8)*(b + a*Cos[c + d*x])*Log[b + a*Cos[c + d*x]]*Sec[c
 + d*x])/(a*b^8*d*(a + b*Sec[c + d*x])) - ((-a^2 + 2*b^2)*(a^4 - 2*a^2*b^2
+ 2*b^4)*(b + a*Cos[c + d*x])*Sec[c + d*x]^2)/(b^7*d*(a + b*Sec[c + d*x]))
- (a*(a^4 - 4*a^2*b^2 + 6*b^4)*(b + a*Cos[c + d*x])*Sec[c + d*x]^3)/(2*b^6*
d*(a + b*Sec[c + d*x])) + ((a^4 - 4*a^2*b^2 + 6*b^4)*(b + a*Cos[c + d*x])*S
ec[c + d*x]^4)/(3*b^5*d*(a + b*Sec[c + d*x])) + (a*(-a + 2*b)*(a + 2*b)*(b
+ a*Cos[c + d*x])*Sec[c + d*x]^5)/(4*b^4*d*(a + b*Sec[c + d*x])) - ((-a + 2
*b)*(a + 2*b)*(b + a*Cos[c + d*x])*Sec[c + d*x]^6)/(5*b^3*d*(a + b*Sec[c +
d*x])) - (a*(b + a*Cos[c + d*x])*Sec[c + d*x]^7)/(6*b^2*d*(a + b*Sec[c + d*
x])) + ((b + a*Cos[c + d*x])*Sec[c + d*x]^8)/(7*b*d*(a + b*Sec[c + d*x]))
```

Maple [A] time = 0.064, size = 460, normalized size = 1.8

$$2\frac{1}{db\left(\cos\left(dx+c\right)\right)^{3}}-\frac{4}{5\,db\left(\cos\left(dx+c\right)\right)^{5}}-\frac{\ln\left(b+a\cos\left(dx+c\right)\right)}{ad}+\frac{1}{7\,db\left(\cos\left(dx+c\right)\right)^{7}}-4\frac{1}{db\cos\left(dx+c\right)}+\frac{a}{3\,db^{5}\left(\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^9/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d/b/cos(d*x+c)^3-4/5/d/b/cos(d*x+c)^5-1/d/a*ln(b+a*cos(d*x+c))+1/7/d/b/co
s(d*x+c)^7-4/d/b/cos(d*x+c)+1/3/d/b^5/cos(d*x+c)^3*a^4-4/3/d/b^3/cos(d*x+c)
^3*a^2+1/d/b^8*a^7*ln(cos(d*x+c))-4/d/b^6*a^5*ln(cos(d*x+c))+6/d/b^4*a^3*ln
(cos(d*x+c))-4/d/b^2*a*ln(cos(d*x+c))-1/d/b^8*a^7*ln(b+a*cos(d*x+c))+1/d/b^
7/cos(d*x+c)*a^6-4/d/b^5/cos(d*x+c)*a^4+6/d/b^3/cos(d*x+c)*a^2-1/4/d/b^4*a^
3/cos(d*x+c)^4+1/d/b^2*a/cos(d*x+c)^4-1/2/d/b^6*a^5/cos(d*x+c)^2+2/d/b^4*a^
3/cos(d*x+c)^2-3/d/b^2*a/cos(d*x+c)^2+4/d/b^6*a^5*ln(b+a*cos(d*x+c))-6/d/b^
4*a^3*ln(b+a*cos(d*x+c))+4/d/b^2*a*ln(b+a*cos(d*x+c))-1/6/d*a/b^2/cos(d*x+c
)^6+1/5/d/b^3/cos(d*x+c)^5*a^2
```

Maxima [A] time = 1.00017, size = 362, normalized size = 1.45

$$\frac{\frac{420\left(a^{7}-4\,a^{5}b^{2}+6\,a^{3}b^{4}-4\,ab^{6}\right)\log(\cos(dx+c))}{b^{8}}-\frac{420\left(a^{8}-4\,a^{6}b^{2}+6\,a^{4}b^{4}-4\,a^{2}b^{6}+b^{8}\right)\log(a\cos(dx+c)+b)}{ab^{8}}-\frac{70\,ab^{5}\cos(dx+c)-420\left(a^{6}-4\,a^{4}b^{2}+6\,a^{2}b^{4}-4\,b^{6}\right)\cos\left(\right.}{}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/420*(420*(a^7 - 4*a^5*b^2 + 6*a^3*b^4 - 4*a*b^6)*log(cos(d*x + c))/b^8 -
420*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*log(a*cos(d*x + c) + b)
/(a*b^8) - (70*a*b^5*cos(d*x + c) - 420*(a^6 - 4*a^4*b^2 + 6*a^2*b^4 - 4*b^
6)*cos(d*x + c)^6 - 60*b^6 + 210*(a^5*b - 4*a^3*b^3 + 6*a*b^5)*cos(d*x + c)
^5 - 140*(a^4*b^2 - 4*a^2*b^4 + 6*b^6)*cos(d*x + c)^4 + 105*(a^3*b^3 - 4*a*
b^5)*cos(d*x + c)^3 - 84*(a^2*b^4 - 4*b^6)*cos(d*x + c)^2)/(b^7*cos(d*x + c
)^7))/d
```

Fricas [A] time = 1.13327, size = 674, normalized size = 2.7

$$-\frac{70\,a^{2}b^{6}\cos\left(dx+c\right)+420\left(a^{8}-4\,a^{6}b^{2}+6\,a^{4}b^{4}-4\,a^{2}b^{6}+b^{8}\right)\cos\left(dx+c\right)^{7}\log\left(a\cos\left(dx+c\right)+b\right)-420\left(a^{8}-4\,a^{6}b^{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/420*(70*a^2*b^6*cos(d*x + c) + 420*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*
b^6 + b^8)*cos(d*x + c)^7*log(a*cos(d*x + c) + b) - 420*(a^8 - 4*a^6*b^2 +
6*a^4*b^4 - 4*a^2*b^6)*cos(d*x + c)^7*log(-cos(d*x + c)) - 60*a*b^7 - 420*(
a^7*b - 4*a^5*b^3 + 6*a^3*b^5 - 4*a*b^7)*cos(d*x + c)^6 + 210*(a^6*b^2 - 4*
a^4*b^4 + 6*a^2*b^6)*cos(d*x + c)^5 - 140*(a^5*b^3 - 4*a^3*b^5 + 6*a*b^7)*c
os(d*x + c)^4 + 105*(a^4*b^4 - 4*a^2*b^6)*cos(d*x + c)^3 - 84*(a^3*b^5 - 4*
a*b^7)*cos(d*x + c)^2)/(a*b^8*d*cos(d*x + c)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**9/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 11.2655, size = 2267, normalized size = 9.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/420*(420*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 -
4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*log(abs(a + b + a*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^2*b^8 - a*
b^9) - 420*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - 420*(a^
7 - 4*a^5*b^2 + 6*a^3*b^4 - 4*a*b^6)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) - 1))/b^8 + (1089*a^7 - 840*a^6*b - 4356*a^5*b^2 + 3080*a^4*b^3 +
6534*a^3*b^4 - 4088*a^2*b^5 - 4356*a*b^6 + 2232*b^7 + 7623*a^7*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) - 5040*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
) - 31332*a^5*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 19040*a^4*b^3*(co
```

s(d*x + c) - 1)/(cos(d*x + c) + 1) + 48258*a^3*b^4*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) - 26096*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 33012
*a*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 14784*b^7*(cos(d*x + c) - 1)
/(cos(d*x + c) + 1) + 22869*a^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 -
12600*a^6*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 95676*a^5*b^2*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 47880*a^4*b^3*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 + 151494*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2 - 67368*a^2*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 107436*a*b
^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 40152*b^7*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 + 38115*a^7*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)
^3 - 16800*a^6*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 160860*a^5*b^2
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 62720*a^4*b^3*(cos(d*x + c) -
1)^3/(cos(d*x + c) + 1)^3 + 258930*a^3*b^4*(cos(d*x + c) - 1)^3/(cos(d*x +
c) + 1)^3 - 86240*a^2*b^5*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 19222
0*a*b^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 53760*b^7*(cos(d*x + c)
- 1)^3/(cos(d*x + c) + 1)^3 + 38115*a^7*(cos(d*x + c) - 1)^4/(cos(d*x + c)
+ 1)^4 - 12600*a^6*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 160860*a^
5*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 45080*a^4*b^3*(cos(d*x +
c) - 1)^4/(cos(d*x + c) + 1)^4 + 258930*a^3*b^4*(cos(d*x + c) - 1)^4/(cos(d
*x + c) + 1)^4 - 56840*a^2*b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 -
192220*a*b^6*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 24360*b^7*(cos(d*x
+ c) - 1)^4/(cos(d*x + c) + 1)^4 + 22869*a^7*(cos(d*x + c) - 1)^5/(cos(d*x
+ c) + 1)^5 - 5040*a^6*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 95676
*a^5*b^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 16800*a^4*b^3*(cos(d*x
+ c) - 1)^5/(cos(d*x + c) + 1)^5 + 151494*a^3*b^4*(cos(d*x + c) - 1)^5/(co
s(d*x + c) + 1)^5 - 18480*a^2*b^5*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5
- 107436*a*b^6*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 6720*b^7*(cos(d
*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 7623*a^7*(cos(d*x + c) - 1)^6/(cos(d*
x + c) + 1)^6 - 840*a^6*b*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 31332
*a^5*b^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 2520*a^4*b^3*(cos(d*x
+ c) - 1)^6/(cos(d*x + c) + 1)^6 + 48258*a^3*b^4*(cos(d*x + c) - 1)^6/(cos(
d*x + c) + 1)^6 - 2520*a^2*b^5*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 -
33012*a*b^6*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 840*b^7*(cos(d*x +
c) - 1)^6/(cos(d*x + c) + 1)^6 + 1089*a^7*(cos(d*x + c) - 1)^7/(cos(d*x + c
) + 1)^7 - 4356*a^5*b^2*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 6534*a^
3*b^4*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 4356*a*b^6*(cos(d*x + c)
- 1)^7/(cos(d*x + c) + 1)^7)/(b^8*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
1)^7))/d

3.287 $\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=170

$$\frac{(a^2-3b^2)\sec^3(c+dx)}{3b^3d}-\frac{a(a^2-3b^2)\sec^2(c+dx)}{2b^4d}+\frac{(-3a^2b^2+a^4+3b^4)\sec(c+dx)}{b^5d}-\frac{(a^2-b^2)^3\log(a+b\sec(c+dx))}{ab^6d}$$

```
[Out] Log[Cos[c + d*x]]/(a*d) - ((a^2 - b^2)^3*Log[a + b*Sec[c + d*x]])/(a*b^6*d)
 + ((a^4 - 3*a^2*b^2 + 3*b^4)*Sec[c + d*x])/(b^5*d) - (a*(a^2 - 3*b^2)*Sec[
c + d*x]^2)/(2*b^4*d) + ((a^2 - 3*b^2)*Sec[c + d*x]^3)/(3*b^3*d) - (a*Sec[c
 + d*x]^4)/(4*b^2*d) + Sec[c + d*x]^5/(5*b*d)
```

Rubi [A] time = 0.139207, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 894}

$$\frac{(a^2-3b^2)\sec^3(c+dx)}{3b^3d}-\frac{a(a^2-3b^2)\sec^2(c+dx)}{2b^4d}+\frac{(-3a^2b^2+a^4+3b^4)\sec(c+dx)}{b^5d}-\frac{(a^2-b^2)^3\log(a+b\sec(c+dx))}{ab^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + b*Sec[c + d*x]),x]
```

```
[Out] Log[Cos[c + d*x]]/(a*d) - ((a^2 - b^2)^3*Log[a + b*Sec[c + d*x]])/(a*b^6*d)
 + ((a^4 - 3*a^2*b^2 + 3*b^4)*Sec[c + d*x])/(b^5*d) - (a*(a^2 - 3*b^2)*Sec[
c + d*x]^2)/(2*b^4*d) + ((a^2 - 3*b^2)*Sec[c + d*x]^3)/(3*b^3*d) - (a*Sec[c
 + d*x]^4)/(4*b^2*d) + Sec[c + d*x]^5/(5*b*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
```

```
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^3}{x(a+x)}\,dx, x, b\sec(c+dx)\right)}{b^6 d}$$

$$= -\frac{\text{Subst}\left(\int \left(-a^4\left(1+\frac{3b^2\left(-a^2+b^2\right)}{a^4}\right)+\frac{b^6}{ax}+a\left(a^2-3b^2\right)x-\left(a^2-3b^2\right)x^2+ax^3-x^4+\frac{\left(a^2-b^2\right)^3}{a(a+x)}\right) d\right.}{b^6 d}$$

$$= \frac{\log(\cos(c+dx))}{ad} - \frac{\left(a^2-b^2\right)^3\log(a+b\sec(c+dx))}{ab^6 d} + \frac{\left(a^4-3a^2b^2+3b^4\right)\sec(c+dx)}{b^5 d} - \frac{a\left(a^2\right.}{}$$

Mathematica [B] time = 6.17435, size = 371, normalized size = 2.18

$$\frac{\left(a^2-3b^2\right)\sec^4(c+dx)(a\cos(c+dx)+b)}{3b^3 d(a+b\sec(c+dx))} + \frac{a\left(3b^2-a^2\right)\sec^3(c+dx)(a\cos(c+dx)+b)}{2b^4 d(a+b\sec(c+dx))} + \frac{\left(-3a^2b^2+a^4+3b^4\right)\sec^2(c+}{b^5 d(a+b\sec(}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((a^5 - 3*a^3*b^2 + 3*a*b^4)*(b + a*Cos[c + d*x])*Log[Cos[c + d*x]]*Sec[c +
 d*x])/(b^6*d*(a + b*Sec[c + d*x])) + ((-a^6 + 3*a^4*b^2 - 3*a^2*b^4 + b^6)
*(b + a*Cos[c + d*x])*Log[b + a*Cos[c + d*x]]*Sec[c + d*x])/(a*b^6*d*(a + b
*Sec[c + d*x])) + ((a^4 - 3*a^2*b^2 + 3*b^4)*(b + a*Cos[c + d*x])*Sec[c + d
*x]^2)/(b^5*d*(a + b*Sec[c + d*x])) + (a*(-a^2 + 3*b^2)*(b + a*Cos[c + d*x]
)*Sec[c + d*x]^3)/(2*b^4*d*(a + b*Sec[c + d*x])) + ((a^2 - 3*b^2)*(b + a*Co
s[c + d*x])*Sec[c + d*x]^4)/(3*b^3*d*(a + b*Sec[c + d*x])) - (a*(b + a*Cos[
c + d*x])*Sec[c + d*x]^5)/(4*b^2*d*(a + b*Sec[c + d*x])) + ((b + a*Cos[c +
d*x])*Sec[c + d*x]^6)/(5*b*d*(a + b*Sec[c + d*x]))
```

Maple [A] time = 0.057, size = 292, normalized size = 1.7

$$-\frac{a^5\ln(b+a\cos(dx+c))}{db^6} + 3\frac{a^3\ln(b+a\cos(dx+c))}{db^4} - 3\frac{a\ln(b+a\cos(dx+c))}{db^2} + \frac{\ln(b+a\cos(dx+c))}{ad} - \frac{}{4\,db^2(\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7/(a+b*sec(d*x+c)),x)
```

```
[Out] -1/d/b^6*a^5*ln(b+a*cos(d*x+c))+3/d/b^4*a^3*ln(b+a*cos(d*x+c))-3/d/b^2*a*ln
(b+a*cos(d*x+c))+1/d/a*ln(b+a*cos(d*x+c))-1/4/d/b^2*a/cos(d*x+c)^4+1/3/d/b^
3/cos(d*x+c)^3*a^2-1/d/b/cos(d*x+c)^3+1/d/b^5/cos(d*x+c)*a^4-3/d/b^3/cos(d*
x+c)*a^2+3/d/b/cos(d*x+c)-1/2/d/b^4*a^3/cos(d*x+c)^2+3/2/d/b^2*a/cos(d*x+c)
^2+1/d/b^6*a^5*ln(cos(d*x+c))-3/d/b^4*a^3*ln(cos(d*x+c))+3/d/b^2*a*ln(cos(d
*x+c))+1/5/d/b/cos(d*x+c)^5
```

Maxima [A] time = 0.992441, size = 247, normalized size = 1.45

$$\frac{\frac{60\left(a^5-3\,a^3b^2+3\,ab^4\right)\log(\cos(dx+c))}{b^6}-\frac{60\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\log(a\cos(dx+c)+b)}{ab^6}-\frac{15\,ab^3\cos(dx+c)-60\left(a^4-3\,a^2b^2+3\,b^4\right)\cos(dx+c)^4-12\,b^4+30\left(a^3b-\right.}{b^5\cos(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(60*(a^5 - 3*a^3*b^2 + 3*a*b^4)*log(cos(d*x + c))/b^6 - 60*(a^6 - 3*a^
4*b^2 + 3*a^2*b^4 - b^6)*log(a*cos(d*x + c) + b)/(a*b^6) - (15*a*b^3*cos(d*
x + c) - 60*(a^4 - 3*a^2*b^2 + 3*b^4)*cos(d*x + c)^4 - 12*b^4 + 30*(a^3*b -
 3*a*b^3)*cos(d*x + c)^3 - 20*(a^2*b^2 - 3*b^4)*cos(d*x + c)^2)/(b^5*cos(d*
x + c)^5))/d
```

Fricas [A] time = 1.25118, size = 473, normalized size = 2.78

$$-\frac{15\,a^2b^4\cos(dx+c)+60\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\cos(dx+c)^5\log(a\cos(dx+c)+b)-60\left(a^6-3\,a^4b^2+3\,a^2b^4\right)\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(15*a^2*b^4*cos(d*x + c) + 60*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos
(d*x + c)^5*log(a*cos(d*x + c) + b) - 60*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cos(
d*x + c)^5*log(-cos(d*x + c)) - 12*a*b^5 - 60*(a^5*b - 3*a^3*b^3 + 3*a*b^5)
```

```
*cos(d*x + c)^4 + 30*(a^4*b^2 - 3*a^2*b^4)*cos(d*x + c)^3 - 20*(a^3*b^3 - 3
*a*b^5)*cos(d*x + c)^2)/(a*b^6*d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**7/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**7/(a + b*sec(c + d*x)), x)
```

Giac [B] time = 7.65217, size = 1299, normalized size = 7.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(60*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*
b^6 + b^7)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^2*b^6 - a*b^7) + 60*log(abs(-(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - 60*(a^5 - 3*a^3*b^2 + 3*a*b^4)*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/b^6 + (137*a^5 - 120*a^4*
b - 411*a^3*b^2 + 320*a^2*b^3 + 411*a*b^4 - 264*b^5 + 685*a^5*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) - 480*a^4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
- 2175*a^3*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1360*a^2*b^3*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 2295*a*b^4*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) - 1200*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1370*a^5*(cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2 - 720*a^4*b*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2 - 4470*a^3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 200
0*a^2*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 5070*a*b^4*(cos(d*x +
 c) - 1)^2/(cos(d*x + c) + 1)^2 - 1920*b^5*(cos(d*x + c) - 1)^2/(cos(d*x +
c) + 1)^2 + 1370*a^5*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 480*a^4*b*
(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 4470*a^3*b^2*(cos(d*x + c) - 1)
^3/(cos(d*x + c) + 1)^3 + 1200*a^2*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) +
 1)^3 + 5070*a*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 720*b^5*(cos
```

```
(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 685*a^5*(cos(d*x + c) - 1)^4/(cos(d
*x + c) + 1)^4 - 120*a^4*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 2175
*a^3*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 240*a^2*b^3*(cos(d*x +
 c) - 1)^4/(cos(d*x + c) + 1)^4 + 2295*a*b^4*(cos(d*x + c) - 1)^4/(cos(d*x
+ c) + 1)^4 - 120*b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*a^5*(
cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 411*a^3*b^2*(cos(d*x + c) - 1)^5
/(cos(d*x + c) + 1)^5 + 411*a*b^4*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5
)/(b^6*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^5))/d
```

3.288 $\int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=108

$$\frac{(a^2-2b^2)\sec(c+dx)}{b^3d}-\frac{(a^2-b^2)^2\log(a+b\sec(c+dx))}{ab^4d}-\frac{a\sec^2(c+dx)}{2b^2d}-\frac{\log(\cos(c+dx))}{ad}+\frac{\sec^3(c+dx)}{3bd}$$

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - ((a^2 - b^2)^2*Log[a + b*Sec[c + d*x]])/(a*b^4
*d) + ((a^2 - 2*b^2)*Sec[c + d*x])/(b^3*d) - (a*Sec[c + d*x]^2)/(2*b^2*d) +
 Sec[c + d*x]^3/(3*b*d)
```

Rubi [A] time = 0.0956663, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 894}

$$\frac{(a^2-2b^2)\sec(c+dx)}{b^3d}-\frac{(a^2-b^2)^2\log(a+b\sec(c+dx))}{ab^4d}-\frac{a\sec^2(c+dx)}{2b^2d}-\frac{\log(\cos(c+dx))}{ad}+\frac{\sec^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - ((a^2 - b^2)^2*Log[a + b*Sec[c + d*x]])/(a*b^4
*d) + ((a^2 - 2*b^2)*Sec[c + d*x])/(b^3*d) - (a*Sec[c + d*x]^2)/(2*b^2*d) +
 Sec[c + d*x]^3/(3*b*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x(a+x)}\,dx, x, b\sec(c+dx)\right)}{b^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{2b^2}{a^2}\right)+\frac{b^4}{ax}-ax+x^2-\frac{\left(a^2-b^2\right)^2}{a(a+x)}\right)dx, x, b\sec(c+dx)\right)}{b^4 d}$$
$$= -\frac{\log(\cos(c+dx))}{ad} - \frac{\left(a^2-b^2\right)^2\log(a+b\sec(c+dx))}{ab^4 d} + \frac{\left(a^2-2b^2\right)\sec(c+dx)}{b^3 d} - \frac{a\sec^2(c+}{2b^2 d}$$

Mathematica [A] time = 0.368648, size = 108, normalized size = 1.

$$\frac{-3a^2b^2\sec^2(c+dx)+6ab\left(a^2-2b^2\right)\sec(c+dx)+6a^2\left(a^2-2b^2\right)\log(\cos(c+dx))-6\left(a^2-b^2\right)^2\log(a\cos(c+dx)+b)}{6ab^4 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] (6*a^2*(a^2 - 2*b^2)*Log[Cos[c + d*x]] - 6*(a^2 - b^2)^2*Log[b + a*Cos[c +
d*x]] + 6*a*b*(a^2 - 2*b^2)*Sec[c + d*x] - 3*a^2*b^2*Sec[c + d*x]^2 + 2*a*b
^3*Sec[c + d*x]^3)/(6*a*b^4*d)
```

Maple [A] time = 0.052, size = 163, normalized size = 1.5

$$-\frac{a^3\ln(b+a\cos(dx+c))}{db^4}+2\frac{a\ln(b+a\cos(dx+c))}{db^2}-\frac{\ln(b+a\cos(dx+c))}{ad}-\frac{a}{2\,db^2\left(\cos(dx+c)\right)^2}+\frac{a^2}{db^3\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*sec(d*x+c)),x)
```

```
[Out] -1/d/b^4*a^3*ln(b+a*cos(d*x+c))+2/d/b^2*a*ln(b+a*cos(d*x+c))-1/d/a*ln(b+a*c
os(d*x+c))-1/2/d/b^2*a/cos(d*x+c)^2+1/d/b^3/cos(d*x+c)*a^2-2/d/b/cos(d*x+c)
+1/d/b^4*a^3*ln(cos(d*x+c))-2/d/b^2*a*ln(cos(d*x+c))+1/3/d/b/cos(d*x+c)^3
```

Maxima [A] time = 0.945829, size = 149, normalized size = 1.38

$$\frac{\frac{6\left(a^{3}-2\,ab^{2}\right)\log(\cos(dx+c))}{b^{4}}-\frac{6\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\log(a\cos(dx+c)+b)}{ab^{4}}-\frac{3\,ab\cos(dx+c)-6\left(a^{2}-2\,b^{2}\right)\cos(dx+c)^{2}-2\,b^{2}}{b^{3}\cos(dx+c)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*(a^3 - 2*a*b^2)*log(cos(d*x + c))/b^4 - 6*(a^4 - 2*a^2*b^2 + b^4)*lo
g(a*cos(d*x + c) + b)/(a*b^4) - (3*a*b*cos(d*x + c) - 6*(a^2 - 2*b^2)*cos(d
*x + c)^2 - 2*b^2)/(b^3*cos(d*x + c)^3))/d
```

Fricas [A] time = 1.12712, size = 305, normalized size = 2.82

$$-\frac{3\,a^{2}b^{2}\cos\left(dx+c\right)+6\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\cos\left(dx+c\right)^{3}\log\left(a\cos\left(dx+c\right)+b\right)-6\left(a^{4}-2\,a^{2}b^{2}\right)\cos\left(dx+c\right)^{3}\log\left(-\cos\left(\right.\right.}{6\,ab^{4}d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(3*a^2*b^2*cos(d*x + c) + 6*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^3*log
(a*cos(d*x + c) + b) - 6*(a^4 - 2*a^2*b^2)*cos(d*x + c)^3*log(-cos(d*x + c)
) - 2*a*b^3 - 6*(a^3*b - 2*a*b^3)*cos(d*x + c)^2)/(a*b^4*d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^{5}\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**5/(a + b*sec(c + d*x)), x)
```

Giac [B] time = 3.62829, size = 636, normalized size = 5.89

$$-\frac{\frac{6\left(a^5-a^4b-2a^3b^2+2a^2b^3+ab^4-b^5\right)\log\left(\left|a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^2b^4-ab^5}-\frac{6\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a}-\frac{6\left(a^3-2ab^2\right)\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{b^4}+\frac{11a^3-}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*log(abs(a + b +
 a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1)))/(a^2*b^4 - a*b^5) - 6*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + 1))/a - 6*(a^3 - 2*a*b^2)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) - 1))/b^4 + (11*a^3 - 12*a^2*b - 22*a*b^2 + 20*b^3 + 33*a^3*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 24*a^2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 - 78*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 48*b^3*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) + 33*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 -
12*a^2*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 78*a*b^2*(cos(d*x + c)
 - 1)^2/(cos(d*x + c) + 1)^2 + 12*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2 + 11*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 22*a*b^2*(cos(d*x
 + c) - 1)^3/(cos(d*x + c) + 1)^3)/(b^4*((cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + 1)^3))/d
```

3.289 $\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=59

$$-\frac{(a^2-b^2)\log(a+b\sec(c+dx))}{ab^2d}+\frac{\log(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{bd}$$

```
[Out] Log[Cos[c + d*x]]/(a*d) - ((a^2 - b^2)*Log[a + b*Sec[c + d*x]])/(a*b^2*d) +
 Sec[c + d*x]/(b*d)
```

Rubi [A] time = 0.0710039, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 894}

$$-\frac{(a^2-b^2)\log(a+b\sec(c+dx))}{ab^2d}+\frac{\log(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] Log[Cos[c + d*x]]/(a*d) - ((a^2 - b^2)*Log[a + b*Sec[c + d*x]])/(a*b^2*d) +
 Sec[c + d*x]/(b*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{x(a+x)}\,dx, x, b\sec(c+dx)\right)}{b^2d}$$
$$= -\frac{\text{Subst}\left(\int \left(-1+\frac{b^2}{ax}+\frac{a^2-b^2}{a(a+x)}\right)dx, x, b\sec(c+dx)\right)}{b^2d}$$
$$= \frac{\log(\cos(c+dx))}{ad} - \frac{(a^2-b^2)\log(a+b\sec(c+dx))}{ab^2d} + \frac{\sec(c+dx)}{bd}$$

Mathematica [A] time = 0.120668, size = 52, normalized size = 0.88

$$\frac{(b^2-a^2)\log(a\cos(c+dx)+b)+a^2\log(\cos(c+dx))+ab\sec(c+dx)}{ab^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] (a^2*Log[Cos[c + d*x]] + (-a^2 + b^2)*Log[b + a*Cos[c + d*x]] + a*b*Sec[c +
 d*x])/(a*b^2*d)
```

Maple [A] time = 0.041, size = 70, normalized size = 1.2

$$-\frac{a\ln(b+a\cos(dx+c))}{db^2}+\frac{\ln(b+a\cos(dx+c))}{ad}+\frac{a\ln(\cos(dx+c))}{db^2}+\frac{1}{db\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*sec(d*x+c)),x)
```

```
[Out] -1/d/b^2*a*ln(b+a*cos(d*x+c))+1/d/a*ln(b+a*cos(d*x+c))+1/d/b^2*a*ln(cos(d*x
+c))+1/d/b/cos(d*x+c)
```

Maxima [A] time = 0.971969, size = 77, normalized size = 1.31

$$\frac{\frac{a\log(\cos(dx+c))}{b^2}-\frac{(a^2-b^2)\log(a\cos(dx+c)+b)}{ab^2}+\frac{1}{b\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*log(cos(d*x + c))/b^2 - (a^2 - b^2)*log(a*cos(d*x + c) + b)/(a*b^2) + 1/
(b*cos(d*x + c)))/d
```

Fricas [A] time = 0.929986, size = 161, normalized size = 2.73

$$\frac{a^2\cos(dx+c)\log(-\cos(dx+c)) - \left(a^2-b^2\right)\cos(dx+c)\log(a\cos(dx+c)+b) + ab}{ab^2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] (a^2*cos(d*x + c)*log(-cos(d*x + c)) - (a^2 - b^2)*cos(d*x + c)*log(a*cos(d
*x + c) + b) + a*b)/(a*b^2*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**3/(a + b*sec(c + d*x)), x)
```

Giac [B] time = 1.97962, size = 267, normalized size = 4.53

$$-\frac{\frac{\left(a^3-a^2b-ab^2+b^3\right)\log\left(\left|a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^2b^2-ab^3}+\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a}-\frac{a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{b^2}+\frac{a-2b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}}{b^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -((a^3 - a^2*b - a*b^2 + b^3)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^2*b^2 - a*b^3) +
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - a*log(abs(-(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) - 1))/b^2 + (a - 2*b + a*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1))/(b^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

3.290 $\int \frac{\tan(c+dx)}{a+b\sec(c+dx)} dx$

Optimal. Leaf size=35

$$-\frac{\log(a+b\sec(c+dx))}{ad} - \frac{\log(\cos(c+dx))}{ad}$$

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - Log[a + b*Sec[c + d*x]]/(a*d)
```

Rubi [A] time = 0.0315418, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3885, 36, 29, 31}

$$-\frac{\log(a+b\sec(c+dx))}{ad} - \frac{\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - Log[a + b*Sec[c + d*x]]/(a*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\sec(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, b\sec(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sec(c+dx)\right)}{ad}$$
$$= -\frac{\log(\cos(c+dx))}{ad} - \frac{\log(a+b\sec(c+dx))}{ad}$$

Mathematica [A] time = 0.0368242, size = 19, normalized size = 0.54

$$-\frac{\log(a\cos(c+dx)+b)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(Log[b + a*Cos[c + d*x]]/(a*d))
```

Maple [A] time = 0.02, size = 35, normalized size = 1.

$$-\frac{\ln(a+b\sec(dx+c))}{ad} + \frac{\ln(\sec(dx+c))}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] -ln(a+b*sec(d*x+c))/d/a+1/d/a*ln(sec(d*x+c))
```

Maxima [A] time = 0.963872, size = 26, normalized size = 0.74

$$-\frac{\log(a\cos(dx+c)+b)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -log(a*cos(d*x + c) + b)/(a*d)
```

Fricas [A] time = 0.927896, size = 43, normalized size = 1.23

$$-\frac{\log\left(a\cos\left(dx+c\right)+b\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -log(a*cos(d*x + c) + b)/(a*d)
```

Sympy [A] time = 6.82994, size = 82, normalized size = 2.34

$$\begin{cases} \frac{\tilde{\infty}x\tan\left(c\right)}{\sec\left(c\right)} & \text{for}\: a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{x\tan\left(c\right)}{a+b\sec\left(c\right)} & \text{for}\: d = 0 \\ -\frac{1}{bd\sec\left(c+dx\right)} & \text{for}\: a = 0 \\ \frac{\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2ad} & \text{for}\: b = 0 \\ -\frac{\log\left(\frac{a}{b}+\sec\left(c+dx\right)\right)}{ad} + \frac{\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2ad} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*tan(c)/sec(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x*tan(c)/
(a + b*sec(c)), Eq(d, 0)), (-1/(b*d*sec(c + d*x)), Eq(a, 0)), (log(tan(c +
d*x)**2 + 1)/(2*a*d), Eq(b, 0)), (-log(a/b + sec(c + d*x))/(a*d) + log(tan(
c + d*x)**2 + 1)/(2*a*d), True))
```

Giac [B] time = 1.35047, size = 131, normalized size = 3.74

$$-\frac{\frac{(a-b)\log\left(\left|a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^2-ab}-\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -((a - b)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^2 - a*b) - log(abs(-(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 1))/a)/d
```

3.291 $\int \frac{\cot(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=94

$$-\frac{b^2\log(a+b\sec(c+dx))}{ad\left(a^2-b^2\right)}+\frac{\log(1-\sec(c+dx))}{2d(a+b)}+\frac{\log(\sec(c+dx)+1)}{2d(a-b)}+\frac{\log(\cos(c+dx))}{ad}$$

```
[Out] Log[Cos[c + d*x]]/(a*d) + Log[1 - Sec[c + d*x]]/(2*(a + b)*d) + Log[1 + Sec
[c + d*x]]/(2*(a - b)*d) - (b^2*Log[a + b*Sec[c + d*x]])/(a*(a^2 - b^2)*d)
```

Rubi [A] time = 0.102287, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3885, 894}

$$-\frac{b^2\log(a+b\sec(c+dx))}{ad\left(a^2-b^2\right)}+\frac{\log(1-\sec(c+dx))}{2d(a+b)}+\frac{\log(\sec(c+dx)+1)}{2d(a-b)}+\frac{\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] Log[Cos[c + d*x]]/(a*d) + Log[1 - Sec[c + d*x]]/(2*(a + b)*d) + Log[1 + Sec
[c + d*x]]/(2*(a - b)*d) - (b^2*Log[a + b*Sec[c + d*x]])/(a*(a^2 - b^2)*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cot(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{x(a+x)\left(b^2-x^2\right)}\,dx, x, b\sec(c+dx)\right)}{d}$$
$$= -\frac{b^2\,\mathrm{Subst}\left(\int \left(\frac{1}{2b^2(a+b)(b-x)} + \frac{1}{ab^2x} + \frac{1}{a(a-b)(a+b)(a+x)} - \frac{1}{2(a-b)b^2(b+x)}\right)dx, x, b\sec(c+dx)\right)}{d}$$
$$= \frac{\log(\cos(c+dx))}{ad} + \frac{\log(1-\sec(c+dx))}{2(a+b)d} + \frac{\log(1+\sec(c+dx))}{2(a-b)d} - \frac{b^2\log(a+b\sec(c+dx))}{a\left(a^2-b^2\right)d}$$

Mathematica [A] time = 0.105076, size = 70, normalized size = 0.74

$$\frac{b^2(-\log(a\cos(c+dx)+b)) + a(a-b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + a(a+b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{ad(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*(a + b)*Log[Cos[(c + d*x)/2]] - b^2*Log[b + a*Cos[c + d*x]] + a*(a - b)*
Log[Sin[(c + d*x)/2]])/(a*(a - b)*(a + b)*d)
```

Maple [A] time = 0.058, size = 80, normalized size = 0.9

$$-\frac{b^2\ln(b+a\cos(dx+c))}{d(a+b)(a-b)a} + \frac{\ln(\cos(dx+c)+1)}{d(2a-2b)} + \frac{\ln(-1+\cos(dx+c))}{d(2a+2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] -1/d*b^2/(a+b)/(a-b)/a*ln(b+a*cos(d*x+c))+1/d/(2*a-2*b)*ln(cos(d*x+c)+1)+1/
d/(2*a+2*b)*ln(-1+cos(d*x+c))
```

Maxima [A] time = 0.956272, size = 92, normalized size = 0.98

$$-\frac{\frac{2b^2\log(a\cos(dx+c)+b)}{a^3-ab^2} - \frac{\log(\cos(dx+c)+1)}{a-b} - \frac{\log(\cos(dx+c)-1)}{a+b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^2*log(a*cos(d*x + c) + b)/(a^3 - a*b^2) - log(cos(d*x + c) + 1)/(
a - b) - log(cos(d*x + c) - 1)/(a + b))/d
```

Fricas [A] time = 1.07341, size = 190, normalized size = 2.02

$$-\frac{2\,b^2\log\left(a\cos\left(dx+c\right)+b\right)-\left(a^2+ab\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(a^2-ab\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\left(a^3-ab^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*log(a*cos(d*x + c) + b) - (a^2 + a*b)*log(1/2*cos(d*x + c) + 1/
2) - (a^2 - a*b)*log(-1/2*cos(d*x + c) + 1/2))/((a^3 - a*b^2)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cot\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.29026, size = 180, normalized size = 1.91

$$-\frac{\frac{2\,b^2\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^3-ab^2}-\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}+\frac{2\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^3 - a*b^2) - log(abs(-cos(d*x + c)
 + 1)/abs(cos(d*x + c) + 1))/(a + b) + 2*log(abs(-(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 1))/a)/d
```

3.292 $\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=157

$$-\frac{b^4\log(a+b\sec(c+dx))}{ad\left(a^2-b^2\right)^2}+\frac{1}{4d(a+b)(1-\sec(c+dx))}+\frac{1}{4d(a-b)(\sec(c+dx)+1)}-\frac{(2a+3b)\log(1-\sec(c+dx))}{4d(a+b)^2}-$$

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - ((2*a + 3*b)*Log[1 - Sec[c + d*x]])/(4*(a + b)
^2*d) - ((2*a - 3*b)*Log[1 + Sec[c + d*x]])/(4*(a - b)^2*d) - (b^4*Log[a +
b*Sec[c + d*x]])/(a*(a^2 - b^2)^2*d) + 1/(4*(a + b)*d*(1 - Sec[c + d*x])) +
 1/(4*(a - b)*d*(1 + Sec[c + d*x]))
```

Rubi [A] time = 0.181046, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 894}

$$-\frac{b^4\log(a+b\sec(c+dx))}{ad\left(a^2-b^2\right)^2}+\frac{1}{4d(a+b)(1-\sec(c+dx))}+\frac{1}{4d(a-b)(\sec(c+dx)+1)}-\frac{(2a+3b)\log(1-\sec(c+dx))}{4d(a+b)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - ((2*a + 3*b)*Log[1 - Sec[c + d*x]])/(4*(a + b)
^2*d) - ((2*a - 3*b)*Log[1 + Sec[c + d*x]])/(4*(a - b)^2*d) - (b^4*Log[a +
b*Sec[c + d*x]])/(a*(a^2 - b^2)^2*d) + 1/(4*(a + b)*d*(1 - Sec[c + d*x])) +
 1/(4*(a - b)*d*(1 + Sec[c + d*x]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
```

```
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{b^4\,\text{Subst}\left(\int \frac{1}{x(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sec(c+dx)\right)}{d}$$
$$= \frac{b^4\,\text{Subst}\left(\int \left(\frac{1}{4b^3(a+b)(b-x)^2}+\frac{2a+3b}{4b^4(a+b)^2(b-x)}+\frac{1}{ab^4x}-\frac{1}{a(a-b)^2(a+b)^2(a+x)}-\frac{1}{4(a-b)b^3(b+x)^2}+\frac{-2a+3b}{4(a-b)^2b^4(b-}\right.\right.}{d}$$
$$= -\frac{\log(\cos(c+dx))}{ad}-\frac{(2a+3b)\log(1-\sec(c+dx))}{4(a+b)^2d}-\frac{(2a-3b)\log(1+\sec(c+dx))}{4(a-b)^2d}-\frac{b^4\log}{}$$

Mathematica [A] time = 1.0088, size = 141, normalized size = 0.9

$$-\frac{8b^4\log(a\cos(c+dx)+b)+a(a-b)^2(a+b)\csc^2\left(\frac{1}{2}(c+dx)\right)+a(a-b)(a+b)^2\sec^2\left(\frac{1}{2}(c+dx)\right)+4a(a-b)^2(2a+3b}{8ad(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(a*(a - b)^2*(a + b)*Csc[(c + d*x)/2]^2 + 4*a*(2*a - 3*b)*(a + b)^2*Log[Co
s[(c + d*x)/2]] + 8*b^4*Log[b + a*Cos[c + d*x]] + 4*a*(a - b)^2*(2*a + 3*b)
*Log[Sin[(c + d*x)/2]] + a*(a - b)*(a + b)^2*Sec[(c + d*x)/2]^2)/(8*a*(a -
b)^2*(a + b)^2*d)
```

Maple [A] time = 0.075, size = 167, normalized size = 1.1

$$-\frac{b^4\ln(b+a\cos(dx+c))}{d(a+b)^2(a-b)^2a}-\frac{1}{d(4a-4b)(\cos(dx+c)+1)}-\frac{\ln(\cos(dx+c)+1)a}{2d(a-b)^2}+\frac{3\ln(\cos(dx+c)+1)b}{4d(a-b)^2}+\frac{}{d(4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*sec(d*x+c)),x)
```

```
[Out] -1/d*b^4/(a+b)^2/(a-b)^2/a*ln(b+a*cos(d*x+c))-1/d/(4*a-4*b)/(cos(d*x+c)+1)-
1/2/d/(a-b)^2*ln(cos(d*x+c)+1)*a+3/4/d/(a-b)^2*ln(cos(d*x+c)+1)*b+1/d/(4*a+
4*b)/(-1+cos(d*x+c))-1/2/d/(a+b)^2*ln(-1+cos(d*x+c))*a-3/4/d/(a+b)^2*ln(-1+
cos(d*x+c))*b
```

Maxima [A] time = 1.0286, size = 194, normalized size = 1.24

$$-\frac{\frac{4\,b^4\log(a\cos(dx+c)+b)}{a^5-2\,a^3b^2+ab^4}+\frac{(2\,a-3\,b)\log(\cos(dx+c)+1)}{a^2-2\,ab+b^2}+\frac{(2\,a+3\,b)\log(\cos(dx+c)-1)}{a^2+2\,ab+b^2}+\frac{2\,(b\cos(dx+c)-a)}{(a^2-b^2)\cos(dx+c)^2-a^2+b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(4*b^4*log(a*cos(d*x + c) + b)/(a^5 - 2*a^3*b^2 + a*b^4) + (2*a - 3*b)
*log(cos(d*x + c) + 1)/(a^2 - 2*a*b + b^2) + (2*a + 3*b)*log(cos(d*x + c) -
 1)/(a^2 + 2*a*b + b^2) + 2*(b*cos(d*x + c) - a)/((a^2 - b^2)*cos(d*x + c)^
2 - a^2 + b^2))/d
```

Fricas [A] time = 1.07828, size = 587, normalized size = 3.74

$$\frac{2\,a^4-2\,a^2b^2-2\left(a^3b-ab^3\right)\cos\left(dx+c\right)-4\left(b^4\cos\left(dx+c\right)^2-b^4\right)\log\left(a\cos\left(dx+c\right)+b\right)+\left(2\,a^4+a^3b-4\,a^2b^2-3\,ab^3\right.}{4\left(\left(a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^4 - 2*a^2*b^2 - 2*(a^3*b - a*b^3)*cos(d*x + c) - 4*(b^4*cos(d*x +
c)^2 - b^4)*log(a*cos(d*x + c) + b) + (2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3
- (2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c
) + 1/2) + (2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3 - (2*a^4 - a^3*b - 4*a^2*b^
2 + 3*a*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^5 - 2*a^3*b^
2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**3/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.45078, size = 346, normalized size = 2.2

$$-\frac{\frac{8\,b^4\log\left(\left|a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^5-2\,a^3b^2+ab^4}+\frac{2\,(2\,a+3\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2+2\,ab+b^2}-\frac{\left(a+b+\frac{4\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{6\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\left(a^2+2\,ab+b^2\right)(\cos(dx+c)-1)}-\frac{8\log\left(\left|-\frac{\cos}{\cos}\right.\right.}{a}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/8*(8*b^4*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^5 - 2*a^3*b^2 + a*b^4) + 2*(2*a + 3
*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^2 + 2*a*b + b^2) -
 (a + b + 4*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 6*b*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/((a^2 + 2*a*b + b^2)*(cos(d*x + c)
 - 1)) - 8*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - (cos(d*
x + c) - 1)/((a - b)*(cos(d*x + c) + 1)))/d
```

3.293 $\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=234

$$-\frac{b^6\log(a+b\sec(c+dx))}{ad\left(a^2-b^2\right)^3}+\frac{\left(8a^2+21ab+15b^2\right)\log(1-\sec(c+dx))}{16d(a+b)^3}+\frac{\left(8a^2-21ab+15b^2\right)\log(\sec(c+dx)+1)}{16d(a-b)^3}-\frac{}{16d}$$

```
[Out] Log[Cos[c + d*x]]/(a*d) + ((8*a^2 + 21*a*b + 15*b^2)*Log[1 - Sec[c + d*x]])
/(16*(a + b)^3*d) + ((8*a^2 - 21*a*b + 15*b^2)*Log[1 + Sec[c + d*x]])/(16*(
a - b)^3*d) - (b^6*Log[a + b*Sec[c + d*x]])/(a*(a^2 - b^2)^3*d) - 1/(16*(a
+ b)*d*(1 - Sec[c + d*x])^2) - (5*a + 7*b)/(16*(a + b)^2*d*(1 - Sec[c + d*x
])) - 1/(16*(a - b)*d*(1 + Sec[c + d*x])^2) - (5*a - 7*b)/(16*(a - b)^2*d*(
1 + Sec[c + d*x]))
```

Rubi [A] time = 0.294619, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 894}

$$-\frac{b^6\log(a+b\sec(c+dx))}{ad\left(a^2-b^2\right)^3}+\frac{\left(8a^2+21ab+15b^2\right)\log(1-\sec(c+dx))}{16d(a+b)^3}+\frac{\left(8a^2-21ab+15b^2\right)\log(\sec(c+dx)+1)}{16d(a-b)^3}-\frac{}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```



```
[Out] Log[Cos[c + d*x]]/(a*d) + ((8*a^2 + 21*a*b + 15*b^2)*Log[1 - Sec[c + d*x]])
/(16*(a + b)^3*d) + ((8*a^2 - 21*a*b + 15*b^2)*Log[1 + Sec[c + d*x]])/(16*(
a - b)^3*d) - (b^6*Log[a + b*Sec[c + d*x]])/(a*(a^2 - b^2)^3*d) - 1/(16*(a
+ b)*d*(1 - Sec[c + d*x])^2) - (5*a + 7*b)/(16*(a + b)^2*d*(1 - Sec[c + d*x
])) - 1/(16*(a - b)*d*(1 + Sec[c + d*x])^2) - (5*a - 7*b)/(16*(a - b)^2*d*(
1 + Sec[c + d*x]))
```


Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{b^6\,\text{Subst}\left(\int \frac{1}{x(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= -\frac{b^6\,\text{Subst}\left(\int \left(\frac{1}{8b^4(a+b)(b-x)^3}+\frac{5a+7b}{16b^5(a+b)^2(b-x)^2}+\frac{8a^2+21ab+15b^2}{16b^6(a+b)^3(b-x)}+\frac{1}{ab^6x}+\frac{1}{a(a-b)^3(a+b)^3(a+x)}+\frac{1}{8b^4(-a}\right.\right.}{d}$$

$$= \frac{\log(\cos(c+dx))}{ad}+\frac{\left(8a^2+21ab+15b^2\right)\log(1-\sec(c+dx))}{16(a+b)^3d}+\frac{\left(8a^2-21ab+15b^2\right)\log(1+s}{16(a-b)^3d}$$

Mathematica [C] time = 6.2418, size = 625, normalized size = 2.67

$$\frac{2i\left(-3a^3b^2+a^5+3ab^4\right)(c+dx)\sec(c+dx)(a\cos(c+dx)+b)}{d(a-b)^3(a+b)^3(a+b\sec(c+dx))}+\frac{\left(-8a^2+21ab-15b^2\right)\sec(c+dx)\log\left(\cos^2\left(\frac{1}{2}(c+dx\right.\right.}{16d(b-a)^3(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((2*I)*(a^5 - 3*a^3*b^2 + 3*a*b^4)*(c + d*x)*(b + a*Cos[c + d*x])*Sec[c + d
*x])/((a - b)^3*(a + b)^3*d*(a + b*Sec[c + d*x])) - ((I/8)*(-8*a^2 + 21*a*b
 - 15*b^2)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])*Sec[c + d*x])/((-a + b
)^3*d*(a + b*Sec[c + d*x])) - ((I/8)*(8*a^2 + 21*a*b + 15*b^2)*ArcTan[Tan[c
 + d*x]]*(b + a*Cos[c + d*x])*Sec[c + d*x])/((a + b)^3*d*(a + b*Sec[c + d*x
])) + ((7*a + 9*b)*(b + a*Cos[c + d*x])*Csc[(c + d*x)/2]^2*Sec[c + d*x])/(3
2*(a + b)^2*d*(a + b*Sec[c + d*x])) - ((b + a*Cos[c + d*x])*Csc[(c + d*x)/2
]^4*Sec[c + d*x])/(64*(a + b)*d*(a + b*Sec[c + d*x])) + ((-8*a^2 + 21*a*b -
 15*b^2)*(b + a*Cos[c + d*x])*Log[Cos[(c + d*x)/2]^2]*Sec[c + d*x])/(16*(-a
 + b)^3*d*(a + b*Sec[c + d*x])) + (b^6*(b + a*Cos[c + d*x])*Log[b + a*Cos[c
 + d*x]]*Sec[c + d*x])/(a*(-a^2 + b^2)^3*d*(a + b*Sec[c + d*x])) + ((8*a^2
+ 21*a*b + 15*b^2)*(b + a*Cos[c + d*x])*Log[Sin[(c + d*x)/2]^2]*Sec[c + d*x
```

```
])/(16*(a + b)^3*d*(a + b*Sec[c + d*x])) + ((7*a - 9*b)*(b + a*Cos[c + d*x]
)*Sec[(c + d*x)/2]^2*Sec[c + d*x])/(32*(-a + b)^2*d*(a + b*Sec[c + d*x])) +
 ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sec[c + d*x])/(64*(-a + b)*d*(a +
 b*Sec[c + d*x]))
```

Maple [A] time = 0.076, size = 308, normalized size = 1.3

$$-\frac{b^6 \ln(b + a\cos(dx+c))}{d(a+b)^3(a-b)^3 a} - \frac{1}{2d(8a-8b)(\cos(dx+c)+1)^2} + \frac{7a}{16d(a-b)^2(\cos(dx+c)+1)} - \frac{9b}{16d(a-b)^2(\cos(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+b*sec(d*x+c)),x)
```

```
[Out] -1/d*b^6/(a+b)^3/(a-b)^3/a*ln(b+a*cos(d*x+c))-1/2/d/(8*a-8*b)/(cos(d*x+c)+1
)^2+7/16/d/(a-b)^2/(cos(d*x+c)+1)*a-9/16/d/(a-b)^2/(cos(d*x+c)+1)*b+1/2/d/(
a-b)^3*ln(cos(d*x+c)+1)*a^2-21/16/d/(a-b)^3*ln(cos(d*x+c)+1)*a*b+15/16/d/(a
-b)^3*ln(cos(d*x+c)+1)*b^2-1/2/d/(8*a+8*b)/(-1+cos(d*x+c))^2-7/16/d/(a+b)^2
/(-1+cos(d*x+c))*a-9/16/d/(a+b)^2/(-1+cos(d*x+c))*b+1/2/d/(a+b)^3*ln(-1+cos
(d*x+c))*a^2+21/16/d/(a+b)^3*ln(-1+cos(d*x+c))*a*b+15/16/d/(a+b)^3*ln(-1+co
s(d*x+c))*b^2
```

Maxima [A] time = 1.01366, size = 390, normalized size = 1.67

$$-\frac{\frac{16 b^6 \log(a\cos(dx+c)+b)}{a^7-3a^5b^2+3a^3b^4-ab^6} - \frac{(8a^2-21ab+15b^2)\log(\cos(dx+c)+1)}{a^3-3a^2b+3ab^2-b^3} - \frac{(8a^2+21ab+15b^2)\log(\cos(dx+c)-1)}{a^3+3a^2b+3ab^2+b^3} - \frac{2\left((5a^2b-9b^3)\cos(dx+c)^3+6a^3-10ab^2-4(2}{(a^4-2a^2b^2+b^4)\cos(dx+c)^4+a^4-2a^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(16*b^6*log(a*cos(d*x + c) + b)/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)
 - (8*a^2 - 21*a*b + 15*b^2)*log(cos(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2
 - b^3) - (8*a^2 + 21*a*b + 15*b^2)*log(cos(d*x + c) - 1)/(a^3 + 3*a^2*b +
3*a*b^2 + b^3) - 2*((5*a^2*b - 9*b^3)*cos(d*x + c)^3 + 6*a^3 - 10*a*b^2 - 4
*(2*a^3 - 3*a*b^2)*cos(d*x + c)^2 - (3*a^2*b - 7*b^3)*cos(d*x + c))/((a^4 -
 2*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b
^2 + b^4)*cos(d*x + c)^2))/d
```

Fricas [B] time = 1.88919, size = 1312, normalized size = 5.61

$$\frac{12\,a^6-32\,a^4b^2+20\,a^2b^4+2\left(5\,a^5b-14\,a^3b^3+9\,ab^5\right)\cos\left(dx+c\right)^3-8\left(2\,a^6-5\,a^4b^2+3\,a^2b^4\right)\cos\left(dx+c\right)^2-2\left(3\,a^5b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(12*a^6 - 32*a^4*b^2 + 20*a^2*b^4 + 2*(5*a^5*b - 14*a^3*b^3 + 9*a*b^5)
*cos(d*x + c)^3 - 8*(2*a^6 - 5*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^2 - 2*(3*a
^5*b - 10*a^3*b^3 + 7*a*b^5)*cos(d*x + c) - 16*(b^6*cos(d*x + c)^4 - 2*b^6*
cos(d*x + c)^2 + b^6)*log(a*cos(d*x + c) + b) + (8*a^6 + 3*a^5*b - 24*a^4*b
^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5 + (8*a^6 + 3*a^5*b - 24*a^4*b^2 - 1
0*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cos(d*x + c)^4 - 2*(8*a^6 + 3*a^5*b - 24
*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cos(d*x + c)^2)*log(1/2*cos(
d*x + c) + 1/2) + (8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 -
 15*a*b^5 + (8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*
b^5)*cos(d*x + c)^4 - 2*(8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2
*b^4 - 15*a*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^7 - 3*a^
5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^4 - 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^
4 - a*b^6)*d*cos(d*x + c)^2 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^5\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**5/(a + b*sec(c + d*x)), x)
```

Giac [B] time = 1.49062, size = 639, normalized size = 2.73

$$-\frac{\frac{64\,b^6\log\left(\left|a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^7-3\,a^5b^2+3\,a^3b^4-ab^6}-\frac{4\left(8\,a^2+21\,ab+15\,b^2\right)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{\frac{12\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{16\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{b(\cos(dx}{(\cos(dx}}{a^2-2\,ab+b^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/64*(64*b^6*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6
) - 4*(8*a^2 + 21*a*b + 15*b^2)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c)
 + 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + (12*a*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) - 16*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(a
^2 - 2*a*b + b^2) + (a^2 + 2*a*b + b^2 + 12*a^2*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) + 28*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 16*b^2*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 48*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 + 126*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 90*b^2*(cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/((a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*(cos(d*x + c) - 1)^2) + 64*log(abs(-(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 1))/a)/d
```

3.294 $\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=198

$$-\frac{a\left(a^2-2b^2\right)\tan(c+dx)}{b^4d}+\frac{\left(-20a^2b^2+8a^4+15b^4\right)\tanh^{-1}(\sin(c+dx))}{8b^5d}+\frac{\left(4a^2-7b^2\right)\tan(c+dx)\sec(c+dx)}{8b^3d}-\frac{a\,\mathrm{ta}}{}$$

```
[Out] -(x/a) + ((8*a^4 - 20*a^2*b^2 + 15*b^4)*ArcTanh[Sin[c + d*x]])/(8*b^5*d) -
(2*(a - b)^(5/2)*(a + b)^(5/2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[
a + b]])/(a*b^5*d) - (a*(a^2 - 2*b^2)*Tan[c + d*x])/(b^4*d) + ((4*a^2 - 7*b
^2)*Sec[c + d*x]*Tan[c + d*x])/(8*b^3*d) - (a*Tan[c + d*x]^3)/(3*b^2*d) + (
Sec[c + d*x]*Tan[c + d*x]^3)/(4*b*d)
```

Rubi [A] time = 0.372717, antiderivative size = 271, normalized size of antiderivative = 1.37, number of steps used = 15, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3898, 2897, 2659, 208, 3770, 3767, 8, 3768}

$$-\frac{a\left(a^2-3b^2\right)\tan(c+dx)}{b^4d}+\frac{\left(a^2-3b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^3d}+\frac{\left(-3a^2b^2+a^4+3b^4\right)\tanh^{-1}(\sin(c+dx))}{b^5d}+\frac{\left(a^2-3b^2\right)\mathrm{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(x/a) + (3*ArcTanh[Sin[c + d*x]])/(8*b*d) + ((a^2 - 3*b^2)*ArcTanh[Sin[c +
 d*x]])/(2*b^3*d) + ((a^4 - 3*a^2*b^2 + 3*b^4)*ArcTanh[Sin[c + d*x]])/(b^5*
d) - (2*(a - b)^(5/2)*(a + b)^(5/2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/
Sqrt[a + b]])/(a*b^5*d) - (a*Tan[c + d*x])/(b^2*d) - (a*(a^2 - 3*b^2)*Tan[c
 + d*x])/(b^4*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(8*b*d) + ((a^2 - 3*b^2)*S
ec[c + d*x]*Tan[c + d*x])/(2*b^3*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(4*b*d)
 - (a*Tan[c + d*x]^3)/(3*b^2*d)
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sin(c+dx)\tan^5(c+dx)}{b+a\cos(c+dx)}\,dx$$

$$= \int\left(-\frac{1}{a} - \frac{\left(a^2-b^2\right)^3}{ab^5(b+a\cos(c+dx))} + \frac{\left(a^4-3a^2b^2+3b^4\right)\sec(c+dx)}{b^5} + \frac{\left(-a^3+3ab^2\right)\sec^2(c+d\right.$$

$$= -\frac{x}{a} - \frac{a\int\sec^4(c+dx)\,dx}{b^2} + \frac{\int\sec^5(c+dx)\,dx}{b} - \frac{\left(a\left(a^2-3b^2\right)\right)\int\sec^2(c+dx)\,dx}{b^4} + \frac{\left(a^2-3b^2\right.}{}$$

$$= -\frac{x}{a} + \frac{\left(a^4-3a^2b^2+3b^4\right)\tanh^{-1}(\sin(c+dx))}{b^5d} + \frac{\left(a^2-3b^2\right)\sec(c+dx)\tan(c+dx)}{2b^3d} + \frac{\sec^3(c}{}$$

$$= -\frac{x}{a} + \frac{\left(a^2-3b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} + \frac{\left(a^4-3a^2b^2+3b^4\right)\tanh^{-1}(\sin(c+dx))}{b^5d} - \frac{2(a-b)^{5/2}}{}$$

$$= -\frac{x}{a} + \frac{3\tanh^{-1}(\sin(c+dx))}{8bd} + \frac{\left(a^2-3b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} + \frac{\left(a^4-3a^2b^2+3b^4\right)\tanh^{-1}(}{b^5d}$$

Mathematica [B] time = 6.16505, size = 907, normalized size = 4.58

$$-\frac{2\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))\sec(c+dx)\left(b^2-a^2\right)^3}{ab^5\sqrt{a^2-b^2}\,d(a+b\sec(c+dx))} - \frac{a(b+a\cos(c+dx))\sec(c+dx)\sin\left(\frac{1}{2}(c\right.}{6b^2d(a+b\sec(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(((c + d*x)*(b + a*Cos[c + d*x])*Sec[c + d*x])/(a*d*(a + b*Sec[c + d*x])))
 - (2*(-a^2 + b^2)^3*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(
b + a*Cos[c + d*x])*Sec[c + d*x])/(a*b^5*Sqrt[a^2 - b^2]*d*(a + b*Sec[c + d
*x])) + ((-8*a^4 + 20*a^2*b^2 - 15*b^4)*(b + a*Cos[c + d*x])*Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]]*Sec[c + d*x])/(8*b^5*d*(a + b*Sec[c + d*x])) + (
(8*a^4 - 20*a^2*b^2 + 15*b^4)*(b + a*Cos[c + d*x])*Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]]*Sec[c + d*x])/(8*b^5*d*(a + b*Sec[c + d*x])) + ((b + a*Cos
[c + d*x])*Sec[c + d*x])/(16*b*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] - S
in[(c + d*x)/2])^4) + ((12*a^2 - 4*a*b - 27*b^2)*(b + a*Cos[c + d*x])*Sec[c
 + d*x])/(48*b^3*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2
])^2) - (a*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[(c + d*x)/2])/(6*b^2*d*(a
```

```
+ b*Sec[c + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) - ((b + a*Cos[c
+ d*x])*Sec[c + d*x])/(16*b*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^4) - (a*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[(c + d*x)/2])/(
6*b^2*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + ((-
12*a^2 + 4*a*b + 27*b^2)*(b + a*Cos[c + d*x])*Sec[c + d*x])/(48*b^3*d*(a +
b*Sec[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + ((b + a*Cos[c +
d*x])*Sec[c + d*x]*(-3*a^3*Sin[(c + d*x)/2] + 7*a*b^2*Sin[(c + d*x)/2]))/(3
*b^4*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + ((b +
a*Cos[c + d*x])*Sec[c + d*x]*(-3*a^3*Sin[(c + d*x)/2] + 7*a*b^2*Sin[(c + d*
x)/2]))/(3*b^4*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
)
```

Maple [B] time = 0.082, size = 785, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/2/d/b/(tan(1/2*d*x+1/2*c)-1)^3-5/8/d/b/(tan(1/2*d*x+1/2*c)-1)^2-15/8/d/b*
ln(tan(1/2*d*x+1/2*c)-1)-7/8/d/b/(tan(1/2*d*x+1/2*c)-1)-1/4/d/b/(tan(1/2*d*
x+1/2*c)+1)^4+1/2/d/b/(tan(1/2*d*x+1/2*c)+1)^3+5/8/d/b/(tan(1/2*d*x+1/2*c)+
1)^2+15/8/d/b*ln(tan(1/2*d*x+1/2*c)+1)-7/8/d/b/(tan(1/2*d*x+1/2*c)+1)+1/4/d
/b/(tan(1/2*d*x+1/2*c)-1)^4-2/d/a*arctan(tan(1/2*d*x+1/2*c))+2/d*b/a/((a+b)
*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+1/d/b^4
/(tan(1/2*d*x+1/2*c)-1)*a^3+1/2/d/b^3/(tan(1/2*d*x+1/2*c)-1)*a^2-2/d/b^2/(t
an(1/2*d*x+1/2*c)-1)*a+1/3/d/b^2/(tan(1/2*d*x+1/2*c)+1)^3*a-1/2/d/b^3/(tan(
1/2*d*x+1/2*c)+1)^2*a^2-1/2/d/b^2/(tan(1/2*d*x+1/2*c)+1)^2*a+1/d/b^5*ln(tan
(1/2*d*x+1/2*c)+1)*a^4-5/2/d/b^3*ln(tan(1/2*d*x+1/2*c)+1)*a^2+1/d/b^4/(tan(
1/2*d*x+1/2*c)+1)*a^3+1/2/d/b^3/(tan(1/2*d*x+1/2*c)+1)*a^2-2/d/b^2/(tan(1/2
*d*x+1/2*c)+1)*a+1/3/d/b^2/(tan(1/2*d*x+1/2*c)-1)^3*a+1/2/d/b^3/(tan(1/2*d*
x+1/2*c)-1)^2*a^2+1/2/d/b^2/(tan(1/2*d*x+1/2*c)-1)^2*a-1/d/b^5*ln(tan(1/2*d
*x+1/2*c)-1)*a^4+5/2/d/b^3*ln(tan(1/2*d*x+1/2*c)-1)*a^2-2/d/b^5*a^5/((a+b)*
(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+6/d/b^3*
a^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2
))-6/d/b*a/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b
))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.8257, size = 1423, normalized size = 7.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/48*(48*b^5*d*x*cos(d*x + c)^4 - 24*(a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b
^2)*cos(d*x + c)^4*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 -
 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*co
s(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 3*(8*a^5 - 20*a^3*b^2 + 15*a*b^
4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) + 3*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)
*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*a^2*b^3*cos(d*x + c) - 6*a*b^
4 + 8*(3*a^4*b - 7*a^2*b^3)*cos(d*x + c)^3 - 3*(4*a^3*b^2 - 9*a*b^4)*cos(d*
x + c)^2)*sin(d*x + c))/(a*b^5*d*cos(d*x + c)^4), -1/48*(48*b^5*d*x*cos(d*x
 + c)^4 + 48*(a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b
^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^4 - 3*(8*
a^5 - 20*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) + 3*(8*a^
5 - 20*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*a^2
*b^3*cos(d*x + c) - 6*a*b^4 + 8*(3*a^4*b - 7*a^2*b^3)*cos(d*x + c)^3 - 3*(4
*a^3*b^2 - 9*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(a*b^5*d*cos(d*x + c)^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**6/(a + b*sec(c + d*x)), x)
```

Giac [B] time = 4.70169, size = 639, normalized size = 3.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(24*(d*x + c)/a - 3*(8*a^4 - 20*a^2*b^2 + 15*b^4)*log(abs(tan(1/2*d*x
 + 1/2*c) + 1))/b^5 + 3*(8*a^4 - 20*a^2*b^2 + 15*b^4)*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/b^5 + 48*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/2*(d
*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan
(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a*b^5) - 2*(24*a^3*
tan(1/2*d*x + 1/2*c)^7 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 48*a*b^2*tan(1/2
*d*x + 1/2*c)^7 - 21*b^3*tan(1/2*d*x + 1/2*c)^7 - 72*a^3*tan(1/2*d*x + 1/2*
c)^5 - 12*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 176*a*b^2*tan(1/2*d*x + 1/2*c)^5 +
 45*b^3*tan(1/2*d*x + 1/2*c)^5 + 72*a^3*tan(1/2*d*x + 1/2*c)^3 - 12*a^2*b*t
an(1/2*d*x + 1/2*c)^3 - 176*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 45*b^3*tan(1/2*d
*x + 1/2*c)^3 - 24*a^3*tan(1/2*d*x + 1/2*c) + 12*a^2*b*tan(1/2*d*x + 1/2*c)
 + 48*a*b^2*tan(1/2*d*x + 1/2*c) - 21*b^3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d
*x + 1/2*c)^2 - 1)^4*b^4))/d
```

3.295 $\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=126

$$\frac{(2a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{2b^3d}-\frac{a\tan(c+dx)}{b^2d}-\frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ab^3d}+\frac{x}{a}+\frac{\tan(c+dx)\,\mathrm{se}}{2bd}$$

```
[Out] x/a + ((2*a^2 - 3*b^2)*ArcTanh[Sin[c + d*x]])/(2*b^3*d) - (2*(a - b)^(3/2)*
(a + b)^(3/2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*b^3*d
) - (a*Tan[c + d*x])/(b^2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.342726, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3898, 2893, 3057, 2659, 208, 3770}

$$\frac{(2a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{2b^3d}-\frac{a\tan(c+dx)}{b^2d}-\frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ab^3d}+\frac{x}{a}+\frac{\tan(c+dx)\,\mathrm{se}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] x/a + ((2*a^2 - 3*b^2)*ArcTanh[Sin[c + d*x]])/(2*b^3*d) - (2*(a - b)^(3/2)*
(a + b)^(3/2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*b^3*d
) - (a*Tan[c + d*x])/(b^2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*b*d)
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
```

```
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3057

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(C*x)/(b*d), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sin(c+dx)\tan^3(c+dx)}{b+a\cos(c+dx)}\,dx$$

$$= -\frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd} - \frac{\int \frac{\left(-2a^2+3b^2-ab\cos(c+dx)-2b^2\cos^2(c+dx)\right)\sec(c+dx)}{b+a\cos(c+dx)}\,dx}{2b^2}$$

$$= \frac{x}{a} - \frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd} - \frac{\left(a^2-b^2\right)^2\int \frac{1}{b+a\cos(c+dx)}\,dx}{ab^3} - \frac{\left(-2a^2+3b^2\right)\int}{2b}$$

$$= \frac{x}{a} + \frac{\left(2a^2-3b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} - \frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd} - \frac{\left(2\left(a^2-b\right.\right.}{}$$

$$= \frac{x}{a} + \frac{\left(2a^2-3b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} - \frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ab^3d} - \frac{a\tan}{}$$

Mathematica [B] time = 2.14776, size = 287, normalized size = 2.28

$$\frac{\sec(c+dx)(a\cos(c+dx)+b)\left(\frac{8\left(a^2-b^2\right)^{3/2}\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ab^3} - \frac{4a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{b^3} + \frac{4a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}\right.\right.}{b^3}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((4*c)/a + (4*d*x)/a + (8*(a^2 - b^2)^(3
/2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*b^3) - (4*a^2*
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/b^3 + (6*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]])/b + (4*a^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/b^3
- (6*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/b + 1/(b*(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^2) - 1/(b*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) - (4*
a*Tan[c + d*x])/b^2))/(4*d*(a + b*Sec[c + d*x]))
```

Maple [B] time = 0.072, size = 374, normalized size = 3.

$$2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{ad} - 2\,\frac{a^3}{db^3\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan\left(1/2\,dx+c/2\right)}{\sqrt{(a+b)(a-b)}}\right) + 4\,\frac{a}{db\sqrt{(a+b)(a-b)}}\mathrm{Artanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d/a*arctan(tan(1/2*d*x+1/2*c))-2/d/b^3*a^3/((a+b)*(a-b))^(1/2)*arctanh((a
-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+4/d/b*a/((a+b)*(a-b))^(1/2)*arc
tanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-2/d*b/a/((a+b)*(a-b))^(1
/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-1/2/d/b/(tan(1/2*
d*x+1/2*c)+1)^2+1/2/d/b/(tan(1/2*d*x+1/2*c)+1)+1/d/b^2/(tan(1/2*d*x+1/2*c)+
1)*a+1/d/b^3*ln(tan(1/2*d*x+1/2*c)+1)*a^2-3/2/d/b*ln(tan(1/2*d*x+1/2*c)+1)+
1/2/d/b/(tan(1/2*d*x+1/2*c)-1)^2+1/2/d/b/(tan(1/2*d*x+1/2*c)-1)+1/d/b^2/(ta
n(1/2*d*x+1/2*c)-1)*a-1/d/b^3*ln(tan(1/2*d*x+1/2*c)-1)*a^2+3/2/d/b*ln(tan(1
/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68004, size = 1048, normalized size = 8.32

$$\left[\frac{4\,b^3dx\cos\left(dx+c\right)^2-2\left(a^2-b^2\right)^{\frac{3}{2}}\cos\left(dx+c\right)^2\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/4*(4*b^3*d*x*cos(d*x + c)^2 - 2*(a^2 - b^2)^(3/2)*cos(d*x + c)^2*log((2*
a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(
d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d
*x + c) + b^2)) + (2*a^3 - 3*a*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) -
```

```
(2*a^3 - 3*a*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(2*a^2*b*cos(d*
x + c) - a*b^2)*sin(d*x + c))/(a*b^3*d*cos(d*x + c)^2), 1/4*(4*b^3*d*x*cos(
d*x + c)^2 - 4*(a^2 - b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos
(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^2 + (2*a^3 - 3*a*b^
2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^3 - 3*a*b^2)*cos(d*x + c)^2*
log(-sin(d*x + c) + 1) - 2*(2*a^2*b*cos(d*x + c) - a*b^2)*sin(d*x + c))/(a*
b^3*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

Giac [B] time = 2.50431, size = 324, normalized size = 2.57

$$\frac{\frac{2(dx+c)}{a} + \frac{(2a^2-3b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^3} - \frac{(2a^2-3b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^3} - \frac{4(a^4-2a^2b^2+b^4)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}d}{}\right)\right)}{\sqrt{-a^2+b^2}ab^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)/a + (2*a^2 - 3*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3
 - (2*a^2 - 3*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^3 - 4*(a^4 - 2*a^2*
b^2 + b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*t
an(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2
 + b^2)*a*b^3) + 2*(2*a*tan(1/2*d*x + 1/2*c)^3 + b*tan(1/2*d*x + 1/2*c)^3 -
 2*a*tan(1/2*d*x + 1/2*c) + b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^
2 - 1)^2*b^2))/d
```

3.296 $\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=76

$$-\frac{2\sqrt{a-b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd}-\frac{x}{a}+\frac{\tanh^{-1}(\sin(c+dx))}{bd}$$

```
[Out] -(x/a) + ArcTanh[Sin[c + d*x]]/(b*d) - (2*Sqrt[a - b]*Sqrt[a + b]*ArcTanh[(
Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*b*d)
```

Rubi [A] time = 0.184586, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3894, 4051, 3770, 3919, 3831, 2659, 208}

$$-\frac{2\sqrt{a-b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd}-\frac{x}{a}+\frac{\tanh^{-1}(\sin(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(x/a) + ArcTanh[Sin[c + d*x]]/(b*d) - (2*Sqrt[a - b]*Sqrt[a + b]*ArcTanh[(
Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*b*d)
```

Rule 3894

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4051

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)), x_Symbol] :> Dist[C/b, Int[Csc[e + f*x], x], x] + Dist[1/b, Int[
(A*b - a*C*Csc[e + f*x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 A, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx &= \int \frac{-1+\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx \\
&= \frac{\int \sec(c+dx)\,dx}{b} + \frac{\int \frac{-b-a\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{b} \\
&= -\frac{x}{a} + \frac{\tanh^{-1}(\sin(c+dx))}{bd} - \left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx \\
&= -\frac{x}{a} + \frac{\tanh^{-1}(\sin(c+dx))}{bd} - \frac{\left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b} \\
&= -\frac{x}{a} + \frac{\tanh^{-1}(\sin(c+dx))}{bd} - \frac{\left(2\left(\frac{a}{b}-\frac{b}{a}\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd} \\
&= -\frac{x}{a} + \frac{\tanh^{-1}(\sin(c+dx))}{bd} - \frac{2\sqrt{a-b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd}
\end{aligned}
$$

Mathematica [A] time = 0.14621, size = 115, normalized size = 1.51

$$
-\frac{-2\sqrt{a^2-b^2}\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right) + a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+
}{abd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((b*c + b*d*x - 2*Sqrt[a^2 - b^2]*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt
[a^2 - b^2]] + a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - a*Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]])/(a*b*d))
```

Maple [B] time = 0.054, size = 153, normalized size = 2.

$$
-2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{ad} - 2\,\frac{a}{db\sqrt{(a+b)(a-b)}}\text{Artanh}\left(\frac{(a-b)\tan\left(1/2\,dx+c/2\right)}{\sqrt{(a+b)(a-b)}}\right) + 2\,\frac{b}{ad\sqrt{(a+b)(a-b)}}\text{Artanh}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/d/a*arctan(tan(1/2*d*x+1/2*c))-2/d/b*a/((a+b)*(a-b))^(1/2)*arctanh((a-b)
*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+2/d*b/a/((a+b)*(a-b))^(1/2)*arctan
h((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+1/d/b*ln(tan(1/2*d*x+1/2*c)
+1)-1/d/b*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.864895, size = 599, normalized size = 7.88

$$\left[-\frac{2\,bdx - a\log\left(\sin\left(dx+c\right)+1\right) + a\log\left(-\sin\left(dx+c\right)+1\right) - \sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c) - \left(a^2-2\,b^2\right)\cos(dx+c)^2 - 2\,\sqrt{a^2-b^2}(b\cos(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*b*d*x - a*log(sin(d*x + c) + 1) + a*log(-sin(d*x + c) + 1) - sqrt(
a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(
a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x +
c)^2 + 2*a*b*cos(d*x + c) + b^2)))/(a*b*d), -1/2*(2*b*d*x - a*log(sin(d*x +
 c) + 1) + a*log(-sin(d*x + c) + 1) + 2*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2
+ b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))))/(a*b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

Giac [B] time = 1.58747, size = 189, normalized size = 2.49

$$-\frac{\frac{dx+c}{a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b}+\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)\left(a^2-b^2\right)}{\sqrt{-a^2+b^2}ab}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)/a - log(abs(tan(1/2*d*x + 1/2*c) + 1))/b + log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/b + 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + ar
ctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*
(a^2 - b^2)/(sqrt(-a^2 + b^2)*a*b))/d
```

3.297 $\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)} dx$

Optimal. Leaf size=106

$$-\frac{a\cot(c+dx)}{d\left(a^2-b^2\right)}+\frac{b\csc(c+dx)}{d\left(a^2-b^2\right)}-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a^2-b^2}\tan\left(\frac{1}{2}(c+dx)\right)}{a+b}\right)}{ad\left(a^2-b^2\right)^{3/2}}-\frac{x}{a}$$

```
[Out] -(x/a) - (2*b^3*ArcTanh[(Sqrt[a^2 - b^2]*Tan[(c + d*x)/2])/(a + b)])/(a*(a^
2 - b^2)^(3/2)*d) - (a*Cot[c + d*x])/((a^2 - b^2)*d) + (b*Csc[c + d*x])/((a
^2 - b^2)*d)
```

Rubi [A] time = 0.243004, antiderivative size = 135, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3898, 2902, 2606, 8, 3473, 2735, 2659, 208}

$$-\frac{a\cot(c+dx)}{d\left(a^2-b^2\right)}+\frac{b\csc(c+dx)}{d\left(a^2-b^2\right)}+\frac{b^2x}{a\left(a^2-b^2\right)}-\frac{ax}{a^2-b^2}-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((a*x)/(a^2 - b^2)) + (b^2*x)/(a*(a^2 - b^2)) - (2*b^3*ArcTanh[(Sqrt[a - b
]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*(a + b)^(3/2)*d) - (a*Co
t[c + d*x])/((a^2 - b^2)*d) + (b*Csc[c + d*x])/((a^2 - b^2)*d)
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
```

```
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cos(c+dx)\cot^2(c+dx)}{b+a\cos(c+dx)}\,dx$$
$$= \frac{a\int \cot^2(c+dx)\,dx}{a^2-b^2} - \frac{b\int \cot(c+dx)\csc(c+dx)\,dx}{a^2-b^2} + \frac{b^2\int \frac{\cos(c+dx)}{b+a\cos(c+dx)}\,dx}{a^2-b^2}$$
$$= \frac{b^2x}{a\left(a^2-b^2\right)} - \frac{a\cot(c+dx)}{\left(a^2-b^2\right)d} - \frac{a\int 1\,dx}{a^2-b^2} - \frac{b^3\int \frac{1}{b+a\cos(c+dx)}\,dx}{a\left(a^2-b^2\right)} + \frac{b\,\mathrm{Subst}(\int 1\,dx, x, \csc(c+dx)}{\left(a^2-b^2\right)d}$$
$$= -\frac{ax}{a^2-b^2} + \frac{b^2x}{a\left(a^2-b^2\right)} - \frac{a\cot(c+dx)}{\left(a^2-b^2\right)d} + \frac{b\csc(c+dx)}{\left(a^2-b^2\right)d} - \frac{\left(2b^3\right)\mathrm{Subst}\left(\int \frac{1}{a+b+(-a+b)x^2}\,dx, x, \mathrm{t}\right.}{a\left(a^2-b^2\right)d}$$
$$= -\frac{ax}{a^2-b^2} + \frac{b^2x}{a\left(a^2-b^2\right)} - \frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}(a+b)^{3/2}d} - \frac{a\cot(c+dx)}{\left(a^2-b^2\right)d} + \frac{b\csc(c+dx)}{\left(a^2-b^2\right)d}$$

Mathematica [A] time = 0.409352, size = 147, normalized size = 1.39

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{a^2-b^2}\left(\left(a^2-b^2\right)(c+dx)\sin(c+dx)+a^2\cos(c+dx)-ab\right)-2b^3\sin(c+dx)\tanh\right.}{2ad(a-b)(a+b)\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(-2*b^3*ArcTanh[((-a + b)*Tan[(c + d*x)
/2])/Sqrt[a^2 - b^2]]*Sin[c + d*x] + Sqrt[a^2 - b^2]*(-(a*b) + a^2*Cos[c +
d*x] + (a^2 - b^2)*(c + d*x)*Sin[c + d*x])))/(2*a*(a - b)*(a + b)*Sqrt[a^2
- b^2]*d)
```

Maple [A] time = 0.074, size = 123, normalized size = 1.2

$$\frac{1}{2\,d\,(a-b)}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - 2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{ad} - 2\,\frac{b^3}{d\,(a-b)\,(a+b)\,a\sqrt{(a+b)\,(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan(1/2}{\sqrt{(a+b)\,(}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/2/d/(a-b)*tan(1/2*d*x+1/2*c)-2/d/a*arctan(tan(1/2*d*x+1/2*c))-2/d/(a-b)/(
a+b)*b^3/a/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b
))^(1/2))-1/2/d/(a+b)/tan(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.969172, size = 815, normalized size = 7.69

$$\left[-\frac{\sqrt{a^2-b^2}b^3\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)\sin\left(dx+c\right)-2\,a^3b+2\,ab^3+2\left(a^4-\right.}{2\left(a^5-2\,a^3b^2+ab^4\right)d\sin\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a^2 - b^2)*b^3*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
+ c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)
/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*a^3*b +
2*a*b^3 + 2*(a^4 - 2*a^2*b^2 + b^4)*d*x*sin(d*x + c) + 2*(a^4 - a^2*b^2)*co
s(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*sin(d*x + c)), -(sqrt(-a^2 + b^2)*
b^3*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)
))*sin(d*x + c) - a^3*b + a*b^3 + (a^4 - 2*a^2*b^2 + b^4)*d*x*sin(d*x + c)
+ (a^4 - a^2*b^2)*cos(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.34272, size = 190, normalized size = 1.79

$$-\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b^3}{\left(a^3-ab^2\right)\sqrt{-a^2+b^2}}+\frac{2(dx+c)}{a}-\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a-b}+\frac{1}{(a+b)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b^3/((a^3 - a
*b^2)*sqrt(-a^2 + b^2)) + 2*(d*x + c)/a - tan(1/2*d*x + 1/2*c)/(a - b) + 1/
((a + b)*tan(1/2*d*x + 1/2*c)))/d
```

3.298 $\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=177

$$-\frac{a\cot^3(c+dx)}{3d\left(a^2-b^2\right)}+\frac{a\left(a^2-2b^2\right)\cot(c+dx)}{d\left(a^2-b^2\right)^2}+\frac{b\csc^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{b\left(a^2-2b^2\right)\csc(c+dx)}{d\left(a^2-b^2\right)^2}-\frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a^2-b^2}\tan\left(\frac{1}{2}(c+dx)\right)}{a+b}\right)}{ad\left(a^2-b^2\right)^{5/2}}$$

```
[Out] x/a - (2*b^5*ArcTanh[(Sqrt[a^2 - b^2]*Tan[(c + d*x)/2])/(a + b)])/(a*(a^2 -
 b^2)^(5/2)*d) + (a*(a^2 - 2*b^2)*Cot[c + d*x])/((a^2 - b^2)^2*d) - (a*Cot[
c + d*x]^3)/(3*(a^2 - b^2)*d) - (b*(a^2 - 2*b^2)*Csc[c + d*x])/((a^2 - b^2)
^2*d) + (b*Csc[c + d*x]^3)/(3*(a^2 - b^2)*d)
```

Rubi [A] time = 0.386905, antiderivative size = 256, normalized size of antiderivative = 1.45, number of steps used = 15, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3898, 2902, 2606, 3473, 8, 2735, 2659, 208}

$$-\frac{a\cot^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{ab^2\cot(c+dx)}{d\left(a^2-b^2\right)^2}+\frac{a\cot(c+dx)}{d\left(a^2-b^2\right)}+\frac{b\csc^3(c+dx)}{3d\left(a^2-b^2\right)}+\frac{b^3\csc(c+dx)}{d\left(a^2-b^2\right)^2}-\frac{b\csc(c+dx)}{d\left(a^2-b^2\right)}+\frac{b^4x}{a\left(a^2-b^2\right)^2}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((a*b^2*x)/(a^2 - b^2)^2) + (b^4*x)/(a*(a^2 - b^2)^2) + (a*x)/(a^2 - b^2)
- (2*b^5*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*(a - b)^(5
/2)*(a + b)^(5/2)*d) - (a*b^2*Cot[c + d*x])/((a^2 - b^2)^2*d) + (a*Cot[c +
d*x])/((a^2 - b^2)*d) - (a*Cot[c + d*x]^3)/(3*(a^2 - b^2)*d) + (b^3*Csc[c +
 d*x])/((a^2 - b^2)^2*d) - (b*Csc[c + d*x])/((a^2 - b^2)*d) + (b*Csc[c + d*
x]^3)/(3*(a^2 - b^2)*d)
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cos(c+dx)\cot^4(c+dx)}{b+a\cos(c+dx)}\,dx$$

$$= \frac{a\int \cot^4(c+dx)\,dx}{a^2-b^2} - \frac{b\int \cot^3(c+dx)\csc(c+dx)\,dx}{a^2-b^2} + \frac{b^2\int \frac{\cos(c+dx)\cot^2(c+dx)}{b+a\cos(c+dx)}\,dx}{a^2-b^2}$$

$$= -\frac{a\cot^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{\left(ab^2\right)\int \cot^2(c+dx)\,dx}{\left(a^2-b^2\right)^2} - \frac{b^3\int \cot(c+dx)\csc(c+dx)\,dx}{\left(a^2-b^2\right)^2} + \frac{b^4\int \frac{\cos(c+dx)}{b+a\cos(c+dx)}}{\left(a^2-b^2\right)^2}$$

$$= \frac{b^4x}{a\left(a^2-b^2\right)^2} - \frac{ab^2\cot(c+dx)}{\left(a^2-b^2\right)^2 d} + \frac{a\cot(c+dx)}{\left(a^2-b^2\right)d} - \frac{a\cot^3(c+dx)}{3\left(a^2-b^2\right)d} - \frac{b\csc(c+dx)}{\left(a^2-b^2\right)d} + \frac{b\csc^3(c+d}{3\left(a^2-b^2\right)}$$

$$= -\frac{ab^2x}{\left(a^2-b^2\right)^2} + \frac{b^4x}{a\left(a^2-b^2\right)^2} + \frac{ax}{a^2-b^2} - \frac{ab^2\cot(c+dx)}{\left(a^2-b^2\right)^2 d} + \frac{a\cot(c+dx)}{\left(a^2-b^2\right)d} - \frac{a\cot^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{b^3}{(}$$

$$= -\frac{ab^2x}{\left(a^2-b^2\right)^2} + \frac{b^4x}{a\left(a^2-b^2\right)^2} + \frac{ax}{a^2-b^2} - \frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}d} - \frac{ab^2\cot(c+dx)}{\left(a^2-b^2\right)^2 d} + \frac{a\,\mathrm{co}}{(a}$$

Mathematica [B] time = 6.16237, size = 416, normalized size = 2.35

$$\frac{2b^5\sec(c+dx)(a\cos(c+dx)+b)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}\left(b^2-a^2\right)^2(a+b\sec(c+dx))} + \frac{(c+dx)\sec(c+dx)(a\cos(c+dx)+b)}{ad(a+b\sec(c+dx))} + \frac{\csc\left(\frac{1}{2}(c+dx)\right)s}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((c + d*x)*(b + a*Cos[c + d*x])*Sec[c + d*x])/(a*d*(a + b*Sec[c + d*x])) +
(2*b^5*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c +
d*x])*Sec[c + d*x])/(a*Sqrt[a^2 - b^2]*(-a^2 + b^2)^2*d*(a + b*Sec[c + d*x]
)) + ((8*a*Cos[(c + d*x)/2] + 11*b*Cos[(c + d*x)/2])*(b + a*Cos[c + d*x])*C
sc[(c + d*x)/2]*Sec[c + d*x])/(12*(a + b)^2*d*(a + b*Sec[c + d*x])) - ((b +
 a*Cos[c + d*x])*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*Sec[c + d*x])/(24*(a +
 b)*d*(a + b*Sec[c + d*x])) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]*Sec[c
+ d*x]*(-8*a*Sin[(c + d*x)/2] + 11*b*Sin[(c + d*x)/2]))/(12*(-a + b)^2*d*(a
 + b*Sec[c + d*x])) - ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sec[c + d*x]
```

```
*Tan[(c + d*x)/2])/(24*(-a + b)*d*(a + b*Sec[c + d*x]))
```

Maple [A] time = 0.085, size = 238, normalized size = 1.3

$$\frac{a}{24\,d\,(a-b)^{2}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}-\frac{b}{24\,d\,(a-b)^{2}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}-\frac{5\,a}{8\,d\,(a-b)^{2}}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{7\,b}{8\,d\,(a-b)^{2}}\tan\left(\frac{dx}{2}+\frac{c}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/24/d/(a-b)^2*tan(1/2*d*x+1/2*c)^3*a-1/24/d/(a-b)^2*b*tan(1/2*d*x+1/2*c)^3
-5/8/d/(a-b)^2*a*tan(1/2*d*x+1/2*c)+7/8/d/(a-b)^2*b*tan(1/2*d*x+1/2*c)+2/d/
a*arctan(tan(1/2*d*x+1/2*c))-2/d/(a+b)^2/(a-b)^2*b^5/a/((a+b)*(a-b))^(1/2)*
arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-1/24/d/(a+b)/tan(1/2*
d*x+1/2*c)^3+5/8/d/(a+b)^2/tan(1/2*d*x+1/2*c)*a+7/8/d/(a+b)^2/tan(1/2*d*x+1
/2*c)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.974179, size = 1623, normalized size = 9.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/6*(4*a^5*b - 14*a^3*b^3 + 10*a*b^5 + 2*(4*a^6 - 11*a^4*b^2 + 7*a^2*b^4)*
cos(d*x + c)^3 + 3*(b^5*cos(d*x + c)^2 - b^5)*sqrt(a^2 - b^2)*log((2*a*b*co
s(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x +
c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c
) + b^2))*sin(d*x + c) - 6*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(d*x + c)^2 - 6
*(a^6 - 3*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c) + 6*((a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6)*d*x*cos(d*x + c)^2 - (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*x)*sin
(d*x + c))/(((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^2 - (a^7
- 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d)*sin(d*x + c)), 1/3*(2*a^5*b - 7*a^3*b^3
 + 5*a*b^5 + (4*a^6 - 11*a^4*b^2 + 7*a^2*b^4)*cos(d*x + c)^3 - 3*(b^5*cos(d
*x + c)^2 - b^5)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c)
+ a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - 3*(a^5*b - 3*a^3*b^3 + 2*a*
b^5)*cos(d*x + c)^2 - 3*(a^6 - 3*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c) + 3*((a^
6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*x*cos(d*x + c)^2 - (a^6 - 3*a^4*b^2 + 3*
a^2*b^4 - b^6)*d*x)*sin(d*x + c))/(((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d
*cos(d*x + c)^2 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d)*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.34975, size = 386, normalized size = 2.18

$$-\frac{\frac{48\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b^5}{\left(a^5-2a^3b^2+ab^4\right)\sqrt{-a^2+b^2}}-\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-15a^2\tan\left(\frac{1}{2}dx\right.}{a^3-3a^2b+3ab^2-b^3}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(48*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*ta
n(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b^5/((a^5 -
 2*a^3*b^2 + a*b^4)*sqrt(-a^2 + b^2)) - (a^2*tan(1/2*d*x + 1/2*c)^3 - 2*a*b
*tan(1/2*d*x + 1/2*c)^3 + b^2*tan(1/2*d*x + 1/2*c)^3 - 15*a^2*tan(1/2*d*x +
 1/2*c) + 36*a*b*tan(1/2*d*x + 1/2*c) - 21*b^2*tan(1/2*d*x + 1/2*c))/(a^3 -
 3*a^2*b + 3*a*b^2 - b^3) - 24*(d*x + c)/a - (15*a*tan(1/2*d*x + 1/2*c)^2 +
 21*b*tan(1/2*d*x + 1/2*c)^2 - a - b)/((a^2 + 2*a*b + b^2)*tan(1/2*d*x + 1/
2*c)^3))/d
```

3.299 $\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=255

$$\frac{(3a^2-4b^2)\sec^4(c+dx)}{4b^4d}-\frac{4a(a^2-2b^2)\sec^3(c+dx)}{3b^5d}+\frac{(-12a^2b^2+5a^4+6b^4)\sec^2(c+dx)}{2b^6d}-\frac{2a(-8a^2b^2+3a^4+6b^4)\,\mathrm{s}}{b^7d}$$

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) + ((a^2 - b^2)^3*(7*a^2 + b^2)*Log[a + b*Sec[c
 + d*x]])/(a^2*b^8*d) - (2*a*(3*a^4 - 8*a^2*b^2 + 6*b^4)*Sec[c + d*x])/(b^7
*d) + ((5*a^4 - 12*a^2*b^2 + 6*b^4)*Sec[c + d*x]^2)/(2*b^6*d) - (4*a*(a^2 -
 2*b^2)*Sec[c + d*x]^3)/(3*b^5*d) + ((3*a^2 - 4*b^2)*Sec[c + d*x]^4)/(4*b^4
*d) - (2*a*Sec[c + d*x]^5)/(5*b^3*d) + Sec[c + d*x]^6/(6*b^2*d) + (a^2 - b^
2)^4/(a*b^8*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.205109, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 894}

$$\frac{(3a^2-4b^2)\sec^4(c+dx)}{4b^4d}-\frac{4a(a^2-2b^2)\sec^3(c+dx)}{3b^5d}+\frac{(-12a^2b^2+5a^4+6b^4)\sec^2(c+dx)}{2b^6d}-\frac{2a(-8a^2b^2+3a^4+6b^4)\,\mathrm{s}}{b^7d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^9/(a + b*Sec[c + d*x])^2,x]
```



```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) + ((a^2 - b^2)^3*(7*a^2 + b^2)*Log[a + b*Sec[c
 + d*x]])/(a^2*b^8*d) - (2*a*(3*a^4 - 8*a^2*b^2 + 6*b^4)*Sec[c + d*x])/(b^7
*d) + ((5*a^4 - 12*a^2*b^2 + 6*b^4)*Sec[c + d*x]^2)/(2*b^6*d) - (4*a*(a^2 -
 2*b^2)*Sec[c + d*x]^3)/(3*b^5*d) + ((3*a^2 - 4*b^2)*Sec[c + d*x]^4)/(4*b^4
*d) - (2*a*Sec[c + d*x]^5)/(5*b^3*d) + Sec[c + d*x]^6/(6*b^2*d) + (a^2 - b^
2)^4/(a*b^8*d*(a + b*Sec[c + d*x]))
```


Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^4}{x(a+x)^2}\,dx, x, b\sec(c+dx)\right)}{b^8 d}$$

$$= \frac{\text{Subst}\left(\int \left(-2a\left(3a^4-8a^2b^2+6b^4\right)+\frac{b^8}{a^2x}+\left(5a^4-12a^2b^2+6b^4\right)x-4a\left(a^2-2b^2\right)x^2+\left(3\right.\right.\right.}{b^8 d}$$

$$= -\frac{\log(\cos(c+dx))}{a^2 d}+\frac{\left(a^2-b^2\right)^3\left(7a^2+b^2\right)\log(a+b\sec(c+dx))}{a^2b^8d}-\frac{2a\left(3a^4-8a^2b^2+6b^4\right)}{b^7 d}$$

Mathematica [B] time = 6.27456, size = 528, normalized size = 2.07

$$\frac{\left(3a^2-4b^2\right)\sec^6(c+dx)(a\cos(c+dx)+b)^2}{4b^4d(a+b\sec(c+dx))^2}+\frac{4a\left(2b^2-a^2\right)\sec^5(c+dx)(a\cos(c+dx)+b)^2}{3b^5d(a+b\sec(c+dx))^2}+\frac{\left(-12a^2b^2+5a^4+6b^4\right)}{2b^6d(a+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^9/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((-a + b)^4*(a + b)^4*(b + a*Cos[c + d*x])*Sec[c + d*x]^2)/(a^2*b^7*d*(a
+ b*Sec[c + d*x])^2)) + ((-7*a^6 + 20*a^4*b^2 - 18*a^2*b^4 + 4*b^6)*(b + a*
Cos[c + d*x])^2*Log[Cos[c + d*x]]*Sec[c + d*x]^2)/(b^8*d*(a + b*Sec[c + d*x
])^2) + ((7*a^8 - 20*a^6*b^2 + 18*a^4*b^4 - 4*a^2*b^6 - b^8)*(b + a*Cos[c +
 d*x])^2*Log[b + a*Cos[c + d*x]]*Sec[c + d*x]^2)/(a^2*b^8*d*(a + b*Sec[c +
d*x])^2) - (2*a*(3*a^4 - 8*a^2*b^2 + 6*b^4)*(b + a*Cos[c + d*x])^2*Sec[c +
d*x]^3)/(b^7*d*(a + b*Sec[c + d*x])^2) + ((5*a^4 - 12*a^2*b^2 + 6*b^4)*(b +
 a*Cos[c + d*x])^2*Sec[c + d*x]^4)/(2*b^6*d*(a + b*Sec[c + d*x])^2) + (4*a*
(-a^2 + 2*b^2)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^5)/(3*b^5*d*(a + b*Sec[c
 + d*x])^2) + ((3*a^2 - 4*b^2)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^6)/(4*b^
4*d*(a + b*Sec[c + d*x])^2) - (2*a*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^7)/(
5*b^3*d*(a + b*Sec[c + d*x])^2) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^8)/(
6*b^2*d*(a + b*Sec[c + d*x])^2)
```

Maple [B] time = 0.072, size = 498, normalized size = 2.

$$-\frac{a^{6}}{db^{7}\left(b+a\cos\left(dx+c\right)\right)}+4\frac{a^{4}}{db^{5}\left(b+a\cos\left(dx+c\right)\right)}-6\frac{a^{2}}{db^{3}\left(b+a\cos\left(dx+c\right)\right)}-\frac{b}{da^{2}\left(b+a\cos\left(dx+c\right)\right)}-7\frac{\ln\left(\cos\left(d\right.\right.}{db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^9/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/d*a^6/b^7/(b+a*cos(d*x+c))+4/d*a^4/b^5/(b+a*cos(d*x+c))-6/d*a^2/b^3/(b+a
*cos(d*x+c))-1/d/a^2*b/(b+a*cos(d*x+c))-7/d/b^8*ln(cos(d*x+c))*a^6+20/d/b^6
*ln(cos(d*x+c))*a^4-18/d/b^4*ln(cos(d*x+c))*a^2-2/5/d/b^3*a/cos(d*x+c)^5-4/
3/d*a^3/b^5/cos(d*x+c)^3+8/3/d*a/b^3/cos(d*x+c)^3-6/d*a^5/b^7/cos(d*x+c)+16
/d*a^3/b^5/cos(d*x+c)-12/d*a/b^3/cos(d*x+c)+7/d/b^8*a^6*ln(b+a*cos(d*x+c))-
20/d/b^6*a^4*ln(b+a*cos(d*x+c))+18/d/b^4*a^2*ln(b+a*cos(d*x+c))+3/4/d/b^4/c
os(d*x+c)^4*a^2+5/2/d/b^6/cos(d*x+c)^2*a^4-6/d/b^4/cos(d*x+c)^2*a^2-4/d/b^2
*ln(b+a*cos(d*x+c))-1/d/a^2*ln(b+a*cos(d*x+c))+4/d/b/(b+a*cos(d*x+c))-1/d/b
^2/cos(d*x+c)^4+3/d/b^2/cos(d*x+c)^2+4/d/b^2*ln(cos(d*x+c))+1/6/d/b^2/cos(d
*x+c)^6
```

Maxima [A] time = 1.02159, size = 433, normalized size = 1.7

$$-\frac{\frac{14\,a^{3}b^{5}\cos(dx+c)-10\,a^{2}b^{6}+60\left(7\,a^{8}-20\,a^{6}b^{2}+18\,a^{4}b^{4}-4\,a^{2}b^{6}+b^{8}\right)\cos(dx+c)^{6}+30\left(7\,a^{7}b-20\,a^{5}b^{3}+18\,a^{3}b^{5}\right)\cos(dx+c)^{5}-10\left(7\,a^{6}b^{2}-20\,a^{4}b^{4}+18\,a^{2}b^{6}\right)\cos(dx+c}{a^{3}b^{7}\cos(dx+c)^{7}+a^{2}b^{8}\cos(dx+c)^{6}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/60*((14*a^3*b^5*cos(d*x + c) - 10*a^2*b^6 + 60*(7*a^8 - 20*a^6*b^2 + 18*
a^4*b^4 - 4*a^2*b^6 + b^8)*cos(d*x + c)^6 + 30*(7*a^7*b - 20*a^5*b^3 + 18*a
^3*b^5)*cos(d*x + c)^5 - 10*(7*a^6*b^2 - 20*a^4*b^4 + 18*a^2*b^6)*cos(d*x +
 c)^4 + 5*(7*a^5*b^3 - 20*a^3*b^5)*cos(d*x + c)^3 - 3*(7*a^4*b^4 - 20*a^2*b
^6)*cos(d*x + c)^2)/(a^3*b^7*cos(d*x + c)^7 + a^2*b^8*cos(d*x + c)^6) + 60*
(7*a^6 - 20*a^4*b^2 + 18*a^2*b^4 - 4*b^6)*log(cos(d*x + c))/b^8 - 60*(7*a^8
 - 20*a^6*b^2 + 18*a^4*b^4 - 4*a^2*b^6 - b^8)*log(a*cos(d*x + c) + b)/(a^2*
b^8))/d
```

Fricas [A] time = 1.55956, size = 946, normalized size = 3.71

$$-\frac{14\,a^3b^6\cos\left(dx+c\right)-10\,a^2b^7+60\left(7\,a^8b-20\,a^6b^3+18\,a^4b^5-4\,a^2b^7+b^9\right)\cos\left(dx+c\right)^6+30\left(7\,a^7b^2-20\,a^5b^4+18\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(14*a^3*b^6*cos(d*x + c) - 10*a^2*b^7 + 60*(7*a^8*b - 20*a^6*b^3 + 18
*a^4*b^5 - 4*a^2*b^7 + b^9)*cos(d*x + c)^6 + 30*(7*a^7*b^2 - 20*a^5*b^4 + 1
8*a^3*b^6)*cos(d*x + c)^5 - 10*(7*a^6*b^3 - 20*a^4*b^5 + 18*a^2*b^7)*cos(d*
x + c)^4 + 5*(7*a^5*b^4 - 20*a^3*b^6)*cos(d*x + c)^3 - 3*(7*a^4*b^5 - 20*a^
2*b^7)*cos(d*x + c)^2 - 60*((7*a^9 - 20*a^7*b^2 + 18*a^5*b^4 - 4*a^3*b^6 -
a*b^8)*cos(d*x + c)^7 + (7*a^8*b - 20*a^6*b^3 + 18*a^4*b^5 - 4*a^2*b^7 - b^
9)*cos(d*x + c)^6)*log(a*cos(d*x + c) + b) + 60*((7*a^9 - 20*a^7*b^2 + 18*a
^5*b^4 - 4*a^3*b^6)*cos(d*x + c)^7 + (7*a^8*b - 20*a^6*b^3 + 18*a^4*b^5 - 4
*a^2*b^7)*cos(d*x + c)^6)*log(-cos(d*x + c)))/(a^3*b^8*d*cos(d*x + c)^7 + a
^2*b^9*d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**9/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 10.865, size = 2290, normalized size = 8.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

[Out] 1/60*(60*(7*a^9 - 7*a^8*b - 20*a^7*b^2 + 20*a^6*b^3 + 18*a^5*b^4 - 18*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 - a*b^8 + b^9)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^3*b^8 - a^2*b^9) + 60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - 60*(7*a^6 - 20*a^4*b^2 + 18*a^2*b^4 - 4*b^6)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/b^8 - 60*(7*a^9 + 9*a^8*b - 18*a^7*b^2 - 26*a^6*b^3 + 12*a^5*b^4 + 24*a^4*b^5 + 2*a^3*b^6 - 6*a^2*b^7 - 3*a*b^8 - b^9 + 7*a^9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 7*a^8*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 20*a^7*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 20*a^6*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 18*a^5*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 18*a^4*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*a^3*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*a^2*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*b^8*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b^9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*a^2*b^8) + (1029*a^6 - 720*a^5*b - 2940*a^4*b^2 + 1760*a^3*b^3 + 2646*a^2*b^4 - 1168*a*b^5 - 588*b^6 + 6174*a^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3600*a^5*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 18240*a^4*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9120*a^3*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 16956*a^2*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 6288*a*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3888*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 15435*a^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 7200*a^5*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 46500*a^4*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 18240*a^3*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 44730*a^2*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 12960*a*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 10740*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 20580*a^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 7200*a^5*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 62400*a^4*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 17600*a^3*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 60840*a^2*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 11680*a*b^5*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 15520*b^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 15435*a^6*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 3600*a^5*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 46500*a^4*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 8160*a^3*b^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 44730*a^2*b^4*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 4560*a*b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 10740*b^6*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 6174*a^6*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 720*a^5*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 18240*a^4*b^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 1440*a^3*b^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 16956*a^2*b^4*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 720*a*b^5*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 3888*b^6*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 1029*a^6*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 2940*a^4*b^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 2646*a^2*b^4*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 588*b^6*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6)/(b^8*((cos(d*x + c) - 1)/(cos(

```
d*x + c) + 1) + 1)^6))/d
```

3.300 $\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=179

$$\frac{3\left(a^2-b^2\right)\sec^2(c+dx)}{2b^4d}-\frac{2a\left(2a^2-3b^2\right)\sec(c+dx)}{b^5d}+\frac{\left(a^2-b^2\right)^3}{ab^6d(a+b\sec(c+dx))}+\frac{\left(a^2-b^2\right)^2\left(5a^2+b^2\right)\log(a+b\sec(c+$$

```
[Out] Log[Cos[c + d*x]]/(a^2*d) + ((a^2 - b^2)^2*(5*a^2 + b^2)*Log[a + b*Sec[c +
d*x]])/(a^2*b^6*d) - (2*a*(2*a^2 - 3*b^2)*Sec[c + d*x])/(b^5*d) + (3*(a^2 -
 b^2)*Sec[c + d*x]^2)/(2*b^4*d) - (2*a*Sec[c + d*x]^3)/(3*b^3*d) + Sec[c +
d*x]^4/(4*b^2*d) + (a^2 - b^2)^3/(a*b^6*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.145637, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 894}

$$\frac{3\left(a^2-b^2\right)\sec^2(c+dx)}{2b^4d}-\frac{2a\left(2a^2-3b^2\right)\sec(c+dx)}{b^5d}+\frac{\left(a^2-b^2\right)^3}{ab^6d(a+b\sec(c+dx))}+\frac{\left(a^2-b^2\right)^2\left(5a^2+b^2\right)\log(a+b\sec(c+$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] Log[Cos[c + d*x]]/(a^2*d) + ((a^2 - b^2)^2*(5*a^2 + b^2)*Log[a + b*Sec[c +
d*x]])/(a^2*b^6*d) - (2*a*(2*a^2 - 3*b^2)*Sec[c + d*x])/(b^5*d) + (3*(a^2 -
 b^2)*Sec[c + d*x]^2)/(2*b^4*d) - (2*a*Sec[c + d*x]^3)/(3*b^3*d) + Sec[c +
d*x]^4/(4*b^2*d) + (a^2 - b^2)^3/(a*b^6*d*(a + b*Sec[c + d*x]))
```


Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
```

```
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^3}{x(a+x)^2}\,dx, x, b\sec(c+dx)\right)}{b^6 d}$$

$$= -\frac{\text{Subst}\left(\int \left(2\left(2a^3-3ab^2\right)+\frac{b^6}{a^2x}-3\left(a^2-b^2\right)x+2ax^2-x^3+\frac{\left(a^2-b^2\right)^3}{a(a+x)^2}-\frac{\left(a^2-b^2\right)^2\left(5a^2+b^2\right)}{a^2(a+x)}\right) d}{b^6 d}$$

$$= \frac{\log(\cos(c+dx))}{a^2 d}+\frac{\left(a^2-b^2\right)^2\left(5a^2+b^2\right)\log(a+b\sec(c+dx))}{a^2b^6d}-\frac{2a\left(2a^2-3b^2\right)\sec(c+d}{b^5d}$$

Mathematica [B] time = 6.19821, size = 383, normalized size = 2.14

$$\frac{2a\left(3b^2-2a^2\right)\sec^3(c+dx)(a\cos(c+dx)+b)^2}{b^5d(a+b\sec(c+dx))^2}+\frac{(b-a)^3(a+b)^3\sec^2(c+dx)(a\cos(c+dx)+b)}{a^2b^5d(a+b\sec(c+dx))^2}+\frac{\left(9a^2b^2-5a^4-3b^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((-a + b)^3*(a + b)^3*(b + a*Cos[c + d*x])*Sec[c + d*x]^2)/(a^2*b^5*d*(a +
b*Sec[c + d*x])^2) + ((-5*a^4 + 9*a^2*b^2 - 3*b^4)*(b + a*Cos[c + d*x])^2*L
og[Cos[c + d*x]]*Sec[c + d*x]^2)/(b^6*d*(a + b*Sec[c + d*x])^2) + ((5*a^6 -
 9*a^4*b^2 + 3*a^2*b^4 + b^6)*(b + a*Cos[c + d*x])^2*Log[b + a*Cos[c + d*x]
]*Sec[c + d*x]^2)/(a^2*b^6*d*(a + b*Sec[c + d*x])^2) + (2*a*(-2*a^2 + 3*b^2
)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^3)/(b^5*d*(a + b*Sec[c + d*x])^2) - (
3*(-a + b)*(a + b)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^4)/(2*b^4*d*(a + b*S
ec[c + d*x])^2) - (2*a*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^5)/(3*b^3*d*(a +
 b*Sec[c + d*x])^2) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^6)/(4*b^2*d*(a +
 b*Sec[c + d*x])^2)
```

Maple [A] time = 0.069, size = 324, normalized size = 1.8

$$-\frac{a^4}{db^5\left(b+a\cos(dx+c)\right)}+3\frac{a^2}{db^3\left(b+a\cos(dx+c)\right)}-3\frac{1}{db\left(b+a\cos(dx+c)\right)}+\frac{b}{da^2\left(b+a\cos(dx+c)\right)}+5\frac{a^4\ln(b+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/d*a^4/b^5/(b+a*cos(d*x+c))+3/d*a^2/b^3/(b+a*cos(d*x+c))-3/d/b/(b+a*cos(d
*x+c))+1/d/a^2*b/(b+a*cos(d*x+c))+5/d/b^6*a^4*ln(b+a*cos(d*x+c))-9/d/b^4*a^
2*ln(b+a*cos(d*x+c))+3/d/b^2*ln(b+a*cos(d*x+c))+1/d/a^2*ln(b+a*cos(d*x+c))+
3/2/d/b^4/cos(d*x+c)^2*a^2-3/2/d/b^2/cos(d*x+c)^2-5/d/b^6*ln(cos(d*x+c))*a^
4+9/d/b^4*ln(cos(d*x+c))*a^2-3/d/b^2*ln(cos(d*x+c))+1/4/d/b^2/cos(d*x+c)^4-
2/3/d*a/b^3/cos(d*x+c)^3-4/d*a^3/b^5/cos(d*x+c)+6/d*a/b^3/cos(d*x+c)
```

Maxima [A] time = 0.984654, size = 306, normalized size = 1.71

$$-\frac{\frac{5\,a^3b^3\cos(dx+c)-3\,a^2b^4+12\left(5\,a^6-9\,a^4b^2+3\,a^2b^4-b^6\right)\cos(dx+c)^4+6\left(5\,a^5b-9\,a^3b^3\right)\cos(dx+c)^3-2\left(5\,a^4b^2-9\,a^2b^4\right)\cos(dx+c)^2}{a^3b^5\cos(dx+c)^5+a^2b^6\cos(dx+c)^4}+\frac{12\left(5\,a^4-9\,a^2b^2+3\,b^4\right)\log(co}{b^6}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/12*((5*a^3*b^3*cos(d*x + c) - 3*a^2*b^4 + 12*(5*a^6 - 9*a^4*b^2 + 3*a^2*
b^4 - b^6)*cos(d*x + c)^4 + 6*(5*a^5*b - 9*a^3*b^3)*cos(d*x + c)^3 - 2*(5*a
^4*b^2 - 9*a^2*b^4)*cos(d*x + c)^2)/(a^3*b^5*cos(d*x + c)^5 + a^2*b^6*cos(d
*x + c)^4) + 12*(5*a^4 - 9*a^2*b^2 + 3*b^4)*log(cos(d*x + c))/b^6 - 12*(5*a
^6 - 9*a^4*b^2 + 3*a^2*b^4 + b^6)*log(a*cos(d*x + c) + b)/(a^2*b^6))/d
```

Fricas [A] time = 1.39557, size = 690, normalized size = 3.85

$$-\frac{5\,a^3b^4\cos(dx+c)-3\,a^2b^5+12\left(5\,a^6b-9\,a^4b^3+3\,a^2b^5-b^7\right)\cos(dx+c)^4+6\left(5\,a^5b^2-9\,a^3b^4\right)\cos(dx+c)^3-2\left(5\,a^4b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/12*(5*a^3*b^4*cos(d*x + c) - 3*a^2*b^5 + 12*(5*a^6*b - 9*a^4*b^3 + 3*a^2
*b^5 - b^7)*cos(d*x + c)^4 + 6*(5*a^5*b^2 - 9*a^3*b^4)*cos(d*x + c)^3 - 2*(
5*a^4*b^3 - 9*a^2*b^5)*cos(d*x + c)^2 - 12*((5*a^7 - 9*a^5*b^2 + 3*a^3*b^4
```

```
+ a*b^6)*cos(d*x + c)^5 + (5*a^6*b - 9*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x +
 c)^4)*log(a*cos(d*x + c) + b) + 12*((5*a^7 - 9*a^5*b^2 + 3*a^3*b^4)*cos(d*
x + c)^5 + (5*a^6*b - 9*a^4*b^3 + 3*a^2*b^5)*cos(d*x + c)^4)*log(-cos(d*x +
 c)))/(a^3*b^6*d*cos(d*x + c)^5 + a^2*b^7*d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**7/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**7/(a + b*sec(c + d*x))**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.301 $\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=121

$$\frac{(a^2-b^2)^2}{ab^4d(a+b\sec(c+dx))}+\frac{(3a^2+b^2)(a^2-b^2)\log(a+b\sec(c+dx))}{a^2b^4d}-\frac{\log(\cos(c+dx))}{a^2d}-\frac{2a\sec(c+dx)}{b^3d}+\frac{\sec^2(c+dx)}{2b^2d}$$

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) + ((a^2 - b^2)*(3*a^2 + b^2)*Log[a + b*Sec[c +
 d*x]])/(a^2*b^4*d) - (2*a*Sec[c + d*x])/(b^3*d) + Sec[c + d*x]^2/(2*b^2*d)
 + (a^2 - b^2)^2/(a*b^4*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.104316, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 894}

$$\frac{(a^2-b^2)^2}{ab^4d(a+b\sec(c+dx))}+\frac{(3a^2+b^2)(a^2-b^2)\log(a+b\sec(c+dx))}{a^2b^4d}-\frac{\log(\cos(c+dx))}{a^2d}-\frac{2a\sec(c+dx)}{b^3d}+\frac{\sec^2(c+dx)}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) + ((a^2 - b^2)*(3*a^2 + b^2)*Log[a + b*Sec[c +
 d*x]])/(a^2*b^4*d) - (2*a*Sec[c + d*x])/(b^3*d) + Sec[c + d*x]^2/(2*b^2*d)
 + (a^2 - b^2)^2/(a*b^4*d*(a + b*Sec[c + d*x]))
```


Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x(a+x)^2}\,dx, x, b\sec(c+dx)\right)}{b^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a+\frac{b^4}{a^2 x}+x-\frac{\left(a^2-b^2\right)^2}{a(a+x)^2}+\frac{\left(a^2-b^2\right)\left(3a^2+b^2\right)}{a^2(a+x)}\right)dx, x, b\sec(c+dx)\right)}{b^4 d}$$
$$= -\frac{\log(\cos(c+dx))}{a^2 d}+\frac{\left(a^2-b^2\right)\left(3a^2+b^2\right)\log(a+b\sec(c+dx))}{a^2 b^4 d}-\frac{2a\sec(c+dx)}{b^3 d}+\frac{\sec^2(c}{2b}$$

Mathematica [A] time = 0.536959, size = 187, normalized size = 1.55

$$\frac{b\left(a^2b^2\sec^2(c+dx)-2\left(a^2\left(3a^2-2b^2\right)\log(\cos(c+dx))+\left(2a^2b^2-3a^4+b^4\right)\log(a\cos(c+dx)+b)-2a^2b^2+3a^4+b^4\right.}{2a^2b^4d(a\cos(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-2*a*Cos[c + d*x]*(a^2*(3*a^2 - 2*b^2)*Log[Cos[c + d*x]] + (-3*a^4 + 2*a^2
*b^2 + b^4)*Log[b + a*Cos[c + d*x]]) + b*(-2*(3*a^4 - 2*a^2*b^2 + b^4 + a^2
*(3*a^2 - 2*b^2)*Log[Cos[c + d*x]] + (-3*a^4 + 2*a^2*b^2 + b^4)*Log[b + a*C
os[c + d*x]]) - 3*a^3*b*Sec[c + d*x] + a^2*b^2*Sec[c + d*x]^2))/(2*a^2*b^4*
d*(b + a*Cos[c + d*x]))
```

Maple [A] time = 0.056, size = 192, normalized size = 1.6

$$-\frac{a^2}{db^3\left(b+a\cos(dx+c)\right)}+2\frac{1}{db\left(b+a\cos(dx+c)\right)}-\frac{b}{da^2\left(b+a\cos(dx+c)\right)}+3\frac{a^2\ln(b+a\cos(dx+c))}{db^4}-2\frac{\ln(b+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/d*a^2/b^3/(b+a*cos(d*x+c))+2/d/b/(b+a*cos(d*x+c))-1/d/a^2*b/(b+a*cos(d*x
+c))+3/d/b^4*a^2*ln(b+a*cos(d*x+c))-2/d/b^2*ln(b+a*cos(d*x+c))-1/d/a^2*ln(b
```

```
+a*cos(d*x+c))-3/d/b^4*ln(cos(d*x+c))*a^2+2/d/b^2*ln(cos(d*x+c))+1/2/d/b^2/
cos(d*x+c)^2-2/d*a/b^3/cos(d*x+c)
```

Maxima [A] time = 0.989591, size = 201, normalized size = 1.66

$$-\frac{\frac{3\,a^3b\cos(dx+c)-a^2b^2+2\left(3\,a^4-2\,a^2b^2+b^4\right)\cos(dx+c)^2}{a^3b^3\cos(dx+c)^3+a^2b^4\cos(dx+c)^2}+\frac{2\left(3\,a^2-2\,b^2\right)\log(\cos(dx+c))}{b^4}-\frac{2\left(3\,a^4-2\,a^2b^2-b^4\right)\log(a\cos(dx+c)+b)}{a^2b^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*a^3*b*cos(d*x + c) - a^2*b^2 + 2*(3*a^4 - 2*a^2*b^2 + b^4)*cos(d*x
 + c)^2)/(a^3*b^3*cos(d*x + c)^3 + a^2*b^4*cos(d*x + c)^2) + 2*(3*a^2 - 2*b
^2)*log(cos(d*x + c))/b^4 - 2*(3*a^4 - 2*a^2*b^2 - b^4)*log(a*cos(d*x + c)
+ b)/(a^2*b^4))/d
```

Fricas [A] time = 1.26654, size = 485, normalized size = 4.01

$$-\frac{3\,a^3b^2\cos\left(dx+c\right)-a^2b^3+2\left(3\,a^4b-2\,a^2b^3+b^5\right)\cos\left(dx+c\right)^2-2\left(\left(3\,a^5-2\,a^3b^2-ab^4\right)\cos\left(dx+c\right)^3+\left(3\,a^4b-2\,a^2b\right.\right.}{2\left(a^3b^4d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(3*a^3*b^2*cos(d*x + c) - a^2*b^3 + 2*(3*a^4*b - 2*a^2*b^3 + b^5)*cos(
d*x + c)^2 - 2*((3*a^5 - 2*a^3*b^2 - a*b^4)*cos(d*x + c)^3 + (3*a^4*b - 2*a
^2*b^3 - b^5)*cos(d*x + c)^2)*log(a*cos(d*x + c) + b) + 2*((3*a^5 - 2*a^3*b
^2)*cos(d*x + c)^3 + (3*a^4*b - 2*a^2*b^3)*cos(d*x + c)^2)*log(-cos(d*x + c
)))/(a^3*b^4*d*cos(d*x + c)^3 + a^2*b^5*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^5\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**5/(a + b*sec(c + d*x))**2, x)
```

Giac [B] time = 3.70442, size = 767, normalized size = 6.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(3*a^5 - 3*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - a*b^4 + b^5)*log(abs(a +
b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1)))/(a^3*b^4 - a^2*b^5) + 2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) + 1))/a^2 - 2*(3*a^2 - 2*b^2)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) - 1))/b^4 + (9*a^2 - 8*a*b - 6*b^2 + 18*a^2*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) - 8*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 16*b^2*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^2*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2 - 6*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(b^4*((cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 1)^2) - 2*(3*a^5 + 5*a^4*b - 4*a^2*b^3 - 3
*a*b^4 - b^5 + 3*a^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a^4*b*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 2*a^3*b^2*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 2*a^2*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*b^4*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) + b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((
a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1))*a^2*b^4))/d
```

3.302 $\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=74

$$\frac{a^2-b^2}{ab^2d(a+b\sec(c+dx))}+\frac{(a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^2d}+\frac{\log(\cos(c+dx))}{a^2d}$$

```
[Out] Log[Cos[c + d*x]]/(a^2*d) + ((a^2 + b^2)*Log[a + b*Sec[c + d*x]])/(a^2*b^2*
d) + (a^2 - b^2)/(a*b^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.0836568, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 894}

$$\frac{a^2-b^2}{ab^2d(a+b\sec(c+dx))}+\frac{(a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^2d}+\frac{\log(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] Log[Cos[c + d*x]]/(a^2*d) + ((a^2 + b^2)*Log[a + b*Sec[c + d*x]])/(a^2*b^2*
d) + (a^2 - b^2)/(a*b^2*d*(a + b*Sec[c + d*x]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{x(a+x)^2}\,dx, x, b\sec(c+dx)\right)}{b^2 d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{a^2x} + \frac{a^2-b^2}{a(a+x)^2} + \frac{-a^2-b^2}{a^2(a+x)}\right)dx, x, b\sec(c+dx)\right)}{b^2 d}$$
$$= \frac{\log(\cos(c+dx))}{a^2 d} + \frac{(a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^2d} + \frac{a^2-b^2}{ab^2d(a+b\sec(c+dx))}$$

Mathematica [A] time = 0.248736, size = 62, normalized size = 0.84

$$-\frac{\frac{b-\frac{b^3}{a^2}}{a\cos(c+dx)+b} - \frac{(a^2+b^2)\log(a\cos(c+dx)+b)}{a^2} + \log(\cos(c+dx))}{b^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((b - b^3/a^2)/(b + a*Cos[c + d*x]) + Log[Cos[c + d*x]] - ((a^2 + b^2)*Lo
g[b + a*Cos[c + d*x]])/a^2)/(b^2*d))
```

Maple [A] time = 0.052, size = 93, normalized size = 1.3

$$-\frac{1}{db(b+a\cos(dx+c))} + \frac{b}{da^2(b+a\cos(dx+c))} + \frac{\ln(b+a\cos(dx+c))}{db^2} + \frac{\ln(b+a\cos(dx+c))}{da^2} - \frac{\ln(\cos(dx+c))}{db^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/d/b/(b+a*cos(d*x+c))+1/d/a^2*b/(b+a*cos(d*x+c))+1/d/b^2*ln(b+a*cos(d*x+c
))+1/d/a^2*ln(b+a*cos(d*x+c))-1/d/b^2*ln(cos(d*x+c))
```

Maxima [A] time = 0.959437, size = 100, normalized size = 1.35

$$-\frac{\frac{a^2-b^2}{a^3b\cos(dx+c)+a^2b^2} + \frac{\log(\cos(dx+c))}{b^2} - \frac{(a^2+b^2)\log(a\cos(dx+c)+b)}{a^2b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((a^2 - b^2)/(a^3*b*cos(d*x + c) + a^2*b^2) + log(cos(d*x + c))/b^2 - (a^2
 + b^2)*log(a*cos(d*x + c) + b)/(a^2*b^2))/d
```

Fricas [A] time = 0.813566, size = 230, normalized size = 3.11

$$-\frac{a^2b - b^3 - \left(a^2b + b^3 + \left(a^3 + ab^2\right)\cos\left(dx + c\right)\right)\log\left(a\cos\left(dx + c\right) + b\right) + \left(a^3\cos\left(dx + c\right) + a^2b\right)\log\left(-\cos\left(dx + c\right)\right)}{a^3b^2d\cos\left(dx + c\right) + a^2b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*b - b^3 - (a^2*b + b^3 + (a^3 + a*b^2)*cos(d*x + c))*log(a*cos(d*x +
c) + b) + (a^3*cos(d*x + c) + a^2*b)*log(-cos(d*x + c)))/(a^3*b^2*d*cos(d*x
 + c) + a^2*b^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**3/(a + b*sec(c + d*x))**2, x)
```

Giac [B] time = 1.98203, size = 423, normalized size = 5.72

$$\frac{\frac{\left(a^3 - a^2b + ab^2 - b^3\right)\log\left(\left|a + b + \frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^3b^2 - a^2b^3} - \frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1\right|\right)}{a^2} - \frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right)}{b^2} - \frac{a^3 + 3\,a^2b + 3\,ab^2 + b^3 + \frac{a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{a^2b(c}{\cos}}{\left(a + b + \frac{a(\cos(dx+c)-1}{\cos(dx+c)+1}\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((a^3 - a^2*b + a*b^2 - b^3)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^3*b^2 - a^2*b^3) -
 log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - log(abs(-(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 1))/b^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3
+ a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a^2*b*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) + a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b^3*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1))/((a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*a^2*b^2))/d
```

3.303 $\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=54

$$-\frac{\log(a+b\sec(c+dx))}{a^2d}-\frac{\log(\cos(c+dx))}{a^2d}+\frac{1}{ad(a+b\sec(c+dx))}$$

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) - Log[a + b*Sec[c + d*x]]/(a^2*d) + 1/(a*d*(a
+ b*Sec[c + d*x]))
```

Rubi [A] time = 0.0428775, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3885, 44}

$$-\frac{\log(a+b\sec(c+dx))}{a^2d}-\frac{\log(\cos(c+dx))}{a^2d}+\frac{1}{ad(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) - Log[a + b*Sec[c + d*x]]/(a^2*d) + 1/(a*d*(a
+ b*Sec[c + d*x]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^2}\,dx, x, b\sec(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x} - \frac{1}{a(a+x)^2} - \frac{1}{a^2(a+x)}\right)dx, x, b\sec(c+dx)\right)}{d}$$
$$= -\frac{\log(\cos(c+dx))}{a^2d} - \frac{\log(a+b\sec(c+dx))}{a^2d} + \frac{1}{ad(a+b\sec(c+dx))}$$

Mathematica [A] time = 0.038492, size = 54, normalized size = 1.

$$-\frac{b\log(a\cos(c+dx)+b) + a\cos(c+dx)\log(a\cos(c+dx)+b) + b}{a^2d(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((b + b*Log[b + a*Cos[c + d*x]] + a*Cos[c + d*x]*Log[b + a*Cos[c + d*x]])/
(a^2*d*(b + a*Cos[c + d*x])))
```

Maple [A] time = 0.023, size = 54, normalized size = 1.

$$-\frac{\ln(a+b\sec(dx+c))}{da^2} + \frac{1}{ad(a+b\sec(dx+c))} + \frac{\ln(\sec(dx+c))}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -ln(a+b*sec(d*x+c))/d/a^2+1/a/d/(a+b*sec(d*x+c))+1/d/a^2*ln(sec(d*x+c))
```

Maxima [A] time = 0.96915, size = 55, normalized size = 1.02

$$-\frac{\frac{b}{a^3\cos(dx+c)+a^2b} + \frac{\log(a\cos(dx+c)+b)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(b/(a^3*cos(d*x + c) + a^2*b) + log(a*cos(d*x + c) + b)/a^2)/d
```

Fricas [A] time = 0.882972, size = 113, normalized size = 2.09

$$-\frac{(a\cos(dx+c)+b)\log(a\cos(dx+c)+b)+b}{a^3d\cos(dx+c)+a^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -((a*cos(d*x + c) + b)*log(a*cos(d*x + c) + b) + b)/(a^3*d*cos(d*x + c) + a
^2*b*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31203, size = 321, normalized size = 5.94

$$-\frac{\frac{(a-b)\log\left(\left|a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^3-a^2b}-\frac{a^2-2\,ab-b^2+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\left(a^3-a^2b\right)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}-\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -((a - b)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^3 - a^2*b) - (a^2 - 2*a*b - b^2 + a^2
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a*b*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) + b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((a^3 - a^2*b)*(a +
b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1))) - log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2)/d
```

3.304 $\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=138

$$\frac{b^2}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{b^2\left(3a^2-b^2\right)\log(a+b\sec(c+dx))}{a^2d\left(a^2-b^2\right)^2}+\frac{\log(\cos(c+dx))}{a^2d}+\frac{\log(1-\sec(c+dx))}{2d(a+b)^2}+\frac{\log(s$$

```
[Out] Log[Cos[c + d*x]]/(a^2*d) + Log[1 - Sec[c + d*x]]/(2*(a + b)^2*d) + Log[1 +
 Sec[c + d*x]]/(2*(a - b)^2*d) - (b^2*(3*a^2 - b^2)*Log[a + b*Sec[c + d*x]]
)/(a^2*(a^2 - b^2)^2*d) + b^2/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.14238, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3885, 894}

$$\frac{b^2}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{b^2\left(3a^2-b^2\right)\log(a+b\sec(c+dx))}{a^2d\left(a^2-b^2\right)^2}+\frac{\log(\cos(c+dx))}{a^2d}+\frac{\log(1-\sec(c+dx))}{2d(a+b)^2}+\frac{\log(s$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] Log[Cos[c + d*x]]/(a^2*d) + Log[1 - Sec[c + d*x]]/(2*(a + b)^2*d) + Log[1 +
 Sec[c + d*x]]/(2*(a - b)^2*d) - (b^2*(3*a^2 - b^2)*Log[a + b*Sec[c + d*x]]
)/(a^2*(a^2 - b^2)^2*d) + b^2/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
```

```
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{b^2 \operatorname{Subst}\left(\int \frac{1}{x(a+x)^2(b^2-x^2)}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= -\frac{b^2 \operatorname{Subst}\left(\int \left(\frac{1}{2b^2(a+b)^2(b-x)} + \frac{1}{a^2b^2x} + \frac{1}{a(a-b)(a+b)(a+x)^2} + \frac{3a^2-b^2}{a^2(a-b)^2(a+b)^2(a+x)} - \frac{1}{2(a-b)^2b^2(b+x)}\right) dx\right.}{d}$$

$$= \frac{\log(\cos(c+dx))}{a^2d} + \frac{\log(1-\sec(c+dx))}{2(a+b)^2d} + \frac{\log(1+\sec(c+dx))}{2(a-b)^2d} - \frac{b^2\left(3a^2-b^2\right)\log(a+b\,\mathrm{s}}{a^2\left(a^2-b^2\right)^2}$$

Mathematica [A] time = 0.325961, size = 189, normalized size = 1.37

$$\frac{a\cos(c+dx)\left(\left(b^4-3a^2b^2\right)\log(a\cos(c+dx)+b)+a^2(a-b)^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+a^2(a+b)^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)}{a^2d(a-b)^2(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a*Cos[c + d*x]*(a^2*(a + b)^2*Log[Cos[(c + d*x)/2]] + (-3*a^2*b^2 + b^4)*L
og[b + a*Cos[c + d*x]] + a^2*(a - b)^2*Log[Sin[(c + d*x)/2]]) + b*(a^2*(a +
 b)^2*Log[Cos[(c + d*x)/2]] + (-3*a^2*b^2 + b^4)*Log[b + a*Cos[c + d*x]] +
(a - b)*(-(b^2*(a + b)) + a^2*(a - b)*Log[Sin[(c + d*x)/2]])))/(a^2*(a - b)
^2*(a + b)^2*d*(b + a*Cos[c + d*x]))
```

Maple [A] time = 0.071, size = 141, normalized size = 1.

$$-\frac{b^3}{da^2(a+b)(a-b)(b+a\cos(dx+c))} - 3\frac{b^2\ln(b+a\cos(dx+c))}{d(a+b)^2(a-b)^2} + \frac{b^4\ln(b+a\cos(dx+c))}{d(a+b)^2(a-b)^2a^2} + \frac{\ln(\cos(dx+c)+1)}{2d(a-b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/d/a^2*b^3/(a+b)/(a-b)/(b+a*cos(d*x+c))-3/d*b^2/(a+b)^2/(a-b)^2*ln(b+a*co
s(d*x+c))+1/d*b^4/(a+b)^2/(a-b)^2/a^2*ln(b+a*cos(d*x+c))+1/2/d/(a-b)^2*ln(c
os(d*x+c)+1)+1/2/d/(a+b)^2*ln(-1+cos(d*x+c))
```

Maxima [A] time = 0.994884, size = 192, normalized size = 1.39

$$-\frac{\frac{2\,b^{3}}{a^{4}b-a^{2}b^{3}+\left(a^{5}-a^{3}b^{2}\right)\cos(dx+c)}+\frac{2\left(3\,a^{2}b^{2}-b^{4}\right)\log(a\cos(dx+c)+b)}{a^{6}-2\,a^{4}b^{2}+a^{2}b^{4}}-\frac{\log(\cos(dx+c)+1)}{a^{2}-2\,ab+b^{2}}-\frac{\log(\cos(dx+c)-1)}{a^{2}+2\,ab+b^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^3/(a^4*b - a^2*b^3 + (a^5 - a^3*b^2)*cos(d*x + c)) + 2*(3*a^2*b^2
 - b^4)*log(a*cos(d*x + c) + b)/(a^6 - 2*a^4*b^2 + a^2*b^4) - log(cos(d*x +
 c) + 1)/(a^2 - 2*a*b + b^2) - log(cos(d*x + c) - 1)/(a^2 + 2*a*b + b^2))/d
```

Fricas [A] time = 1.26706, size = 525, normalized size = 3.8

$$-\frac{2\,a^{2}b^{3}-2\,b^{5}+2\left(3\,a^{2}b^{3}-b^{5}+\left(3\,a^{3}b^{2}-ab^{4}\right)\cos\left(dx+c\right)\right)\log\left(a\cos\left(dx+c\right)+b\right)-\left(a^{4}b+2\,a^{3}b^{2}+a^{2}b^{3}+\left(a^{5}+2\,a^{4}b\right.\right.}{2\left(\left(a^{7}-2\,a^{5}b^{2}+a^{3}b^{4}\right)d\,\mathrm{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*b^3 - 2*b^5 + 2*(3*a^2*b^3 - b^5 + (3*a^3*b^2 - a*b^4)*cos(d*x
+ c))*log(a*cos(d*x + c) + b) - (a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4
*b + a^3*b^2)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - (a^4*b - 2*a^3*b^
2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c))*log(-1/2*cos(d*x + c)
 + 1/2))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 +
 a^2*b^5)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

Giac [B] time = 1.32971, size = 409, normalized size = 2.96

$$-\frac{\frac{2\left(3a^2b^2-b^4\right)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^6-2a^4b^2+a^2b^4}-\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2+2ab+b^2}-\frac{2\left(3a^2b^2+4ab^3+b^4+\frac{3a^2b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b^4(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\left(a^5+a^4b-a^3b^2-a^2b^3\right)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}+\frac{2\log\left(\left|-\ldots\right.\right.}{\ldots}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(3*a^2*b^2 - b^4)*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^6 - 2*a^4*b^2 + a^2*
b^4) - log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^2 + 2*a*b + b^2
) - 2*(3*a^2*b^2 + 4*a*b^3 + b^4 + 3*a^2*b^2*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) - b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((a^5 + a^4*b - a^3*b^
2 - a^2*b^3)*(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1))) + 2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 1))/a^2)/d
```

3.305 $\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=197

$$\frac{b^4}{ad\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{b^4\left(5a^2-b^2\right)\log(a+b\sec(c+dx))}{a^2d\left(a^2-b^2\right)^3}-\frac{\log(\cos(c+dx))}{a^2d}+\frac{1}{4d(a+b)^2(1-\sec(c+dx))}$$

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) - ((a + 2*b)*Log[1 - Sec[c + d*x]])/(2*(a + b)
^3*d) - ((a - 2*b)*Log[1 + Sec[c + d*x]])/(2*(a - b)^3*d) - (b^4*(5*a^2 - b
^2)*Log[a + b*Sec[c + d*x]])/(a^2*(a^2 - b^2)^3*d) + 1/(4*(a + b)^2*d*(1 -
Sec[c + d*x])) + 1/(4*(a - b)^2*d*(1 + Sec[c + d*x])) + b^4/(a*(a^2 - b^2)^
2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.228771, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 894}

$$\frac{b^4}{ad\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{b^4\left(5a^2-b^2\right)\log(a+b\sec(c+dx))}{a^2d\left(a^2-b^2\right)^3}-\frac{\log(\cos(c+dx))}{a^2d}+\frac{1}{4d(a+b)^2(1-\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) - ((a + 2*b)*Log[1 - Sec[c + d*x]])/(2*(a + b)
^3*d) - ((a - 2*b)*Log[1 + Sec[c + d*x]])/(2*(a - b)^3*d) - (b^4*(5*a^2 - b
^2)*Log[a + b*Sec[c + d*x]])/(a^2*(a^2 - b^2)^3*d) + 1/(4*(a + b)^2*d*(1 -
Sec[c + d*x])) + 1/(4*(a - b)^2*d*(1 + Sec[c + d*x])) + b^4/(a*(a^2 - b^2)^
2*d*(a + b*Sec[c + d*x]))
```


Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{b^4\,\mathrm{Subst}\left(\int \frac{1}{x(a+x)^2\left(b^2-x^2\right)^2}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= \frac{b^4\,\mathrm{Subst}\left(\int \left(\frac{1}{4b^3(a+b)^2(b-x)^2} + \frac{a+2b}{2b^4(a+b)^3(b-x)} + \frac{1}{a^2b^4x} - \frac{1}{a(a-b)^2(a+b)^2(a+x)^2} + \frac{-5a^2+b^2}{a^2(a-b)^3(a+b)^3(a+x)} - \right.\right.}{d}$$

$$= -\frac{\log(\cos(c+dx))}{a^2d} - \frac{(a+2b)\log(1-\sec(c+dx))}{2(a+b)^3d} - \frac{(a-2b)\log(1+\sec(c+dx))}{2(a-b)^3d} - \frac{b^4\left(5a\right.}{}$$

Mathematica [C] time = 2.05403, size = 351, normalized size = 1.78

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)\left(-\frac{16i\left(-3a^2b^2+a^4-2b^4\right)(c+dx)(a\cos(c+dx)+b)}{(a-b)^3(a+b)^3} + \frac{8b^4\left(b^2-5a^2\right)(a\cos(c+dx)+b)\log(a\cos(c+dx)+b)}{a^2\left(a^2-b^2\right)^3} - \frac{8b^5}{a^2(a-b)^2(a+}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*((-8*b^5)/(a^2*(a - b)^2*(a + b)^2) - ((16*I)*(a^4 -
3*a^2*b^2 - 2*b^4)*(c + d*x)*(b + a*Cos[c + d*x]))/((a - b)^3*(a + b)^3) +
((8*I)*(a - 2*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x]))/(a - b)^3 + ((8
*I)*(a + 2*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x]))/(a + b)^3 - ((b +
a*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/(a + b)^2 + (4*(a - 2*b)*(b + a*Cos[c +
 d*x])*Log[Cos[(c + d*x)/2]^2])/(-a + b)^3 + (8*b^4*(-5*a^2 + b^2)*(b + a*C
os[c + d*x])*Log[b + a*Cos[c + d*x]])/(a^2*(a^2 - b^2)^3) - (4*(a + 2*b)*(b
 + a*Cos[c + d*x])*Log[Sin[(c + d*x)/2]^2])/(a + b)^3 - ((b + a*Cos[c + d*x
])*Sec[(c + d*x)/2]^2)/(a - b)^2)*Sec[c + d*x]^2)/(8*d*(a + b*Sec[c + d*x])
^2)
```

Maple [A] time = 0.084, size = 226, normalized size = 1.2

$$-\frac{b^5}{da^2(a+b)^2(a-b)^2(b+a\cos(dx+c))}-5\frac{b^4\ln(b+a\cos(dx+c))}{d(a+b)^3(a-b)^3}+\frac{b^6\ln(b+a\cos(dx+c))}{d(a+b)^3(a-b)^3a^2}-\frac{1}{4d(a-b)^2(\cos(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/d*b^5/a^2/(a+b)^2/(a-b)^2/(b+a*cos(d*x+c))-5/d*b^4/(a+b)^3/(a-b)^3*ln(b+
a*cos(d*x+c))+1/d*b^6/(a+b)^3/(a-b)^3/a^2*ln(b+a*cos(d*x+c))-1/4/d/(a-b)^2/
(cos(d*x+c)+1)-1/2/d/(a-b)^3*ln(cos(d*x+c)+1)*a+1/d/(a-b)^3*ln(cos(d*x+c)+1
)*b+1/4/d/(a+b)^2/(-1+cos(d*x+c))-1/2/d/(a+b)^3*ln(-1+cos(d*x+c))*a-1/d/(a+
b)^3*ln(-1+cos(d*x+c))*b
```

Maxima [A] time = 1.0238, size = 409, normalized size = 2.08

$$-\frac{\frac{2\left(5a^2b^4-b^6\right)\log(a\cos(dx+c)+b)}{a^8-3a^6b^2+3a^4b^4-a^2b^6}+\frac{(a-2b)\log(\cos(dx+c)+1)}{a^3-3a^2b+3ab^2-b^3}+\frac{(a+2b)\log(\cos(dx+c)-1)}{a^3+3a^2b+3ab^2+b^3}+\frac{a^4b+a^2b^3+2b^5-2\left(a^4b+b^5\right)\cos(dx+c}{a^6b-2a^4b^3+a^2b^5-\left(a^7-2a^5b^2+a^3b^4\right)\cos(dx+c)^3-\left(a^6b-2a^4b^3\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(5*a^2*b^4 - b^6)*log(a*cos(d*x + c) + b)/(a^8 - 3*a^6*b^2 + 3*a^4*
b^4 - a^2*b^6) + (a - 2*b)*log(cos(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3) + (a + 2*b)*log(cos(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + (
a^4*b + a^2*b^3 + 2*b^5 - 2*(a^4*b + b^5)*cos(d*x + c)^2 + (a^5 - a^3*b^2)*
cos(d*x + c))/(a^6*b - 2*a^4*b^3 + a^2*b^5 - (a^7 - 2*a^5*b^2 + a^3*b^4)*co
s(d*x + c)^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c)^2 + (a^7 - 2*a^5*
b^2 + a^3*b^4)*cos(d*x + c)))/d
```

Fricas [B] time = 1.51635, size = 1455, normalized size = 7.39

$$\frac{a^6b+a^2b^5-2b^7-2\left(a^6b-a^4b^3+a^2b^5-b^7\right)\cos(dx+c)^2+\left(a^7-2a^5b^2+a^3b^4\right)\cos(dx+c)+2\left(5a^2b^5-b^7-\left(5a^3b^4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^6*b + a^2*b^5 - 2*b^7 - 2*(a^6*b - a^4*b^3 + a^2*b^5 - b^7)*cos(d*x
+ c)^2 + (a^7 - 2*a^5*b^2 + a^3*b^4)*cos(d*x + c) + 2*(5*a^2*b^5 - b^7 - (5
*a^3*b^4 - a*b^6)*cos(d*x + c)^3 - (5*a^2*b^5 - b^7)*cos(d*x + c)^2 + (5*a^
3*b^4 - a*b^6)*cos(d*x + c))*log(a*cos(d*x + c) + b) + (a^6*b + a^5*b^2 - 3
*a^4*b^3 - 5*a^3*b^4 - 2*a^2*b^5 - (a^7 + a^6*b - 3*a^5*b^2 - 5*a^4*b^3 - 2
*a^3*b^4)*cos(d*x + c)^3 - (a^6*b + a^5*b^2 - 3*a^4*b^3 - 5*a^3*b^4 - 2*a^2
*b^5)*cos(d*x + c)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 5*a^4*b^3 - 2*a^3*b^4)*co
s(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (a^6*b - a^5*b^2 - 3*a^4*b^3 + 5*
a^3*b^4 - 2*a^2*b^5 - (a^7 - a^6*b - 3*a^5*b^2 + 5*a^4*b^3 - 2*a^3*b^4)*cos
(d*x + c)^3 - (a^6*b - a^5*b^2 - 3*a^4*b^3 + 5*a^3*b^4 - 2*a^2*b^5)*cos(d*x
 + c)^2 + (a^7 - a^6*b - 3*a^5*b^2 + 5*a^4*b^3 - 2*a^3*b^4)*cos(d*x + c))*l
og(-1/2*cos(d*x + c) + 1/2))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos
(d*x + c)^3 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*cos(d*x + c)^2 -
(a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c) - (a^8*b - 3*a^6*b^3
 + 3*a^4*b^5 - a^2*b^7)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**3/(a + b*sec(c + d*x))**2, x)
```

Giac [B] time = 1.39264, size = 886, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*(4*(a + 2*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^3 +
3*a^2*b + 3*a*b^2 + b^3) + 8*(5*a^2*b^4 - b^6)*log(abs(-a - b - a*(cos(d*x
```

```
+ c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a
^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6) - (a^5 - a^4*b - a^3*b^2 + a^2*b^3 +
3*a^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a^4*b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 3*a^3*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^2*
b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 20*a*b^4*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) + 4*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*a^5*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 4*a^4*b*(cos(d*x + c) - 1)^2/(cos(d*
x + c) + 1)^2 - 2*a^3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 12*a^
2*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 4*a*b^4*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2 - 4*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)
/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2 - b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)) - (cos(d*x + c) -
1)/((a^2 - 2*a*b + b^2)*(cos(d*x + c) + 1)) - 8*log(abs(-(cos(d*x + c) - 1)
/(cos(d*x + c) + 1) + 1))/a^2)/d
```

3.306 $\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=278

$$\frac{b^6}{ad\left(a^2-b^2\right)^3(a+b\sec(c+dx))}-\frac{b^6\left(7a^2-b^2\right)\log(a+b\sec(c+dx))}{a^2d\left(a^2-b^2\right)^4}+\frac{\left(4a^2+13ab+12b^2\right)\log(1-\sec(c+dx))}{8d(a+b)^4}+\ldots$$

```
[Out] Log[Cos[c + d*x]]/(a^2*d) + ((4*a^2 + 13*a*b + 12*b^2)*Log[1 - Sec[c + d*x]
])/(8*(a + b)^4*d) + ((4*a^2 - 13*a*b + 12*b^2)*Log[1 + Sec[c + d*x]])/(8*(
a - b)^4*d) - (b^6*(7*a^2 - b^2)*Log[a + b*Sec[c + d*x]])/(a^2*(a^2 - b^2)^
4*d) - 1/(16*(a + b)^2*d*(1 - Sec[c + d*x])^2) - (5*a + 9*b)/(16*(a + b)^3*
d*(1 - Sec[c + d*x])) - 1/(16*(a - b)^2*d*(1 + Sec[c + d*x])^2) - (5*a - 9*
b)/(16*(a - b)^3*d*(1 + Sec[c + d*x])) + b^6/(a*(a^2 - b^2)^3*d*(a + b*Sec[
c + d*x]))
```

Rubi [A] time = 0.371301, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 894}

$$\frac{b^6}{ad\left(a^2-b^2\right)^3(a+b\sec(c+dx))}-\frac{b^6\left(7a^2-b^2\right)\log(a+b\sec(c+dx))}{a^2d\left(a^2-b^2\right)^4}+\frac{\left(4a^2+13ab+12b^2\right)\log(1-\sec(c+dx))}{8d(a+b)^4}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] Log[Cos[c + d*x]]/(a^2*d) + ((4*a^2 + 13*a*b + 12*b^2)*Log[1 - Sec[c + d*x]
])/(8*(a + b)^4*d) + ((4*a^2 - 13*a*b + 12*b^2)*Log[1 + Sec[c + d*x]])/(8*(
a - b)^4*d) - (b^6*(7*a^2 - b^2)*Log[a + b*Sec[c + d*x]])/(a^2*(a^2 - b^2)^
4*d) - 1/(16*(a + b)^2*d*(1 - Sec[c + d*x])^2) - (5*a + 9*b)/(16*(a + b)^3*
d*(1 - Sec[c + d*x])) - 1/(16*(a - b)^2*d*(1 + Sec[c + d*x])^2) - (5*a - 9*
b)/(16*(a - b)^3*d*(1 + Sec[c + d*x])) + b^6/(a*(a^2 - b^2)^3*d*(a + b*Sec[
c + d*x]))
```


Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
```

```
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{b^6\,\mathrm{Subst}\left(\int \frac{1}{x(a+x)^2\left(b^2-x^2\right)^3}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= -\frac{b^6\,\mathrm{Subst}\left(\int \left(\frac{1}{8b^4(a+b)^2(b-x)^3} + \frac{5a+9b}{16b^5(a+b)^3(b-x)^2} + \frac{4a^2+13ab+12b^2}{8b^6(a+b)^4(b-x)} + \frac{1}{a^2b^6x} + \frac{1}{a(a-b)^3(a+b)^3(a+x)^2} + \frac{}{a^2}\right.\right.}{d}$$

$$= \frac{\log(\cos(c+dx))}{a^2d} + \frac{\left(4a^2+13ab+12b^2\right)\log(1-\sec(c+dx))}{8(a+b)^4d} + \frac{\left(4a^2-13ab+12b^2\right)\log(1+}{8(a-b)^4d}$$

Mathematica [C] time = 3.03477, size = 473, normalized size = 1.7

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)\left(\frac{128i\left(-4a^4b^2+6a^2b^4+a^6+3b^6\right)(c+dx)(a\cos(c+dx)+b)}{(a-b)^4(a+b)^4} + \frac{8\left(4a^2-13ab+12b^2\right)\log\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b)}{(a-b)^4} + \right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*((64*b^7)/(a^2*(-a + b)^3*(a + b)^3) + ((128*I)*(a^6
- 4*a^4*b^2 + 6*a^2*b^4 + 3*b^6)*(c + d*x)*(b + a*Cos[c + d*x]))/((a - b)^4
*(a + b)^4) - ((16*I)*(4*a^2 - 13*a*b + 12*b^2)*ArcTan[Tan[c + d*x]]*(b + a
*Cos[c + d*x]))/(a - b)^4 - ((16*I)*(4*a^2 + 13*a*b + 12*b^2)*ArcTan[Tan[c
+ d*x]]*(b + a*Cos[c + d*x]))/(a + b)^4 + (2*(7*a + 11*b)*(b + a*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/(a + b)^3 - ((b + a*Cos[c + d*x])*Csc[(c + d*x)/2]^
4)/(a + b)^2 + (8*(4*a^2 - 13*a*b + 12*b^2)*(b + a*Cos[c + d*x])*Log[Cos[(c
 + d*x)/2]^2])/(a - b)^4 + (64*(-7*a^2*b^6 + b^8)*(b + a*Cos[c + d*x])*Log[
b + a*Cos[c + d*x]])/(a^2*(a^2 - b^2)^4) + (8*(4*a^2 + 13*a*b + 12*b^2)*(b
```

```
+ a*Cos[c + d*x])*Log[Sin[(c + d*x)/2]^2])/(a + b)^4 + (2*(7*a - 11*b)*(b +
 a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a - b)^3 - ((b + a*Cos[c + d*x])*Sec[
(c + d*x)/2]^4)/(a - b)^2)*Sec[c + d*x]^2)/(64*d*(a + b*Sec[c + d*x])^2)
```

Maple [A] time = 0.09, size = 367, normalized size = 1.3

$$-\frac{b^7}{da^2(a+b)^3(a-b)^3(b+a\cos(dx+c))}-7\frac{b^6\ln(b+a\cos(dx+c))}{d(a+b)^4(a-b)^4}+\frac{b^8\ln(b+a\cos(dx+c))}{d(a+b)^4(a-b)^4a^2}-\frac{1}{16\,d(a-b)^2(\cos(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/d*b^7/a^2/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))-7/d*b^6/(a+b)^4/(a-b)^4*ln(b+
a*cos(d*x+c))+1/d*b^8/(a+b)^4/(a-b)^4/a^2*ln(b+a*cos(d*x+c))-1/16/d/(a-b)^2
/(cos(d*x+c)+1)^2+7/16/d/(a-b)^3/(cos(d*x+c)+1)*a-11/16/d/(a-b)^3/(cos(d*x+
c)+1)*b+1/2/d/(a-b)^4*ln(cos(d*x+c)+1)*a^2-13/8/d/(a-b)^4*ln(cos(d*x+c)+1)*
a*b+3/2/d/(a-b)^4*ln(cos(d*x+c)+1)*b^2-1/16/d/(a+b)^2/(-1+cos(d*x+c))^2-7/1
6/d/(a+b)^3/(-1+cos(d*x+c))*a-11/16/d/(a+b)^3/(-1+cos(d*x+c))*b+1/2/d/(a+b)
^4*ln(-1+cos(d*x+c))*a^2+13/8/d/(a+b)^4*ln(-1+cos(d*x+c))*a*b+3/2/d/(a+b)^4
*ln(-1+cos(d*x+c))*b^2
```

Maxima [B] time = 1.11149, size = 753, normalized size = 2.71

$$-\frac{\frac{8\left(7a^2b^6-b^8\right)\log(a\cos(dx+c)+b)}{a^{10}-4a^8b^2+6a^6b^4-4a^4b^6+a^2b^8}-\frac{\left(4a^2-13ab+12b^2\right)\log(\cos(dx+c)+1)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}-\frac{\left(4a^2+13ab+12b^2\right)\log(\cos(dx+c)-1)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{2\left(3a^6b-6\cdots\right.}{a^8b-3a^6b^3+3a^4b^5-a^2b^7+\left(a^9-3a^7\cdots\right.}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(8*(7*a^2*b^6 - b^8)*log(a*cos(d*x + c) + b)/(a^10 - 4*a^8*b^2 + 6*a^6
*b^4 - 4*a^4*b^6 + a^2*b^8) - (4*a^2 - 13*a*b + 12*b^2)*log(cos(d*x + c) +
1)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (4*a^2 + 13*a*b + 12*b^2)*
log(cos(d*x + c) - 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 2*(3*a^
6*b - 6*a^4*b^3 - 5*a^2*b^5 - 4*b^7 + (5*a^6*b - 13*a^4*b^3 - 4*b^7)*cos(d*
x + c)^4 - (4*a^7 - 11*a^5*b^2 + 7*a^3*b^4)*cos(d*x + c)^3 - (7*a^6*b - 17*
a^4*b^3 - 6*a^2*b^5 - 8*b^7)*cos(d*x + c)^2 + 3*(a^7 - 3*a^5*b^2 + 2*a^3*b^
4)*cos(d*x + c))/(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7 + (a^9 - 3*a^7*b^
```

```
2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)^5 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 -
a^2*b^7)*cos(d*x + c)^4 - 2*(a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x
 + c)^3 - 2*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x + c)^2 + (a^9
 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)))/d
```

Fricas [B] time = 2.64518, size = 3028, normalized size = 10.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(6*a^8*b - 18*a^6*b^3 + 2*a^4*b^5 + 2*a^2*b^7 + 8*b^9 + 2*(5*a^8*b - 18
*a^6*b^3 + 13*a^4*b^5 - 4*a^2*b^7 + 4*b^9)*cos(d*x + c)^4 - 2*(4*a^9 - 15*a
^7*b^2 + 18*a^5*b^4 - 7*a^3*b^6)*cos(d*x + c)^3 - 2*(7*a^8*b - 24*a^6*b^3 +
 11*a^4*b^5 - 2*a^2*b^7 + 8*b^9)*cos(d*x + c)^2 + 6*(a^9 - 4*a^7*b^2 + 5*a^
5*b^4 - 2*a^3*b^6)*cos(d*x + c) - 8*(7*a^2*b^7 - b^9 + (7*a^3*b^6 - a*b^8)*
cos(d*x + c)^5 + (7*a^2*b^7 - b^9)*cos(d*x + c)^4 - 2*(7*a^3*b^6 - a*b^8)*c
os(d*x + c)^3 - 2*(7*a^2*b^7 - b^9)*cos(d*x + c)^2 + (7*a^3*b^6 - a*b^8)*co
s(d*x + c))*log(a*cos(d*x + c) + b) + (4*a^8*b + 3*a^7*b^2 - 16*a^6*b^3 - 1
4*a^5*b^4 + 24*a^4*b^5 + 35*a^3*b^6 + 12*a^2*b^7 + (4*a^9 + 3*a^8*b - 16*a^
7*b^2 - 14*a^6*b^3 + 24*a^5*b^4 + 35*a^4*b^5 + 12*a^3*b^6)*cos(d*x + c)^5 +
 (4*a^8*b + 3*a^7*b^2 - 16*a^6*b^3 - 14*a^5*b^4 + 24*a^4*b^5 + 35*a^3*b^6 +
 12*a^2*b^7)*cos(d*x + c)^4 - 2*(4*a^9 + 3*a^8*b - 16*a^7*b^2 - 14*a^6*b^3
+ 24*a^5*b^4 + 35*a^4*b^5 + 12*a^3*b^6)*cos(d*x + c)^3 - 2*(4*a^8*b + 3*a^7
*b^2 - 16*a^6*b^3 - 14*a^5*b^4 + 24*a^4*b^5 + 35*a^3*b^6 + 12*a^2*b^7)*cos(
d*x + c)^2 + (4*a^9 + 3*a^8*b - 16*a^7*b^2 - 14*a^6*b^3 + 24*a^5*b^4 + 35*a
^4*b^5 + 12*a^3*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (4*a^8*b -
 3*a^7*b^2 - 16*a^6*b^3 + 14*a^5*b^4 + 24*a^4*b^5 - 35*a^3*b^6 + 12*a^2*b^7
 + (4*a^9 - 3*a^8*b - 16*a^7*b^2 + 14*a^6*b^3 + 24*a^5*b^4 - 35*a^4*b^5 + 1
2*a^3*b^6)*cos(d*x + c)^5 + (4*a^8*b - 3*a^7*b^2 - 16*a^6*b^3 + 14*a^5*b^4
+ 24*a^4*b^5 - 35*a^3*b^6 + 12*a^2*b^7)*cos(d*x + c)^4 - 2*(4*a^9 - 3*a^8*b
 - 16*a^7*b^2 + 14*a^6*b^3 + 24*a^5*b^4 - 35*a^4*b^5 + 12*a^3*b^6)*cos(d*x
+ c)^3 - 2*(4*a^8*b - 3*a^7*b^2 - 16*a^6*b^3 + 14*a^5*b^4 + 24*a^4*b^5 - 35
*a^3*b^6 + 12*a^2*b^7)*cos(d*x + c)^2 + (4*a^9 - 3*a^8*b - 16*a^7*b^2 + 14*
a^6*b^3 + 24*a^5*b^4 - 35*a^4*b^5 + 12*a^3*b^6)*cos(d*x + c))*log(-1/2*cos(
d*x + c) + 1/2))/((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*co
s(d*x + c)^5 + (a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos
(d*x + c)^4 - 2*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(
d*x + c)^3 - 2*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos
(d*x + c)^2 + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*
```

```
x + c) + (a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**5/(a + b*sec(c + d*x))**2, x)
```

Giac [B] time = 1.49443, size = 1073, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/64*(8*(4*a^2 + 13*a*b + 12*b^2)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x +
c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 64*(7*a^2*b^6 - b^8)
*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1)))/(a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*
b^8) - (12*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 32*a*b*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 20*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a
^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 2*a*b*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 + b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(a^4 -
 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (a^2 + 2*a*b + b^2 + 12*a^2*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 32*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 20*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 48*a^2*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2 + 156*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2 + 144*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^
2/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(cos(d*x + c) - 1)^2) + 64*(
7*a^3*b^6 + 5*a^2*b^7 - 3*a*b^8 - b^9 + 7*a^3*b^6*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) - 7*a^2*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*b^8*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + b^9*(cos(d*x + c) - 1)/(cos(d*x + c) +
1))/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*(a + b + a*(cos(d
```

```
*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))
 - 64*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2)/d
```

3.307 $\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=200

$$\frac{(3a^2-2b^2)\tan(c+dx)}{b^4d}-\frac{a(4a^2-5b^2)\tanh^{-1}(\sin(c+dx))}{b^5d}+\frac{(a^2-b^2)^2\sin(c+dx)}{ab^4d(a\cos(c+dx)+b)}+\frac{2(a-b)^{3/2}(a+b)^{3/2}(4a^2+b^2}{a^2b}$$

```
[Out] -(x/a^2) - (a*(4*a^2 - 5*b^2)*ArcTanh[Sin[c + d*x]])/(b^5*d) + (2*(a - b)^(
3/2)*(a + b)^(3/2)*(4*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqr
t[a + b]])/(a^2*b^5*d) + ((a^2 - b^2)^2*Sin[c + d*x])/(a*b^4*d*(b + a*Cos[c
 + d*x])) + ((3*a^2 - 2*b^2)*Tan[c + d*x])/(b^4*d) - (a*Sec[c + d*x]*Tan[c
+ d*x])/(b^3*d) + Tan[c + d*x]^3/(3*b^2*d)
```

Rubi [A] time = 0.430917, antiderivative size = 283, normalized size of antiderivative = 1.42, number of steps used = 16, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3898, 2897, 2664, 12, 2659, 208, 3770, 3767, 8, 3768}

$$\frac{3(a^2-b^2)\tan(c+dx)}{b^4d}-\frac{2a(2a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{b^5d}+\frac{(a^2-b^2)^2\sin(c+dx)}{ab^4d(a\cos(c+dx)+b)}-\frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}(}{a^2b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - (a*ArcTanh[Sin[c + d*x]])/(b^3*d) - (2*a*(2*a^2 - 3*b^2)*ArcTanh
[Sin[c + d*x]])/(b^5*d) - (2*(a - b)^(3/2)*(a + b)^(3/2)*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*b^3*d) + (4*(a - b)^(3/2)*(a + b)^(
3/2)*(2*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^
2*b^5*d) + ((a^2 - b^2)^2*Sin[c + d*x])/(a*b^4*d*(b + a*Cos[c + d*x])) + Ta
n[c + d*x]/(b^2*d) + (3*(a^2 - b^2)*Tan[c + d*x])/(b^4*d) - (a*Sec[c + d*x]
*Tan[c + d*x])/(b^3*d) + Tan[c + d*x]^3/(3*b^2*d)
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx=\int \frac{\sin^2(c+dx)\tan^4(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$=\int\left(-\frac{1}{a^2}+\frac{\left(a^2-b^2\right)^3}{a^2b^4(b+a\cos(c+dx))^2}+\frac{2\left(2a^6-3a^4b^2+b^6\right)}{a^2b^5(b+a\cos(c+dx))}+\frac{2\left(-2a^3+3ab^2\right)\sec(c+d\right.$$

$$=-\frac{x}{a^2}-\frac{(2a)\int\sec^3(c+dx)\,dx}{b^3}+\frac{\int\sec^4(c+dx)\,dx}{b^2}-\frac{\left(2a\left(2a^2-3b^2\right)\right)\int\sec(c+dx)\,dx}{b^5}+\frac{(}{}$$

$$=-\frac{x}{a^2}-\frac{2a\left(2a^2-3b^2\right)\tanh^{-1}(\sin(c+dx))}{b^5d}+\frac{\left(a^2-b^2\right)^2\sin(c+dx)}{ab^4d(b+a\cos(c+dx))}-\frac{a\sec(c+dx)\tan(c}{b^3d}$$

$$=-\frac{x}{a^2}-\frac{a\tanh^{-1}(\sin(c+dx))}{b^3d}-\frac{2a\left(2a^2-3b^2\right)\tanh^{-1}(\sin(c+dx))}{b^5d}+\frac{4(a-b)^{3/2}(a+b)^{3/2}}{}$$

$$=-\frac{x}{a^2}-\frac{a\tanh^{-1}(\sin(c+dx))}{b^3d}-\frac{2a\left(2a^2-3b^2\right)\tanh^{-1}(\sin(c+dx))}{b^5d}+\frac{4(a-b)^{3/2}(a+b)^{3/2}}{}$$

$$=-\frac{x}{a^2}-\frac{a\tanh^{-1}(\sin(c+dx))}{b^3d}-\frac{2a\left(2a^2-3b^2\right)\tanh^{-1}(\sin(c+dx))}{b^5d}-\frac{2(a-b)^{3/2}(a+b)^{3/2}}{}$$

Mathematica [B] time = 6.22672, size = 865, normalized size = 4.32

$$\frac{(b+a\cos(c+dx))^2\sin\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)}{6b^2d(a+b\sec(c+dx))^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{(b+a\cos(c+dx))^2\left(9a^2\sin\left(\frac{1}{2}(c+dx)\right)-7b^2\sin\left(\frac{1}{2}(c\right.\right.}{3b^4d(a+b\sec(c+dx))^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((c + d*x)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^2)/(a^2*d*(a + b*Sec[c + d
*x])^2)) - (2*(-a^2 + b^2)^2*(4*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/
2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^2)/(a^2*b^5*Sqrt[a
^2 - b^2]*d*(a + b*Sec[c + d*x])^2) + ((4*a^3 - 5*a*b^2)*(b + a*Cos[c + d*x
])^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sec[c + d*x]^2)/(b^5*d*(a + b
*Sec[c + d*x])^2) + ((-4*a^3 + 5*a*b^2)*(b + a*Cos[c + d*x])^2*Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]]*Sec[c + d*x]^2)/(b^5*d*(a + b*Sec[c + d*x])^2)
 + ((-6*a + b)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^2)/(12*b^3*d*(a + b*Sec[
c + d*x])^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + ((b + a*Cos[c + d*x]
)^2*Sec[c + d*x]^2*Sin[(c + d*x)/2])/(6*b^2*d*(a + b*Sec[c + d*x])^2*(Cos[(
c + d*x)/2] - Sin[(c + d*x)/2])^3) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2
*Sin[(c + d*x)/2])/(6*b^2*d*(a + b*Sec[c + d*x])^2*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^3) + ((6*a - b)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^2)/(12*b^
3*d*(a + b*Sec[c + d*x])^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + ((b +
 a*Cos[c + d*x])^2*Sec[c + d*x]^2*(9*a^2*Sin[(c + d*x)/2] - 7*b^2*Sin[(c +
d*x)/2]))/(3*b^4*d*(a + b*Sec[c + d*x])^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)
/2])) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*(9*a^2*Sin[(c + d*x)/2] - 7*
b^2*Sin[(c + d*x)/2]))/(3*b^4*d*(a + b*Sec[c + d*x])^2*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])) + ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*(a^4*Sin[c + d*x]
 - 2*a^2*b^2*Sin[c + d*x] + b^4*Sin[c + d*x]))/(a*b^4*d*(a + b*Sec[c + d*x]
)^2)
```

Maple [B] time = 0.092, size = 723, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/d/a^2*arctan(tan(1/2*d*x+1/2*c))-2/d/b^4*a^3*tan(1/2*d*x+1/2*c)/(tan(1/2
*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)+4/d/b^2*a*tan(1/2*d*x+1/2*c)/(t
an(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-2/d/a*tan(1/2*d*x+1/2*c)/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)+8/d/b^5*a^4/((a+b)*(a-b
))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-14/d/b^3*a^2
/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+
4/d/b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1
/2))+2/d*b/a^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*
(a-b))^(1/2))-1/3/d/b^2/(tan(1/2*d*x+1/2*c)+1)^3+1/2/d/b^2/(tan(1/2*d*x+1/2
*c)+1)^2+1/d/b^3/(tan(1/2*d*x+1/2*c)+1)^2*a-3/d/b^4/(tan(1/2*d*x+1/2*c)+1)*
```

```
a^2-1/d/b^3/(tan(1/2*d*x+1/2*c)+1)*a+2/d/b^2/(tan(1/2*d*x+1/2*c)+1)-4/d*a^3
/b^5*ln(tan(1/2*d*x+1/2*c)+1)+5/d*a/b^3*ln(tan(1/2*d*x+1/2*c)+1)-1/3/d/b^2/
(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/b^2/(tan(1/2*d*x+1/2*c)-1)^2-1/d/b^3/(tan(1/
2*d*x+1/2*c)-1)^2*a-3/d/b^4/(tan(1/2*d*x+1/2*c)-1)*a^2-1/d/b^3/(tan(1/2*d*x
+1/2*c)-1)*a+2/d/b^2/(tan(1/2*d*x+1/2*c)-1)+4/d*a^3/b^5*ln(tan(1/2*d*x+1/2*
c)-1)-5/d*a/b^3*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.39632, size = 1893, normalized size = 9.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(6*a*b^5*d*x*cos(d*x + c)^4 + 6*b^6*d*x*cos(d*x + c)^3 + 3*((4*a^5 -
3*a^3*b^2 - a*b^4)*cos(d*x + c)^4 + (4*a^4*b - 3*a^2*b^3 - b^5)*cos(d*x + c
)^3)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2
 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*
cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 3*((4*a^6 - 5*a^4*b^2)*cos(d*
x + c)^4 + (4*a^5*b - 5*a^3*b^3)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 3*
((4*a^6 - 5*a^4*b^2)*cos(d*x + c)^4 + (4*a^5*b - 5*a^3*b^3)*cos(d*x + c)^3)
*log(-sin(d*x + c) + 1) + 2*(2*a^3*b^3*cos(d*x + c) - a^2*b^4 - (12*a^5*b -
 13*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 - (6*a^4*b^2 - 7*a^2*b^4)*cos(d*x + c
)^2)*sin(d*x + c))/(a^3*b^5*d*cos(d*x + c)^4 + a^2*b^6*d*cos(d*x + c)^3), -
1/6*(6*a*b^5*d*x*cos(d*x + c)^4 + 6*b^6*d*x*cos(d*x + c)^3 - 6*((4*a^5 - 3*
a^3*b^2 - a*b^4)*cos(d*x + c)^4 + (4*a^4*b - 3*a^2*b^3 - b^5)*cos(d*x + c)^
3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b
^2)*sin(d*x + c))) + 3*((4*a^6 - 5*a^4*b^2)*cos(d*x + c)^4 + (4*a^5*b - 5*a
^3*b^3)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 3*((4*a^6 - 5*a^4*b^2)*cos(
```

```
d*x + c)^4 + (4*a^5*b - 5*a^3*b^3)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) +
 2*(2*a^3*b^3*cos(d*x + c) - a^2*b^4 - (12*a^5*b - 13*a^3*b^3 + 3*a*b^5)*co
s(d*x + c)^3 - (6*a^4*b^2 - 7*a^2*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(a^3*b
^5*d*cos(d*x + c)^4 + a^2*b^6*d*cos(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**6/(a + b*sec(c + d*x))**2, x)
```

Giac [B] time = 5.04026, size = 555, normalized size = 2.78

$$-\frac{\frac{3(dx+c)}{a^2} + \frac{3\left(4a^3-5ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^5} - \frac{3\left(4a^3-5ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^5} + \frac{6\left(a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b^4\tan\left(\frac{1}{2}dx+\right.\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a-b\right)ab^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)/a^2 + 3*(4*a^3 - 5*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) +
1))/b^5 - 3*(4*a^3 - 5*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^5 + 6*(a
^4*tan(1/2*d*x + 1/2*c) - 2*a^2*b^2*tan(1/2*d*x + 1/2*c) + b^4*tan(1/2*d*x
+ 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)*a*
b^4) - 6*(4*a^6 - 7*a^4*b^2 + 2*a^2*b^4 + b^6)*(pi*floor(1/2*(d*x + c)/pi +
 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1
/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^2*b^5) + 2*(9*a^2*tan(1/2*d*x
 + 1/2*c)^5 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 - 6*b^2*tan(1/2*d*x + 1/2*c)^5 -
 18*a^2*tan(1/2*d*x + 1/2*c)^3 + 16*b^2*tan(1/2*d*x + 1/2*c)^3 + 9*a^2*tan(
1/2*d*x + 1/2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) - 6*b^2*tan(1/2*d*x + 1/2*c))
/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*b^4))/d
```

3.308 $\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=150

$$\frac{(2a^2-b^2)\sin(c+dx)}{ab^2d(a\cos(c+dx)+b)}+\frac{2\sqrt{a-b}\sqrt{a+b}\,(2a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2b^3d}+\frac{x}{a^2}-\frac{2a\tanh^{-1}(\sin(c+dx))}{b^3d}+\frac{\text{t}}{bd(a\,\text{c}}$$

```
[Out] x/a^2 - (2*a*ArcTanh[Sin[c + d*x]])/(b^3*d) + (2*Sqrt[a - b]*Sqrt[a + b]*(2
*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*b^3*d
) + ((2*a^2 - b^2)*Sin[c + d*x])/(a*b^2*d*(b + a*Cos[c + d*x])) + Tan[c + d
*x]/(b*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 0.325731, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3898, 2890, 3057, 2659, 208, 3770}

$$\frac{(2a^2-b^2)\sin(c+dx)}{ab^2d(a\cos(c+dx)+b)}+\frac{2\sqrt{a-b}\sqrt{a+b}\,(2a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2b^3d}+\frac{x}{a^2}-\frac{2a\tanh^{-1}(\sin(c+dx))}{b^3d}+\frac{\text{t}}{bd(a\,\text{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] x/a^2 - (2*a*ArcTanh[Sin[c + d*x]])/(b^3*d) + (2*Sqrt[a - b]*Sqrt[a + b]*(2
*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*b^3*d
) + ((2*a^2 - b^2)*Sin[c + d*x])/(a*b^2*d*(b + a*Cos[c + d*x])) + Tan[c + d
*x]/(b*d*(b + a*Cos[c + d*x]))
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rule 2890

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
```

```
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[((a^2*(n + 1) - b^2*(m + n + 2))*Cos[e +
f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^2*b*d^2*f*(n
 + 1)*(m + 1)), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3057

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(C*x)/(b*d), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$
$$= \frac{(2a^2-b^2)\sin(c+dx)}{ab^2d(b+a\cos(c+dx))} + \frac{\tan(c+dx)}{bd(b+a\cos(c+dx))} + \frac{\int \frac{(-2a^2-ab\cos(c+dx)+b^2\cos^2(c+dx))\sec(c+dx)}{b+a\cos(c+dx)}\,d}{ab^2}$$
$$= \frac{x}{a^2} + \frac{(2a^2-b^2)\sin(c+dx)}{ab^2d(b+a\cos(c+dx))} + \frac{\tan(c+dx)}{bd(b+a\cos(c+dx))} - \frac{(2a)\int\sec(c+dx)\,dx}{b^3} - \frac{(-2a^4+a}{}$$
$$= \frac{x}{a^2} - \frac{2a\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{(2a^2-b^2)\sin(c+dx)}{ab^2d(b+a\cos(c+dx))} + \frac{\tan(c+dx)}{bd(b+a\cos(c+dx))} - \frac{(2(-2a}{}$$
$$= \frac{x}{a^2} - \frac{2a\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{2\sqrt{a-b}\sqrt{a+b}(2a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2b^3d} + \frac{(2}{ab^2}$$

Mathematica [B] time = 1.57522, size = 327, normalized size = 2.18

$$\sec^2(c+dx)(a\cos(c+dx)+b)\left(\frac{(a^2-b^2)\sin(c+dx)}{ab^2} + \frac{2(a^2b^2-2a^4+b^4)(a\cos(c+dx)+b)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^3\sqrt{a^2-b^2}} + \frac{(c+dx)(a\cos(c+dx)+b)}{a^2} + \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*(((c + d*x)*(b + a*Cos[c + d*x]))/a^2
+ (2*(-2*a^4 + a^2*b^2 + b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]]*(b + a*Cos[c + d*x]))/(a^2*b^3*Sqrt[a^2 - b^2]) + (2*a*(b + a*Cos[c
 + d*x])*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/b^3 - (2*a*(b + a*Cos[c
+ d*x])*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/b^3 + ((b + a*Cos[c + d*x
])*Sin[(c + d*x)/2])/(b^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + ((b + a*
Cos[c + d*x])*Sin[(c + d*x)/2])/(b^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
 + ((a^2 - b^2)*Sin[c + d*x])/(a*b^2)))/(d*(a + b*Sec[c + d*x])^2)
```

Maple [B] time = 0.075, size = 353, normalized size = 2.4

$$2\,\frac{\arctan(\tan(1/2\,dx+c/2))}{da^2} - 2\,\frac{a\tan(1/2\,dx+c/2)}{db^2\left((\tan(1/2\,dx+c/2))^2a-(\tan(1/2\,dx+c/2))^2b-a-b\right)} + 2\,\frac{}{ad\left((\tan(1/2\,dx+c/2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*arctan(tan(1/2*d*x+1/2*c))-2/d/b^2*a*tan(1/2*d*x+1/2*c)/(tan(1/2*d*
x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)+2/d/a*tan(1/2*d*x+1/2*c)/(tan(1/2*
d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)+4/d/b^3*a^2/((a+b)*(a-b))^(1/2)*
arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-2/d/b/((a+b)*(a-b))^(
1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-2/d*b/a^2/((a+b)
*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-1/d/b^2
/(tan(1/2*d*x+1/2*c)+1)-2/d*a/b^3*ln(tan(1/2*d*x+1/2*c)+1)-1/d/b^2/(tan(1/2
*d*x+1/2*c)-1)+2/d*a/b^3*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.18755, size = 1374, normalized size = 9.16

$$\left[\frac{2\,ab^3dx\cos(dx+c)^2 + 2\,b^4dx\cos(dx+c) + \left(\left(2\,a^3+ab^2\right)\cos(dx+c)^2 + \left(2\,a^2b+b^3\right)\cos(dx+c)\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^3*d*x*cos(d*x + c)^2 + 2*b^4*d*x*cos(d*x + c) + ((2*a^3 + a*b^2
)*cos(d*x + c)^2 + (2*a^2*b + b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b
*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x
 + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x
+ c) + b^2)) - 2*(a^4*cos(d*x + c)^2 + a^3*b*cos(d*x + c))*log(sin(d*x + c)
```

```
 + 1) + 2*(a^4*cos(d*x + c)^2 + a^3*b*cos(d*x + c))*log(-sin(d*x + c) + 1)
+ 2*(a^2*b^2 + (2*a^3*b - a*b^3)*cos(d*x + c))*sin(d*x + c))/(a^3*b^3*d*cos
(d*x + c)^2 + a^2*b^4*d*cos(d*x + c)), (a*b^3*d*x*cos(d*x + c)^2 + b^4*d*x*
cos(d*x + c) + ((2*a^3 + a*b^2)*cos(d*x + c)^2 + (2*a^2*b + b^3)*cos(d*x +
c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 -
b^2)*sin(d*x + c))) - (a^4*cos(d*x + c)^2 + a^3*b*cos(d*x + c))*log(sin(d*x
 + c) + 1) + (a^4*cos(d*x + c)^2 + a^3*b*cos(d*x + c))*log(-sin(d*x + c) +
1) + (a^2*b^2 + (2*a^3*b - a*b^3)*cos(d*x + c))*sin(d*x + c))/(a^3*b^3*d*co
s(d*x + c)^2 + a^2*b^4*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

Giac [B] time = 2.60112, size = 397, normalized size = 2.65

$$\frac{\frac{dx+c}{a^2} - \frac{2a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^3} + \frac{2a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^3} - \frac{2\left(2a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 2a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)/a^2 - 2*a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 + 2*a*log(abs(t
an(1/2*d*x + 1/2*c) - 1))/b^3 - 2*(2*a^2*tan(1/2*d*x + 1/2*c)^3 - a*b*tan(1
/2*d*x + 1/2*c)^3 - b^2*tan(1/2*d*x + 1/2*c)^3 - 2*a^2*tan(1/2*d*x + 1/2*c)
 - a*b*tan(1/2*d*x + 1/2*c) + b^2*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1
/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)*a*
b^2) + 2*(2*a^4 - a^2*b^2 - b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a
 + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a
```

```
^2 + b^2)))/(sqrt(-a^2 + b^2)*a^2*b^3))/d
```

3.309 $\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=85

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{x}{a^2}+\frac{\tan(c+dx)}{ad(a+b\sec(c+dx))}$$

```
[Out] -(x/a^2) + (2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*S
qrt[a - b]*Sqrt[a + b]*d) + Tan[c + d*x]/(a*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.145879, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3894, 4061, 12, 3783, 2659, 208}

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{x}{a^2}+\frac{\tan(c+dx)}{ad(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```



```
[Out] -(x/a^2) + (2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*S
qrt[a - b]*Sqrt[a + b]*d) + Tan[c + d*x]/(a*d*(a + b*Sec[c + d*x]))
```


Rule 3894

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4061

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[
e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

```
&& LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3783

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{-1+\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\tan(c+dx)}{ad(a+b\sec(c+dx))} - \frac{\int \frac{a^2-b^2}{a+b\sec(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$
$$= \frac{\tan(c+dx)}{ad(a+b\sec(c+dx))} - \frac{\int \frac{1}{a+b\sec(c+dx)}\,dx}{a}$$
$$= -\frac{x}{a^2} + \frac{\tan(c+dx)}{ad(a+b\sec(c+dx))} + \frac{\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a^2}$$
$$= -\frac{x}{a^2} + \frac{\tan(c+dx)}{ad(a+b\sec(c+dx))} + \frac{2\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^2d}$$
$$= -\frac{x}{a^2} + \frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}d} + \frac{\tan(c+dx)}{ad(a+b\sec(c+dx))}$$

Mathematica [A] time = 0.255037, size = 80, normalized size = 0.94

$$-\frac{\frac{2b\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{a\sin(c+dx)}{a\cos(c+dx)+b} + c + dx}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((c + d*x + (2*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqr
t[a^2 - b^2] - (a*Sin[c + d*x])/(b + a*Cos[c + d*x]))/(a^2*d))
```

Maple [A] time = 0.065, size = 120, normalized size = 1.4

$$-2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^2} - 2\,\frac{\tan\left(1/2\,dx+c/2\right)}{ad\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^2a - \left(\tan\left(1/2\,dx+c/2\right)\right)^2b - a - b\right)} + 2\,\frac{b}{da^2\sqrt{(a+b)(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/d/a^2*arctan(tan(1/2*d*x+1/2*c))-2/d/a*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1
/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)+2/d*b/a^2/((a+b)*(a-b))^(1/2)*arctanh
((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.898244, size = 845, normalized size = 9.94

$$\left[-\frac{2\left(a^3-ab^2\right)dx\cos\left(dx+c\right)+2\left(a^2b-b^3\right)dx-\left(ab\cos\left(dx+c\right)+b^2\right)\sqrt{a^2-b^2}\log\left(\frac{2ab\cos(dx+c)-\left(a^2-2b^2\right)\cos(dx+c)^2+2\sqrt{a^2-b^2}}{a^2\cos(dx+c)^2+2ab\cos}\right.}{2\left(\left(a^5-a^3b^2\right)d\cos\left(dx+c\right)+\left(a^4b-a^2b^3\right)d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^3 - a*b^2)*d*x*cos(d*x + c) + 2*(a^2*b - b^3)*d*x - (a*b*cos(d*
x + c) + b^2)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d
*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b
^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(a^3 - a*b^2)*sin(
d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x + c) + (a^4*b - a^2*b^3)*d), -((a^3 -
a*b^2)*d*x*cos(d*x + c) + (a^2*b - b^3)*d*x - (a*b*cos(d*x + c) + b^2)*sqrt
(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin
(d*x + c))) - (a^3 - a*b^2)*sin(d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x + c) +
 (a^4*b - a^2*b^3)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

Giac [A] time = 1.49275, size = 194, normalized size = 2.28

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b}{\sqrt{-a^2+b^2}\,a^2}-\frac{dx+c}{a^2}-\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a-b\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b/(sqrt(-a^2 + b^2
)*a^2) - (d*x + c)/a^2 - 2*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2
- b*tan(1/2*d*x + 1/2*c)^2 - a - b)*a))/d
```

3.310 $\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=227

$$\frac{b^4\sin(c+dx)}{ad\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}-\frac{4b^3\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}-\frac{x}{a^2}-\frac{\cdots}{2d(a\cdots}$$

```
[Out] -(x/a^2) - (2*b^5*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2
*(a - b)^(5/2)*(a + b)^(5/2)*d) - (4*b^3*(2*a^2 - b^2)*ArcTanh[(Sqrt[a - b]
*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(5/2)*(a + b)^(5/2)*d) - Sin[
c + d*x]/(2*(a + b)^2*d*(1 - Cos[c + d*x])) + Sin[c + d*x]/(2*(a - b)^2*d*(
1 + Cos[c + d*x])) + (b^4*Sin[c + d*x])/(a*(a^2 - b^2)^2*d*(b + a*Cos[c + d
*x]))
```

Rubi [A] time = 0.411295, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3898, 2897, 2648, 2664, 12, 2659, 208}

$$\frac{b^4\sin(c+dx)}{ad\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}-\frac{4b^3\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}-\frac{x}{a^2}-\frac{\cdots}{2d(a\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - (2*b^5*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2
*(a - b)^(5/2)*(a + b)^(5/2)*d) - (4*b^3*(2*a^2 - b^2)*ArcTanh[(Sqrt[a - b]
*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(5/2)*(a + b)^(5/2)*d) - Sin[
c + d*x]/(2*(a + b)^2*d*(1 - Cos[c + d*x])) + Sin[c + d*x]/(2*(a - b)^2*d*(
1 + Cos[c + d*x])) + (b^4*Sin[c + d*x])/(a*(a^2 - b^2)^2*d*(b + a*Cos[c + d
*x]))
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
```

```
IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= \int \left(-\frac{1}{a^2} - \frac{1}{2(a-b)^2(-1-\cos(c+dx))} + \frac{1}{2(a+b)^2(1-\cos(c+dx))} + \frac{b^4}{a^2\left(a^2-b^2\right)(-b-a\cos\right.$$

$$= -\frac{x}{a^2} - \frac{\int \frac{1}{-1-\cos(c+dx)}\,dx}{2(a-b)^2} + \frac{\int \frac{1}{1-\cos(c+dx)}\,dx}{2(a+b)^2} + \frac{b^4\int \frac{1}{(-b-a\cos(c+dx))^2}\,dx}{a^2\left(a^2-b^2\right)} + \frac{\left(2b^3\left(2a^2-b^2\right)\right)\int}{a^2\left(a^2-b^2\right.}$$

$$= -\frac{x}{a^2} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))} + \frac{b^4\sin(c+dx)}{a\left(a^2-b^2\right)^2 d(b+a\cos(\right.}$$

$$= -\frac{x}{a^2} - \frac{4b^3\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin}{2(a-b)^2d(}$$

$$= -\frac{x}{a^2} - \frac{4b^3\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin}{2(a-b)^2d(}$$

$$= -\frac{x}{a^2} - \frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{4b^3\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{s}{2(a+b)^2}$$

Mathematica [A] time = 1.73764, size = 209, normalized size = 0.92

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)\left(-\frac{4b^3\left(b^2-4a^2\right)(a\cos(c+dx)+b)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{5/2}} - \frac{2(c+dx)(a\cos(c+dx)+b)}{a^2} + \frac{2b^4\sin(c+dx)}{a(a-b)^2(a+b)^2} + \frac{\tan\left(\frac{1}{2}(\right.}{}\right.}{2d(a+b\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*((-2*(c + d*x)*(b + a*Cos[c + d*x]))/a
^2 - (4*b^3*(-4*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]]*(b + a*Cos[c + d*x]))/(a^2*(a^2 - b^2)^(5/2)) - ((b + a*Cos[c + d*x])*
Cot[(c + d*x)/2])/(a + b)^2 + (2*b^4*Sin[c + d*x])/(a*(a - b)^2*(a + b)^2)
```

```
+ ((b + a*Cos[c + d*x])*Tan[(c + d*x)/2])/(a - b)^2))/(2*d*(a + b*Sec[c + d
*x])^2)
```

Maple [A] time = 0.092, size = 255, normalized size = 1.1

$$\frac{1}{2\,d\left(a^{2}-2\,ab+b^{2}\right)}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^{2}}-2\,\frac{b^{4}\tan\left(1/2\,dx+c/2\right)}{d\left(a+b\right)^{2}\left(a-b\right)^{2}a\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}a-(\mathrm{tar}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/2/d/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-2/d/a^2*arctan(tan(1/2*d*x+1/2*c))
-2/d*b^4/(a+b)^2/(a-b)^2/a*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b-a-b)-8/d*b^3/(a+b)^2/(a-b)^2/((a+b)*(a-b))^(1/2)*arctanh(
(a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+2/d*b^5/(a+b)^2/(a-b)^2/a^2/(
(a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-1/
2/d/(a+b)^2/tan(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.881407, size = 1507, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(4*a^5*b^2 - 2*a^3*b^4 - 2*a*b^6 - (4*a^2*b^4 - b^6 + (4*a^3*b^3 - a*b
^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*c
os(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2
 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(
a^7 - a*b^6)*cos(d*x + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c)
- 2*((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*x*cos(d*x + c) + (a^6*b - 3*a^
4*b^3 + 3*a^2*b^5 - b^7)*d*x)*sin(d*x + c))/(((a^9 - 3*a^7*b^2 + 3*a^5*b^4
- a^3*b^6)*d*cos(d*x + c) + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d)*si
n(d*x + c)), (2*a^5*b^2 - a^3*b^4 - a*b^6 - (4*a^2*b^4 - b^6 + (4*a^3*b^3 -
 a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x
+ c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (a^7 - a*b^6)*cos(d*x
+ c)^2 + (a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c) - ((a^7 - 3*a^5*b^2 + 3
*a^3*b^4 - a*b^6)*d*x*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*
d*x)*sin(d*x + c))/(((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)
 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d)*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(c + dx)}{(a + b\sec(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

Giac [A] time = 1.35958, size = 448, normalized size = 1.97

$$-\frac{\frac{4\left(4\,a^2b^3-b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-2\,a^4b^2+a^2b^4\right)\sqrt{-a^2+b^2}}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2-2\,ab+b^2}+\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^2b^2}{\left(a^5-2\,a^3b^2+ab^4\right)\left(a\tan\left(\frac{1}{2}\,dx+\right.\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*(4*a^2*b^3 - b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b)
 + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^
```

```
2)))/((a^6 - 2*a^4*b^2 + a^2*b^4)*sqrt(-a^2 + b^2)) - tan(1/2*d*x + 1/2*c)/
(a^2 - 2*a*b + b^2) + (a^4*tan(1/2*d*x + 1/2*c)^2 - 3*a^3*b*tan(1/2*d*x + 1
/2*c)^2 + 3*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - a*b^3*tan(1/2*d*x + 1/2*c)^2 +
 4*b^4*tan(1/2*d*x + 1/2*c)^2 - a^4 + a^3*b + a^2*b^2 - a*b^3)/((a^5 - 2*a^
3*b^2 + a*b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 - a*tan
(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))) + 2*(d*x + c)/a^2)/d
```

3.311 $\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=360

$$\frac{b^6\sin(c+dx)}{ad\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{2b^7\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{7/2}(a+b)^{7/2}}-\frac{4b^5\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{7/2}(a+b)^{7/2}}+\frac{x}{a^2}+\frac{(\ldots}{4d(\ldots}$$

```
[Out] x/a^2 - (2*b^7*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a
 - b)^(7/2)*(a + b)^(7/2)*d) - (4*b^5*(3*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Ta
n[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(7/2)*(a + b)^(7/2)*d) - Sin[c +
 d*x]/(12*(a + b)^2*d*(1 - Cos[c + d*x])^2) - Sin[c + d*x]/(12*(a + b)^2*d*
(1 - Cos[c + d*x])) + ((3*a + 5*b)*Sin[c + d*x])/(4*(a + b)^3*d*(1 - Cos[c
+ d*x])) + Sin[c + d*x]/(12*(a - b)^2*d*(1 + Cos[c + d*x])^2) - ((3*a - 5*b
)*Sin[c + d*x])/(4*(a - b)^3*d*(1 + Cos[c + d*x])) + Sin[c + d*x]/(12*(a -
b)^2*d*(1 + Cos[c + d*x])) + (b^6*Sin[c + d*x])/(a*(a^2 - b^2)^3*d*(b + a*C
os[c + d*x]))
```

Rubi [A] time = 0.567013, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3898, 2897, 2650, 2648, 2664, 12, 2659, 208}

$$\frac{b^6\sin(c+dx)}{ad\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{2b^7\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{7/2}(a+b)^{7/2}}-\frac{4b^5\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{7/2}(a+b)^{7/2}}+\frac{x}{a^2}+\frac{(\ldots}{4d(\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] x/a^2 - (2*b^7*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a
 - b)^(7/2)*(a + b)^(7/2)*d) - (4*b^5*(3*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Ta
n[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(7/2)*(a + b)^(7/2)*d) - Sin[c +
 d*x]/(12*(a + b)^2*d*(1 - Cos[c + d*x])^2) - Sin[c + d*x]/(12*(a + b)^2*d*
(1 - Cos[c + d*x])) + ((3*a + 5*b)*Sin[c + d*x])/(4*(a + b)^3*d*(1 - Cos[c
+ d*x])) + Sin[c + d*x]/(12*(a - b)^2*d*(1 + Cos[c + d*x])^2) - ((3*a - 5*b
)*Sin[c + d*x])/(4*(a - b)^3*d*(1 + Cos[c + d*x])) + Sin[c + d*x]/(12*(a -
b)^2*d*(1 + Cos[c + d*x])) + (b^6*Sin[c + d*x])/(a*(a^2 - b^2)^3*d*(b + a*C
os[c + d*x]))
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= \int \left(\frac{1}{a^2} + \frac{1}{4(a-b)^2(-1-\cos(c+dx))^2} + \frac{3a-5b}{4(a-b)^3(-1-\cos(c+dx))} + \frac{1}{4(a+b)^2(1-\cos(c\right.$$

$$= \frac{x}{a^2} + \frac{(3a-5b)\int \frac{1}{-1-\cos(c+dx)}\,dx}{4(a-b)^3} + \frac{\int \frac{1}{(-1-\cos(c+dx))^2}\,dx}{4(a-b)^2} + \frac{\int \frac{1}{(1-\cos(c+dx))^2}\,dx}{4(a+b)^2} - \frac{(3a+5b)\int \frac{}{1-c}}{4(a+b}$$

$$= \frac{x}{a^2} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2} + \frac{(3a+5b)\sin(c+dx)}{4(a+b)^3d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{12(a-b)^2d(1+\cos(c+}$$

$$= \frac{x}{a^2} - \frac{4b^5\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2} - \frac{\sin}{12(a+b)^2}$$

$$= \frac{x}{a^2} - \frac{4b^5\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2} - \frac{\sin}{12(a+b)^2}$$

$$= \frac{x}{a^2} - \frac{2b^7\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{7/2}(a+b)^{7/2}d} - \frac{4b^5\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\text{si}}{12(a+b)^2}$$

Mathematica [A] time = 2.4544, size = 303, normalized size = 0.84

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)\left(-\frac{48b^5\left(b^2-6a^2\right)(a\cos(c+dx)+b)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{7/2}} + \frac{24(c+dx)(a\cos(c+dx)+b)}{a^2} + \frac{24b^6\sin(c+dx)}{a(a-b)^3(a+b)^3} + \frac{4(7b}{}\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*((24*(c + d*x)*(b + a*Cos[c + d*x]))/a
^2 - (48*b^5*(-6*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 -
b^2]]*(b + a*Cos[c + d*x]))/(a^2*(a^2 - b^2)^(7/2)) + (4*(4*a + 7*b)*(b + a
*Cos[c + d*x])*Cot[(c + d*x)/2])/(a + b)^3 - ((b + a*Cos[c + d*x])*Cot[(c +
 d*x)/2]*Csc[(c + d*x)/2]^2)/(a + b)^2 + (24*b^6*Sin[c + d*x])/(a*(a - b)^3
*(a + b)^3) + (4*(-4*a + 7*b)*(b + a*Cos[c + d*x])*Tan[(c + d*x)/2])/(a - b
)^3 + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a - b)^2)
)/(24*d*(a + b*Sec[c + d*x])^2)
```

Maple [A] time = 0.099, size = 416, normalized size = 1.2

$$\frac{a}{24\,d\left(a^2-2\,ab+b^2\right)(a-b)}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{b}{24\,d\left(a^2-2\,ab+b^2\right)(a-b)}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{5\,a}{8\,d\left(a^2-2\,ab+b^2\right)(a-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/24/d/(a^2-2*a*b+b^2)/(a-b)*tan(1/2*d*x+1/2*c)^3*a-1/24/d/(a^2-2*a*b+b^2)/
(a-b)*b*tan(1/2*d*x+1/2*c)^3-5/8/d/(a^2-2*a*b+b^2)/(a-b)*a*tan(1/2*d*x+1/2*
c)+9/8/d/(a^2-2*a*b+b^2)/(a-b)*b*tan(1/2*d*x+1/2*c)+2/d/a^2*arctan(tan(1/2*
d*x+1/2*c))-2/d*b^6/(a+b)^3/(a-b)^3/a*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c
)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-12/d*b^5/(a+b)^3/(a-b)^3/((a+b)*(a-b))^(1
/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+2/d*b^7/(a+b)^3/(
a-b)^3/a^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b
))^(1/2))-1/24/d/(a+b)^2/tan(1/2*d*x+1/2*c)^3+5/8/d/(a+b)^3/tan(1/2*d*x+1/2
*c)*a+9/8/d/(a+b)^3/tan(1/2*d*x+1/2*c)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.37913, size = 3191, normalized size = 8.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/6*(8*a^7*b^2 - 40*a^5*b^4 + 26*a^3*b^6 + 6*a*b^8 + 2*(4*a^9 - 13*a^7*b^2
 + 2*a^5*b^4 + 4*a^3*b^6 + 3*a*b^8)*cos(d*x + c)^4 - 2*(2*a^8*b - 11*a^6*b^
3 + 16*a^4*b^5 - 7*a^2*b^7)*cos(d*x + c)^3 - 3*(6*a^2*b^6 - b^8 - (6*a^3*b^
5 - a*b^7)*cos(d*x + c)^3 - (6*a^2*b^6 - b^8)*cos(d*x + c)^2 + (6*a^3*b^5 -
 a*b^7)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^
2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2
*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) -
 6*(a^9 - 2*a^7*b^2 - 7*a^5*b^4 + 6*a^3*b^6 + 2*a*b^8)*cos(d*x + c)^2 + 2*(
a^8*b - 8*a^6*b^3 + 13*a^4*b^5 - 6*a^2*b^7)*cos(d*x + c) + 6*((a^9 - 4*a^7*
b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*x*cos(d*x + c)^3 + (a^8*b - 4*a^6*b^
3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*x*cos(d*x + c)^2 - (a^9 - 4*a^7*b^2 + 6*
a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*x*cos(d*x + c) - (a^8*b - 4*a^6*b^3 + 6*a^4*
b^5 - 4*a^2*b^7 + b^9)*d*x)*sin(d*x + c))/(((a^11 - 4*a^9*b^2 + 6*a^7*b^4 -
 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + (a^10*b - 4*a^8*b^3 + 6*a^6*b^5 -
4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 - (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a
^5*b^6 + a^3*b^8)*d*cos(d*x + c) - (a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*
b^7 + a^2*b^9)*d)*sin(d*x + c)), 1/3*(4*a^7*b^2 - 20*a^5*b^4 + 13*a^3*b^6 +
 3*a*b^8 + (4*a^9 - 13*a^7*b^2 + 2*a^5*b^4 + 4*a^3*b^6 + 3*a*b^8)*cos(d*x +
 c)^4 - (2*a^8*b - 11*a^6*b^3 + 16*a^4*b^5 - 7*a^2*b^7)*cos(d*x + c)^3 + 3*
(6*a^2*b^6 - b^8 - (6*a^3*b^5 - a*b^7)*cos(d*x + c)^3 - (6*a^2*b^6 - b^8)*c
os(d*x + c)^2 + (6*a^3*b^5 - a*b^7)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-
sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x +
 c) - 3*(a^9 - 2*a^7*b^2 - 7*a^5*b^4 + 6*a^3*b^6 + 2*a*b^8)*cos(d*x + c)^2
+ (a^8*b - 8*a^6*b^3 + 13*a^4*b^5 - 6*a^2*b^7)*cos(d*x + c) + 3*((a^9 - 4*a
^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*x*cos(d*x + c)^3 + (a^8*b - 4*a^6
*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*x*cos(d*x + c)^2 - (a^9 - 4*a^7*b^2 +
 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*x*cos(d*x + c) - (a^8*b - 4*a^6*b^3 + 6*a
^4*b^5 - 4*a^2*b^7 + b^9)*d*x)*sin(d*x + c))/(((a^11 - 4*a^9*b^2 + 6*a^7*b^
4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + (a^10*b - 4*a^8*b^3 + 6*a^6*b^5
 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 - (a^11 - 4*a^9*b^2 + 6*a^7*b^4 -
4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c) - (a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a
^4*b^7 + a^2*b^9)*d)*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

Giac [A] time = 1.38352, size = 657, normalized size = 1.82

$$-\frac{\frac{48\,b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^7-3\,a^5b^2+3\,a^3b^4-ab^6\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)}-\frac{48\left(6\,a^2b^5-b^7\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6\right)\sqrt{-a^2+b^2}}\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(48*b^6*tan(1/2*d*x + 1/2*c)/((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*(
a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - 48*(6*a^2*b
^5 - b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^8 - 3*a^6
*b^2 + 3*a^4*b^4 - a^2*b^6)*sqrt(-a^2 + b^2)) - (a^4*tan(1/2*d*x + 1/2*c)^3
 - 4*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*
b^3*tan(1/2*d*x + 1/2*c)^3 + b^4*tan(1/2*d*x + 1/2*c)^3 - 15*a^4*tan(1/2*d*
x + 1/2*c) + 72*a^3*b*tan(1/2*d*x + 1/2*c) - 126*a^2*b^2*tan(1/2*d*x + 1/2*
c) + 96*a*b^3*tan(1/2*d*x + 1/2*c) - 27*b^4*tan(1/2*d*x + 1/2*c))/(a^6 - 6*
a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6) - 24*(d*x + c
)/a^2 - (15*a*tan(1/2*d*x + 1/2*c)^2 + 27*b*tan(1/2*d*x + 1/2*c)^2 - a - b)
/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(1/2*d*x + 1/2*c)^3))/d
```

3.312 $\int \frac{(e\tan(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=761

$$-\frac{e^{5/2}\left(a^2-b^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d}+\frac{e^{5/2}\left(a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ab^2d}+\frac{e^{5/2}\left(a^2-b^2\right)\log\left(\sqrt{e}\tan(c+dx)-\right.}{2\sqrt{2}ab^2d}$$

```
[Out] (a*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*b^2
*d) - ((a^2 - b^2)*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e
]])/(Sqrt[2]*a*b^2*d) - (a*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]]
)/Sqrt[e]])/(Sqrt[2]*b^2*d) + ((a^2 - b^2)*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt
[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*b^2*d) - (a*e^(5/2)*Log[Sqrt[e] + Sq
rt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*b^2*d) + ((a
^2 - b^2)*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c
 + d*x]]])/(2*Sqrt[2]*a*b^2*d) + (a*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d
*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*b^2*d) - ((a^2 - b^2)*e^(5/
2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*S
qrt[2]*a*b^2*d) + (2*Sqrt[2]*Sqrt[a - b]*Sqrt[a + b]*e^2*Sqrt[Cos[c + d*x]]
*EllipticPi[-(Sqrt[a - b]/Sqrt[a + b]), ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 +
Cos[c + d*x]]], -1]*Sqrt[e*Tan[c + d*x]])/(a*b*d*Sqrt[Sin[c + d*x]]) - (2*S
qrt[2]*Sqrt[a - b]*Sqrt[a + b]*e^2*Sqrt[Cos[c + d*x]]*EllipticPi[Sqrt[a - b
]/Sqrt[a + b], ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 + Cos[c + d*x]]], -1]*Sqrt[
e*Tan[c + d*x]])/(a*b*d*Sqrt[Sin[c + d*x]]) - (2*e^2*Cos[c + d*x]*EllipticE
[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(b*d*Sqrt[Sin[2*c + 2*d*x]]) + (2
*e*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(b*d)
```

Rubi [A] time = 1.17476, antiderivative size = 761, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 22, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.88, Rules used = {3891, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639, 3890, 2733, 2730, 2906, 2905, 490, 1213, 537}

$$-\frac{e^{5/2}\left(a^2-b^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d}+\frac{e^{5/2}\left(a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ab^2d}+\frac{e^{5/2}\left(a^2-b^2\right)\log\left(\sqrt{e}\tan(c+dx)-\right.}{2\sqrt{2}ab^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(5/2)/(a + b*Sec[c + d*x]),x]
```


```
[Out] (a*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*b^2
*d) - ((a^2 - b^2)*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e
```

```
]])/(Sqrt[2]*a*b^2*d) - (a*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]]
)/Sqrt[e]])/(Sqrt[2]*b^2*d) + ((a^2 - b^2)*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt
[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*b^2*d) - (a*e^(5/2)*Log[Sqrt[e] + Sq
rt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*b^2*d) + ((a
^2 - b^2)*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c
 + d*x]]])/(2*Sqrt[2]*a*b^2*d) + (a*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d
*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*b^2*d) - ((a^2 - b^2)*e^(5/
2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*S
qrt[2]*a*b^2*d) + (2*Sqrt[2]*Sqrt[a - b]*Sqrt[a + b]*e^2*Sqrt[Cos[c + d*x]]
*EllipticPi[-(Sqrt[a - b]/Sqrt[a + b]), ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 +
Cos[c + d*x]]], -1]*Sqrt[e*Tan[c + d*x]])/(a*b*d*Sqrt[Sin[c + d*x]]) - (2*S
qrt[2]*Sqrt[a - b]*Sqrt[a + b]*e^2*Sqrt[Cos[c + d*x]]*EllipticPi[Sqrt[a - b
]/Sqrt[a + b], ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 + Cos[c + d*x]]], -1]*Sqrt[
e*Tan[c + d*x]])/(a*b*d*Sqrt[Sin[c + d*x]]) - (2*e^2*Cos[c + d*x]*EllipticE
[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(b*d*Sqrt[Sin[2*c + 2*d*x]]) + (2
*e*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(b*d)
```

Rule 3891

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Dist[e^2/b^2, Int[(e*Cot[c + d*x])^(m - 2)*(a - b*Csc[c
 + d*x]), x], x] + Dist[(e^2*(a^2 - b^2))/b^2, Int[(e*Cot[c + d*x])^(m - 2)
/(a + b*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2
, 0] && IGtQ[m - 1/2, 0]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
```

```
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3890

```
Int[Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Dist[1/a, Int[Sqrt[e*Cot[c + d*x]], x], x] - Dist[b/a, In
t[Sqrt[e*Cot[c + d*x]]/(b + a*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2733

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2730

```
Int[1/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(g_)*tan[(e_.) + (f_.)*(
x_)]]), x_Symbol] :> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[g*Tan
[e + f*x]]), Int[Sqrt[Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x]
)), x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = -\frac{e^2\int (a-b\sec(c+dx))\sqrt{e\tan(c+dx)}\,dx}{b^2} + \frac{\left(\left(a^2-b^2\right)e^2\right)\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx}{b^2}$$

$$= -\frac{\left(ae^2\right)\int \sqrt{e\tan(c+dx)}\,dx}{b^2} + \frac{e^2\int \sec(c+dx)\sqrt{e\tan(c+dx)}\,dx}{b} + \frac{\left(\left(a^2-b^2\right)e^2\right)\int \sqrt{e\tan(c}{ab^2}$$

$$= \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{bd} - \frac{\left(2e^2\right)\int \cos(c+dx)\sqrt{e\tan(c+dx)}\,dx}{b} - \frac{\left(ae^3\right)\text{Subst}\left(\int \frac{}{e^2}\right.}{}$$

$$= \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{bd} - \frac{\left(2ae^3\right)\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{b^2d} + \frac{\left(2\left(a^2-b^2\right.\right.}{}$$

$$= \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{bd} + \frac{\left(ae^3\right)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{b^2d} - \frac{\left(ae^3\right)\text{Subst}}{}$$

$$= -\frac{2e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{bd\sqrt{\sin(2c+2dx)}} + \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{bd} - \frac{\left(ae^{5/2}\right.}{}$$

$$= -\frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}b^2d} + \frac{\left(a^2-b^2\right)e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+\right.}{2\sqrt{2}ab^2d}$$

$$= \frac{ae^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}b^2d} - \frac{\left(a^2-b^2\right)e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d} - \frac{ae^{5/2}\tan^{-1}\left(1+\right.}{\sqrt{2}b^2}$$

$$= \frac{ae^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}b^2d} - \frac{\left(a^2-b^2\right)e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d} - \frac{ae^{5/2}\tan^{-1}\left(1+\right.}{\sqrt{2}b^2}$$

Mathematica [C] time = 26.2116, size = 1846, normalized size = 2.43

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Tan[c + d*x])^(5/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Cot[c + d*x]*(e*Tan[c + d*x])^(5/2))/(b*d*(a + b*Se
c[c + d*x])) - ((b + a*Cos[c + d*x])*Sec[c + d*x]*(e*Tan[c + d*x])^(5/2)*((
4*a*Sec[c + d*x]^2*((-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Tan[c + d*x]])/(-a
^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Tan[c + d*x]])/(-a^2
+ b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[b]*(-a^2 + b^2)^(1/4)*S
```

```
qrt[Tan[c + d*x]] + b*Tan[c + d*x]] - Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[b
]*(-a^2 + b^2)^(1/4)*Sqrt[Tan[c + d*x]] + b*Tan[c + d*x]])/(4*Sqrt[2]*Sqrt[
b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, -Tan[c + d*x]^2, (b^
2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(3/2))/(3*a^2 - 3*b^2))*(a + b*
Sqrt[1 + Tan[c + d*x]^2]))/((b + a*Cos[c + d*x])*(1 + Tan[c + d*x]^2)^(3/2)
) - (b*Sec[c + d*x]*(6*Sqrt[2]*(a^2 - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c +
d*x]]] - 6*Sqrt[2]*a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + 6*Sqrt[2]*b
^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] - (6 + 6*I)*Sqrt[b]*(a^2 - b^2)^(
3/4)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)] + (
6 + 6*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Tan[c +
 d*x]])/(a^2 - b^2)^(1/4)] - 3*Sqrt[2]*a^2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x
]] + Tan[c + d*x]] + 3*Sqrt[2]*b^2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan
[c + d*x]] + 3*Sqrt[2]*a^2*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]] - 3*Sqrt[2]*b^2*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + (3
+ 3*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[b]*(a^2
 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]] - (3 + 3*I)*Sqrt[b]*(a
^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqr
t[Tan[c + d*x]] + I*b*Tan[c + d*x]] + 8*a*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan
[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(3/2))*(a + b*S
qrt[1 + Tan[c + d*x]^2]))/(4*(a^3 - a*b^2)*(b + a*Cos[c + d*x])*(1 + Tan[c
+ d*x]^2)) + (Cos[2*(c + d*x)]*Sec[c + d*x]^2*(-84*Sqrt[2]*b*ArcTan[1 - Sqr
t[2]*Sqrt[Tan[c + d*x]]] + 84*Sqrt[2]*b*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x
]]] + ((42 + 42*I)*(-a^2 + 2*b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c +
d*x]])/(a^2 - b^2)^(1/4)])/(Sqrt[b]*(a^2 - b^2)^(1/4)) + ((42 + 42*I)*(a^2
- 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)]
)/(Sqrt[b]*(a^2 - b^2)^(1/4)) + 42*Sqrt[2]*b*Log[1 - Sqrt[2]*Sqrt[Tan[c + d
*x]] + Tan[c + d*x]] - 42*Sqrt[2]*b*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Ta
n[c + d*x]] + ((21 + 21*I)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt
[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]])/(Sqrt[b]*(a^2
 - b^2)^(1/4)) + ((21 + 21*I)*(-a^2 + 2*b^2)*Log[Sqrt[a^2 - b^2] + (1 + I)*
Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]])/(Sqrt[b]*
(a^2 - b^2)^(1/4)) + (112*a^3*AppellF1[3/4, 1/2, 1, 7/4, -Tan[c + d*x]^2, (
b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(3/2))/(a^2 - b^2) - (168*a*b
^2*AppellF1[3/4, 1/2, 1, 7/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 -
b^2)]*Tan[c + d*x]^(3/2))/(a^2 - b^2) - (24*a*b^2*AppellF1[7/4, 1/2, 1, 11/
4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(7/2))/(
a^2 - b^2) - (168*a*Tan[c + d*x]^(3/2))/Sqrt[1 + Tan[c + d*x]^2])*(a + b*Sq
rt[1 + Tan[c + d*x]^2]))/(84*a*(b + a*Cos[c + d*x])*(-1 + Tan[c + d*x]^2)*S
qrt[1 + Tan[c + d*x]^2])))/(b*d*(a + b*Sec[c + d*x])*Tan[c + d*x]^(5/2))
```

Maple [B] time = 0.288, size = 3747, normalized size = 4.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*tan(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x)

[Out] -1/d*2^(1/2)/b/((a^2-b^2)^(1/2)-a+b)/((a^2-b^2)^(1/2)+a-b)/a*(a-b)*(-1+cos(
d*x+c))^2*(-cos(d*x+c)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*E
llipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2
))*b^2-cos(d*x+c)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*Ellipt
icPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^
2-2*2^(1/2)*a*b+2*cos(d*x+c)*2^(1/2)*a*b-(a^2-b^2)^(1/2)*((-1+cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*b-4*((-1+cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2),1/2*2^(1/2))*a*b+2*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*
2^(1/2))*a*b-I*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticP
i(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b^2+I
*cos(d*x+c)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^2+(a^2
-b^2)^(1/2)*cos(d*x+c)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*E
llipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a+b)*(a
-b))^(1/2)),1/2*2^(1/2))*a+(a^2-b^2)^(1/2)*cos(d*x+c)*((-1+cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2),(a-b)/(a-b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*b-(a^2-b^2)^(1/2)*c
os(d*x+c)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)*(a-b))^(1/2))
,1/2*2^(1/2))*a-(a^2-b^2)^(1/2)*cos(d*x+c)*((-1+cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),
-(a-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*b-4*cos(d*x+c)*((-1+cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2),1/2*2^(1/2))*a*b+2*cos(d*x+c)*((-1+cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*

```
x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2),1/2*2^(1/2))*a*b-I*cos(d*x+c)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I
,1/2*2^(1/2))*b^2-((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*Ellipt
icPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b^
2-((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^2-((-1+cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2),(a-b)/(a-b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a^2+((-1+cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((
1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*b^2-((
-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2)
)*a^2+((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2
*2^(1/2))*b^2-cos(d*x+c)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a+b)*
(a-b))^(1/2)),1/2*2^(1/2))*a^2+cos(d*x+c)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(
a-b)/(a-b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*b^2-cos(d*x+c)*((-1+cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a^2+cos(d*x+
c)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^
(1/2))*b^2+I*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^2+(a^
2-b^2)^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a+b)*(a-b))^(1/2)
),1/2*2^(1/2))*a+(a^2-b^2)^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b
+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*b-(a^2-b^2)^(1/2)*((-1+cos(d*x+c)+sin(d*
```

```
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2),-(a-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a)*cos(d*x+c)^2*(c
os(d*x+c)+1)^2*(e*sin(d*x+c)/cos(d*x+c))^(5/2)/sin(d*x+c)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^{\frac{5}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*tan(d*x + c))^(5/2)/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(5/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \tan (dx + c))^{\frac{5}{2}}}{b \sec (dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^(5/2)/(b*sec(d*x + c) + a), x)
```

3.313 $\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=740

$$\frac{e^2\sqrt{\sin(2c+2dx)}\sec(c+dx)\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{bd\sqrt{e\tan(c+dx)}}-\frac{e^{3/2}\left(a^2-b^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d}+\frac{e^{3/2}\left(a^2-b^2\right)\tan^{-1}(}{\sqrt{2}a}$$

```
[Out] (a*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*b^2
*d) - ((a^2 - b^2)*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e
]])/(Sqrt[2]*a*b^2*d) - (a*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]]
)/Sqrt[e]])/(Sqrt[2]*b^2*d) + ((a^2 - b^2)*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt
[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*b^2*d) + (a*e^(3/2)*Log[Sqrt[e] + Sq
rt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*b^2*d) - ((a
^2 - b^2)*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c
 + d*x]]])/(2*Sqrt[2]*a*b^2*d) - (a*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d
*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*b^2*d) + ((a^2 - b^2)*e^(3/
2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*S
qrt[2]*a*b^2*d) - (2*Sqrt[2]*Sqrt[a^2 - b^2]*e^2*EllipticPi[b/(a - Sqrt[a^2
 - b^2]), ArcSin[Sqrt[-Cos[c + d*x]]/Sqrt[1 + Sin[c + d*x]]], -1]*Sqrt[Sin[
c + d*x]])/(a*b*d*Sqrt[-Cos[c + d*x]]*Sqrt[e*Tan[c + d*x]]) + (2*Sqrt[2]*Sq
rt[a^2 - b^2]*e^2*EllipticPi[b/(a + Sqrt[a^2 - b^2]), ArcSin[Sqrt[-Cos[c +
d*x]]/Sqrt[1 + Sin[c + d*x]]], -1]*Sqrt[Sin[c + d*x]])/(a*b*d*Sqrt[-Cos[c +
 d*x]]*Sqrt[e*Tan[c + d*x]]) + (e^2*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*
x]*Sqrt[Sin[2*c + 2*d*x]])/(b*d*Sqrt[e*Tan[c + d*x]])
```

Rubi [A] time = 1.01498, antiderivative size = 740, normalized size of antiderivative = 1., number of steps used = 35, number of rules used = 19, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.76, Rules used = {3891, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641, 3892, 2733, 2729, 2907, 1213, 537}

$$-\frac{e^{3/2}\left(a^2-b^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d}+\frac{e^{3/2}\left(a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ab^2d}-\frac{e^{3/2}\left(a^2-b^2\right)\log\left(\sqrt{e}\tan(c+dx)\right.}{2\sqrt{2}ab^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*b^2
*d) - ((a^2 - b^2)*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e
]])/(Sqrt[2]*a*b^2*d) - (a*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]]
```

```
)/Sqrt[e]])/(Sqrt[2]*b^2*d) + ((a^2 - b^2)*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt
[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*b^2*d) + (a*e^(3/2)*Log[Sqrt[e] + Sq
rt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*b^2*d) - ((a
^2 - b^2)*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c
 + d*x]]])/(2*Sqrt[2]*a*b^2*d) - (a*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d
*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*b^2*d) + ((a^2 - b^2)*e^(3/
2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*S
qrt[2]*a*b^2*d) - (2*Sqrt[2]*Sqrt[a^2 - b^2]*e^2*EllipticPi[b/(a - Sqrt[a^2
 - b^2]), ArcSin[Sqrt[-Cos[c + d*x]]/Sqrt[1 + Sin[c + d*x]]], -1]*Sqrt[Sin[
c + d*x]])/(a*b*d*Sqrt[-Cos[c + d*x]]*Sqrt[e*Tan[c + d*x]]) + (2*Sqrt[2]*Sq
rt[a^2 - b^2]*e^2*EllipticPi[b/(a + Sqrt[a^2 - b^2]), ArcSin[Sqrt[-Cos[c +
d*x]]/Sqrt[1 + Sin[c + d*x]]], -1]*Sqrt[Sin[c + d*x]])/(a*b*d*Sqrt[-Cos[c +
 d*x]]*Sqrt[e*Tan[c + d*x]]) + (e^2*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*
x]*Sqrt[Sin[2*c + 2*d*x]])/(b*d*Sqrt[e*Tan[c + d*x]])
```

Rule 3891

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Dist[e^2/b^2, Int[(e*Cot[c + d*x])^(m - 2)*(a - b*Csc[c
 + d*x]), x], x] + Dist[(e^2*(a^2 - b^2))/b^2, Int[(e*Cot[c + d*x])^(m - 2)
/(a + b*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2
, 0] && IGtQ[m - 1/2, 0]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
```

```
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3892

```
Int[1/(Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Dist[1/a, Int[1/Sqrt[e*Cot[c + d*x]], x], x] - Dist[b
/a, Int[1/(Sqrt[e*Cot[c + d*x]]*(b + a*Sin[c + d*x])), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2733

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2729

```
Int[Sqrt[(g_.)*tan[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[g*Tan[e + f*x]])/Sqrt[Sin[e
 + f*x]], Int[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a + b*Sin[e + f*x])),
 x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = -\frac{e^2\int \frac{a-b\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{b^2}+\frac{\left(\left(a^2-b^2\right)e^2\right)\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx}{b^2}$$

$$= -\frac{\left(ae^2\right)\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{b^2}+\frac{e^2\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{b}+\frac{\left(\left(a^2-b^2\right)e^2\right)\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{ab^2}-\frac{\left(\left(a^2-b^2\right)e^2\right.}{}$$

$$= -\frac{\left(ae^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(e^2+x^2\right)}\,dx,x,e\tan(c+dx)\right)}{b^2d}+\frac{\left(\left(a^2-b^2\right)e^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(e^2+x^2\right)}\,dx,x,e\,\text{ta}\right.}{ab^2d}$$

$$= -\frac{\left(2ae^3\right)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{b^2d}+\frac{\left(2\left(a^2-b^2\right)e^3\right)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx,x,\sqrt{e\tan}\right.}{ab^2d}$$

$$= \frac{e^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{bd\sqrt{e\tan(c+dx)}}-\frac{\left(ae^2\right)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{b^2d}$$

$$= \frac{e^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{bd\sqrt{e\tan(c+dx)}}+\frac{\left(ae^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(}\right.}{2\sqrt{2}b^2d}$$

$$= \frac{ae^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}b^2d}-\frac{\left(a^2-b^2\right)e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+d\right.}{2\sqrt{2}ab^2d}$$

$$= \frac{ae^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}b^2d}-\frac{\left(a^2-b^2\right)e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d}-\frac{ae^{3/2}\tan^{-1}\left(1+\right.}{\sqrt{2}b^2}$$

Mathematica [C] time = 6.92316, size = 202, normalized size = 0.27

$$\frac{2e\sqrt{1-\tan\left(\frac{1}{2}(c+dx)\right)}\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{e\tan(c+dx)}\sqrt{\frac{-\sin(c+dx)+\cos(c+dx)-1}{\cos(c+dx)+1}}\left(-\Pi\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}};\sin^{-1}\left(\sqrt{-\tan\left(\frac{1}{2}(c+dx)\right)}\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Tan[c + d*x])^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*e*Cot[(c + d*x)/2]*(EllipticPi[-I, ArcSin[Sqrt[-Tan[(c + d*x)/2]]], -1]
+ EllipticPi[I, ArcSin[Sqrt[-Tan[(c + d*x)/2]]], -1] - EllipticPi[-(Sqrt[a
- b]/Sqrt[a + b]), ArcSin[Sqrt[-Tan[(c + d*x)/2]]], -1] - EllipticPi[Sqrt[a
 - b]/Sqrt[a + b], ArcSin[Sqrt[-Tan[(c + d*x)/2]]], -1])*Sqrt[(-1 + Cos[c +
 d*x] - Sin[c + d*x])/(1 + Cos[c + d*x])]*Sqrt[1 - Tan[(c + d*x)/2]]*Sqrt[e
*Tan[c + d*x]])/(a*d)
```

Maple [B] time = 0.252, size = 1801, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/2/d*2^(1/2)/((a^2-b^2)^(1/2)-a+b)/((a^2-b^2)^(1/2)+a-b)/(a^2-b^2)^(1/2)/a
*(cos(d*x+c)+1)^2*(e*sin(d*x+c)/cos(d*x+c))^(3/2)*(-1+cos(d*x+c))*cos(d*x+c
)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-EllipticPi(((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*
a^2-EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2
^(1/2))*(a^2-b^2)^(1/2)*b^2-EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^2-EllipticPi(((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*b^2
-2*(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),
(a-b)/(a-b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a^2+2*(a^2-b^2)^(1/2)*Elliptic
Pi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a+b)*(a-b))^(1
/2)),1/2*2^(1/2))*b^2-2*(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a^2+2*(
a^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-
b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*b^2-2*EllipticPi(((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*
```

```
a^2*b-2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+
((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a*b^2+2*EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a^2*b
+2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a
+b)*(a-b))^(1/2)),1/2*2^(1/2))*a*b^2-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)+I*EllipticPi(((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^
(3/2)+2*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I
,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a*b-2*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a*b+2*EllipticPi(
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^
2)^(1/2)*a*b+2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+
1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a*b-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^2-I*Elliptic
Pi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2
-b^2)^(1/2)*b^2+EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2
-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)+EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)+2*EllipticPi(((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a+b)*(a-b))^(1/2)),1/2*
2^(1/2))*a^3+2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b
)/(a-b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*b^3-2*EllipticPi(((1-cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*
a^3-2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+
((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*b^3+I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^2+I*EllipticP
i(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-
b^2)^(1/2)*b^2)/sin(d*x+c)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^{\frac{3}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*tan(d*x + c))^(3/2)/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \tan(c + dx))^{\frac{3}{2}}}{a + b \sec(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((e*tan(c + d*x))**(3/2)/(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \tan(dx + c))^{\frac{3}{2}}}{b \sec(dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^(3/2)/(b*sec(d*x + c) + a), x)
```

3.314 $\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=415

$$\frac{2\sqrt{2}b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\Pi\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right)\middle|-1\right)}{ad\sqrt{a-b}\sqrt{a+b}\sqrt{\sin(c+dx)}}-\frac{2\sqrt{2}b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\Pi\left(\frac{\sqrt{a-b}}{\sqrt{a+b}};\sin\right.}{ad\sqrt{a-b}\sqrt{a+b}\sqrt{\sin(c+}}$$

```
[Out] -((Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
)) + (Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
a*d) + (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*a*d) - (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sq
rt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) + (2*Sqrt[2]*b*Sqrt[Cos[c + d*
x]]*EllipticPi[-(Sqrt[a - b]/Sqrt[a + b]), ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1
 + Cos[c + d*x]]], -1]*Sqrt[e*Tan[c + d*x]])/(a*Sqrt[a - b]*Sqrt[a + b]*d*S
qrt[Sin[c + d*x]]) - (2*Sqrt[2]*b*Sqrt[Cos[c + d*x]]*EllipticPi[Sqrt[a - b]
/Sqrt[a + b], ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 + Cos[c + d*x]]], -1]*Sqrt[e
*Tan[c + d*x]])/(a*Sqrt[a - b]*Sqrt[a + b]*d*Sqrt[Sin[c + d*x]])
```

Rubi [A] time = 0.704931, antiderivative size = 415, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {3890, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2733, 2730, 2906, 2905, 490, 1213, 537}

$$\frac{2\sqrt{2}b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\Pi\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right)\middle|-1\right)}{ad\sqrt{a-b}\sqrt{a+b}\sqrt{\sin(c+dx)}}-\frac{2\sqrt{2}b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\Pi\left(\frac{\sqrt{a-b}}{\sqrt{a+b}};\sin\right.}{ad\sqrt{a-b}\sqrt{a+b}\sqrt{\sin(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Tan[c + d*x]]/(a + b*Sec[c + d*x]),x]
```


```
[Out] -((Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
)) + (Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
a*d) + (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*a*d) - (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sq
rt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) + (2*Sqrt[2]*b*Sqrt[Cos[c + d*
x]]*EllipticPi[-(Sqrt[a - b]/Sqrt[a + b]), ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1
 + Cos[c + d*x]]], -1]*Sqrt[e*Tan[c + d*x]])/(a*Sqrt[a - b]*Sqrt[a + b]*d*S
qrt[Sin[c + d*x]]) - (2*Sqrt[2]*b*Sqrt[Cos[c + d*x]]*EllipticPi[Sqrt[a - b]
/Sqrt[a + b], ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 + Cos[c + d*x]]], -1]*Sqrt[e
```

```
*Tan[c + d*x]])/(a*Sqrt[a - b]*Sqrt[a + b]*d*Sqrt[Sin[c + d*x]])
```

Rule 3890

```
Int[Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Dist[1/a, Int[Sqrt[e*Cot[c + d*x]], x], x] - Dist[b/a, In
t[Sqrt[e*Cot[c + d*x]]/(b + a*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2733

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2730

```
Int[1/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(g_)*tan[(e_.) + (f_.)*(
x_)]]), x_Symbol] :> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[g*Tan
[e + f*x]]), Int[Sqrt[Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x]
)), x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
```

```
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx = \frac{\int \sqrt{e\tan(c+dx)}\,dx}{a} - \frac{b\int \frac{\sqrt{e\tan(c+dx)}}{b+a\cos(c+dx)}\,dx}{a}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\tan(c+dx)\right)}{ad} - \frac{\left(b\sqrt{e\cot(c+dx)}\sqrt{e\tan(c+dx)}\right)\int \frac{1}{(b+a\cos(c+dx))\sqrt{e\cot\cdots}}}{a}$$

$$= \frac{(2e)\,\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} - \frac{\left(b\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int \frac{\sqrt{\sin(\cdots}}{\sqrt{-\cos(c+dx)}(b\cdots}}{a\sqrt{\sin(c+dx)}}$$

$$= -\frac{e\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} + \frac{e\,\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} - \frac{\left(b\sqrt{\cos\cdots}\right.}{\cdots}$$

$$= \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} + \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+\cdots}\right.}{2\sqrt{2}ad}$$

$$= \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} - \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\,\mathrm{t}\cdots}\right.}{2\sqrt{2}ad}$$

$$= -\frac{\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{\sqrt{e}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+d\cdots\right.}{2\sqrt{2}a\cdots}$$

$$= -\frac{\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{\sqrt{e}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+d\cdots\right.}{2\sqrt{2}a\cdots}$$

Mathematica [C] time = 5.27994, size = 232, normalized size = 0.56

$$-\frac{4\sqrt{\tan\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{e\tan(c+dx)}(a\cos(c+dx)+b)\left(\frac{b\left(\Pi\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}};-\sin^{-1}\left(\sqrt{\tan\left(\frac{1}{2}(c+dx)\right)}\right)\middle|-1\right)-\Pi\left(\frac{\sqrt{a-b}}{\sqrt{a+b}};-\sin^{-1}\left(\sqrt{\mathrm{ta}\cdots}\right.\right.\right.}{\sqrt{a-b}\sqrt{a+b}}\cdots}{ad\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}(\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Tan[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-4*(b + a*Cos[c + d*x])*Csc[c + d*x]*(I*EllipticPi[-I, -ArcSin[Sqrt[Tan[(c
 + d*x)/2]]], -1] - I*EllipticPi[I, -ArcSin[Sqrt[Tan[(c + d*x)/2]]], -1] +
(b*(EllipticPi[-(Sqrt[a - b]/Sqrt[a + b]), -ArcSin[Sqrt[Tan[(c + d*x)/2]]],
```

```
-1] - EllipticPi[Sqrt[a - b]/Sqrt[a + b], -ArcSin[Sqrt[Tan[(c + d*x)/2]]],
 -1]))/(Sqrt[a - b]*Sqrt[a + b]))*Sqrt[Tan[(c + d*x)/2]]*Sqrt[e*Tan[c + d*x
]])/(a*d*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*(a + b*Sec[c + d*x]))
```

Maple [B] time = 0.254, size = 874, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -1/d*2^(1/2)*b/((a^2-b^2)^(1/2)-a+b)/((a^2-b^2)^(1/2)+a-b)/a*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(e*sin(d
*x+c)/cos(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*(cos
(d*x+c)+1)^2*(I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1
/2-1/2*I,1/2*2^(1/2))*a-I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c
))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b-I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a+I*EllipticPi((-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b-EllipticPi((-(-1+cos(
d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a+EllipticPi((-
(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b-Ellip
ticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))
*a+EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*
2^(1/2))*b-(a^2-b^2)^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+
c))^(1/2),(a-b)/(a-b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))+(a^2-b^2)^(1/2)*Elli
pticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)*(
a-b))^(1/2)),1/2*2^(1/2))+EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c
))^(1/2),(a-b)/(a-b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a-b*EllipticPi((-(-1+
cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a+b)*(a-b))^(1/2)),1/
2*2^(1/2))+EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)
/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a-b*EllipticPi((-(-1+cos(d*x+c)-si
n(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2)))
*(-1+cos(d*x+c))/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\tan\left(dx+c\right)}}{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*tan(d*x + c))/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\tan{(c+dx)}}}{a+b\sec{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(1/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(e*tan(c + d*x))/(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\tan{(dx+c)}}}{b\sec{(dx+c)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*tan(d*x + c))/(b*sec(d*x + c) + a), x)
```

3.315 $\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx$

Optimal. Leaf size=422

$$-\frac{2\sqrt{2}b\sqrt{\sin(c+dx)}\Pi\left(\frac{b}{a-\sqrt{a^2-b^2}};\sin^{-1}\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right)\middle|-1\right)}{ad\sqrt{a^2-b^2}\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}+\frac{2\sqrt{2}b\sqrt{\sin(c+dx)}\Pi\left(\frac{b}{a+\sqrt{a^2-b^2}};\sin^{-1}\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right)\middle|-1\right)}{ad\sqrt{a^2-b^2}\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}-$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*Sqrt[e]))
 + ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*Sqrt[e])
 - Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]]/(2*Sq
rt[2]*a*d*Sqrt[e]) + Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Ta
n[c + d*x]]]/(2*Sqrt[2]*a*d*Sqrt[e]) - (2*Sqrt[2]*b*EllipticPi[b/(a - Sqrt[
a^2 - b^2]), ArcSin[Sqrt[-Cos[c + d*x]]/Sqrt[1 + Sin[c + d*x]]], -1]*Sqrt[S
in[c + d*x]])/(a*Sqrt[a^2 - b^2]*d*Sqrt[-Cos[c + d*x]]*Sqrt[e*Tan[c + d*x]]
) + (2*Sqrt[2]*b*EllipticPi[b/(a + Sqrt[a^2 - b^2]), ArcSin[Sqrt[-Cos[c + d
*x]]/Sqrt[1 + Sin[c + d*x]]], -1]*Sqrt[Sin[c + d*x]])/(a*Sqrt[a^2 - b^2]*d*
Sqrt[-Cos[c + d*x]]*Sqrt[e*Tan[c + d*x]])
```

Rubi [A] time = 0.564216, antiderivative size = 422, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.56, Rules used = {3892, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2733, 2729, 2907, 1213, 537}

$$-\frac{2\sqrt{2}b\sqrt{\sin(c+dx)}\Pi\left(\frac{b}{a-\sqrt{a^2-b^2}};\sin^{-1}\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right)\middle|-1\right)}{ad\sqrt{a^2-b^2}\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}+\frac{2\sqrt{2}b\sqrt{\sin(c+dx)}\Pi\left(\frac{b}{a+\sqrt{a^2-b^2}};\sin^{-1}\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right)\middle|-1\right)}{ad\sqrt{a^2-b^2}\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]),x]
```


```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*Sqrt[e]))
 + ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*Sqrt[e])
 - Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]]/(2*Sq
rt[2]*a*d*Sqrt[e]) + Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Ta
n[c + d*x]]]/(2*Sqrt[2]*a*d*Sqrt[e]) - (2*Sqrt[2]*b*EllipticPi[b/(a - Sqrt[
a^2 - b^2]), ArcSin[Sqrt[-Cos[c + d*x]]/Sqrt[1 + Sin[c + d*x]]], -1]*Sqrt[S
in[c + d*x]])/(a*Sqrt[a^2 - b^2]*d*Sqrt[-Cos[c + d*x]]*Sqrt[e*Tan[c + d*x]]
) + (2*Sqrt[2]*b*EllipticPi[b/(a + Sqrt[a^2 - b^2]), ArcSin[Sqrt[-Cos[c + d
*x]]/Sqrt[1 + Sin[c + d*x]]], -1]*Sqrt[Sin[c + d*x]])/(a*Sqrt[a^2 - b^2]*d*
```

```
Sqrt[-Cos[c + d*x]]*Sqrt[e*Tan[c + d*x]])
```

Rule 3892

```
Int[1/(Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Dist[1/a, Int[1/Sqrt[e*Cot[c + d*x]], x], x] - Dist[b
/a, Int[1/(Sqrt[e*Cot[c + d*x]]*(b + a*Sin[c + d*x])), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2733

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2729

```
Int[Sqrt[(g_.)*tan[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[g*Tan[e + f*x]])/Sqrt[Sin[e
 + f*x]], Int[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a + b*Sin[e + f*x])),
 x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx = \frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a} - \frac{b\int \frac{1}{(b+a\cos(c+dx))\sqrt{e\tan(c+dx)}}\,dx}{a}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x, e\tan(c+dx)\right)}{ad} - \frac{b\int \frac{\sqrt{e\cot(c+dx)}}{b+a\cos(c+dx)}\,dx}{a\sqrt{e\cot(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$= \frac{(2e)\,\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} - \frac{\left(b\sqrt{\sin(c+dx)}\right)\int \frac{\sqrt{-\cos(c}}{(b+a\cos(c+dx))}}{a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c}}$$

$$= \frac{\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} + \frac{\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx}\right.}{ad}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2ad} + \frac{\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e}\right.}{2ad}$$

$$= -\frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad\sqrt{e}} + \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx\right.}{2\sqrt{2}a}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad\sqrt{e}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad\sqrt{e}} - \frac{\log\left(\sqrt{e}+\sqrt{e}\tan\right.}{}$$

Mathematica [C] time = 12.3246, size = 254, normalized size = 0.6

$$\frac{4\left(a(a-b)\text{EllipticF}\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{2}(c+dx)\right)}}\right),-1\right)+\left(a^2-b^2\right)\Pi\left(-i;-\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{2}(c+dx)\right)}}\right)\middle|-1\right)+a^2\Pi\left(i;-\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{2}(c+dx)\right)}}\right.\right.\right.}{ad\left(a^2-b^2\right)\sqrt{\tan\left(\frac{1}{2}(c+\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]),x]
```

```
[Out] (4*(a*(a - b)*EllipticF[ArcSin[1/Sqrt[Tan[(c + d*x)/2]]], -1] + (a^2 - b^2)
*EllipticPi[-I, -ArcSin[1/Sqrt[Tan[(c + d*x)/2]]], -1] + a^2*EllipticPi[I,
-ArcSin[1/Sqrt[Tan[(c + d*x)/2]]], -1] - b^2*EllipticPi[I, -ArcSin[1/Sqrt[T
an[(c + d*x)/2]]], -1] + b^2*EllipticPi[-(Sqrt[a + b]/Sqrt[a - b]), -ArcSin
[1/Sqrt[Tan[(c + d*x)/2]]], -1] + b^2*EllipticPi[Sqrt[a + b]/Sqrt[a - b], -
ArcSin[1/Sqrt[Tan[(c + d*x)/2]]], -1]))/(a*(a^2 - b^2)*d*Sqrt[1 + Cot[(c +
d*x)/2]]*Sqrt[-1 + Tan[(c + d*x)/2]]*Sqrt[e*Tan[c + d*x]])
```

Maple [B] time = 0.232, size = 2313, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d*2^(1/2)/((a^2-b^2)^(1/2)-a+b)/((a^2-b^2)^(1/2)+a-b)/(a^2-b^2)^(1/2)/
(a-b)/a*(EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,
1/2*2^(1/2))*(a^2-b^2)^(3/2)*a-EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)*b-EllipticPi(((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^
3+EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(
1/2))*(a^2-b^2)^(1/2)*b^3+EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)*a-EllipticPi(((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)*b-Ellip
ticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(
a^2-b^2)^(1/2)*a^3+EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),
1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*b^3-2*(a^2-b^2)^(1/2)*EllipticPi(((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a+b)*(a-b))^(1/2)),1
```

/2*2^(1/2))*b^3-2*(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*b^3-2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a^2*b^2+4*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a*b^3+2*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^3-2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*b^4+2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*b^4+2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a^2*b^2-4*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a*b^3-2*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(a^2-b^2)^(3/2)*a+I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)*a+I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)*b+I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*b^3+I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^3-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)*b-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)*a-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^3-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*b^3-4*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^2*b+2*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(a^2-b^2)^(1/2)*a*b^2+3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^2*b-3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a*b^2+3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^2*b-3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a*b^2+2*(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a*b^2+2*(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a*b^2+3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^2*b-3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a*b^2-3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^2*b+3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a*b^2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-1+cos(d*x+c))/sin(d*x+c)^2/cos(d*x+c)*(cos(d*x+c)+1)^2/(e*sin(d*x+c)/cos(d*x+c))^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)\sqrt{e\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sqrt(e*tan(d*x + c))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\tan(c+dx)}\,(a+b\sec(c+dx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*tan(c + d*x))*(a + b*sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)\sqrt{e\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sqrt(e*tan(d*x + c))), x)
```

3.316 $\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=863

$$\frac{2\sqrt{2}\sqrt{\cos(c+dx)}\,\Pi\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right)\middle|-1\right)\sqrt{e\tan(c+dx)}\,b^3}{a(a-b)^{3/2}(a+b)^{3/2}de^2\sqrt{\sin(c+dx)}}-\frac{2\sqrt{2}\sqrt{\cos(c+dx)}\,\Pi\left(\frac{\sqrt{a-b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right.\right.}{a(a-b)^{3/2}(a+b)^{3/2}de^2\sqrt{\sin(}}$$

```
[Out] (a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 - b^2)
*d*e^(3/2)) - (b^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqr
t[2]*a*(a^2 - b^2)*d*e^(3/2)) - (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]]
)/Sqrt[e]])/(Sqrt[2]*(a^2 - b^2)*d*e^(3/2)) + (b^2*ArcTan[1 + (Sqrt[2]*Sqrt
[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*(a^2 - b^2)*d*e^(3/2)) - (a*Log[Sqrt
[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*(a^2
 - b^2)*d*e^(3/2)) + (b^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt
[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*(a^2 - b^2)*d*e^(3/2)) + (a*Log[Sqrt[e] + S
qrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*(a^2 - b^2)
*d*e^(3/2)) - (b^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[
c + d*x]]])/(2*Sqrt[2]*a*(a^2 - b^2)*d*e^(3/2)) - (2*(a - b*Sec[c + d*x]))/
((a^2 - b^2)*d*e*Sqrt[e*Tan[c + d*x]]) + (2*Sqrt[2]*b^3*Sqrt[Cos[c + d*x]]*
EllipticPi[-(Sqrt[a - b]/Sqrt[a + b]), ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 + C
os[c + d*x]]], -1]*Sqrt[e*Tan[c + d*x]])/(a*(a - b)^(3/2)*(a + b)^(3/2)*d*e
^2*Sqrt[Sin[c + d*x]]) - (2*Sqrt[2]*b^3*Sqrt[Cos[c + d*x]]*EllipticPi[Sqrt[
a - b]/Sqrt[a + b], ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 + Cos[c + d*x]]], -1]*
Sqrt[e*Tan[c + d*x]])/(a*(a - b)^(3/2)*(a + b)^(3/2)*d*e^2*Sqrt[Sin[c + d*x
]]) + (2*b*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/
((a^2 - b^2)*d*e^2*Sqrt[Sin[2*c + 2*d*x]]) - (2*b*Cos[c + d*x]*(e*Tan[c + d
*x])^(3/2))/((a^2 - b^2)*d*e^3)
```

Rubi [A] time = 1.24995, antiderivative size = 863, normalized size of antiderivative = 1., number of steps used = 39, number of rules used = 23, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.92, Rules used = {3893, 3882, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639, 3890, 2733, 2730, 2906, 2905, 490, 1213, 537}

$$\frac{2\sqrt{2}\sqrt{\cos(c+dx)}\,\Pi\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right)\middle|-1\right)\sqrt{e\tan(c+dx)}\,b^3}{a(a-b)^{3/2}(a+b)^{3/2}de^2\sqrt{\sin(c+dx)}}-\frac{2\sqrt{2}\sqrt{\cos(c+dx)}\,\Pi\left(\frac{\sqrt{a-b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right.\right.}{a(a-b)^{3/2}(a+b)^{3/2}de^2\sqrt{\sin(}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2)),x]
```

```
[Out] (a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 - b^2)
*d*e^(3/2)) - (b^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqr
t[2]*a*(a^2 - b^2)*d*e^(3/2)) - (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]]
)/Sqrt[e]])/(Sqrt[2]*(a^2 - b^2)*d*e^(3/2)) + (b^2*ArcTan[1 + (Sqrt[2]*Sqrt
[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*(a^2 - b^2)*d*e^(3/2)) - (a*Log[Sqrt
[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*(a^2
 - b^2)*d*e^(3/2)) + (b^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt
[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*(a^2 - b^2)*d*e^(3/2)) + (a*Log[Sqrt[e] + S
qrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*(a^2 - b^2)
*d*e^(3/2)) - (b^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[
c + d*x]]])/(2*Sqrt[2]*a*(a^2 - b^2)*d*e^(3/2)) - (2*(a - b*Sec[c + d*x]))/
((a^2 - b^2)*d*e*Sqrt[e*Tan[c + d*x]]) + (2*Sqrt[2]*b^3*Sqrt[Cos[c + d*x]]*
EllipticPi[-(Sqrt[a - b]/Sqrt[a + b]), ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 + C
os[c + d*x]]], -1]*Sqrt[e*Tan[c + d*x]])/(a*(a - b)^(3/2)*(a + b)^(3/2)*d*e
^2*Sqrt[Sin[c + d*x]]) - (2*Sqrt[2]*b^3*Sqrt[Cos[c + d*x]]*EllipticPi[Sqrt[
a - b]/Sqrt[a + b], ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 + Cos[c + d*x]]], -1]*
Sqrt[e*Tan[c + d*x]])/(a*(a - b)^(3/2)*(a + b)^(3/2)*d*e^2*Sqrt[Sin[c + d*x
]]) + (2*b*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/
((a^2 - b^2)*d*e^2*Sqrt[Sin[2*c + 2*d*x]]) - (2*b*Cos[c + d*x]*(e*Tan[c + d
*x])^(3/2))/((a^2 - b^2)*d*e^3)
```

Rule 3893

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Dist[1/(a^2 - b^2), Int[(e*Cot[c + d*x])^m*(a - b*Csc[c
+ d*x]), x], x] + Dist[b^2/(e^2*(a^2 - b^2)), Int[(e*Cot[c + d*x])^(m + 2)/
(a + b*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2,
 0] && ILtQ[m + 1/2, 0]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3890

```
Int[Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Dist[1/a, Int[Sqrt[e*Cot[c + d*x]], x], x] - Dist[b/a, In
t[Sqrt[e*Cot[c + d*x]]/(b + a*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2733

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
```

```
*(g*Tan[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2730

```
Int[1/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(g_)*tan[(e_.) + (f_.)*(
x_)]]), x_Symbol] :> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[g*Tan
[e + f*x]]), Int[Sqrt[Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x]
)), x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
```

```
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \frac{\int \frac{a-b\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx}{a^2-b^2} + \frac{b^2\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx}{\left(a^2-b^2\right)e^2}$$

$$= -\frac{2(a-b\sec(c+dx))}{\left(a^2-b^2\right)de\sqrt{e\tan(c+dx)}} + \frac{2\int \left(-\frac{a}{2}-\frac{1}{2}b\sec(c+dx)\right)\sqrt{e\tan(c+dx)}\,d}{\left(a^2-b^2\right)e^2}$$

$$= -\frac{2(a-b\sec(c+dx))}{\left(a^2-b^2\right)de\sqrt{e\tan(c+dx)}} - \frac{a\int \sqrt{e\tan(c+dx)}\,dx}{\left(a^2-b^2\right)e^2} - \frac{b\int \sec(c+dx)\sqrt{e}}{\left(a^2-b\right.}$$

$$= -\frac{2(a-b\sec(c+dx))}{\left(a^2-b^2\right)de\sqrt{e\tan(c+dx)}} - \frac{2b\cos(c+dx)(e\tan(c+dx))^{3/2}}{\left(a^2-b^2\right)de^3} + \frac{(2b)\int c}{}$$

$$= -\frac{2(a-b\sec(c+dx))}{\left(a^2-b^2\right)de\sqrt{e\tan(c+dx)}} - \frac{2b\cos(c+dx)(e\tan(c+dx))^{3/2}}{\left(a^2-b^2\right)de^3} - \frac{(2a)\,\mathrm{Sub}}{}$$

$$= -\frac{2(a-b\sec(c+dx))}{\left(a^2-b^2\right)de\sqrt{e\tan(c+dx)}} - \frac{2b\cos(c+dx)(e\tan(c+dx))^{3/2}}{\left(a^2-b^2\right)de^3} + \frac{b^2\,\mathrm{Subs}}{}$$

$$= \frac{b^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a\left(a^2-b^2\right)de^{3/2}} - \frac{b^2\log\left(\sqrt{e}+\sqrt{e}\tan(\right.}{2\sqrt{2}a}$$

$$= -\frac{b^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\left(a^2-b^2\right)de^{3/2}} + \frac{b^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\left(a^2-b^2\right)de^{3/2}} - \frac{a\log\left(\sqrt{e}\right.}{}$$

$$= \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2-b^2\right)de^{3/2}} - \frac{b^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\left(a^2-b^2\right)de^{3/2}} - \frac{a\tan^{-1}\left(1+\right.}{\sqrt{2}\left(a^2-\right.}$$

Mathematica [C] time = 27.1128, size = 1571, normalized size = 1.82

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((-2*(b - a*Cos[c + d*x])*Csc[c + d*x])/
(-a^2 + b^2) + (2*b*Sin[c + d*x])/(-a^2 + b^2))*Tan[c + d*x]^2)/(d*(a + b*S
ec[c + d*x])*(e*Tan[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])*Sec[c + d*x]*T
an[c + d*x]^(3/2)*(-((-a^2 + 3*b^2)*Sec[c + d*x]*(6*Sqrt[2]*(a^2 - b^2)*Arc
Tan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 6*Sqrt[2]*a^2*ArcTan[1 + Sqrt[2]*Sqrt
[Tan[c + d*x]]] + 6*Sqrt[2]*b^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] - (6
 + 6*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c +
d*x]])/(a^2 - b^2)^(1/4)] + (6 + 6*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 +
((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)] - 3*Sqrt[2]*a^2*Log
[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 3*Sqrt[2]*b^2*Log[1 - Sqr
t[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 3*Sqrt[2]*a^2*Log[1 + Sqrt[2]*Sqr
t[Tan[c + d*x]] + Tan[c + d*x]] - 3*Sqrt[2]*b^2*Log[1 + Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]] + (3 + 3*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2
- b^2] - (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c +
 d*x]] - (3 + 3*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + (1 + I)*
Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]] + 8*a*b*Ap
pellF1[3/4, 1/2, 1, 7/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]
*Tan[c + d*x]^(3/2))*(a + b*Sqrt[1 + Tan[c + d*x]^2]))/(12*(a^3 - a*b^2)*(b
 + a*Cos[c + d*x])*(1 + Tan[c + d*x]^2)) + (b*Cos[2*(c + d*x)]*Sec[c + d*x]
^2*(-84*Sqrt[2]*b*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 84*Sqrt[2]*b*Arc
Tan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + ((42 + 42*I)*(-a^2 + 2*b^2)*ArcTan[1
- ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)])/(Sqrt[b]*(a^2 -
b^2)^(1/4)) + ((42 + 42*I)*(a^2 - 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[T
an[c + d*x]])/(a^2 - b^2)^(1/4)])/(Sqrt[b]*(a^2 - b^2)^(1/4)) + 42*Sqrt[2]*
b*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - 42*Sqrt[2]*b*Log[1 +
 Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + ((21 + 21*I)*(a^2 - 2*b^2)*Lo
g[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] +
I*b*Tan[c + d*x]])/(Sqrt[b]*(a^2 - b^2)^(1/4)) + ((21 + 21*I)*(-a^2 + 2*b^2
)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]
] + I*b*Tan[c + d*x]])/(Sqrt[b]*(a^2 - b^2)^(1/4)) + (112*a^3*AppellF1[3/4,
 1/2, 1, 7/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c + d*
x]^(3/2))/(a^2 - b^2) - (168*a*b^2*AppellF1[3/4, 1/2, 1, 7/4, -Tan[c + d*x]
^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(3/2))/(a^2 - b^2) - (24
*a*b^2*AppellF1[7/4, 1/2, 1, 11/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a
^2 - b^2)]*Tan[c + d*x]^(7/2))/(a^2 - b^2) - (168*a*Tan[c + d*x]^(3/2))/Sqr
t[1 + Tan[c + d*x]^2])*(a + b*Sqrt[1 + Tan[c + d*x]^2]))/(84*a*(b + a*Cos[c
 + d*x])*(-1 + Tan[c + d*x]^2)*Sqrt[1 + Tan[c + d*x]^2])))/((a - b)*(a + b)
*d*(a + b*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2))
```

Maple [B] time = 0.251, size = 6426, normalized size = 7.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)\,(e\tan(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*tan(d*x + c))^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\tan(c+dx))^{\frac{3}{2}}\,(a+b\sec(c+dx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((e*tan(c + d*x))**(3/2)*(a + b*sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)\,(e\tan(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*tan(d*x + c))^(3/2)), x)
```

3.317 $\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=836

$$-\frac{2\sqrt{2}\Pi\left(\frac{b}{a-\sqrt{a^2-b^2}};\sin^{-1}\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right)\middle|-1\right)\sqrt{\sin(c+dx)}b^3}{a\left(a^2-b^2\right)^{3/2}de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}+\frac{2\sqrt{2}\Pi\left(\frac{b}{a+\sqrt{a^2-b^2}};\sin^{-1}\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right)\middle|-1\right)\sqrt{\sin(c+dx)}}{a\left(a^2-b^2\right)^{3/2}de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

```
[Out] (a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 - b^2)
*d*e^(5/2)) - (b^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqr
t[2]*a*(a^2 - b^2)*d*e^(5/2)) - (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]]
)/Sqrt[e]])/(Sqrt[2]*(a^2 - b^2)*d*e^(5/2)) + (b^2*ArcTan[1 + (Sqrt[2]*Sqrt
[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*(a^2 - b^2)*d*e^(5/2)) + (a*Log[Sqrt
[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*(a^2
 - b^2)*d*e^(5/2)) - (b^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt
[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*(a^2 - b^2)*d*e^(5/2)) - (a*Log[Sqrt[e] + S
qrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*(a^2 - b^2)
*d*e^(5/2)) + (b^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[
c + d*x]]])/(2*Sqrt[2]*a*(a^2 - b^2)*d*e^(5/2)) - (2*(a - b*Sec[c + d*x]))/
(3*(a^2 - b^2)*d*e*(e*Tan[c + d*x])^(3/2)) - (2*Sqrt[2]*b^3*EllipticPi[b/(a
 - Sqrt[a^2 - b^2]), ArcSin[Sqrt[-Cos[c + d*x]]/Sqrt[1 + Sin[c + d*x]]], -1
]*Sqrt[Sin[c + d*x]])/(a*(a^2 - b^2)^(3/2)*d*e^2*Sqrt[-Cos[c + d*x]]*Sqrt[e
*Tan[c + d*x]]) + (2*Sqrt[2]*b^3*EllipticPi[b/(a + Sqrt[a^2 - b^2]), ArcSin
[Sqrt[-Cos[c + d*x]]/Sqrt[1 + Sin[c + d*x]]], -1]*Sqrt[Sin[c + d*x]])/(a*(a
^2 - b^2)^(3/2)*d*e^2*Sqrt[-Cos[c + d*x]]*Sqrt[e*Tan[c + d*x]]) + (b*Ellipt
icF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*(a^2 - b^2)*
d*e^2*Sqrt[e*Tan[c + d*x]])
```

Rubi [A] time = 1.06347, antiderivative size = 836, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 20, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {3893, 3882, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641, 3892, 2733, 2729, 2907, 1213, 537}

$$-\frac{2\sqrt{2}\Pi\left(\frac{b}{a-\sqrt{a^2-b^2}};\sin^{-1}\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right)\middle|-1\right)\sqrt{\sin(c+dx)}b^3}{a\left(a^2-b^2\right)^{3/2}de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}+\frac{2\sqrt{2}\Pi\left(\frac{b}{a+\sqrt{a^2-b^2}};\sin^{-1}\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right)\middle|-1\right)\sqrt{\sin(c+dx)}}{a\left(a^2-b^2\right)^{3/2}de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)),x]
```

```
[Out] (a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 - b^2)
*d*e^(5/2)) - (b^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqr
t[2]*a*(a^2 - b^2)*d*e^(5/2)) - (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]]
)/Sqrt[e]])/(Sqrt[2]*(a^2 - b^2)*d*e^(5/2)) + (b^2*ArcTan[1 + (Sqrt[2]*Sqrt
[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*(a^2 - b^2)*d*e^(5/2)) + (a*Log[Sqrt
[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*(a^2
 - b^2)*d*e^(5/2)) - (b^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt
[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*(a^2 - b^2)*d*e^(5/2)) - (a*Log[Sqrt[e] + S
qrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*(a^2 - b^2)
*d*e^(5/2)) + (b^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[
c + d*x]]])/(2*Sqrt[2]*a*(a^2 - b^2)*d*e^(5/2)) - (2*(a - b*Sec[c + d*x]))/
(3*(a^2 - b^2)*d*e*(e*Tan[c + d*x])^(3/2)) - (2*Sqrt[2]*b^3*EllipticPi[b/(a
 - Sqrt[a^2 - b^2]), ArcSin[Sqrt[-Cos[c + d*x]]/Sqrt[1 + Sin[c + d*x]]], -1
]*Sqrt[Sin[c + d*x]])/(a*(a^2 - b^2)^(3/2)*d*e^2*Sqrt[-Cos[c + d*x]]*Sqrt[e
*Tan[c + d*x]]) + (2*Sqrt[2]*b^3*EllipticPi[b/(a + Sqrt[a^2 - b^2]), ArcSin
[Sqrt[-Cos[c + d*x]]/Sqrt[1 + Sin[c + d*x]]], -1]*Sqrt[Sin[c + d*x]])/(a*(a
^2 - b^2)^(3/2)*d*e^2*Sqrt[-Cos[c + d*x]]*Sqrt[e*Tan[c + d*x]]) + (b*Ellipt
icF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*(a^2 - b^2)*
d*e^2*Sqrt[e*Tan[c + d*x]])
```

Rule 3893

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Dist[1/(a^2 - b^2), Int[(e*Cot[c + d*x])^m*(a - b*Csc[c
+ d*x]), x], x] + Dist[b^2/(e^2*(a^2 - b^2)), Int[(e*Cot[c + d*x])^(m + 2)/
(a + b*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2,
 0] && ILtQ[m + 1/2, 0]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
```

```
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3892

```
Int[1/(Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Dist[1/a, Int[1/Sqrt[e*Cot[c + d*x]], x], x] - Dist[b
/a, Int[1/(Sqrt[e*Cot[c + d*x]]*(b + a*Sin[c + d*x])), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2733

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2729

```
Int[Sqrt[(g_.)*tan[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[g*Tan[e + f*x]])/Sqrt[Sin[e
 + f*x]], Int[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a + b*Sin[e + f*x])),
 x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \frac{\int \frac{a-b\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx}{a^2-b^2} + \frac{b^2\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx}{\left(a^2-b^2\right)e^2}$$

$$= -\frac{2(a-b\sec(c+dx))}{3\left(a^2-b^2\right)de(e\tan(c+dx))^{3/2}} + \frac{2\int \frac{-\frac{3a}{2}+\frac{1}{2}b\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3\left(a^2-b^2\right)e^2} + \frac{b^2\int \frac{1}{\sqrt{e\tan(c+dx)}}}{a\left(a^2-b^2\right)e^2}$$

$$= -\frac{2(a-b\sec(c+dx))}{3\left(a^2-b^2\right)de(e\tan(c+dx))^{3/2}} - \frac{a\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{\left(a^2-b^2\right)e^2} + \frac{b\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3\left(a^2-b^2\right)e^2} +$$

$$= -\frac{2(a-b\sec(c+dx))}{3\left(a^2-b^2\right)de(e\tan(c+dx))^{3/2}} - \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(e^2+x^2\right)}\,dx, x, e\tan(c+dx)\right)}{\left(a^2-b^2\right)de}$$

$$= -\frac{2(a-b\sec(c+dx))}{3\left(a^2-b^2\right)de(e\tan(c+dx))^{3/2}} + \frac{b^2\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{a\left(a^2-b^2\right)de^2}$$

$$= -\frac{2(a-b\sec(c+dx))}{3\left(a^2-b^2\right)de(e\tan(c+dx))^{3/2}} + \frac{bF\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+}}{3\left(a^2-b^2\right)de^2\sqrt{e\tan(c+dx)}}$$

$$= -\frac{b^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a\left(a^2-b^2\right)de^{5/2}} + \frac{b^2\log\left(\sqrt{e}+\sqrt{e}\tan(\right.}{2\sqrt{2}a}$$

$$= -\frac{b^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\left(a^2-b^2\right)de^{5/2}} + \frac{b^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\left(a^2-b^2\right)de^{5/2}} + \frac{a\log\left(\sqrt{e}+\right.}{}$$

$$= \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2-b^2\right)de^{5/2}} - \frac{b^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\left(a^2-b^2\right)de^{5/2}} - \frac{a\tan^{-1}\left(1+\sqrt{\right.}}{\sqrt{2}\left(a^2-\right.}$$

Mathematica [C] time = 23.6592, size = 2169, normalized size = 2.59

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])*((2*a)/(3*(a^2 - b^2)) - (2*(-a + b*Cos[c + d*x])*Csc
[c + d*x]^2)/(3*(-a^2 + b^2)))*Sec[c + d*x]*Tan[c + d*x]^3)/(d*(a + b*Sec[c
 + d*x])*(e*Tan[c + d*x])^(5/2)) - ((b + a*Cos[c + d*x])*Sec[c + d*x]*Tan[c
 + d*x]^(5/2)*((2*(3*a^2 - 5*b^2)*Sec[c + d*x]^3*(a + b*Sqrt[1 + Tan[c + d*
x]^2])*(((-1/8 + I/8)*a*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/
(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2
 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sq
rt[Tan[c + d*x]] + I*b*Tan[c + d*x]] - Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[b
]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]]))/(Sqrt[b]*(a^2
- b^2)^(3/4)) + (5*b*(-a^2 + b^2)*AppellF1[1/4, -1/2, 1, 5/4, -Tan[c + d*x]
^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Tan[c + d*x]]*Sqrt[1 + Tan[c + d
*x]^2])/((5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, -Tan[c + d*x]^2, (b^2*T
an[c + d*x]^2)/(a^2 - b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, -Tan[c +
 d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2,
 1, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2
)*(a^2 - b^2*(1 + Tan[c + d*x]^2)))))/((b + a*Cos[c + d*x])*(1 + Tan[c + d*
x]^2)^2) + (8*a*b*Sec[c + d*x]^2*(a + b*Sqrt[1 + Tan[c + d*x]^2])*((Sqrt[b]
*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Tan[c + d*x]])/(-a^2 + b^2)^(1/4)] +
2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Tan[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log
[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Tan[c + d*x]] +
 b*Tan[c + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[b]*(-a^2 + b^2)^(1/4
)*Sqrt[Tan[c + d*x]] + b*Tan[c + d*x]]))/(4*Sqrt[2]*(-a^2 + b^2)^(3/4)) + (
5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*
x]^2)/(a^2 - b^2)]*Sqrt[Tan[c + d*x]])/(Sqrt[1 + Tan[c + d*x]^2]*(-5*(a^2 -
 b^2)*AppellF1[1/4, 1/2, 1, 5/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
 - b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c
+ d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, -Tan[c + d
*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)*(-a^2 + b^2*(1 +
Tan[c + d*x]^2)))))/((b + a*Cos[c + d*x])*(1 + Tan[c + d*x]^2)^(3/2)) + ((3
*a^2 - 3*b^2)*Cos[2*(c + d*x)]*Sec[c + d*x]^3*(a + b*Sqrt[1 + Tan[c + d*x]^
2])*((-20*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/a + (20*Sqrt[2]*A
rcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/a + ((10 - 10*I)*(a^2 - 2*b^2)*ArcTa
n[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(
a^2 - b^2)^(3/4)) - ((10 - 10*I)*(a^2 - 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*
Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - (10
*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/a + (10*Sqrt[2
]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/a + ((5 - 5*I)*(a^2 -
 2*b^2)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c
+ d*x]] + I*b*Tan[c + d*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - ((5 - 5*I)*(a^
2 - 2*b^2)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan
[c + d*x]] + I*b*Tan[c + d*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - (8*b*Appell
F1[5/4, 1/2, 1, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan
[c + d*x]^(5/2))/(-a^2 + b^2) - (200*b*(-a^2 + b^2)*AppellF1[1/4, 1/2, 1, 5
/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Tan[c + d*x]])/
(Sqrt[1 + Tan[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, -Tan[c
```

```
+ d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(2*b^2*AppellF1[5/4, 1/2, 2
, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*Ap
pellF1[5/4, 3/2, 1, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]
)*Tan[c + d*x]^2)*(-a^2 + b^2*(1 + Tan[c + d*x]^2)))))/(20*(b + a*Cos[c + d
*x])*(1 - Tan[c + d*x]^2)*(1 + Tan[c + d*x]^2))))/(6*(a - b)*(a + b)*d*(a +
 b*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2))
```

Maple [B] time = 0.459, size = 16178, normalized size = 19.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)\left(e\tan\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*tan(d*x + c))^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)\left(e\tan\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*tan(d*x + c))^(5/2)), x)
```

3.318 $\int \sqrt{a+b\sec(c+dx)}\tan^5(c+dx)\,dx$

Optimal. Leaf size=169

$$\frac{2\left(3a^2-2b^2\right)(a+b\sec(c+dx))^{5/2}}{5b^4d}-\frac{2a\left(a^2-2b^2\right)(a+b\sec(c+dx))^{3/2}}{3b^4d}+\frac{2(a+b\sec(c+dx))^{9/2}}{9b^4d}-\frac{6a(a+b\sec(c+dx)}{7b^4d}$$

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/d + (2*Sqrt[a + b*Se
c[c + d*x]])/d - (2*a*(a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(3/2))/(3*b^4*d) +
 (2*(3*a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(5/2))/(5*b^4*d) - (6*a*(a + b*Sec
[c + d*x])^(7/2))/(7*b^4*d) + (2*(a + b*Sec[c + d*x])^(9/2))/(9*b^4*d)
```

Rubi [A] time = 0.17029, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3885, 898, 1261, 207}

$$\frac{2\left(3a^2-2b^2\right)(a+b\sec(c+dx))^{5/2}}{5b^4d}-\frac{2a\left(a^2-2b^2\right)(a+b\sec(c+dx))^{3/2}}{3b^4d}+\frac{2(a+b\sec(c+dx))^{9/2}}{9b^4d}-\frac{6a(a+b\sec(c+dx)}{7b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^5,x]
```


```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/d + (2*Sqrt[a + b*Se
c[c + d*x]])/d - (2*a*(a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(3/2))/(3*b^4*d) +
 (2*(3*a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(5/2))/(5*b^4*d) - (6*a*(a + b*Sec
[c + d*x])^(7/2))/(7*b^4*d) + (2*(a + b*Sec[c + d*x])^(9/2))/(9*b^4*d)
```


Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
```

```
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+x}\left(b^2-x^2\right)^2}{x}\,dx, x, b\sec(c+dx)\right)}{b^4 d}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x^2\left(-a^2+b^2+2ax^2-x^4\right)^2}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^4 d}$$

$$= \frac{2\,\text{Subst}\left(\int \left(b^4-a\left(a^2-2b^2\right)x^2+\left(3a^2-2b^2\right)x^4-3ax^6+x^8+\frac{ab^4}{-a+x^2}\right)dx, x, \sqrt{\cdots}\right.}{b^4 d}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}}{d} - \frac{2a\left(a^2-2b^2\right)(a+b\sec(c+dx))^{3/2}}{3b^4 d} + \frac{2\left(3a^2-2b^2\right)(a+\cdots}{5b\cdots}$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2\sqrt{a+b\sec(c+dx)}}{d} - \frac{2a\left(a^2-2b^2\right)(a+b\sec\cdots}{3b^4 d}$$

Mathematica [A] time = 6.2912, size = 254, normalized size = 1.5

$$\frac{\sqrt{a+b\sec(c+dx)}\left(-\frac{4\left(a^2+21b^2\right)\sec^2(c+dx)}{105b^2} - \frac{4a\left(21b^2-4a^2\right)\sec(c+dx)}{315b^3} + \frac{2\left(84a^2b^2-16a^4+315b^4\right)}{315b^4} + \frac{2a\sec^3(c+dx)}{63b} + \frac{2}{9}\sec^4(c+dx)\right)}{d} - \frac{\text{si}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^5,x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((2*(-16*a^4 + 84*a^2*b^2 + 315*b^4))/(315*b^4) -
 (4*a*(-4*a^2 + 21*b^2)*Sec[c + d*x])/(315*b^3) - (4*(a^2 + 21*b^2)*Sec[c +
 d*x]^2)/(105*b^2) + (2*a*Sec[c + d*x]^3)/(63*b) + (2*Sec[c + d*x]^4)/9))/d
 - (Sqrt[a*Cos[c + d*x]]*(-Log[1 - Sqrt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c +
d*x]]] + Log[1 + Sqrt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]])*Sqrt[a + b
*Sec[c + d*x]]*Sin[c + d*x]^2)/(d*Sqrt[b + a*Cos[c + d*x]]*(1 - Cos[c + d*x
]^2))
```

Maple [B] time = 0.818, size = 3268, normalized size = 19.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x)
```

```
[Out] -1/2520/d/(a-b)^(1/2)/b^4*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(
d*x+c)+1)*(-1+cos(d*x+c))^4*(968*cos(d*x+c)^4*((b+a*cos(d*x+c))*cos(d*x+c)/
(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(1/2)*a*b^3+144*cos(d*x+c)^3*((b+a*cos(d*x+c)
)*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(1/2)*a^2*b^2-120*cos(d*x+c)^2*(
(b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(1/2)*a*b^3-24*co
s(d*x+c)^7*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(1/2)
*a^2*b^2-64*cos(d*x+c)^6*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/
2)*(a-b)^(1/2)*a^3*b+336*cos(d*x+c)^6*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x
+c)+1)^2)^(3/2)*(a-b)^(1/2)*a*b^3+1260*cos(d*x+c)^7*(a-b)^(1/2)*a^(3/2)*ln(
4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4
*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2
)+2*b)*b^4+1260*cos(d*x+c)^6*(a-b)^(1/2)*a^(1/2)*ln(4*cos(d*x+c)*((b+a*cos(
d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)
*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b^5-72*cos(d*x+c
)^6*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(1/2)*a^2*b^
2-192*cos(d*x+c)^5*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(a-
b)^(1/2)*a^3*b+1008*cos(d*x+c)^5*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1
)^2)^(3/2)*(a-b)^(1/2)*a*b^3+120*cos(d*x+c)^4*((b+a*cos(d*x+c))*cos(d*x+c)/
(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(1/2)*a^2*b^2-64*cos(d*x+c)^3*((b+a*cos(d*x+c
))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(1/2)*a^3*b+216*cos(d*x+c)^3*((
b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(1/2)*a*b^3+48*cos
(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(1/2)*
a^2*b^2-40*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*
(a-b)^(1/2)*a*b^3+128*cos(d*x+c)^6*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)
```

+1)^2)^(1/2)*(a-b)^(1/2)*a^4*b-648*cos(d*x+c)^7*((b+a*cos(d*x+c))*cos(d*x+c
)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)*a^3*b^2-2121*cos(d*x+c)^7*((b+a*cos(d
*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)*a*b^4-648*cos(d*x+c)^
6*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)*a^3*b^2-
648*cos(d*x+c)^6*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)
^(1/2)*a^2*b^3-2121*cos(d*x+c)^6*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1
)^2)^(1/2)*(a-b)^(1/2)*a*b^4+128*cos(d*x+c)^5*((b+a*cos(d*x+c))*cos(d*x+c)/
(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)*a^4*b-648*cos(d*x+c)^5*((b+a*cos(d*x+c)
)*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)*a^2*b^3-24*cos(d*x+c)^5*((
b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(1/2)*a^2*b^2-192*
cos(d*x+c)^4*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(1/
2)*a^3*b-630*cos(d*x+c)^6*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*(
(b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x
+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-
b)^(1/2)-b)/sin(d*x+c)^2)*b^6+630*cos(d*x+c)^6*ln(-2/(a-b)^(1/2)*(-1+cos(d*
x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a
-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d
x+c)+1)^2)^(1/2)(a-b)^(1/2)-b)/sin(d*x+c)^2)*b^6-280*(a-b)^(1/2)*((b+a*co
s(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*b^4+168*cos(d*x+c)^2*((b+a*cos
(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(1/2)*b^4+1260*cos(d*x+c)
^7*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*
x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+
a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^
2)*a^2*b^4-630*cos(d*x+c)^7*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)
*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d
*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(
a-b)^(1/2)-b)/sin(d*x+c)^2)*a*b^5-1260*cos(d*x+c)^7*ln(-2/(a-b)^(1/2)*(-1+c
os(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/
2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(
cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^2*b^4+630*cos(d*x+c)^
7*ln(-2/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x
+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a
*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2
)*a*b^5+2744*cos(d*x+c)^3*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3
/2)*(a-b)^(1/2)*b^4-840*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)
+1)^2)^(3/2)*(a-b)^(1/2)*b^4-189*cos(d*x+c)^5*((b+a*cos(d*x+c))*cos(d*x+c)/
(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(1/2)*b^4+128*cos(d*x+c)^6*((b+a*cos(d*x+c))*
cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)*a^5-2121*cos(d*x+c)^6*((b+a*
cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)*b^5+128*cos(d*x+
c)^7*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)*a^5-2
121*cos(d*x+c)^5*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)
^(1/2)*b^5-399*cos(d*x+c)^7*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^
(3/2)*(a-b)^(1/2)*b^4-1197*cos(d*x+c)^6*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d
x+c)+1)^2)^(3/2)(a-b)^(1/2)*b^4+1260*cos(d*x+c)^6*ln(-1/(a-b)^(1/2)*(-1+c
os(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/

```
2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(
cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a*b^5-1260*cos(d*x+c)^6
*ln(-2/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+
c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*
cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)
*a*b^5+2625*cos(d*x+c)^4*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/
2)*(a-b)^(1/2)*b^4)/sin(d*x+c)^8/cos(d*x+c)^4/((b+a*cos(d*x+c))*cos(d*x+c)/
(cos(d*x+c)+1)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.7358, size = 1048, normalized size = 6.2

$$\left[\frac{315\,\sqrt{a}b^4\cos\left(dx+c\right)^4\log\left(-8\,a^2\cos\left(dx+c\right)^2-8\,ab\cos\left(dx+c\right)-b^2+4\left(2\,a\cos\left(dx+c\right)^2+b\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(}{\cos(}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] [1/630*(315*sqrt(a)*b^4*cos(d*x + c)^4*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*co
s(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*
cos(d*x + c) + b)/cos(d*x + c))) + 4*(5*a*b^3*cos(d*x + c) - (16*a^4 - 84*a
^2*b^2 - 315*b^4)*cos(d*x + c)^4 + 35*b^4 + 2*(4*a^3*b - 21*a*b^3)*cos(d*x
+ c)^3 - 6*(a^2*b^2 + 21*b^4)*cos(d*x + c)^2)*sqrt((a*cos(d*x + c) + b)/cos
(d*x + c)))/(b^4*d*cos(d*x + c)^4), 1/315*(315*sqrt(-a)*b^4*arctan(2*sqrt(-
a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) +
```

```
b))*cos(d*x + c)^4 + 2*(5*a*b^3*cos(d*x + c) - (16*a^4 - 84*a^2*b^2 - 315*
b^4)*cos(d*x + c)^4 + 35*b^4 + 2*(4*a^3*b - 21*a*b^3)*cos(d*x + c)^3 - 6*(a
^2*b^2 + 21*b^4)*cos(d*x + c)^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(
b^4*d*cos(d*x + c)^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sec(c + dx)} \tan^5(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)*tan(d*x+c)**5,x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*tan(c + d*x)**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx + c) + a} \tan(dx + c)^5\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^5, x)
```

3.319 $\int \sqrt{a+b\sec(c+dx)}\tan^3(c+dx)\,dx$

Optimal. Leaf size=100

$$\frac{2(a+b\sec(c+dx))^{5/2}}{5b^2d}-\frac{2a(a+b\sec(c+dx))^{3/2}}{3b^2d}-\frac{2\sqrt{a+b\sec(c+dx)}}{d}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[a + b*Sec
[c + d*x]])/d - (2*a*(a + b*Sec[c + d*x])^(3/2))/(3*b^2*d) + (2*(a + b*Sec[
c + d*x])^(5/2))/(5*b^2*d)
```

Rubi [A] time = 0.112788, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3885, 898, 1261, 207}

$$\frac{2(a+b\sec(c+dx))^{5/2}}{5b^2d}-\frac{2a(a+b\sec(c+dx))^{3/2}}{3b^2d}-\frac{2\sqrt{a+b\sec(c+dx)}}{d}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^3,x]
```


```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[a + b*Sec
[c + d*x]])/d - (2*a*(a + b*Sec[c + d*x])^(3/2))/(3*b^2*d) + (2*(a + b*Sec[
c + d*x])^(5/2))/(5*b^2*d)
```


Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
```

```
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\tan^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{a+x}\left(b^2-x^2\right)}{x}\,dx, x, b\sec(c+dx)\right)}{b^2d}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{x^2\left(-a^2+b^2+2ax^2-x^4\right)}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^2d}$$
$$= -\frac{2\,\text{Subst}\left(\int \left(b^2+ax^2-x^4+\frac{ab^2}{-a+x^2}\right)dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^2d}$$
$$= -\frac{2\sqrt{a+b\sec(c+dx)}}{d} - \frac{2a(a+b\sec(c+dx))^{3/2}}{3b^2d} + \frac{2(a+b\sec(c+dx))^{5/2}}{5b^2d} - \frac{(2}{}$$
$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{a+b\sec(c+dx)}}{d} - \frac{2a(a+b\sec(c+dx))^{3/2}}{3b^2d}$$

Mathematica [A] time = 6.2172, size = 194, normalized size = 1.94

$$\frac{\sqrt{a+b\sec(c+dx)}\left(-\frac{2\left(2a^2+15b^2\right)}{15b^2}+\frac{2a\sec(c+dx)}{15b}+\frac{2}{5}\sec^2(c+dx)\right)}{d} + \frac{\sin^2(c+dx)\sqrt{a\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\left(\log\left(\right.\right.}{d\left(1-\cos^2(c+dx)\right)\sqrt{a\cos}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^3,x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((-2*(2*a^2 + 15*b^2))/(15*b^2) + (2*a*Sec[c + d*
x])/(15*b) + (2*Sec[c + d*x]^2)/5))/d + (Sqrt[a*Cos[c + d*x]]*(-Log[1 - Sqr
t[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]] + Log[1 + Sqrt[b + a*Cos[c + d*
x]]/Sqrt[a*Cos[c + d*x]]])*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x]^2)/(d*Sqrt
[b + a*Cos[c + d*x]]*(1 - Cos[c + d*x]^2))
```

Maple [B] time = 0.497, size = 2342, normalized size = 23.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x)
```

```
[Out] 1/60/d/b^2/(a-b)^(1/2)*(cos(d*x+c)+1)*(-1+cos(d*x+c))^4*(30*cos(d*x+c)^4*ln
(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/
(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos
(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a*
b^3-30*cos(d*x+c)^4*ln(-2/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*c
os(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*
cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/
2)-b)/sin(d*x+c)^2)*a*b^3+30*cos(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos
(d*x+c)+1)^2)^(3/2)*(a-b)^(1/2)*b^2-8*cos(d*x+c)^4*((b+a*cos(d*x+c))*cos(d*
x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)*a^3-54*cos(d*x+c)^4*((b+a*cos(d*x+
c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)*b^3-8*cos(d*x+c)^5*((b+a
*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)*a^3-54*cos(d*x+
c)^3*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)*b^3-6
*cos(d*x+c)^5*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3
/2)*b^2-6*cos(d*x+c)^3*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)
*(a-b)^(1/2)*b^2+36*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^
2)^(3/2)*(a-b)^(1/2)*b^2+30*cos(d*x+c)^5*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*
(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1
/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+
1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^2*b^2-15*cos(d*x+c)^5*ln(-1/(a-b
)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x
+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))
*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a*b^3-30*c
os(d*x+c)^5*ln(-2/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c
))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+
c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/si
n(d*x+c)^2)*a^2*b^2+15*cos(d*x+c)^5*ln(-2/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*co
s(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2
```

```
*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)
^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a*b^3-18*cos(d*x+c)^4*((b+a*cos(d*x+c))
*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(1/2)*b^2-54*cos(d*x+c)^4*((b+a*c
os(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)*a*b^2-8*cos(d*x+c
)^4*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)*a^2*b-
8*cos(d*x+c)^3*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(
1/2)*a^2*b-54*cos(d*x+c)^5*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(
1/2)*(a-b)^(1/2)*a*b^2+30*cos(d*x+c)^5*(a-b)^(1/2)*a^(3/2)*ln(4*cos(d*x+c)*
((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)
+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b^2+30
*cos(d*x+c)^4*(a-b)^(1/2)*a^(1/2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x
+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+
c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b^3+12*cos(d*x+c)^3*((b+a*cos(d
*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(1/2)*a*b+4*cos(d*x+c)*((b+
a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(1/2)*a*b+4*cos(d*x+
c)^4*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(1/2)*a*b+1
2*cos(d*x+c)^2*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(
3/2)*a*b-15*cos(d*x+c)^4*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((
b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+
c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b
)^(1/2)-b)/sin(d*x+c)^2)*b^4+15*cos(d*x+c)^4*ln(-2/(a-b)^(1/2)*(-1+cos(d*x+
c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b
)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x
+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*b^4+12*((b+a*cos(d*x+c))*cos(d
*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(1/2)*b^2)*((b+a*cos(d*x+c))/cos(d*x+c)
)^(1/2)*4^(1/2)/sin(d*x+c)^8/cos(d*x+c)^2/((b+a*cos(d*x+c))*cos(d*x+c)/(cos
(d*x+c)+1)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.62269, size = 779, normalized size = 7.79

$$\left[\frac{15\sqrt{a}b^2\cos(dx+c)^2\log\left(-8a^2\cos(dx+c)^2-8ab\cos(dx+c)-b^2-4\left(2a\cos(dx+c)^2+b\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx}{\cos(dx}}\right.}{30b^2d\cos(dx+c)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(a)*b^2*cos(d*x + c)^2*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(
d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*co
s(d*x + c) + b)/cos(d*x + c))) + 4*(a*b*cos(d*x + c) - (2*a^2 + 15*b^2)*cos
(d*x + c)^2 + 3*b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(b^2*d*cos(d*
x + c)^2), -1/15*(15*sqrt(-a)*b^2*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) +
b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b))*cos(d*x + c)^2 - 2*(a
*b*cos(d*x + c) - (2*a^2 + 15*b^2)*cos(d*x + c)^2 + 3*b^2)*sqrt((a*cos(d*x
+ c) + b)/cos(d*x + c)))/(b^2*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sec(c+dx)}\tan^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)*tan(d*x+c)**3,x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*tan(c + d*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec(dx+c)+a}\tan(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^3, x)
```

3.320 $\int \sqrt{a+b\sec(c+dx)}\tan(c+dx)\,dx$

Optimal. Leaf size=51

$$\frac{2\sqrt{a+b\sec(c+dx)}}{d}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}$$

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/d + (2*Sqrt[a + b*Se
c[c + d*x]])/d
```

Rubi [A] time = 0.0476281, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3885, 50, 63, 207}

$$\frac{2\sqrt{a+b\sec(c+dx)}}{d}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/d + (2*Sqrt[a + b*Se
c[c + d*x]])/d
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\sec(c+dx)}\tan(c+dx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\sqrt{a+x}}{x}\,dx, x, b\sec(c+dx)\right)}{d}\\ &= \frac{2\sqrt{a+b\sec(c+dx)}}{d}+\frac{a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\sec(c+dx)\right)}{d}\\ &= \frac{2\sqrt{a+b\sec(c+dx)}}{d}+\frac{(2a)\operatorname{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}\\ &= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{2\sqrt{a+b\sec(c+dx)}}{d}\end{aligned}$$

Mathematica [B] time = 0.248529, size = 137, normalized size = 2.69

$$\frac{\sqrt{a+b\sec(c+dx)}\left(2\sqrt{a\cos(c+dx)+b}+\sqrt{a\cos(c+dx)}\log\left(1-\frac{\sqrt{a\cos(c+dx)+b}}{\sqrt{a\cos(c+dx)}}\right)-\sqrt{a\cos(c+dx)}\log\left(\frac{\sqrt{a\cos(c+dx)+b}}{\sqrt{a\cos(c+dx)}}+\right.\right.}{d\sqrt{a\cos(c+dx)+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x],x]
```

```
[Out] ((2*Sqrt[b + a*Cos[c + d*x]] + Sqrt[a*Cos[c + d*x]]*Log[1 - Sqrt[b + a*Cos[
c + d*x]]/Sqrt[a*Cos[c + d*x]]] - Sqrt[a*Cos[c + d*x]]*Log[1 + Sqrt[b + a*C
os[c + d*x]]/Sqrt[a*Cos[c + d*x]]])*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[b + a
*Cos[c + d*x]])
```

Maple [A] time = 0.049, size = 42, normalized size = 0.8

$$\frac{1}{d}\left(2\sqrt{a+b\sec\left(dx+c\right)}-2\sqrt{a}\operatorname{Artanh}\left(\frac{\sqrt{a+b\sec\left(dx+c\right)}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)*tan(d*x+c),x)
```

```
[Out] 1/d*(2*(a+b*sec(d*x+c))^(1/2)-2*a^(1/2)*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1
/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.00288, size = 489, normalized size = 9.59

$$\left[\frac{\sqrt{a}\log\left(-8\,a^{2}\cos\left(dx+c\right)^{2}-8\,ab\cos\left(dx+c\right)-b^{2}+4\left(2\,a\cos\left(dx+c\right)^{2}+b\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\right)+4\sqrt{\frac{a\cos(}{\cos(}}}{2\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a
*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x
 + c))) + 4*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/d, (sqrt(-a)*arctan(2*
sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x
```

```
+ c) + b)) + 2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sec(c + dx)}\tan(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)*tan(d*x+c),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*tan(c + d*x), x)
```

Giac [B] time = 1.38951, size = 250, normalized size = 4.9

$$\frac{2\left(\frac{a\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a+b}+\sqrt{a-b}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{2\,b}{\sqrt{a-b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-b\tan\left(\frac{1}{2}\right.}}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="giac")
```

```
[Out] 2*(a*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x +
 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)
+ sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*b/(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2
 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d
*x + 1/2*c)^2 + a + b) - sqrt(a - b)))*sgn(cos(d*x + c))/d
```

3.321 $\int \cot(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=106

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/d - (Sqrt[a - b]*ArcT
anh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]])/d - (Sqrt[a + b]*ArcTanh[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]])/d
```

Rubi [A] time = 0.158867, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3885, 898, 1287, 206, 207}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/d - (Sqrt[a - b]*ArcT
anh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]])/d - (Sqrt[a + b]*ArcTanh[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]])/d
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
```

```
, p] && FractionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \cot(c+dx)\sqrt{a+b\sec(c+dx)}\,dx &= -\frac{b^2\,\text{Subst}\left(\int \frac{\sqrt{a+x}}{x\left(b^2-x^2\right)}\,dx, x, b\sec(c+dx)\right)}{d} \\
&= -\frac{\left(2b^2\right)\text{Subst}\left(\int \frac{x^2}{\left(-a+x^2\right)\left(-a^2+b^2+2ax^2-x^4\right)}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} \\
&= -\frac{\left(2b^2\right)\text{Subst}\left(\int \left(-\frac{a}{b^2\left(a-x^2\right)}+\frac{a+b}{2b^2\left(a+b-x^2\right)}+\frac{-a+b}{2b^2\left(-a+b+x^2\right)}\right)dx, x, \sqrt{a+b\sec(c+dx}\right.}{d} \\
&= \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}+\frac{(a-b)\,\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x,\right.}{d} \\
&= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d}-\frac{\sqrt{a+b}\tanh^{-1}\left(\right.}{d}
\end{aligned}$$

Mathematica [B] time = 5.66549, size = 343, normalized size = 3.24

$$-\frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{a+b\sec(c+dx)}\left(-\sqrt{b}\sqrt{b-a}\sqrt{a+b}\sqrt{\frac{a\cos(c+dx)+b}{b\cos(c+dx)+b}}\sin^{-1}\left(\frac{\sqrt{b-a}\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}}{\sqrt{b}}\right)-2\sqrt{a}\sqrt{a+\cdots}\right.}{d\sqrt{a+b}(a\cos(c+dx)\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*((a + b)*ArcTa
n[(Sqrt[a + b]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2])/Sqrt[-((b + a*Cos[c +
 d*x])*Sec[(c + d*x)/2]^2)]]*Sqrt[(-b - a*Cos[c + d*x])/(1 + Cos[c + d*x])]
 - 2*Sqrt[a]*Sqrt[a + b]*ArcTanh[(Sqrt[a]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])])/Sqrt[(b + a*Cos[c + d*x])/(1 + Cos[c + d*x])]]*Sqrt[(b + a*Cos[c + d*
x])/(1 + Cos[c + d*x])] - Sqrt[b]*Sqrt[-a + b]*Sqrt[a + b]*ArcSin[(Sqrt[-a
+ b]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])])/Sqrt[b]]*Sqrt[(b + a*Cos[c + d*
x])/(b + b*Cos[c + d*x])])*Sqrt[a + b*Sec[c + d*x]])/(Sqrt[a + b]*d*(b + a*
Cos[c + d*x]))
```

Maple [B] time = 0.298, size = 576, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/4/d/(a-b)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*4^(1/2)*cos(d*x+c)*(
-1+cos(d*x+c))*(2*a^(1/2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos
(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(
d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*(a-b)^(1/2)-(a+b)^(1/2)*ln(-2*(2*((b+a*
cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a+b)^(1/2)*cos(d*x+c)+2*(a+
b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*a*cos(d*x+c
)+b*cos(d*x+c)+b)/(-1+cos(d*x+c)))*(a-b)^(1/2)+ln(-1/(a-b)^(1/2)*(-1+cos(d*
x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a
-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d
*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a-ln(-1/(a-b)^(1/2)*(-1+cos(
d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*
(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos
```

```
(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*b)/sin(d*x+c)^2/((b+a*cos(
d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec (dx + c) + a} \cot (dx + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cot(d*x + c), x)
```

Fricas [B] time = 3.97924, size = 5389, normalized size = 50.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(2
*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d
*x + c))) + sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 -
4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 +
2*cos(d*x + c) + 1)) + sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)
^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((
a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*
x + c)^2 - 2*cos(d*x + c) + 1)))/d, 1/4*(2*sqrt(-a - b)*arctan(2*sqrt(-a -
b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x
+ c) + b)) + 2*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2
 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b
)/cos(d*x + c))) + sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 +
 b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*co
s(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x +
c)^2 + 2*cos(d*x + c) + 1)))/d, -1/4*(2*sqrt(-a + b)*arctan(-2*sqrt(-a + b)
*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x +
c) + b)) - 2*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 -
```

```
 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/
cos(d*x + c))) - sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b
^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(
d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)
^2 - 2*cos(d*x + c) + 1)))/d, -1/2*(sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqr
t((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) +
 b)) - sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x
 + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - sqrt(a)*log(-8*a^2*cos(
d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x +
 c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))))/d, -1/4*(4*sqrt(-a)*
arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a
*cos(d*x + c) + b)) - sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^
2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a
*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x
 + c)^2 + 2*cos(d*x + c) + 1)) - sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*co
s(d*x + c)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a +
 b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c
))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/d, -1/4*(4*sqrt(-a)*arctan(2*sqr
t(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c
) + b)) - 2*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/co
s(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - sqrt(a - b)*log(-(
(8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 +
b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*
a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/d, -1/4*
(4*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d
*x + c)/(2*a*cos(d*x + c) + b)) + 2*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqr
t((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) +
 b)) - sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2
*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) +
 b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos
(d*x + c) + 1)))/d, -1/2*(2*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
 + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) + sqrt(-a + b)*arc
tan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((
2*a - b)*cos(d*x + c) + b)) - sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*co
s(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)))/d
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sec(c + dx)} \cot(c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*cot(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)+a}\cot(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cot(d*x + c), x)
```

3.322 $\int \cot^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=215

$$-\frac{\cot^2(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4d\sqrt{a-b}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}$$

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/d + (a*ArcTanh[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a - b]])/(Sqrt[a - b]*d) - (3*b*ArcTanh[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a - b]])/(4*Sqrt[a - b]*d) + (a*ArcTanh[Sqrt[a + b*Sec[
c + d*x]]/Sqrt[a + b]])/(Sqrt[a + b]*d) + (3*b*ArcTanh[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]])/(4*Sqrt[a + b]*d) - (Cot[c + d*x]^2*Sqrt[a + b*Sec[c + d
*x]])/(2*d)
```

Rubi [A] time = 0.294987, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3885, 898, 1315, 1178, 12, 1093, 206, 1170, 207}

$$-\frac{\cot^2(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4d\sqrt{a-b}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/d + (a*ArcTanh[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a - b]])/(Sqrt[a - b]*d) - (3*b*ArcTanh[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a - b]])/(4*Sqrt[a - b]*d) + (a*ArcTanh[Sqrt[a + b*Sec[
c + d*x]]/Sqrt[a + b]])/(Sqrt[a + b]*d) + (3*b*ArcTanh[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]])/(4*Sqrt[a + b]*d) - (Cot[c + d*x]^2*Sqrt[a + b*Sec[c + d
*x]])/(2*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1315

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[f^2/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^(
m - 2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^p, x], x] - Dist[(d*e*f^2)/(c*d^
2 - b*d*e + a*e^2), Int[((f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1))/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
p, -1] && GtQ[m, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{b^4\,\text{Subst}\left(\int \frac{\sqrt{a+x}}{x\left(b^2-x^2\right)^2}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= \frac{\left(2b^4\right)\text{Subst}\left(\int \frac{x^2}{\left(-a+x^2\right)\left(-a^2+b^2+2ax^2-x^4\right)^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}$$

$$= \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{-a^2+b^2+ax^2}{\left(-a^2+b^2+2ax^2-x^4\right)^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} + \frac{\left(2ab^2\right)\text{Subst}\left(\int \ldots\right.}{}$$

$$= -\frac{b^2\sqrt{a+b\sec(c+dx)}}{2d\left(a^2-b^2-2a(a+b\sec(c+dx))+(a+b\sec(c+dx))^2\right)} + \frac{\left(2ab^2\right)\text{Subst}\left(\int\left(\ldots\right.\right.}{}$$

$$= -\frac{b^2\sqrt{a+b\sec(c+dx)}}{2d\left(a^2-b^2-2a(a+b\sec(c+dx))+(a+b\sec(c+dx))^2\right)} + \frac{a\,\text{Subst}\left(\int \frac{1}{a+b-x^2}\ldots\right.}{}$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d} + \frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+\ldots}}{\sqrt{a+b}}\right.}{\sqrt{a+b}\,d}$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d} - \frac{3b\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c\ldots}}{\sqrt{a-b}}\right.}{4\sqrt{a-b}\,d}$$

Mathematica [B] time = 19.0386, size = 937, normalized size = 4.36

$$\frac{\sqrt{a+b\sec(c+dx)}\left(\frac{1}{2}-\frac{1}{2}\csc^2(c+dx)\right)}{d}+\frac{\left(\frac{2\left(4\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{-a\cos(c+dx)}}\right)-\sqrt{a}\left(\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{b+a\cos(c+dx)}}{\sqrt{a-b}\sqrt{-a\cos(c+dx)}}\right)+\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{b+a\cdots}}{\sqrt{a+b}\sqrt{-a\cdots}}\right.\right.\right.}{\sqrt{a-b}\sqrt{a+b}\left(a^2-2b^2-2(b+a\cos(c+dx))^2+4b(b+a\cos\cdots\right.}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((1/2 - Csc[c + d*x]^2/2)*Sqrt[a + b*Sec[c + d*x]])/d + (((3*a*b*(-(Sqrt[-a
^2]*Sqrt[a + b]*Log[-Sqrt[-a + b] + Sqrt[b + a*Cos[c + d*x]]]) + Sqrt[-a^2]
*Sqrt[a + b]*Log[Sqrt[-a + b] + Sqrt[b + a*Cos[c + d*x]]] - a*Sqrt[-a + b]*
Log[-Sqrt[a + b] + Sqrt[b + a*Cos[c + d*x]]] + a*Sqrt[-a + b]*Log[Sqrt[a +
b] + Sqrt[b + a*Cos[c + d*x]]] + Sqrt[-a^2]*Sqrt[a + b]*Log[b + Sqrt[a]*Sqr
t[-(a*Cos[c + d*x])] - Sqrt[-a + b]*Sqrt[b + a*Cos[c + d*x]]] - Sqrt[-a^2]*
Sqrt[a + b]*Log[b + Sqrt[a]*Sqrt[-(a*Cos[c + d*x])] + Sqrt[-a + b]*Sqrt[b +
 a*Cos[c + d*x]]] + a*Sqrt[-a + b]*Log[b + Sqrt[-a]*Sqrt[-(a*Cos[c + d*x])]
 - Sqrt[a + b]*Sqrt[b + a*Cos[c + d*x]]] - a*Sqrt[-a + b]*Log[b + Sqrt[-a]*
Sqrt[-(a*Cos[c + d*x])] + Sqrt[a + b]*Sqrt[b + a*Cos[c + d*x]]]))/(2*(-a)^(
3/2)*Sqrt[-a + b]*Sqrt[a + b]*Sqrt[-(a*Cos[c + d*x])]*Sqrt[Sec[c + d*x]]) +
 (2*Sqrt[a]*(Sqrt[a - b]*(a + b)*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c + d*x]])/
(Sqrt[a - b]*Sqrt[-(a*Cos[c + d*x])])] + (a - b)*Sqrt[a + b]*ArcTan[(Sqrt[a
]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a + b]*Sqrt[-(a*Cos[c + d*x])])])*Sqrt[-(
a*Cos[c + d*x])]*Sqrt[Sec[c + d*x]])/((a - b)*(a + b)) + (2*a^2*(4*Sqrt[a -
 b]*Sqrt[a + b]*ArcTan[Sqrt[b + a*Cos[c + d*x]]/Sqrt[-(a*Cos[c + d*x])]] -
Sqrt[a]*(Sqrt[a + b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a - b]
*Sqrt[-(a*Cos[c + d*x])])] + Sqrt[a - b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c +
 d*x]])/(Sqrt[a + b]*Sqrt[-(a*Cos[c + d*x])])]))*Sqrt[-(a*Cos[c + d*x])]*Co
s[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - 2*b^2 +
4*b*(b + a*Cos[c + d*x]) - 2*(b + a*Cos[c + d*x])^2)))*Sqrt[a + b*Sec[c + d
*x]])/(4*d*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Maple [B] time = 0.296, size = 2844, normalized size = 13.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/16/d/(a+b)^(3/2)/(a-b)^(3/2)*(-1+cos(d*x+c))*(4*(a-b)^(3/2)*ln(-2*(2*((b
+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a+b)^(1/2)*cos(d*x+c)+2*
(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*a*cos(d*
x+c)+b*cos(d*x+c)+b)/(-1+cos(d*x+c)))*4^(1/2)*a^2+3*(a-b)^(3/2)*ln(-2*(2*((
b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a+b)^(1/2)*cos(d*x+c)+2
*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*a*cos(d
*x+c)+b*cos(d*x+c)+b)/(-1+cos(d*x+c)))*4^(1/2)*b^2-4*(a+b)^(1/2)*ln(-1/(a-b
)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x
+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))
*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*4^(1/2)*a^
3+4*(a-b)^(3/2)*cos(d*x+c)^2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(
d*x+c)+1)^2)^(1/2)*4^(1/2)*a+8*(a-b)^(3/2)*a^(1/2)*cos(d*x+c)^2*(a+b)^(1/2)
*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/
2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^
(1/2)+2*b)*4^(1/2)*b-3*(a+b)^(1/2)*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos
(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*
a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^
(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*4^(1/2)*b^3-16*(a-b)^(3/2)*cos(d*x+c)^2*
(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)-32*(a-b)^(
3/2)*cos(d*x+c)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^
(3/2)-4*(a-b)^(3/2)*cos(d*x+c)^2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(
cos(d*x+c)+1)^2)^(1/2)*4^(1/2)*b-16*(a-b)^(3/2)*(a+b)^(1/2)*((b+a*cos(d*x+c
))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)-8*(a-b)^(3/2)*a^(3/2)*(a+b)^(1/2)*ln(
4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4
*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2
)+2*b)*4^(1/2)-4*(a-b)^(3/2)*cos(d*x+c)^2*ln(-2*(2*((b+a*cos(d*x+c))*cos(d*
x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a+b)^(1/2)*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos
(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+b)/
(-1+cos(d*x+c)))*4^(1/2)*a^2-3*(a-b)^(3/2)*cos(d*x+c)^2*ln(-2*(2*((b+a*cos(
d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a+b)^(1/2)*cos(d*x+c)+2*(a+b)^(
1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*a*cos(d*x+c)+b*
cos(d*x+c)+b)/(-1+cos(d*x+c)))*4^(1/2)*b^2+4*cos(d*x+c)^2*(a+b)^(1/2)*ln(-1
/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(co
s(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*
x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*4^(1/
2)*a^3+3*cos(d*x+c)^2*(a+b)^(1/2)*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(
d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a
*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(
1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*4^(1/2)*b^3+7*(a-b)^(3/2)*ln(-2*(2*((b+a*
cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a+b)^(1/2)*cos(d*x+c)+2*(a+
b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*a*cos(d*x+c
)+b*cos(d*x+c)+b)/(-1+cos(d*x+c)))*4^(1/2)*a*b+3*(a+b)^(1/2)*ln(-1/(a-b)^(1
/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+
1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos
(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*4^(1/2)*a^2*b+
```

```
4*(a+b)^(1/2)*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x
+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*
x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/
sin(d*x+c)^2)*4^(1/2)*a*b^2-4*(a-b)^(3/2)*cos(d*x+c)*(a+b)^(1/2)*((b+a*cos(
d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*4^(1/2)*a+4*(a-b)^(3/2)*cos(d*x+
c)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*4^(1/2)
*b+8*(a-b)^(3/2)*a^(3/2)*cos(d*x+c)^2*(a+b)^(1/2)*ln(4*cos(d*x+c)*((b+a*cos
(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2
)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*4^(1/2)-7*(a-b)
^(3/2)*cos(d*x+c)^2*ln(-2*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)
^(1/2)*(a+b)^(1/2)*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(c
os(d*x+c)+1)^2)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+b)/(-1+cos(d*x+c)))*4^(1/
2)*a*b-8*(a-b)^(3/2)*a^(1/2)*(a+b)^(1/2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*
cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*c
os(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*4^(1/2)*b-3*cos(d*x+c)^2
*(a+b)^(1/2)*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+
c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x
+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/s
in(d*x+c)^2)*4^(1/2)*a^2*b-4*cos(d*x+c)^2*(a+b)^(1/2)*ln(-1/(a-b)^(1/2)*(-1
+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(
1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)
/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*4^(1/2)*a*b^2)*((b+a*
cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)/sin(d*x+c)^4/((b+a*cos(d*x+c))*cos
(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)+a}\cot(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cot(d*x + c)^3, x)
```

Fricas [B] time = 11.7887, size = 8409, normalized size = 39.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/16*(8*(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)^2
 + 8*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(a)*log(-8*a^2*cos(d*x +
c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*s
qrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) - ((4*a^2 + a*b - 3*b^2)*co
s(d*x + c)^2 - 4*a^2 - a*b + 3*b^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)
*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(
a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x
+ c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + ((4*a^2 - a*b - 3*b^2)*cos(d
*x + c)^2 - 4*a^2 + a*b + 3*b^2)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*co
s(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a +
 b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c
))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^2 - b^2)*d*cos(d*x + c)^2 -
(a^2 - b^2)*d), 1/16*(8*(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))
*cos(d*x + c)^2 - 2*((4*a^2 - a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 + a*b + 3
*b^2)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x
+ c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) + 8*((a^2 - b^2)*cos(d*x +
 c)^2 - a^2 + b^2)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) -
 b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)
 + b)/cos(d*x + c))) - ((4*a^2 + a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 - a*b
+ 3*b^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*(
(2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c)
 + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*c
os(d*x + c) + 1)))/((a^2 - b^2)*d*cos(d*x + c)^2 - (a^2 - b^2)*d), 1/16*(8*
(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)^2 + 2*((4*
a^2 + a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 - a*b + 3*b^2)*sqrt(-a + b)*arcta
n(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*
a - b)*cos(d*x + c) + b)) + 8*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt
(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x +
 c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) +
((4*a^2 - a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 + a*b + 3*b^2)*sqrt(a + b)*lo
g(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^
2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2
*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^
2 - b^2)*d*cos(d*x + c)^2 - (a^2 - b^2)*d), 1/8*(4*(a^2 - b^2)*sqrt((a*cos(
d*x + c) + b)/cos(d*x + c))*cos(d*x + c)^2 + ((4*a^2 + a*b - 3*b^2)*cos(d*x
 + c)^2 - 4*a^2 - a*b + 3*b^2)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*
cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b))
- ((4*a^2 - a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 + a*b + 3*b^2)*sqrt(-a - b)
*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)
/((2*a + b)*cos(d*x + c) + b)) + 4*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)
*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(
d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)
```

```
)))/((a^2 - b^2)*d*cos(d*x + c)^2 - (a^2 - b^2)*d), 1/16*(8*(a^2 - b^2)*sqr
t((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)^2 + 16*((a^2 - b^2)*cos(d
*x + c)^2 - a^2 + b^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)
/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - ((4*a^2 + a*b - 3*b^2
)*cos(d*x + c)^2 - 4*a^2 - a*b + 3*b^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b +
b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*s
qrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(
d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + ((4*a^2 - a*b - 3*b^2)*c
os(d*x + c)^2 - 4*a^2 + a*b + 3*b^2)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2
)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt
(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x
 + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^2 - b^2)*d*cos(d*x + c)^
2 - (a^2 - b^2)*d), 1/16*(8*(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
 c))*cos(d*x + c)^2 + 16*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(-a)*
arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a
*cos(d*x + c) + b)) - 2*((4*a^2 - a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 + a*b
 + 3*b^2)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(
d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - ((4*a^2 + a*b - 3*b^
2)*cos(d*x + c)^2 - 4*a^2 - a*b + 3*b^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b +
 b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*
sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos
(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/((a^2 - b^2)*d*cos(d*x +
 c)^2 - (a^2 - b^2)*d), 1/16*(8*(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/cos(d
*x + c))*cos(d*x + c)^2 + 16*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(
-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/
(2*a*cos(d*x + c) + b)) + 2*((4*a^2 + a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 -
 a*b + 3*b^2)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)
/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) + ((4*a^2 - a*b -
 3*b^2)*cos(d*x + c)^2 - 4*a^2 + a*b + 3*b^2)*sqrt(a + b)*log(-((8*a^2 + 8*
a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x +
 c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2
)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^2 - b^2)*d*cos(
d*x + c)^2 - (a^2 - b^2)*d), 1/8*(4*(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/c
os(d*x + c))*cos(d*x + c)^2 + 8*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sq
rt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x +
c)/(2*a*cos(d*x + c) + b)) + ((4*a^2 + a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2
- a*b + 3*b^2)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b
)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - ((4*a^2 - a*b
- 3*b^2)*cos(d*x + c)^2 - 4*a^2 + a*b + 3*b^2)*sqrt(-a - b)*arctan(2*sqrt(-
a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(
d*x + c) + b)))/((a^2 - b^2)*d*cos(d*x + c)^2 - (a^2 - b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sec(c + dx)}\cot^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*cot(c + d*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx + c) + a}\cot(dx + c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cot(d*x + c)^3, x)
```

3.323 $\int \sqrt{a+b\sec(c+dx)}\tan^2(c+dx)\,dx$

Optimal. Leaf size=344

$$-\frac{2\sqrt{a+b}(a+2b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3bd}-\frac{2a(a-b)\sqrt{a+b}\,\mathrm{co}}{}$$

```
[Out] (-2*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) - (2*Sqrt[a + b]*(a + 2*b
)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
 b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*
x]))/(a - b))])/(3*b*d) + (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a,
 ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*S
qrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.385066, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3894, 4057, 4058, 3921, 3784, 3832, 4004}

$$-\frac{2a(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d}+\frac{2\tan(c+dx)\sqrt{a+b\sec(c}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^2,x]
```


```
[Out] (-2*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) - (2*Sqrt[a + b]*(a + 2*b
)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
 b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*
x]))/(a - b))])/(3*b*d) + (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a,
 ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*S
qrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```


Rule 3894

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4057

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)
/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*
A*(m + 1) + (A*b*(m + 1) + b*C*m)*Csc[e + f*x] + a*C*m*Csc[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\tan^2(c+dx)\,dx = \int \sqrt{a+b\sec(c+dx)}\left(-1+\sec^2(c+dx)\right)\,dx$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d} + \frac{2}{3}\int \frac{-\frac{3a}{2} - b\sec(c+dx) + \frac{1}{2}a\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d} + \frac{2}{3}\int \frac{-\frac{3a}{2} + \left(-\frac{a}{2} - b\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx + \frac{1}{3}a\int$$

$$= -\frac{2a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b}{}}}{3b^2d}$$

$$= -\frac{2a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b}{}}}{3b^2d}$$

Mathematica [C] time = 17.673, size = 692, normalized size = 2.01

$$\frac{\sqrt{a+b\sec(c+dx)}\left(\frac{2a\sin(c+dx)}{3b} + \frac{2}{3}\tan(c+dx)\right)}{d} - \frac{2\sqrt{\frac{-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b\tan^2\left(\frac{1}{2}(c+dx)\right)+b}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}}\sqrt{a+b\sec(c+dx)}\left(2ib(a-b)\sqrt{\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^2,x]
```

```
[Out] (-2*Sqrt[a + b*Sec[c + d*x]]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*((-I)*a*(a - b)*EllipticE[I*ArcSinh[
Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*(a - b)*b*EllipticF[I*ArcSinh[Sqrt[(
```

```
-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)
/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(a + b)] - (6*I)*a*b*EllipticPi[-((a + b)/(a - b)), I*ArcSi
nh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[
(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
 + b*Tan[(c + d*x)/2]^2)/(a + b)] + a*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/
2]*(b - b*Tan[(c + d*x)/2]^4 + a*(-1 + Tan[(c + d*x)/2]^2)^2)))/(3*b*Sqrt[(
-a + b)/(a + b)]*d*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(1 + Ta
n[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(b - b*Tan[(c + d*x)/2]^4 + a*(
-1 + Tan[(c + d*x)/2]^2)^2)) + (Sqrt[a + b*Sec[c + d*x]]*((2*a*Sin[c + d*x]
)/(3*b) + (2*Tan[c + d*x])/3))/d
```

Maple [B] time = 0.377, size = 1109, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x)
```

```
[Out] -2/3/d/b*(-1+cos(d*x+c))^2*(4*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-2*sin(d*x+c)*cos(d*x+c)^2*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-sin(d*x
+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a^2-sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a*b-6*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b+4*sin(d*x+c)*cos(d*x+c
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-2*si
n(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*b^2-sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a^2-sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-6*sin(d*x+c)*cos(d*x+c)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
```

```
)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b+cos(d*x
+c)^3*a^2+cos(d*x+c)^3*a*b-cos(d*x+c)^2*a^2+cos(d*x+c)^2*a*b+cos(d*x+c)^2*b
^2-2*a*b*cos(d*x+c)-b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)
^2/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)+a}\tan(dx+c)^2\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sec(dx+c)+a}\tan(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec(c+dx)}\tan^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)*tan(d*x+c)**2,x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*tan(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec (dx + c) + a} \tan (dx + c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^2, x)
```

3.324 $\int \sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=125

$$-\frac{2\cot(c+dx)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(\sec(c+dx)+1)}{a+b\sec(c+dx)}}(a+b\sec(c+dx))\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right)|\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}$$

```
[Out] (-2*Cot[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sec[c
+ d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sec[c + d*x]))/(a + b*Sec[c + d*x
]))]*Sqrt[(b*(1 + Sec[c + d*x]))/(a + b*Sec[c + d*x])]*(a + b*Sec[c + d*x])
)/(Sqrt[a + b]*d)
```

Rubi [A] time = 0.0244018, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3780}

$$-\frac{2\cot(c+dx)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(\sec(c+dx)+1)}{a+b\sec(c+dx)}}(a+b\sec(c+dx))\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right)|\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] (-2*Cot[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sec[c
+ d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sec[c + d*x]))/(a + b*Sec[c + d*x
]))]*Sqrt[(b*(1 + Sec[c + d*x]))/(a + b*Sec[c + d*x])]*(a + b*Sec[c + d*x])
)/(Sqrt[a + b]*d)
```


Rule 3780

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*(a + b
*Csc[c + d*x])*Sqrt[(b*(1 + Csc[c + d*x]))/(a + b*Csc[c + d*x])]*Sqrt[-((b*
(1 - Csc[c + d*x]))/(a + b*Csc[c + d*x]))]*EllipticPi[a/(a + b), ArcSin[Rt[
a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)/(a + b)])/(d*Rt[a + b, 2]*Cot[
c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\,dx = -\frac{2\cot(c+dx)\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right)|\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(1+\sec(c+dx))}{a+b\sec(c+dx)}}(a+b\sec(c+}{\sqrt{a+b}d}$$

Mathematica [A] time = 0.250522, size = 153, normalized size = 1.22

$$-\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\sqrt{a+b\sec(c+dx)}\left((a-b)\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{a-b}{a+b}\right)+\right.}{d(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*((a - b)*EllipticF[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)])*Sqrt[a + b*Sec[c + d*x]])/(d*(b + a*Cos[c + d*x]))
```

Maple [A] time = 0.259, size = 215, normalized size = 1.7

$$-2\,\frac{(\cos(dx+c)+1)^2(-1+\cos(dx+c))}{d\,(b+a\cos(dx+c))(\sin(dx+c))^2}\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\left(\text{EllipticF}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1)^2*(EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*a*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,((a-b)/(a+b))^(1/2)))*(-1+cos(d*x+c))/(b+a*cos(d*x+c))/sin(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sec (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec (dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a), x)
```

3.325 $\int \cot^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=246

$$\frac{\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}-\frac{\cot(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

```
[Out] (Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[c + d*x]))/(a - b))])/d - (Cot[c + d*x]*Sqrt[a + b*Sec[c + d*x]])/d +
(2*Cot[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sec[c +
 d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sec[c + d*x]))/(a + b*Sec[c + d*x]
))]*Sqrt[(b*(1 + Sec[c + d*x]))/(a + b*Sec[c + d*x])]*(a + b*Sec[c + d*x]))
/(Sqrt[a + b]*d)
```

Rubi [A] time = 0.211957, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3896, 3780, 3875, 3832}

$$-\frac{\cot(c+dx)\sqrt{a+b\sec(c+dx)}}{d}+\frac{\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}+\frac{2\,\text{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] (Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[c + d*x]))/(a - b))])/d - (Cot[c + d*x]*Sqrt[a + b*Sec[c + d*x]])/d +
(2*Cot[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sec[c +
 d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sec[c + d*x]))/(a + b*Sec[c + d*x]
))]*Sqrt[(b*(1 + Sec[c + d*x]))/(a + b*Sec[c + d*x])]*(a + b*Sec[c + d*x]))
/(Sqrt[a + b]*d)
```


Rule 3896

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-(m/2)), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &
```

```
& ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Rule 3780

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*(a + b
*Csc[c + d*x])*Sqrt[(b*(1 + Csc[c + d*x]))/(a + b*Csc[c + d*x])]*Sqrt[-((b*
(1 - Csc[c + d*x]))/(a + b*Csc[c + d*x]))]*EllipticPi[a/(a + b), ArcSin[Rt[
a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)/(a + b)])/(d*Rt[a + b, 2]*Cot[
c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3875

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(Tan[e + f*x]*(a + b*Csc[e + f*x])^m)/f, x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \left(-\sqrt{a+b\sec(c+dx)}+\csc^2(c+dx)\sqrt{a+b\sec(c+dx)}\right)dx$$

$$= -\int \sqrt{a+b\sec(c+dx)}\,dx + \int \csc^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$= -\frac{\cot(c+dx)\sqrt{a+b\sec(c+dx)}}{d} + \frac{2\cot(c+dx)\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right)|\frac{a}{a}\right.}{}$$

$$= \frac{\sqrt{a+b}\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d}$$

Mathematica [A] time = 3.3656, size = 156, normalized size = 0.63

$$\frac{\sqrt{a+b\sec(c+dx)}\left(-\frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{1}{\sec(c+dx)+1}}\sqrt{\frac{a+b\sec(c+dx)}{(a+b)(\sec(c+dx)+1)}}\left((b-2a)\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{a-b}{a+b}\right)-4a\Pi\left(-1;-\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\right.\right.}{a\cos(c+dx)+b}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*(-Cot[c + d*x] - (2*Cos[(c + d*x)/2]^2*((-2*a + b
)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 4*a*EllipticPi[-1,
 -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sqrt[(1 + Sec[c + d*x])^(-1)]
*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))])/(b + a*Cos[c + d*
x])))/d
```

Maple [B] time = 0.296, size = 628, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(-1+cos(d*x+c))^2*(2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a-EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-4*a*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)+2
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)*a-(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(
d*x+c)-4*a*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1
/2))*sin(d*x+c)+a*cos(d*x+c)^2+b*cos(d*x+c))*(cos(d*x+c)+1)^2*((b+a*cos(d*x
+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)+a}\cot(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cot(d*x + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec(c+dx)}\cot^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*cot(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)+a}\cot(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cot(d*x + c)^2, x)
```

3.326 $\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=148

$$\frac{2\left(3a^2-2b^2\right)(a+b\sec(c+dx))^{3/2}}{3b^4d}-\frac{2a\left(a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}}{b^4d}+\frac{2(a+b\sec(c+dx))^{7/2}}{7b^4d}-\frac{6a(a+b\sec(c+dx))}{5b^4d}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - (2*a*(a^2 - 2*
b^2)*Sqrt[a + b*Sec[c + d*x]])/(b^4*d) + (2*(3*a^2 - 2*b^2)*(a + b*Sec[c +
d*x])^(3/2))/(3*b^4*d) - (6*a*(a + b*Sec[c + d*x])^(5/2))/(5*b^4*d) + (2*(a
 + b*Sec[c + d*x])^(7/2))/(7*b^4*d)
```

Rubi [A] time = 0.143342, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3885, 898, 1153, 207}

$$\frac{2\left(3a^2-2b^2\right)(a+b\sec(c+dx))^{3/2}}{3b^4d}-\frac{2a\left(a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}}{b^4d}+\frac{2(a+b\sec(c+dx))^{7/2}}{7b^4d}-\frac{6a(a+b\sec(c+dx))}{5b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - (2*a*(a^2 - 2*
b^2)*Sqrt[a + b*Sec[c + d*x]])/(b^4*d) + (2*(3*a^2 - 2*b^2)*(a + b*Sec[c +
d*x])^(3/2))/(3*b^4*d) - (6*a*(a + b*Sec[c + d*x])^(5/2))/(5*b^4*d) + (2*(a
 + b*Sec[c + d*x])^(7/2))/(7*b^4*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
```

```
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x\sqrt{a+x}}\,dx, x, b\sec(c+dx)\right)}{b^4d}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{\left(-a^2+b^2+2ax^2-x^4\right)^2}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^4d}$$
$$= \frac{2\,\text{Subst}\left(\int \left(-a^3+2ab^2+\left(3a^2-2b^2\right)x^2-3ax^4+x^6+\frac{b^4}{-a+x^2}\right)dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^4d}$$
$$= -\frac{2a\left(a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}}{b^4d} + \frac{2\left(3a^2-2b^2\right)(a+b\sec(c+dx))^{3/2}}{3b^4d} - \frac{6a(a+b\sec(c+d}{5b^4d}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{2a\left(a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}}{b^4d} + \frac{2\left(3a^2-2b^2\right)(a+b\sec(c+}{3b^4d}$$

Mathematica [A] time = 6.30109, size = 248, normalized size = 1.68

$$\frac{\sec(c+dx)(a\cos(c+dx)+b)\left(-\frac{4\left(35b^2-12a^2\right)\sec(c+dx)}{105b^3}+\frac{8a\left(35b^2-12a^2\right)}{105b^4}-\frac{12a\sec^2(c+dx)}{35b^2}+\frac{2\sec^3(c+dx)}{7b}\right)}{d\sqrt{a+b\sec(c+dx)}} - \frac{\sin(c+dx)\tan(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((8*a*(-12*a^2 + 35*b^2))/(105*b^4) - (4
*(-12*a^2 + 35*b^2)*Sec[c + d*x])/(105*b^3) - (12*a*Sec[c + d*x]^2)/(35*b^2
) + (2*Sec[c + d*x]^3)/(7*b)))/(d*Sqrt[a + b*Sec[c + d*x]]) - (Sqrt[a*Cos[c
 + d*x]]*Sqrt[b + a*Cos[c + d*x]]*(-Log[1 - Sqrt[b + a*Cos[c + d*x]]/Sqrt[a
*Cos[c + d*x]]] + Log[1 + Sqrt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]])*S
in[c + d*x]*Tan[c + d*x])/(a*d*(1 - Cos[c + d*x]^2)*Sqrt[a + b*Sec[c + d*x]
])
```

Maple [B] time = 0.705, size = 4997, normalized size = 33.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/420/d/b^4/a/(a-b)^(3/2)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos
(d*x+c)+1)*(-1+cos(d*x+c))^4*(-420*cos(d*x+c)^5*ln(-1/(a-b)^(1/2)*(-1+cos(d
*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(
a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(
d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^3*b^4+315*cos(d*x+c)^5*ln
(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/
(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos
(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^
2*b^5-105*cos(d*x+c)^5*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+
a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)
+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^
(1/2)-b)/sin(d*x+c)^2)*a*b^6-60*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)
^2)^(3/2)*(a-b)^(3/2)*a*b^3-210*cos(d*x+c)^6*ln(-2/(a-b)^(1/2)*(-1+cos(d*x+
c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b
)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x
+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^6*b-105*cos(d*x+c)^6*ln(-2/(
a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(
d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+
c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^5*b^2
+420*cos(d*x+c)^6*ln(-2/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos
(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*co
s(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)
```

```
-b)/sin(d*x+c)^2)*a^4*b^3-315*cos(d*x+c)^6*ln(-2/(a-b)^(1/2)*(-1+cos(d*x+c)
)*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^
(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c
)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^3*b^4+105*cos(d*x+c)^6*ln(-2/(
a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(
d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+
c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^2*b^5
+210*cos(d*x+c)^6*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos
(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*co
s(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)
-b)/sin(d*x+c)^2)*a^6*b+105*cos(d*x+c)^6*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*
(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1
/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+
1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^5*b^2+105*cos(d*x+c)^6*ln(-2/(a-
b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*
x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c)
)*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^7-105*c
os(d*x+c)^6*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c
))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+
c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/si
n(d*x+c)^2)*a^7+420*cos(d*x+c)^5*ln(-2/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d
*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*
cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1
/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^3*b^4-315*cos(d*x+c)^5*ln(-2/(a-b)^(1/2)
*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^
2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*
x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^2*b^5+105*cos(d
*x+c)^5*ln(-2/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*c
os(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2
*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*
x+c)^2)*a*b^6-105*cos(d*x+c)^5*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x
+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*co
s(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2
)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^6*b+210*cos(d*x+c)^5*ln(-1/(a-b)^(1/2)*(-1
+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(
1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)
/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^5*b^2+192*cos(d*x+c
)^6*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^5+19
2*cos(d*x+c)^5*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(
1/2)*a^5+105*cos(d*x+c)^5*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*(
(b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x
+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-
b)^(1/2)-b)/sin(d*x+c)^2)*a^4*b^3-420*cos(d*x+c)^6*ln(-1/(a-b)^(1/2)*(-1+co
s(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2
)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(c
```

os(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^4*b^3+315*cos(d*x+c)^6
ln(-1/(a-b)^(1/2)(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+
c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*
cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)
*a^3*b^4-105*cos(d*x+c)^6*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*(
(b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x
+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-
b)^(1/2)-b)/sin(d*x+c)^2)*a^2*b^5+105*cos(d*x+c)^5*ln(-2/(a-b)^(1/2)*(-1+co
s(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2
)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(c
os(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^6*b-210*cos(d*x+c)^5*l
n(-2/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)
/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*co
s(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a
^5*b^2-105*cos(d*x+c)^5*ln(-2/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b
+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c
)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)
^(1/2)-b)/sin(d*x+c)^2)*a^4*b^3+192*cos(d*x+c)^5*(a-b)^(3/2)*((b+a*cos(d*x+
c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^4*b-524*cos(d*x+c)^5*(a-b)^(3/2)*(
(b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^3*b^2-524*cos(d*x+c)^
5*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2*b^3-
524*cos(d*x+c)^6*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)
^(1/2)*a^3*b^2+210*cos(d*x+c)^6*(a-b)^(3/2)*a^(3/2)*ln(4*cos(d*x+c)*((b+a*c
os(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1
/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b^4+210*cos(d
x+c)^5(a-b)^(3/2)*a^(1/2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(c
os(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*co
s(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b^5-108*cos(d*x+c)^5*(a-b)^(3/2)*((b+
a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*a^2*b^2-288*cos(d*x+c)^4*(
a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*a^3*b+840*c
os(d*x+c)^4*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2
)*a*b^3+180*cos(d*x+c)^3*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+
c)+1)^2)^(3/2)*a^2*b^2-96*cos(d*x+c)^2*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*
x+c)/(cos(d*x+c)+1)^2)^(3/2)*a^3*b+100*cos(d*x+c)^2*(a-b)^(3/2)*((b+a*cos(d
*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*a*b^3+72*cos(d*x+c)*(a-b)^(3/2)*(
(b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*a^2*b^2+192*cos(d*x+c)^
4*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^4*b-18
0*cos(d*x+c)*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/
2)*a*b^3-524*cos(d*x+c)^4*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x
+c)+1)^2)^(1/2)*a^2*b^3-36*cos(d*x+c)^6*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d
*x+c)/(cos(d*x+c)+1)^2)^(3/2)*a^2*b^2-96*cos(d*x+c)^5*(a-b)^(3/2)*((b+a*cos
(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*a^3*b+280*cos(d*x+c)^5*(a-b)^(3
/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*a*b^3-36*cos(d*x+c
)^4*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*a^2*b^
2-288*cos(d*x+c)^3*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^

```
2)^(3/2)*a^3*b+780*cos(d*x+c)^3*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(c
os(d*x+c)+1)^2)^(3/2)*a*b^3+216*cos(d*x+c)^2*(a-b)^(3/2)*((b+a*cos(d*x+c))*
cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*a^2*b^2)/cos(d*x+c)^3/sin(d*x+c)^8/((b+a
*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.08129, size = 945, normalized size = 6.39

$$\left[\frac{105\sqrt{a}b^4\cos(dx+c)^3\log\left(-8a^2\cos(dx+c)^2-8ab\cos(dx+c)-b^2+4\left(2a\cos(dx+c)^2+b\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(\cdots}{\cos(\cdots}}\cdots\right.}{210ab^4d\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*sqrt(a)*b^4*cos(d*x + c)^3*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*co
s(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*
cos(d*x + c) + b)/cos(d*x + c))) - 4*(18*a^2*b^2*cos(d*x + c) - 15*a*b^3 +
4*(12*a^4 - 35*a^2*b^2)*cos(d*x + c)^3 - 2*(12*a^3*b - 35*a*b^3)*cos(d*x +
c)^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(a*b^4*d*cos(d*x + c)^3), 1/
105*(105*sqrt(-a)*b^4*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
 c))*cos(d*x + c)/(2*a*cos(d*x + c) + b))*cos(d*x + c)^3 - 2*(18*a^2*b^2*co
s(d*x + c) - 15*a*b^3 + 4*(12*a^4 - 35*a^2*b^2)*cos(d*x + c)^3 - 2*(12*a^3*
b - 35*a*b^3)*cos(d*x + c)^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(a*b
^4*d*cos(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**5/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(dx+c)^5}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^5/sqrt(b*sec(d*x + c) + a), x)
```

3.327 $\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=79

$$\frac{2(a+b\sec(c+dx))^{3/2}}{3b^2d}-\frac{2a\sqrt{a+b\sec(c+dx)}}{b^2d}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[Out] (2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - (2*a*Sqrt[a + b
*Sec[c + d*x]])/(b^2*d) + (2*(a + b*Sec[c + d*x])^(3/2))/(3*b^2*d)
```

Rubi [A] time = 0.0984735, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3885, 898, 1153, 207}

$$\frac{2(a+b\sec(c+dx))^{3/2}}{3b^2d}-\frac{2a\sqrt{a+b\sec(c+dx)}}{b^2d}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - (2*a*Sqrt[a + b
*Sec[c + d*x]])/(b^2*d) + (2*(a + b*Sec[c + d*x])^(3/2))/(3*b^2*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{x\sqrt{a+x}}\,dx, x, b\sec(c+dx)\right)}{b^2 d}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{-a^2+b^2+2ax^2-x^4}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^2 d}$$

$$= -\frac{2\,\text{Subst}\left(\int \left(a-x^2+\frac{b^2}{-a+x^2}\right)dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^2 d}$$

$$= -\frac{2a\sqrt{a+b\sec(c+dx)}}{b^2 d} + \frac{2(a+b\sec(c+dx))^{3/2}}{3b^2 d} - \frac{2\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+a}\right.}{d}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{2a\sqrt{a+b\sec(c+dx)}}{b^2 d} + \frac{2(a+b\sec(c+dx))^{3/2}}{3b^2 d}$$

Mathematica [B] time = 1.18511, size = 194, normalized size = 2.46

$$\frac{\sec(c+dx)(a\cos(c+dx)+b)\left(\frac{2\sec(c+dx)}{3b}-\frac{4a}{3b^2}\right)}{d\sqrt{a+b\sec(c+dx)}} + \frac{\sin(c+dx)\tan(c+dx)\sqrt{a\cos(c+dx)}\sqrt{a\cos(c+dx)+b}\left(\log\left(\frac{\sqrt{a\,\text{co}}}{\sqrt{a\,\text{c}}}\right.\right.}{ad\left(1-\cos^2(c+dx)\right)\sqrt{a+b\sec(c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((-4*a)/(3*b^2) + (2*Sec[c + d*x])/(3*b)
))/(d*Sqrt[a + b*Sec[c + d*x]]) + (Sqrt[a*Cos[c + d*x]]*Sqrt[b + a*Cos[c +
```

```
d*x]]*(-Log[1 - Sqrt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]] + Log[1 + Sq
rt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]])*Sin[c + d*x]*Tan[c + d*x])/(a
*d*(1 - Cos[c + d*x]^2)*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.443, size = 3003, normalized size = 38.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/12/d/b^2/a/(a-b)^(3/2)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+c
os(d*x+c))^3*(-8*cos(d*x+c)^4*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos
(d*x+c)+1)^2)^(1/2)*a^3-8*cos(d*x+c)^3*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*
x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^3+6*cos(d*x+c)^4*ln(-2/(a-b)^(1/2)*(-1+cos(d
*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(
a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(
d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^4*b-6*cos(d*x+c)^4*ln(-2/
(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos
(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x
+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^3*b^
2+3*cos(d*x+c)^4*ln(-2/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(
d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos
(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-
b)/sin(d*x+c)^2)*a^2*b^3-6*cos(d*x+c)^4*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(
2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/
2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1
)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^4*b+6*cos(d*x+c)^4*ln(-1/(a-b)^(1
/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+
1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos
(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^3*b^2-3*cos(
d*x+c)^4*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*
cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+
2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d
*x+c)^2)*a^2*b^3-3*cos(d*x+c)^3*ln(-2/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*
x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*c
os(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/
2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^4*b+6*cos(d*x+c)^3*ln(-2/(a-b)^(1/2)*(-1+
cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1
/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/
(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^3*b^2-6*cos(d*x+c)^3
```

```
*ln(-2/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+
c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*
cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)
*a^2*b^3+3*cos(d*x+c)^3*ln(-2/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b
+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c
)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)
^(1/2)-b)/sin(d*x+c)^2)*a*b^4+3*cos(d*x+c)^3*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+
c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b
)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x
+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^4*b-6*cos(d*x+c)^3*ln(-1/(a-
b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*
x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c)
)*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^3*b^2+6
*cos(d*x+c)^3*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x
+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*
x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/
sin(d*x+c)^2)*a^2*b^3-3*cos(d*x+c)^3*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*c
os(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-
2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2
)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a*b^4+4*((b+a*cos(d*x+c))*cos(d*x+c)/(
cos(d*x+c)+1)^2)^(3/2)*(a-b)^(3/2)*a*b-8*cos(d*x+c)^3*(a-b)^(3/2)*((b+a*cos
(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2*b-8*cos(d*x+c)^2*(a-b)^(3/2
)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2*b+6*cos(d*x+c)^4
*(a-b)^(3/2)*a^(3/2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+
c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c
)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b^2+6*cos(d*x+c)^3*(a-b)^(3/2)*a^(1/2)*ln(4*
cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a
*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+
2*b)*b^3+12*cos(d*x+c)^2*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+
c)+1)^2)^(3/2)*a*b+4*cos(d*x+c)^3*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/
(cos(d*x+c)+1)^2)^(3/2)*a*b+12*cos(d*x+c)*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos
(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*a*b-3*cos(d*x+c)^4*ln(-2/(a-b)^(1/2)*(-1+co
s(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2
)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(c
os(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^5+3*cos(d*x+c)^4*ln(-1
/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(co
s(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*
x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^5)/
sin(d*x+c)^6/cos(d*x+c)/((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.93079, size = 676, normalized size = 8.56

$$\left[\frac{3\sqrt{a}b^2\cos(dx+c)\log\left(-8a^2\cos(dx+c)^2-8ab\cos(dx+c)-b^2-4\left(2a\cos(dx+c)^2+b\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c}{\cos(dx+c}}}{6ab^2d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(a)*b^2*cos(d*x + c)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x
+ c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*
x + c) + b)/cos(d*x + c))) - 4*(2*a^2*cos(d*x + c) - a*b)*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c)))/(a*b^2*d*cos(d*x + c)), -1/3*(3*sqrt(-a)*b^2*arctan
(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d
*x + c) + b))*cos(d*x + c) + 2*(2*a^2*cos(d*x + c) - a*b)*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c)))/(a*b^2*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**3/sqrt(a + b*sec(c + d*x)), x)
```

Giac [B] time = 2.26838, size = 350, normalized size = 4.43

$$-\frac{2\left(\frac{3\arctan\left(-\frac{\sqrt{a+b}-\sqrt{a-b-\frac{2a}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}+\frac{a}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}+\frac{b}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}}+\frac{\sqrt{a+b}}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}{2\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\left(3\left(\sqrt{a-b-\frac{2a}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}+\frac{a}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}+\frac{b}{\tan\left(\frac{1}{2}dx+\cdots\right.}\cdots\right.\right.}{\left(\sqrt{a+b}+\sqrt{a-b-\frac{2a}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}+\frac{a}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}+\cdots}\right.}\right.}{3\,d\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*arctan(-1/2*(sqrt(a + b) - sqrt(a - b - 2*a/tan(1/2*d*x + 1/2*c)^2
+ a/tan(1/2*d*x + 1/2*c)^4 + b/tan(1/2*d*x + 1/2*c)^4) + sqrt(a + b)/tan(1/
2*d*x + 1/2*c)^2)/sqrt(-a))/sqrt(-a) + 2*(3*(sqrt(a - b - 2*a/tan(1/2*d*x +
 1/2*c)^2 + a/tan(1/2*d*x + 1/2*c)^4 + b/tan(1/2*d*x + 1/2*c)^4) - sqrt(a +
 b)/tan(1/2*d*x + 1/2*c)^2)^2 - 3*a + b)/(sqrt(a + b) + sqrt(a - b - 2*a/ta
n(1/2*d*x + 1/2*c)^2 + a/tan(1/2*d*x + 1/2*c)^4 + b/tan(1/2*d*x + 1/2*c)^4)
 - sqrt(a + b)/tan(1/2*d*x + 1/2*c)^2)^3)/(d*sgn(tan(1/2*d*x + 1/2*c)^2 - 1
))
```

3.328 $\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=31

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d)
```

Rubi [A] time = 0.0426453, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3885, 63, 207}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\sec(c+dx)\right)}{d}\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}\end{aligned}$$

Mathematica [B] time = 0.192134, size = 108, normalized size = 3.48

$$\frac{\sqrt{a\cos(c+dx)+b}\left(\log\left(1-\frac{\sqrt{a\cos(c+dx)+b}}{\sqrt{a\cos(c+dx)}}\right)-\log\left(\frac{\sqrt{a\cos(c+dx)+b}}{\sqrt{a\cos(c+dx)}}+1\right)\right)}{d\sqrt{a\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[b + a*Cos[c + d*x]]*(Log[1 - Sqrt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c +
d*x]]] - Log[1 + Sqrt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]]))/(d*Sqrt[a
*Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Maple [A] time = 0.048, size = 26, normalized size = 0.8

$$-2\,\frac{1}{d\sqrt{a}}\text{Artanh}\left(\frac{\sqrt{a+b\sec(dx+c)}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.88349, size = 373, normalized size = 12.03

$$\left[\frac{\log\left(-8\,a^2\cos\left(dx+c\right)^2-8\,ab\cos\left(dx+c\right)-b^2+4\left(2\,a\cos\left(dx+c\right)^2+b\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\right)}{2\sqrt{a}d}, \frac{\sqrt{-a}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x
+ c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(
sqrt(a)*d), sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
c))*cos(d*x + c)/(2*a*cos(d*x + c) + b))/(a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(c+dx\right)}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

Giac [B] time = 1.55222, size = 147, normalized size = 4.74

$$-\frac{2\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a+b}+\sqrt{a-b}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}d\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1
/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b) +
sqrt(a - b))/sqrt(-a))/(sqrt(-a)*d*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))
```

3.329 $\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=106

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}$$

```
[Out] (2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - ArcTanh[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d) - ArcTanh[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Rubi [A] time = 0.134064, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3885, 898, 1170, 206, 207}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - ArcTanh[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d) - ArcTanh[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
```

```
, p] && FractionQ[m]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx &= -\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+x}\left(b^2-x^2\right)}\,dx, x, b\sec(c+dx)\right)}{d} \\
&= -\frac{\left(2b^2\right)\mathrm{Subst}\left(\int \frac{1}{\left(-a+x^2\right)\left(-a^2+b^2+2ax^2-x^4\right)}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} \\
&= -\frac{\left(2b^2\right)\mathrm{Subst}\left(\int \left(-\frac{1}{b^2\left(a-x^2\right)}+\frac{1}{2b^2\left(a+b-x^2\right)}-\frac{1}{2b^2\left(-a+b+x^2\right)}\right)dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} \\
&= -\frac{\mathrm{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}+\frac{\mathrm{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} \\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d}
\end{aligned}
$$

Mathematica [B] time = 6.28259, size = 218, normalized size = 2.06

$$\frac{\sqrt{a\cos(c+dx)+b}\left(\sqrt{a}\left(\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{a\cos(c+dx)+b}}{\sqrt{a-b}\sqrt{-a\cos(c+dx)}}\right)+\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{a\cos(c+dx)+b}}{\sqrt{a+b}\sqrt{-a\cos(c+dx)}}\right)\right)-2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{}}\right.\right.}{d\sqrt{a-b}\sqrt{a+b}\sqrt{-a\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((-2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[Sqrt[b + a*Cos[c + d*x]]/Sqrt[-(a*Cos[c
 + d*x])]] + Sqrt[a]*(Sqrt[a + b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c + d*x]])
/(Sqrt[a - b]*Sqrt[-(a*Cos[c + d*x])])] + Sqrt[a - b]*ArcTan[(Sqrt[a]*Sqrt[
b + a*Cos[c + d*x]])/(Sqrt[a + b]*Sqrt[-(a*Cos[c + d*x])])]))*Sqrt[b + a*Co
s[c + d*x]])/(Sqrt[a - b]*Sqrt[a + b]*d*Sqrt[-(a*Cos[c + d*x])]*Sqrt[a + b*
Sec[c + d*x]])
```

Maple [B] time = 0.31, size = 691, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/4/d/(a-b)^(3/2)/(a+b)/a*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*4^(1/2)*cos(
d*x+c)*(2*(a-b)^(3/2)*a^(3/2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/
(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*
cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)+2*(a-b)^(3/2)*a^(1/2)*ln(4*cos(d*x+
c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x
+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b-(
a-b)^(3/2)*(a+b)^(1/2)*ln(-2*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)
^2)^(1/2)*(a+b)^(1/2)*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)
/(cos(d*x+c)+1)^2)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+b)/(-1+cos(d*x+c)))*a+
ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c
)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*c
os(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*
a^3-ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d
*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b
+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)
^2)*a*b^2)*(-1+cos(d*x+c))/sin(d*x+c)^2/((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d
```

```
*x+c)+1)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(dx+c)}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [B] time = 11.2856, size = 5951, normalized size = 56.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2 - b^2)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c)
- b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c
) + b)/cos(d*x + c))) + (a^2 + a*b)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)
*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(
a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x
+ c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + (a^2 - a*b)*sqrt(a + b)*log(
-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2
+ b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(
4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^3
- a*b^2)*d), -1/4*(4*(a^2 - b^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
 + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - (a^2 + a*b)
*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b
)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/co
s(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x +
 c) + 1)) - (a^2 - a*b)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c
)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt(
(a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d
*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^3 - a*b^2)*d), -1/4*(2*(a^2 + a*b)*sq
rt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*c
```

```
os(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - 2*(a^2 - b^2)*sqrt(a)*log(-8*a^
2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos
(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) - (a^2 - a*b)*s
qrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b)*
cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(
d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c
) + 1)))/((a^3 - a*b^2)*d), -1/4*(4*(a^2 - b^2)*sqrt(-a)*arctan(2*sqrt(-a)*
sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)
) + 2*(a^2 + a*b)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c)
+ b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - (a^2 - a*b)
*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b
)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/co
s(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x +
 c) + 1)))/((a^3 - a*b^2)*d), 1/4*(2*(a^2 - a*b)*sqrt(-a - b)*arctan(2*sqrt
(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*co
s(d*x + c) + b)) + 2*(a^2 - b^2)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*
cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((
a*cos(d*x + c) + b)/cos(d*x + c))) + (a^2 + a*b)*sqrt(a - b)*log(-((8*a^2 -
 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*
x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*
b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/((a^3 - a*b^2)*d
), -1/4*(4*(a^2 - b^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)
/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - 2*(a^2 - a*b)*sqrt(-a
 - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x
 + c)/((2*a + b)*cos(d*x + c) + b)) - (a^2 + a*b)*sqrt(a - b)*log(-((8*a^2
- 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d
*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3
*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/((a^3 - a*b^2)*
d), -1/2*((a^2 + a*b)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - (a^2 -
a*b)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
 c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - (a^2 - b^2)*sqrt(a)*log(-
8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b
*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))))/((a^3 - a*
b^2)*d), -1/2*(2*(a^2 - b^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c
) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) + (a^2 + a*b)*sqr
t(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*co
s(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - (a^2 - a*b)*sqrt(-a - b)*arctan(
2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a +
 b)*cos(d*x + c) + b)))/((a^3 - a*b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(dx+c)}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

3.330 $\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=260

$$\frac{\sqrt{a+b\sec(c+dx)}}{4d(a+b)(1-\sec(c+dx))}+\frac{\sqrt{a+b\sec(c+dx)}}{4d(a-b)(\sec(c+dx)+1)}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{3/2}}+\frac{\tanh^{-1}}{}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) + ArcTanh[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d) - (b*ArcTanh[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(3/2)*d) + (b*ArcTanh[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]])/(4*(a + b)^(3/2)*d) + ArcTanh[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]]/(Sqrt[a + b]*d) + Sqrt[a + b*Sec[c + d*x]]/(4*(a + b)*d*(1 -
 Sec[c + d*x])) + Sqrt[a + b*Sec[c + d*x]]/(4*(a - b)*d*(1 + Sec[c + d*x]))
```

Rubi [A] time = 0.263382, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3885, 898, 1238, 206, 199, 207}

$$\frac{\sqrt{a+b\sec(c+dx)}}{4d(a+b)(1-\sec(c+dx))}+\frac{\sqrt{a+b\sec(c+dx)}}{4d(a-b)(\sec(c+dx)+1)}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{3/2}}+\frac{\tanh^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) + ArcTanh[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d) - (b*ArcTanh[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(3/2)*d) + (b*ArcTanh[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]])/(4*(a + b)^(3/2)*d) + ArcTanh[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]]/(Sqrt[a + b]*d) + Sqrt[a + b*Sec[c + d*x]]/(4*(a + b)*d*(1 -
 Sec[c + d*x])) + Sqrt[a + b*Sec[c + d*x]]/(4*(a - b)*d*(1 + Sec[c + d*x]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{b^4\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}\left(b^2-x^2\right)^2}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= \frac{\left(2b^4\right)\text{Subst}\left(\int \frac{1}{\left(-a+x^2\right)\left(-a^2+b^2+2ax^2-x^4\right)^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}$$

$$= \frac{\left(2b^4\right)\text{Subst}\left(\int \left(-\frac{1}{b^4\left(a-x^2\right)} + \frac{1}{4b^3\left(a+b-x^2\right)^2} + \frac{1}{2b^4\left(a+b-x^2\right)} - \frac{1}{4b^3\left(-a+b+x^2\right)^2} - \frac{1}{2b^4\left(-a+b+x^2\right)}\right)dx, x, \sqrt{a}\right.}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} - \frac{\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} - $$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d} + \frac{\sqrt{a+b\,\text{s}}}{4(a+b)d(1}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{3/2}d} + \frac{b\tanh^{-1}\left(\right.}{4(a}$$

Mathematica [B] time = 6.90347, size = 1022, normalized size = 3.93

$$\frac{(b+a\cos(c+dx))\left(\frac{(a-b\cos(c+dx))\csc^2(c+dx)}{2\left(b^2-a^2\right)} + \frac{a}{2\left(a^2-b^2\right)}\right)\sec(c+dx)}{d\sqrt{a+b\sec(c+dx)}} - \frac{\sqrt{b+a\cos(c+dx)}\left(-\frac{b\left(-\sqrt{-a^2}\sqrt{a+b}\log\left(\sqrt{b+a\cos(c+dx)}-\sqrt{b-}\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] -(Sqrt[b + a*Cos[c + d*x]]*(-(a^2*b*(-(Sqrt[-a^2]*Sqrt[a + b]*Log[-Sqrt[-a
+ b] + Sqrt[b + a*Cos[c + d*x]]]) + Sqrt[-a^2]*Sqrt[a + b]*Log[Sqrt[-a + b]
 + Sqrt[b + a*Cos[c + d*x]]] - a*Sqrt[-a + b]*Log[-Sqrt[a + b] + Sqrt[b + a
*Cos[c + d*x]]] + a*Sqrt[-a + b]*Log[Sqrt[a + b] + Sqrt[b + a*Cos[c + d*x]]
] + Sqrt[-a^2]*Sqrt[a + b]*Log[b + Sqrt[a]*Sqrt[-(a*Cos[c + d*x])] - Sqrt[-
a + b]*Sqrt[b + a*Cos[c + d*x]]] - Sqrt[-a^2]*Sqrt[a + b]*Log[b + Sqrt[a]*S
qrt[-(a*Cos[c + d*x])] + Sqrt[-a + b]*Sqrt[b + a*Cos[c + d*x]]] + a*Sqrt[-a
 + b]*Log[b + Sqrt[-a]*Sqrt[-(a*Cos[c + d*x])] - Sqrt[a + b]*Sqrt[b + a*Cos
[c + d*x]]] - a*Sqrt[-a + b]*Log[b + Sqrt[-a]*Sqrt[-(a*Cos[c + d*x])] + Sqr
t[a + b]*Sqrt[b + a*Cos[c + d*x]]]))/(2*(-a)^(3/2)*Sqrt[-a + b]*Sqrt[a + b]
```

```
*Sqrt[-(a*Cos[c + d*x])]*Sqrt[Sec[c + d*x]]) - ((2*a^2 - 3*b^2)*(Sqrt[a - b
]*(a + b)*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a - b]*Sqrt[-(a*C
os[c + d*x])])] + (a - b)*Sqrt[a + b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c + d*
x]])/(Sqrt[a + b]*Sqrt[-(a*Cos[c + d*x])])])*Sqrt[-(a*Cos[c + d*x])]*Sqrt[S
ec[c + d*x]])/(Sqrt[a]*(a - b)*(a + b)) - (a*(2*a^2 - 2*b^2)*(4*Sqrt[a - b]
*Sqrt[a + b]*ArcTan[Sqrt[b + a*Cos[c + d*x]]/Sqrt[-(a*Cos[c + d*x])]] - Sqr
t[a]*(Sqrt[a + b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a - b]*Sq
rt[-(a*Cos[c + d*x])])] + Sqrt[a - b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c + d*
x]])/(Sqrt[a + b]*Sqrt[-(a*Cos[c + d*x])])]))*Sqrt[-(a*Cos[c + d*x])]*Cos[2
*(c + d*x)]*Sqrt[Sec[c + d*x]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - 2*b^2 + 4*b
*(b + a*Cos[c + d*x]) - 2*(b + a*Cos[c + d*x])^2)))*Sqrt[Sec[c + d*x]])/(4*
(a - b)*(a + b)*d*Sqrt[a + b*Sec[c + d*x]]) + ((b + a*Cos[c + d*x])*(a/(2*(
a^2 - b^2)) + ((a - b*Cos[c + d*x])*Csc[c + d*x]^2)/(2*(-a^2 + b^2)))*Sec[c
 + d*x])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.409, size = 4203, normalized size = 16.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/16/d/a/(a-b)^(5/2)/(a+b)^(5/2)*(-1+cos(d*x+c))*(4*4^(1/2)*ln(-2*(2*((b+a
*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a+b)^(1/2)*cos(d*x+c)+2*(a
+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*a*cos(d*x+
c)+b*cos(d*x+c)+b)/(-1+cos(d*x+c)))*(a-b)^(3/2)*a^4-16*((b+a*cos(d*x+c))*co
s(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(3/2)*(a+b)^(1/2)*a^2-4*4^(1/2)*ln(-
1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(c
os(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d
*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*(a+b
)^(1/2)*a^5+8*cos(d*x+c)^2*4^(1/2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*
x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x
+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*(a-b)^(3/2)*a^(7/2)*(a+b)^(1/2
)-4*cos(d*x+c)*4^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)
*(a-b)^(3/2)*(a+b)^(1/2)*a^3-8*cos(d*x+c)^2*4^(1/2)*ln(4*cos(d*x+c)*((b+a*c
os(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1
/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*(a-b)^(3/2)*a
^(1/2)*(a+b)^(1/2)*b^3-12*cos(d*x+c)^2*4^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)
/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(3/2)*(a+b)^(1/2)*a^2*b+8*cos(d*x+c)*4^(1/2)
*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(3/2)*(a+b)^(1/
2)*a^2*b-4*cos(d*x+c)*4^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2
```

```
)^(1/2)*(a-b)^(3/2)*(a+b)^(1/2)*a*b^2+8*cos(d*x+c)^2*4^(1/2)*ln(4*cos(d*x+c
)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+
c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*(a-b
)^(3/2)*a^(5/2)*(a+b)^(1/2)*b-8*cos(d*x+c)^2*4^(1/2)*ln(4*cos(d*x+c)*((b+a*
cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(
1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*(a-b)^(3/2)*
a^(3/2)*(a+b)^(1/2)*b^2+4*cos(d*x+c)^2*4^(1/2)*ln(-1/(a-b)^(1/2)*(-1+cos(d*
x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a
-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d
*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*(a+b)^(1/2)*a^5-32*cos(d*x+c
)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(3/2)*(a+b)^(1
/2)*a^2+5*4^(1/2)*ln(-2*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(
1/2)*(a+b)^(1/2)*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos
(d*x+c)+1)^2)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+b)/(-1+cos(d*x+c)))*(a-b)^(
3/2)*a^3*b-4*4^(1/2)*ln(-2*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2
)^(1/2)*(a+b)^(1/2)*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(
cos(d*x+c)+1)^2)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+b)/(-1+cos(d*x+c)))*(a-b
)^(3/2)*a^2*b^2-5*4^(1/2)*ln(-2*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)
+1)^2)^(1/2)*(a+b)^(1/2)*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x
+c)/(cos(d*x+c)+1)^2)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+b)/(-1+cos(d*x+c)))
*(a-b)^(3/2)*a*b^3+16*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*
(a-b)^(3/2)*(a+b)^(1/2)*a*b+4^(1/2)*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*co
s(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2
*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)
^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*(a+b)^(1/2)*a^4*b+9*4^(1/2)*ln(-1/(a-b)
^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+
c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*
cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*(a+b)^(1/2)
*a^3*b^2-4^(1/2)*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(
d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos
(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-
b)/sin(d*x+c)^2)*(a+b)^(1/2)*a^2*b^3-5*4^(1/2)*ln(-1/(a-b)^(1/2)*(-1+cos(d*
x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a
-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d
*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*(a+b)^(1/2)*a*b^4+4*4^(1/2)*
((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(3/2)*(a+b)^(1/2
)*a^2*b+4*4^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b
)^(3/2)*(a+b)^(1/2)*a*b^2+8*4^(1/2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d
*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*
x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*(a-b)^(3/2)*a^(1/2)*(a+b)^(1/
2)*b^3-8*4^(1/2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1
)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(c
os(d*x+c)+1)^2)^(1/2)+2*b)*(a-b)^(3/2)*a^(5/2)*(a+b)^(1/2)*b-5*cos(d*x+c)^2
*4^(1/2)*ln(-2*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a+b
)^(1/2)*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1
```

```
)^2)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+b)/(-1+cos(d*x+c)))*(a-b)^(3/2)*a^3*
b+4*cos(d*x+c)^2*4^(1/2)*ln(-2*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+
1)^2)^(1/2)*(a+b)^(1/2)*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+
c)/(cos(d*x+c)+1)^2)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+b)/(-1+cos(d*x+c)))*
(a-b)^(3/2)*a^2*b^2+5*cos(d*x+c)^2*4^(1/2)*ln(-2*(2*((b+a*cos(d*x+c))*cos(d
*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a+b)^(1/2)*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*co
s(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+b)
/(-1+cos(d*x+c)))*(a-b)^(3/2)*a*b^3+16*cos(d*x+c)^2*((b+a*cos(d*x+c))*cos(d
*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(3/2)*(a+b)^(1/2)*a*b+8*4^(1/2)*ln(4*co
s(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*c
os(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*
b)*(a-b)^(3/2)*a^(3/2)*(a+b)^(1/2)*b^2-cos(d*x+c)^2*4^(1/2)*ln(-1/(a-b)^(1/
2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1
)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(
d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*(a+b)^(1/2)*a^4
*b-9*cos(d*x+c)^2*4^(1/2)*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*(
(b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x
+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-
b)^(1/2)-b)/sin(d*x+c)^2)*(a+b)^(1/2)*a^3*b^2+cos(d*x+c)^2*4^(1/2)*ln(-1/(a
-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d
*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c
))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*(a+b)^(1
/2)*a^2*b^3+5*cos(d*x+c)^2*4^(1/2)*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos
(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*
a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^
(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*(a+b)^(1/2)*a*b^4+32*cos(d*x+c)*((b+a*co
s(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(3/2)*(a+b)^(1/2)*a*b+4*
cos(d*x+c)^2*4^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(
a-b)^(3/2)*(a+b)^(1/2)*a^3-8*4^(1/2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(
d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d
*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*(a-b)^(3/2)*a^(7/2)*(a+b)^(1
/2)-4*cos(d*x+c)^2*4^(1/2)*ln(-2*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c
)+1)^2)^(1/2)*(a+b)^(1/2)*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*
x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+b)/(-1+cos(d*x+c))
)*(a-b)^(3/2)*a^4-16*cos(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+
1)^2)^(3/2)*(a-b)^(3/2)*(a+b)^(1/2)*a^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2
)*cos(d*x+c)/((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)/sin(d*x+c
)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(dx+c)^3}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [B] time = 145.021, size = 10157, normalized size = 39.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*
sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d
*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))
) + (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*a^3*b - 6*a^2*b^2 -
 5*a*b^3)*cos(d*x + c)^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x +
 c)^2 + b^2 + 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqr
t((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos
(d*x + c)^2 + 2*cos(d*x + c) + 1)) + (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3
 - (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(a + b)*log(
-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2
+ b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(
4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)) - 8*((a
^4 - a^2*b^2)*cos(d*x + c)^2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*
x + c) + b)/cos(d*x + c)))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a
^5 - 2*a^3*b^2 + a*b^4)*d), -1/16*(16*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2
*b^2 + b^4)*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
 + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) + (4*a^4 + 3*a^3*b
 - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3)*cos(d*x +
c)^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*
a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) +
b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)) + (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3 - (4*a^4 - 3*a^3*b
- 6*a^2*b^2 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b
^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sq
rt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d
*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)) - 8*((a^4 - a^2*b^2)*cos(d*
x + c)^2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x
+ c)))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b
```

^4)*d), -1/16*(2*(4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3)*cos(d*x + c)^2)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) + 8*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) + (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3 - (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)) - 8*((a^4 - a^2*b^2)*cos(d*x + c)^2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d), -1/16*(16*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) + 2*(4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3)*cos(d*x + c)^2)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) + (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3 - (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)) - 8*((a^4 - a^2*b^2)*cos(d*x + c)^2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d), 1/16*(2*(4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3 - (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - 8*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) - (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3)*cos(d*x + c)^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 8*((a^4 - a^2*b^2)*cos(d*x + c)^2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d), -1/16*(16*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - 2*(4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3 - (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) + (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3)*cos(d*x + c)^2)*sqrt(a - b)*log(-((8*a^2 -

```
8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x
 + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b
^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 8*((a^4 - a^2*b^
2)*cos(d*x + c)^2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)
/cos(d*x + c)))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*
b^2 + a*b^4)*d), -1/8*((4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*
a^3*b - 6*a^2*b^2 - 5*a*b^3)*cos(d*x + c)^2)*sqrt(-a + b)*arctan(-2*sqrt(-a
 + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d
*x + c) + b)) - (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3 - (4*a^4 - 3*a^3*b -
 6*a^2*b^2 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sq
rt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c)
+ b)) + 4*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*
sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d
*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))
) - 4*((a^4 - a^2*b^2)*cos(d*x + c)^2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt(
(a*cos(d*x + c) + b)/cos(d*x + c)))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x +
c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d), -1/8*(8*(a^4 - 2*a^2*b^2 + b^4 - (a^4
- 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d
*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) + (4*a^4 +
3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3)*cos
(d*x + c)^2)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/
cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - (4*a^4 - 3*a^3*b
 - 6*a^2*b^2 + 5*a*b^3 - (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3)*cos(d*x +
c)^2)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x
+ c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - 4*((a^4 - a^2*b^2)*cos(d
*x + c)^2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x
 + c)))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*
b^4)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**3/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(dx+c)^3}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

3.331 $\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=404

$$\frac{2\sqrt{a+b}\left(-8a^2+2ab+21b^2\right)\cot(c+dx)\sqrt{-\frac{b(\sec(c+dx)-1)}{a+b}}\sqrt{\frac{b(\sec(c+dx)+1)}{b-a}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{15b^3d}-\frac{2(a-b)}{}$$

```
[Out] (-2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d) - (2*(a - b)*Sqrt[a + b]*(8
*a^2 - 21*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[-((b*(-1 + Sec[c + d*x]))/(a + b))]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^4*d) + (2*Sqrt[a + b]*(-8*a^2 + 2*a*
b + 21*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
 b]], (a + b)/(a - b)]*Sqrt[-((b*(-1 + Sec[c + d*x]))/(a + b))]*Sqrt[(b*(1
+ Sec[c + d*x]))/(-a + b)])/(15*b^3*d) - (8*a*Sqrt[a + b*Sec[c + d*x]]*Tan[
c + d*x])/(15*b^2*d) + (2*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x
])/(5*b*d)
```

Rubi [A] time = 0.763477, antiderivative size = 610, normalized size of antiderivative = 1.51, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3895, 3784, 3837, 3832, 4004, 3860, 4082, 4005}

$$-\frac{2\sqrt{a+b}\left(8a^2-2ab+9b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^3d}-\frac{2(a-b)\sqrt{a+b}\left(8a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (4*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*(a - b)*Sqrt[a + b]*(8*a^
2 + 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + S
ec[c + d*x]))/(a - b))])/(15*b^4*d) + (4*Sqrt[a + b]*Cot[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) -
(2*Sqrt[a + b]*(8*a^2 - 2*a*b + 9*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
```

```
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^3*d) - (2*Sqrt[a
 + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b
*(1 + Sec[c + d*x]))/(a - b))])/(a*d) - (8*a*Sqrt[a + b*Sec[c + d*x]]*Tan[c
 + d*x])/(15*b^2*d) + (2*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]
)/(5*b*d)
```

Rule 3895

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Csc[c + d
*x]^2)^(m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && I
GtQ[m/2, 0] && IntegerQ[n - 1/2]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3837

```
Int[csc[(e_.) + (f_.)*(x_)]^2/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x
_Symbol] :> -Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x] + Int[(Csc[e + f
*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x] /; FreeQ[{a, b, e, f},
 x] && NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
```

```
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 3860

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*S
qrt[a + b*Csc[e + f*x]])/(b*f*(2*n - 3)), x] + Dist[d^3/(b*(2*n - 3)), Int[
((d*Csc[e + f*x])^(n - 3)*Simp[2*a*(n - 3) + b*(2*n - 5)*Csc[e + f*x] - 2*a
*(n - 2)*Csc[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \left(\frac{1}{\sqrt{a+b\sec(c+dx)}} - \frac{2\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}} + \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\right)dx$$

$$= -\left(2\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx\right) + \int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx + \int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= -\frac{2\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad} + \frac{2}{}$$

$$= \frac{4(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^2d} + \frac{4}{}$$

$$= \frac{4(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^2d} + \frac{4}{}$$

$$= \frac{4(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^2d} - \frac{2}{}$$

Mathematica [B] time = 16.8896, size = 839, normalized size = 2.08

$$\frac{(b+a\cos(c+dx))\sec(c+dx)\left(-\frac{2(21b^2-8a^2)\sin(c+dx)}{15b^3} + \frac{2\sec(c+dx)\tan(c+dx)}{5b} - \frac{8a\tan(c+dx)}{15b^2}\right)}{d\sqrt{a+b\sec(c+dx)}} - \frac{2\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^4/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(1 - Tan[(c + d*x)/2]^
2)^(-1)]*(8*a^3*Tan[(c + d*x)/2] + 8*a^2*b*Tan[(c + d*x)/2] - 21*a*b^2*Tan[
(c + d*x)/2] - 21*b^3*Tan[(c + d*x)/2] - 16*a^3*Tan[(c + d*x)/2]^3 + 42*a*b
^2*Tan[(c + d*x)/2]^3 + 8*a^3*Tan[(c + d*x)/2]^5 - 8*a^2*b*Tan[(c + d*x)/2]
^5 - 21*a*b^2*Tan[(c + d*x)/2]^5 + 21*b^3*Tan[(c + d*x)/2]^5 + 30*b^3*Ellip
ticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
 + 30*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c
 + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^
2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (8*a^3 + 8*a^2*b - 21*a*b^2 - 21*b^3)*
```

```
EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)
/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(a + b)] - 2*b*(4*a^2 + a*b - 18*b^2)*EllipticF[ArcSin[Tan[
(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c +
d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
b)]))/(15*b^3*d*Sqrt[a + b*Sec[c + d*x]]*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqr
t[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/
2]^2)]) + ((b + a*Cos[c + d*x])*Sec[c + d*x]*((-2*(-8*a^2 + 21*b^2)*Sin[c +
 d*x])/(15*b^3) - (8*a*Tan[c + d*x])/(15*b^2) + (2*Sec[c + d*x]*Tan[c + d*x
])/(5*b)))/(d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.566, size = 1780, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/15/d/b^3*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x
+c))^2*(4*a^2*b*cos(d*x+c)^2+8*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-21*cos(d*x+c)^3*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-a*b^2
*cos(d*x+c)+3*b^3+8*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-21*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-8*cos(d*x+c)
^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x
+c)*a^2*b-2*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*sin(d*x+c)*a*b^2+8*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-21*cos(d*x+c)^2*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^
2-8*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*sin(d*x+c)*a^2*b-2*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
```

```
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-8*cos(d*x+c)^3*a^2*b+4*cos(d*
x+c)^4*a^2*b+21*cos(d*x+c)^4*a*b^2-20*cos(d*x+c)^3*a*b^2+8*a^3*cos(d*x+c)^3
+21*cos(d*x+c)^3*b^3-8*cos(d*x+c)^4*a^3-24*cos(d*x+c)^2*b^3-30*cos(d*x+c)^3
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d
*x+c)*b^3+36*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*sin(d*x+c)*b^3+8*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-21*cos(d*x+c)^2*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-30*
cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^
(1/2))*sin(d*x+c)*b^3+36*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3)/(b+a*cos(d*x+c))/cos(d*x+c)^2/s
in(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(dx+c)^4}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tan(dx+c)^4}{\sqrt{b\sec(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(tan(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**4/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(dx+c)^4}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

3.332 $\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=310

$$-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(b*d) + (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a +
 b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d)
```

Rubi [A] time = 0.24768, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3894, 4059, 3921, 3784, 3832, 4004}

$$-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec}{a-}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(b*d) + (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a +
 b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d)
```


Rule 3894

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
```

```
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4059

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f
*x]], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{-1+\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \int \frac{-1-\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx + \int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= -\frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^2d} -$$
$$= -\frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^2d} -$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] $Aborted
```

Maple [B] time = 0.348, size = 823, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d/b*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))
^2*(2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)*b-cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a-cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*cos(d*x+c)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x
```

```
+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b+2*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)-(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*sin(d*x+c)
-(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x
+c)-2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+
b))^(1/2))*b+a*cos(d*x+c)^2-a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^5/(b+a*
cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan{(dx+c)}^2}{\sqrt{b\sec{(dx+c)}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tan{(dx+c)}^2}{\sqrt{b\sec{(dx+c)}+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(tan(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2{(c+dx)}}{\sqrt{a+b\sec{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

3.333 $\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=106

$$-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}$$

```
[Out] (-2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d)
```

Rubi [A] time = 0.0208312, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3784}

$$-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d)
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx=-\frac{2\sqrt{a+b}\cot(c+dx)\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad}$$

Mathematica [A] time = 0.220036, size = 140, normalized size = 1.32

$$-\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sec(c+dx)\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\left(\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{a-b}{a+b}\right)+2\Pi\left(-1;-\text{s}\right.\right.}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*(EllipticF[ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)] + 2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/
(a + b)])*Sec[c + d*x])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [A] time = 0.269, size = 178, normalized size = 1.7

$$-2\,\frac{\left(\cos\left(dx+c\right)+1\right)^2\left(-1+\cos\left(dx+c\right)\right)}{d\left(b+a\cos\left(dx+c\right)\right)\left(\sin\left(dx+c\right)\right)^2}\sqrt{\frac{b+a\cos\left(dx+c\right)}{\cos\left(dx+c\right)}}\sqrt{\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\sqrt{\frac{b+a\cos\left(dx+c\right)}{\left(a+b\right)\left(\cos\left(dx+c\right)+1\right)}}\left(\text{EllipticF}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1)^2*(EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-2*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2)))*(-1+cos(d*x+c))/(b+a*cos(d*x+c))/sin
(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sec(d*x + c) + a), x)
```

3.334 $\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=361

$$-\frac{\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d\sqrt{a+b}}+\frac{b^2\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{}{d\cdots}$$

```
[Out] (Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x
]))/(a - b))])/(Sqrt[a + b]*d) - (Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(Sqrt[a + b]*d) + (2*Sqrt[a
 + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b
*(1 + Sec[c + d*x]))/(a - b))])/(a*d) - Cot[c + d*x]/(d*Sqrt[a + b*Sec[c +
d*x]]) + (b^2*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.420432, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3896, 3784, 3875, 3833, 21, 3829, 3832, 4004}

$$\frac{b^2\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\cot(c+dx)}{d\sqrt{a+b\sec(c+dx)}}-\frac{\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\cdots\right.}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] (Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x
]))/(a - b))])/(Sqrt[a + b]*d) - (Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(Sqrt[a + b]*d) + (2*Sqrt[a
 + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b
*(1 + Sec[c + d*x]))/(a - b))])/(a*d) - Cot[c + d*x]/(d*Sqrt[a + b*Sec[c +
d*x]]) + (b^2*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 3896

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-(m/2)), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &
& ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3875

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(Tan[e + f*x]*(a + b*Csc[e + f*x])^m)/f, x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3833

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3829

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[a - b, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[b, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x],
x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \left(-\frac{1}{\sqrt{a+b\sec(c+dx)}}+\frac{\csc^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\right)dx$$

$$= -\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx + \int \frac{\csc^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad} - \frac{}{d\sqrt{}}$$

$$= \frac{2\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad} - \frac{}{d\sqrt{}}$$

$$= \frac{2\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad} - \frac{}{d\sqrt{}}$$

$$= \frac{2\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad} - \frac{}{d\sqrt{}}$$

$$= \frac{\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{\sqrt{a+b}d} - \frac{\cot(c+dx)F\left(\sin\right.}{}$$

Mathematica [C] time = 18.735, size = 1198, normalized size = 3.32

$$\frac{(b+a\cos(c+dx))\sec(c+dx)\left(\frac{(a\cos(c+dx)-b)\csc(c+dx)}{b^2-a^2}+\frac{b\sin(c+dx)}{b^2-a^2}\right)}{d\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-b^2\sqrt{\frac{b-a}{a+b}}\tan^5\left(\frac{1}{2}(c\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*(((-b + a*Cos[c + d*x])*Csc[c + d*x])/(-
a^2 + b^2) + (b*Sin[c + d*x])/(-a^2 + b^2)))/(d*Sqrt[a + b*Sec[c + d*x]]) -
 (Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(a*b*Sqrt[(-a + b)/(a + b)]*T
an[(c + d*x)/2] + b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 2*a*b*Sqrt[
(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c +
d*x)/2]^5 - b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + (4*I)*a^2*Ellip
ticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]
], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d
*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (4*I)*b^2*EllipticPi[-((a + b)/
(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a -
b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan
[(c + d*x)/2]^2)/(a + b)] + (4*I)*a^2*EllipticPi[-((a + b)/(a - b)), I*ArcS
inh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x
)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*
Tan[(c + d*x)/2]^2)/(a + b)] - (4*I)*b^2*EllipticPi[-((a + b)/(a - b)), I*A
rcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c +
d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
 b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b)*b*EllipticE[I*ArcSinh[Sqrt[(-a
+ b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(a + b)] - I*(2*a^2 - a*b - b^2)*EllipticF[I*ArcSinh[Sqrt[(-a
+ b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(a + b)]))/(Sqrt[(-a + b)/(a + b)]*(a^2 - b^2)*d*Sqrt[a + b*Se
c[c + d*x]]*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*Sqrt[(a
 + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2
)]*(-1 + Tan[(c + d*x)/2]^4))
```

Maple [B] time = 0.335, size = 1408, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x)

[Out] -1/d/(a-b)/(a+b)*(-1+cos(d*x+c))^2*(sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)*cos(d*x+c)-4*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x
+c)*cos(d*x+c)+4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+
b))^(1/2))*b^2*sin(d*x+c)*cos(d*x+c)+2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)*cos(d*x+c)-sin(d*x+c)*cos(d*x+c
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-3*si
n(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*b^2+EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)*a*b+EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)-4*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(
a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+4*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b-3*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)
^2*a^2-cos(d*x+c)^2*a*b+a*b*cos(d*x+c)-cos(d*x+c)*b^2)*(cos(d*x+c)+1)^2*((b
+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot (dx + c)^2}{\sqrt{b \sec (dx + c) + a}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2 (c + dx)}{\sqrt{a + b \sec (c + dx)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot (dx + c)^2}{\sqrt{b \sec (dx + c) + a}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

3.335 $\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=148

$$\frac{2\left(3a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}}{b^4d}+\frac{2\left(a^2-b^2\right)^2}{ab^4d\sqrt{a+b\sec(c+dx)}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2(a+b\sec(c+dx))^{5/2}}{5b^4d}-\frac{2a(a-$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (2*(a^2 - b^2)
^2)/(a*b^4*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^2 - 2*b^2)*Sqrt[a + b*Sec[
c + d*x]])/(b^4*d) - (2*a*(a + b*Sec[c + d*x])^(3/2))/(b^4*d) + (2*(a + b*S
ec[c + d*x])^(5/2))/(5*b^4*d)
```

Rubi [A] time = 0.171534, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3885, 898, 1261, 206}

$$\frac{2\left(3a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}}{b^4d}+\frac{2\left(a^2-b^2\right)^2}{ab^4d\sqrt{a+b\sec(c+dx)}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2(a+b\sec(c+dx))^{5/2}}{5b^4d}-\frac{2a(a-$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (2*(a^2 - b^2)
^2)/(a*b^4*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^2 - 2*b^2)*Sqrt[a + b*Sec[
c + d*x]])/(b^4*d) - (2*a*(a + b*Sec[c + d*x])^(3/2))/(b^4*d) + (2*(a + b*S
ec[c + d*x])^(5/2))/(5*b^4*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
```

```
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x(a+x)^{3/2}}\,dx, x, b\sec(c+dx)\right)}{b^4 d}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{\left(-a^2+b^2+2ax^2-x^4\right)^2}{x^2\left(-a+x^2\right)}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^4 d}$$

$$= \frac{2\,\text{Subst}\left(\int \left(3a^2\left(1-\frac{2b^2}{3a^2}\right)-\frac{\left(a^2-b^2\right)^2}{ax^2}-3ax^2+x^4-\frac{b^4}{a\left(a-x^2\right)}\right)dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^4 d}$$

$$= \frac{2\left(a^2-b^2\right)^2}{ab^4 d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}}{b^4 d}-\frac{2a(a+b\sec(c+dx))^{3/2}}{b^4 d}+$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2\left(a^2-b^2\right)^2}{ab^4 d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}}{b^4 d}$$

Mathematica [A] time = 6.37868, size = 263, normalized size = 1.78

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)^2\left(-\frac{2\left(b^2-a^2\right)^2}{a^2b^3(a\cos(c+dx)+b)}+\frac{2\left(-20a^2b^2+16a^4+5b^4\right)}{5a^2b^4}-\frac{6a\sec(c+dx)}{5b^3}+\frac{2\sec^2(c+dx)}{5b^2}\right)}{d(a+b\sec(c+dx))^{3/2}}-\frac{\tan^2(c+dx)\sqrt{a\,\mathrm{c}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*(16*a^4 - 20*a^2*b^2 + 5*b^4))/(
5*a^2*b^4) - (2*(-a^2 + b^2)^2)/(a^2*b^3*(b + a*Cos[c + d*x])) - (6*a*Sec[c
 + d*x])/(5*b^3) + (2*Sec[c + d*x]^2)/(5*b^2)))/(d*(a + b*Sec[c + d*x])^(3/
2)) - (Sqrt[a*Cos[c + d*x]]*(b + a*Cos[c + d*x])^(3/2)*(-Log[1 - Sqrt[b + a
*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]] + Log[1 + Sqrt[b + a*Cos[c + d*x]]/Sqr
t[a*Cos[c + d*x]]])*Tan[c + d*x]^2)/(a^2*d*(1 - Cos[c + d*x]^2)*(a + b*Sec[
c + d*x])^(3/2))
```

Maple [B] time = 0.848, size = 6612, normalized size = 44.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.2865, size = 1098, normalized size = 7.42

$$\left[\frac{5\left(ab^4\cos(dx+c)^3+b^5\cos(dx+c)^2\right)\sqrt{a}\log\left(-8\,a^2\cos(dx+c)^2-8\,ab\cos(dx+c)-b^2+4\left(2\,a\cos(dx+c)^2+b\cos\right.\right.}{10\left(a^3b^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/10*(5*(a*b^4*cos(d*x + c)^3 + b^5*cos(d*x + c)^2)*sqrt(a)*log(-8*a^2*cos
(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x
+ c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) - 4*(2*a^3*b^2*cos(d
*x + c) - a^2*b^3 - (16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^3 - 2*(4*a
^4*b - 5*a^2*b^3)*cos(d*x + c)^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/
(a^3*b^4*d*cos(d*x + c)^3 + a^2*b^5*d*cos(d*x + c)^2), 1/5*(5*(a*b^4*cos(d*
x + c)^3 + b^5*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - 2*(2*a^3*b^2*
cos(d*x + c) - a^2*b^3 - (16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^3 - 2
*(4*a^4*b - 5*a^2*b^3)*cos(d*x + c)^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
c)))/(a^3*b^4*d*cos(d*x + c)^3 + a^2*b^5*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**5/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(dx+c)^5}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^5/(b*sec(d*x + c) + a)^(3/2), x)
```

3.336 $\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=88

$$\frac{2(a^2-b^2)}{ab^2d\sqrt{a+b\sec(c+dx)}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2\sqrt{a+b\sec(c+dx)}}{b^2d}$$

```
[Out] (2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (2*(a^2 - b^2))
/(a*b^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*Sqrt[a + b*Sec[c + d*x]])/(b^2*d)
```

Rubi [A] time = 0.123803, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3885, 898, 1261, 206}

$$\frac{2(a^2-b^2)}{ab^2d\sqrt{a+b\sec(c+dx)}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2\sqrt{a+b\sec(c+dx)}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (2*(a^2 - b^2))
/(a*b^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*Sqrt[a + b*Sec[c + d*x]])/(b^2*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{x(a+x)^{3/2}}\,dx,x,b\sec(c+dx)\right)}{b^2d}\\ &= -\frac{2\,\text{Subst}\left(\int \frac{-a^2+b^2+2ax^2-x^4}{x^2\left(-a+x^2\right)}\,dx,x,\sqrt{a+b\sec(c+dx)}\right)}{b^2d}\\ &= -\frac{2\,\text{Subst}\left(\int \left(-1+\frac{a^2-b^2}{ax^2}-\frac{b^2}{a\left(a-x^2\right)}\right)dx,x,\sqrt{a+b\sec(c+dx)}\right)}{b^2d}\\ &= \frac{2\left(a^2-b^2\right)}{ab^2d\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{a+b\sec(c+dx)}}{b^2d}+\frac{2\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,\sqrt{a+b\sec(c+dx)}\right)}{ad}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2\left(a^2-b^2\right)}{ab^2d\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{a+b\sec(c+dx)}}{b^2d}\end{aligned}$$

Mathematica [A] time = 1.00991, size = 167, normalized size = 1.9

$$\frac{4a^2-\frac{b^2\sqrt{a\cos(c+dx)+b}\log\left(1-\frac{\sqrt{a\cos(c+dx)+b}}{\sqrt{a\cos(c+dx)}}\right)}{\sqrt{a\cos(c+dx)}}+\frac{b^2\sqrt{a\cos(c+dx)+b}\log\left(\frac{\sqrt{a\cos(c+dx)+b}}{\sqrt{a\cos(c+dx)}}+1\right)}{\sqrt{a\cos(c+dx)}}+2ab\sec(c+dx)-2b^2}{ab^2d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (4*a^2 - 2*b^2 - (b^2*Sqrt[b + a*Cos[c + d*x]]*Log[1 - Sqrt[b + a*Cos[c + d
*x]]/Sqrt[a*Cos[c + d*x]]])/Sqrt[a*Cos[c + d*x]] + (b^2*Sqrt[b + a*Cos[c +
d*x]]*Log[1 + Sqrt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]])/Sqrt[a*Cos[c
+ d*x]] + 2*a*b*Sec[c + d*x])/(a*b^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.377, size = 2830, normalized size = 32.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d/b^2/(a-b)^(3/2)/a^2*(-1+cos(d*x+c))^3*(4*((b+a*cos(d*x+c))*cos(d*x+c
)/(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(3/2)*a^3-2*cos(d*x+c)^2*ln(-2/(a-b)^(1/2)*
(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2
)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x
+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^4*b^2+2*cos(d*x+
c)^2*ln(-2/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(
d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((
b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c
)^2)*a^3*b^3-4*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(
3/2)*a*b^2+cos(d*x+c)*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a
*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+
b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(
1/2)-b)/sin(d*x+c)^2)*a^3*b^3-cos(d*x+c)*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*
(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1
/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+
1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^2*b^4-cos(d*x+c)*ln(-2/(a-b)^(1/
2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1
)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(
d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^3*b^3+cos(d*x
+c)*ln(-2/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d
*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b
+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)
^2)*a^2*b^4+12*cos(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^
(3/2)*(a-b)^(3/2)*a^3+cos(d*x+c)^3*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos
(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*
a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^
(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^5*b-cos(d*x+c)^3*ln(-1/(a-b)^(1/2)*(-1
+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(
1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)
```

```
/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^4*b^2-cos(d*x+c)^3*
ln(-2/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c
)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*c
os(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*
a^5*b+cos(d*x+c)^3*ln(-2/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*co
s(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*c
os(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2
)-b)/sin(d*x+c)^2)*a^4*b^2+2*cos(d*x+c)^2*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))
*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(
1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)
+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^4*b^2-2*cos(d*x+c)^2*ln(-1/(a-b
)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x
+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))
*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^3*b^3+4*
(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2*b^2+4*
cos(d*x+c)^3*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(3/
2)*a^4+4*cos(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*
(a-b)^(3/2)*a^4+12*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2
)^(3/2)*(a-b)^(3/2)*a^3+4*cos(d*x+c)^3*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*
x+c)+1)^2)^(3/2)*(a-b)^(3/2)*a^3+8*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/
(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(3/2)*a^3*b+4*cos(d*x+c)*((b+a*cos(d*x+c))*co
s(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(3/2)*a^2*b^2+8*cos(d*x+c)^2*((b+a*c
os(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(3/2)*a^3*b-4*cos(d*x+c
)^3*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(3/2)*a*b^2-
12*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(3
/2)*a*b^2+2*cos(d*x+c)^3*(a-b)^(3/2)*a^(5/2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+
c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b
+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b^2+4*cos(d*x+c)^2*(
a-b)^(3/2)*a^(3/2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)
+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/
(cos(d*x+c)+1)^2)^(1/2)+2*b)*b^3-12*cos(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+
c)/(cos(d*x+c)+1)^2)^(3/2)*(a-b)^(3/2)*a*b^2+2*cos(d*x+c)*(a-b)^(3/2)*a^(1/
2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(
1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2
)^(1/2)+2*b)*b^4)*cos(d*x+c)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*4^(1/2)/(b
+a*cos(d*x+c))/((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)/sin(d*x
+c)^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.91271, size = 763, normalized size = 8.67

$$\left[\frac{\left(ab^2\cos\left(dx+c\right)+b^3\right)\sqrt{a}\log\left(-8\,a^2\cos\left(dx+c\right)^2-8\,ab\cos\left(dx+c\right)-b^2-4\left(2\,a\cos\left(dx+c\right)^2+b\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\cdots}\right.}{2\left(a^3b^2d\cos\left(dx+c\right)+a^2b^3d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*b^2*cos(d*x + c) + b^3)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*
cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((
a*cos(d*x + c) + b)/cos(d*x + c))) + 4*(a^2*b + (2*a^3 - a*b^2)*cos(d*x + c
))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(a^3*b^2*d*cos(d*x + c) + a^2*b
^3*d), -((a*b^2*cos(d*x + c) + b^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(
d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - 2*(a^2*b
 + (2*a^3 - a*b^2)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(
a^3*b^2*d*cos(d*x + c) + a^2*b^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**3/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(dx+c)^3}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^3/(b*sec(d*x + c) + a)^(3/2), x)
```

3.337 $\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=54

$$\frac{2}{ad\sqrt{a+b\sec(c+dx)}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + 2/(a*d*Sqrt[a
+ b*Sec[c + d*x]])
```

Rubi [A] time = 0.0528239, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3885, 51, 63, 207}

$$\frac{2}{ad\sqrt{a+b\sec(c+dx)}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + 2/(a*d*Sqrt[a
+ b*Sec[c + d*x]])
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^{3/2}}\,dx, x, b\sec(c+dx)\right)}{d}\\ &= \frac{2}{ad\sqrt{a+b\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\sec(c+dx)\right)}{ad}\\ &= \frac{2}{ad\sqrt{a+b\sec(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{ad}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2}{ad\sqrt{a+b\sec(c+dx)}}\end{aligned}$$

Mathematica [B] time = 0.387311, size = 128, normalized size = 2.37

$$\frac{\sec(c+dx)\left(\sqrt{a\cos(c+dx)}\sqrt{a\cos(c+dx)+b}\left(\log\left(1-\frac{\sqrt{a\cos(c+dx)+b}}{\sqrt{a\cos(c+dx)}}\right)-\log\left(\frac{\sqrt{a\cos(c+dx)+b}}{\sqrt{a\cos(c+dx)}}+1\right)\right)+2a\cos(c+dx)\right)}{a^2d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((2*a*Cos[c + d*x] + Sqrt[a*Cos[c + d*x]]*Sqrt[b + a*Cos[c + d*x]]*(Log[1 -
 Sqrt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]] - Log[1 + Sqrt[b + a*Cos[c
+ d*x]]/Sqrt[a*Cos[c + d*x]]]))*Sec[c + d*x])/(a^2*d*Sqrt[a + b*Sec[c + d*x
```

```
]])
```

Maple [A] time = 0.039, size = 45, normalized size = 0.8

$$\frac{1}{d}\left(-2\,\frac{1}{a^{3/2}}\mathrm{Artanh}\left(\frac{\sqrt{a+b\sec(dx+c)}}{\sqrt{a}}\right)+2\,\frac{1}{a\sqrt{a+b\sec(dx+c)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d*(-2/a^(3/2)*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))+2/a/(a+b*sec(d*x+c)
)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.29228, size = 664, normalized size = 12.3

$$\left[\frac{4\,a\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\cos(dx+c)+(a\cos(dx+c)+b)\sqrt{a}\log\left(-8\,a^{2}\cos(dx+c)^{2}-8\,ab\cos(dx+c)-b^{2}+4\left(2\,a\cos(dx\right.\right.}{2\left(a^{3}d\cos(dx+c)+a^{2}bd\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(4*a*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c) + (a*cos(d*x
 + c) + b)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4
*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/co
s(d*x + c))))/(a^3*d*cos(d*x + c) + a^2*b*d), ((a*cos(d*x + c) + b)*sqrt(-a
)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2
*a*cos(d*x + c) + b)) + 2*a*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x
 + c))/(a^3*d*cos(d*x + c) + a^2*b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.338 $\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{2b^2}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d(a-b)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}$$

```
[Out] (2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - ArcTanh[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a - b]]/((a - b)^(3/2)*d) - ArcTanh[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]]/((a + b)^(3/2)*d) + (2*b^2)/(a*(a^2 - b^2)*d*Sqrt[a +
 b*Sec[c + d*x]])
```

Rubi [A] time = 0.196904, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3885, 898, 1287, 206}

$$\frac{2b^2}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d(a-b)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - ArcTanh[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a - b]]/((a - b)^(3/2)*d) - ArcTanh[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]]/((a + b)^(3/2)*d) + (2*b^2)/(a*(a^2 - b^2)*d*Sqrt[a +
 b*Sec[c + d*x]])
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
```

```
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{b^2\operatorname{Subst}\left(\int \frac{1}{x(a+x)^{3/2}\left(b^2-x^2\right)}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= -\frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{1}{x^2\left(-a+x^2\right)\left(-a^2+b^2+2ax^2-x^4\right)}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}$$

$$= -\frac{\left(2b^2\right)\operatorname{Subst}\left(\int \left(\frac{1}{a\left(a^2-b^2\right)x^2} - \frac{1}{ab^2\left(a-x^2\right)} + \frac{1}{2(a-b)b^2\left(a-b-x^2\right)} + \frac{1}{2b^2(a+b)\left(a+b-x^2\right)}\right) dx, x, \sqrt{a+b\operatorname{se}}\right.}{d}$$

$$= \frac{2b^2}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{ad} - \frac{\operatorname{Subst}\left(\int \right.}{}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d} + \frac{}{a\left(a^2-b^2\right)}$$

Mathematica [B] time = 6.93043, size = 1020, normalized size = 7.18

$$\frac{(b+a\cos(c+dx))^2\left(-\frac{2b^3}{a^2\left(a^2-b^2\right)(b+a\cos(c+dx))} - \frac{2b^2}{a^2\left(b^2-a^2\right)}\right)\sec^2(c+dx)}{d(a+b\sec(c+dx))^{3/2}} - \frac{(b+a\cos(c+dx))^{3/2}\left(\frac{b\left(-\sqrt{-a^2}\sqrt{a+b}\log\left(\sqrt{b+a\cos(c+dx)}\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] -((b + a*Cos[c + d*x])^(3/2)*((a^2*b*(-(Sqrt[-a^2]*Sqrt[a + b]*Log[-Sqrt[-a
 + b] + Sqrt[b + a*Cos[c + d*x]]]) + Sqrt[-a^2]*Sqrt[a + b]*Log[Sqrt[-a + b
] + Sqrt[b + a*Cos[c + d*x]]] - a*Sqrt[-a + b]*Log[-Sqrt[a + b] + Sqrt[b +
a*Cos[c + d*x]]] + a*Sqrt[-a + b]*Log[Sqrt[a + b] + Sqrt[b + a*Cos[c + d*x]
]] + Sqrt[-a^2]*Sqrt[a + b]*Log[b + Sqrt[a]*Sqrt[-(a*Cos[c + d*x])] - Sqrt[
-a + b]*Sqrt[b + a*Cos[c + d*x]]] - Sqrt[-a^2]*Sqrt[a + b]*Log[b + Sqrt[a]*
Sqrt[-(a*Cos[c + d*x])] + Sqrt[-a + b]*Sqrt[b + a*Cos[c + d*x]]] + a*Sqrt[-
a + b]*Log[b + Sqrt[-a]*Sqrt[-(a*Cos[c + d*x])] - Sqrt[a + b]*Sqrt[b + a*Co
s[c + d*x]]] - a*Sqrt[-a + b]*Log[b + Sqrt[-a]*Sqrt[-(a*Cos[c + d*x])] + Sq
rt[a + b]*Sqrt[b + a*Cos[c + d*x]]]))/((-a)^(3/2)*Sqrt[-a + b]*Sqrt[a + b]*
Sqrt[-(a*Cos[c + d*x])]*Sqrt[Sec[c + d*x]]) - ((a^2 + b^2)*(Sqrt[a - b]*(a
+ b)*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a - b]*Sqrt[-(a*Cos[c
+ d*x])])] + (a - b)*Sqrt[a + b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c + d*x]])/
(Sqrt[a + b]*Sqrt[-(a*Cos[c + d*x])])])*Sqrt[-(a*Cos[c + d*x])]*Sqrt[Sec[c
+ d*x]])/(Sqrt[a]*(a - b)*(a + b)) - (a*(a^2 - b^2)*(4*Sqrt[a - b]*Sqrt[a +
 b]*ArcTan[Sqrt[b + a*Cos[c + d*x]]/Sqrt[-(a*Cos[c + d*x])]] - Sqrt[a]*(Sqr
t[a + b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a - b]*Sqrt[-(a*Co
s[c + d*x])])] + Sqrt[a - b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c + d*x]])/(Sqr
t[a + b]*Sqrt[-(a*Cos[c + d*x])])]))*Sqrt[-(a*Cos[c + d*x])]*Cos[2*(c + d*x
)]*Sqrt[Sec[c + d*x]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - 2*b^2 + 4*b*(b + a*C
os[c + d*x]) - 2*(b + a*Cos[c + d*x])^2)))*Sec[c + d*x]^(3/2))/(2*a*(-a + b
)*(a + b)*d*(a + b*Sec[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])^2*((-2*b^2)
/(a^2*(-a^2 + b^2)) - (2*b^3)/(a^2*(a^2 - b^2)*(b + a*Cos[c + d*x])))*Sec[c
 + d*x]^2)/(d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.263, size = 2766, normalized size = 19.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d/(a-b)^(5/2)/(a+b)^2/a^2*(-1+cos(d*x+c))*(2*ln(4*cos(d*x+c)*((b+a*cos
(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2
)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*cos(d*x+c)*(a-b
)^(3/2)*a^(9/2)+2*cos(d*x+c)*(a-b)^(3/2)*a^(7/2)*ln(4*cos(d*x+c)*((b+a*cos(
d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)
```

```
*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b-2*ln(4*cos(d*x
+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*
x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*co
s(d*x+c)*(a-b)^(3/2)*a^(5/2)*b^2+2*(a-b)^(3/2)*a^(7/2)*ln(4*cos(d*x+c)*((b+
a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a
^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b-2*cos(d*
x+c)*(a-b)^(3/2)*a^(3/2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(
d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d
*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b^3+2*(a-b)^(3/2)*a^(5/2)*ln(4*cos(d*x+c
)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+
c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b^2-
ln(-2*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a+b)^(1/2)*c
os(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2
)+2*a*cos(d*x+c)+b*cos(d*x+c)+b)/(-1+cos(d*x+c)))*cos(d*x+c)*(a-b)^(3/2)*(a
+b)^(1/2)*a^4+ln(-2*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)
*(a+b)^(1/2)*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x
+c)+1)^2)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+b)/(-1+cos(d*x+c)))*cos(d*x+c)*
(a-b)^(3/2)*(a+b)^(1/2)*a^3*b-2*(a-b)^(3/2)*a^(3/2)*ln(4*cos(d*x+c)*((b+a*c
os(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1
/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b^3+4*cos(d*x
+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(3/2)*a^2*b^
2+4*cos(d*x+c)*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(
1/2)*a*b^3-2*(a-b)^(3/2)*a^(1/2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+
c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c
))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b^4-(a-b)^(3/2)*(a+b)^(1/2)*ln(-
2*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a+b)^(1/2)*cos(d
*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*
a*cos(d*x+c)+b*cos(d*x+c)+b)/(-1+cos(d*x+c)))*a^3*b+(a-b)^(3/2)*(a+b)^(1/2)
*ln(-2*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a+b)^(1/2)*
cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/
2)+2*a*cos(d*x+c)+b*cos(d*x+c)+b)/(-1+cos(d*x+c)))*a^2*b^2+ln(-1/(a-b)^(1/2
)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)
^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d
*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*cos(d*x+c)*a^6+c
os(d*x+c)*ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))
*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)
+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(
d*x+c)^2)*a^5*b-ln(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d
*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(
d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b
)/sin(d*x+c)^2)*cos(d*x+c)*a^4*b^2-cos(d*x+c)*ln(-1/(a-b)^(1/2)*(-1+cos(d*x
+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-
b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*
x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^3*b^3+4*(a-b)^(3/2)*((b+a*c
os(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2*b^2+4*(a-b)^(3/2)*((b+a*c
```

```
os(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a*b^3+ln(-1/(a-b)^(1/2)*(-1+c
os(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/
2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(
cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^5*b+ln(-1/(a-b)^(1/2)
*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^
2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*
x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^4*b^2-ln(-1/(a-
b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*
x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c)
)*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a^3*b^3-l
n(-1/(a-b)^(1/2)*(-1+cos(d*x+c))*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)
/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*co
s(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2)*a
^2*b^4)*cos(d*x+c)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*4^(1/2)/((b+a*cos(d*
x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(dx+c)}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 104.274, size = 9288, normalized size = 65.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(8*(a^3*b^2 - a*b^4)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x +
 c) + 2*(a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*
sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(2*a*cos(d
*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))
```

) - (a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4*b + a^3*b^2)*cos(d*x + c))*
sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a - b)
*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos
(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x +
c) + 1)) + (a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x
 + c))*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2
*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) +
 b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos
(d*x + c) + 1)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a
^4*b^3 + a^2*b^5)*d), -1/4*(4*(a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 +
 a*b^4)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/
cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - 8*(a^3*b^2 - a*b^4)*sq
rt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c) + (a^4*b + 2*a^3*b^2 + a
^2*b^3 + (a^5 + 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(a - b)*log(-((8*a^2 -
 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*
x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*
b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - (a^4*b - 2*a^3*
b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(a + b)*log(-((
8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b
*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a
*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^7 - 2
*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), -1/4
(2(a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4*b + a^3*b^2)*cos(d*x + c))*
sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))
*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - 8*(a^3*b^2 - a*b^4)*sqrt((a*c
os(d*x + c) + b)/cos(d*x + c))*cos(d*x + c) - 2*(a^4*b - 2*a^2*b^3 + b^5 +
(a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 -
 8*a*b*cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)
*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) - (a^4*b - 2*a^3*b^2 + a^2*b^3 +
(a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(a + b)*log(-((8*a^2 + 8*a*b +
b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*s
qrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(
d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^7 - 2*a^5*b^2 + a^3*b
^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), -1/4*(4*(a^4*b - 2*a
^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*sqrt(-a)*arctan(2*sq
rt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x +
c) + b)) + 2*(a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4*b + a^3*b^2)*cos(d
*x + c))*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(
d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - 8*(a^3*b^2 - a*b^4)*
sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c) - (a^4*b - 2*a^3*b^2 +
 a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(a + b)*log(-((8*a^2
 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(
d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b +
3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^7 - 2*a^5*
b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), 1/4*(2*(a

```
^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(-
a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*
x + c)/((2*a + b)*cos(d*x + c) + b)) + 8*(a^3*b^2 - a*b^4)*sqrt((a*cos(d*x
+ c) + b)/cos(d*x + c))*cos(d*x + c) + 2*(a^4*b - 2*a^2*b^3 + b^5 + (a^5 -
2*a^3*b^2 + a*b^4)*cos(d*x + c))*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*
cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((
a*cos(d*x + c) + b)/cos(d*x + c))) - (a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 +
2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*co
s(d*x + c)^2 + b^2 + 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a -
 b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c
))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*c
os(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), -1/4*(4*(a^4*b - 2*a^2*b^3
+ b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*
sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)
) - 2*(a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c)
)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)
)*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - 8*(a^3*b^2 - a*b^4)*sqrt((a*
cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c) + (a^4*b + 2*a^3*b^2 + a^2*b^3
 + (a^5 + 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(a - b)*log(-((8*a^2 - 8*a*b
 + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c)
)*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*c
os(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/((a^7 - 2*a^5*b^2 + a^
3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), -1/2*((a^4*b + 2*
a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(-a + b)*ar
ctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(
(2*a - b)*cos(d*x + c) + b)) - (a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*
b + a^3*b^2)*cos(d*x + c))*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d
*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - 4*(
a^3*b^2 - a*b^4)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c) - (a^
4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*sqrt(a)*log
(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 +
 b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))))/((a^7 -
2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), -1/
2*(2*(a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*sqr
t(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c
)/(2*a*cos(d*x + c) + b)) + (a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4*b +
 a^3*b^2)*cos(d*x + c))*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x
 + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - (a^4*
b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(-a -
 b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x +
 c)/((2*a + b)*cos(d*x + c) + b)) - 4*(a^3*b^2 - a*b^4)*sqrt((a*cos(d*x + c
) + b)/cos(d*x + c))*cos(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x +
 c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot (c+dx)}{(a+b\sec (c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot (dx+c)}{(b\sec (dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.339 $\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=236

$$\frac{2b^4}{ad\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{\sqrt{a+b\sec(c+dx)}}{4d(a+b)^2(1-\sec(c+dx))}+\frac{\sqrt{a+b\sec(c+dx)}}{4d(a-b)^2(\sec(c+dx)+1}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + ((4*a - 7*b)*A
rcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(5/2)*d) + ((4*a +
 7*b)*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]])/(4*(a + b)^(5/2)*d) +
(2*b^4)/(a*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]]) + Sqrt[a + b*Sec[c + d
*x]]/(4*(a + b)^2*d*(1 - Sec[c + d*x])) + Sqrt[a + b*Sec[c + d*x]]/(4*(a -
b)^2*d*(1 + Sec[c + d*x]))
```

Rubi [A] time = 0.384817, antiderivative size = 316, normalized size of antiderivative = 1.34, number of steps used = 11, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3885, 898, 1335, 206, 199}

$$\frac{2b^4}{ad\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{\sqrt{a+b\sec(c+dx)}}{4d(a+b)^2(1-\sec(c+dx))}+\frac{\sqrt{a+b\sec(c+dx)}}{4d(a-b)^2(\sec(c+dx)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + ((2*a - 3*b)*A
rcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]])/(2*(a - b)^(5/2)*d) - (b*ArcT
anh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(5/2)*d) + (b*ArcTanh
[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]])/(4*(a + b)^(5/2)*d) + ((2*a + 3*b)*
ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]])/(2*(a + b)^(5/2)*d) + (2*b^4
)/(a*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]]) + Sqrt[a + b*Sec[c + d*x]]/(
4*(a + b)^2*d*(1 - Sec[c + d*x])) + Sqrt[a + b*Sec[c + d*x]]/(4*(a - b)^2*d
*(1 + Sec[c + d*x]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
```

```
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{b^4\,\text{Subst}\left(\int \frac{1}{x(a+x)^{3/2}\left(b^2-x^2\right)^2}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= \frac{\left(2b^4\right)\text{Subst}\left(\int \frac{1}{x^2\left(-a+x^2\right)\left(-a^2+b^2+2ax^2-x^4\right)^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}$$

$$= \frac{\left(2b^4\right)\text{Subst}\left(\int\left(-\frac{1}{a(a-b)^2(a+b)^2x^2} - \frac{1}{ab^4\left(a-x^2\right)} - \frac{1}{4(a-b)b^3\left(a-b-x^2\right)^2} + \frac{2a-3b}{4(a-b)^2b^4\left(a-b-x^2\right)} + \frac{1}{4b^3(a+b)(a}\right.\right.}{d}$$

$$= \frac{2b^4}{a\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} - \frac{2\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{ad} + \frac{(2a-3b}{}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{2(a-b)^{5/2}d} + \frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\,\text{s}}}{\sqrt{}}\right.}{2(a+b)^{5/2}d}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{2(a-b)^{5/2}d} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{5/2}d}$$

Mathematica [B] time = 7.16121, size = 1114, normalized size = 4.72

$$\frac{(b+a\cos(c+dx))^2\left(-\frac{2b^5}{a^2\left(a^2-b^2\right)^2(b+a\cos(c+dx))} + \frac{\left(-a^2+2b\cos(c+dx)a-b^2\right)\csc^2(c+dx)}{2\left(b^2-a^2\right)^2} + \frac{a^4+b^2a^2+4b^4}{2a^2\left(b^2-a^2\right)^2}\right)\sec^2(c+dx)}{d(a+b\sec(c+dx))^{3/2}} - \frac{(b+a\cos(c+}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] -((b + a*Cos[c + d*x])^(3/2)*(-(a*(-(a^3*b) + 7*a*b^3)*(-(Sqrt[-a^2]*Sqrt[a
 + b]*Log[-Sqrt[-a + b] + Sqrt[b + a*Cos[c + d*x]]]) + Sqrt[-a^2]*Sqrt[a +
b]*Log[Sqrt[-a + b] + Sqrt[b + a*Cos[c + d*x]]] - a*Sqrt[-a + b]*Log[-Sqrt[
a + b] + Sqrt[b + a*Cos[c + d*x]]] + a*Sqrt[-a + b]*Log[Sqrt[a + b] + Sqrt[
b + a*Cos[c + d*x]]] + Sqrt[-a^2]*Sqrt[a + b]*Log[b + Sqrt[a]*Sqrt[-(a*Cos[
c + d*x])] - Sqrt[-a + b]*Sqrt[b + a*Cos[c + d*x]]] - Sqrt[-a^2]*Sqrt[a + b
]*Log[b + Sqrt[a]*Sqrt[-(a*Cos[c + d*x])] + Sqrt[-a + b]*Sqrt[b + a*Cos[c +
 d*x]]] + a*Sqrt[-a + b]*Log[b + Sqrt[-a]*Sqrt[-(a*Cos[c + d*x])] - Sqrt[a
```

```
+ b]*Sqrt[b + a*Cos[c + d*x]]] - a*Sqrt[-a + b]*Log[b + Sqrt[-a]*Sqrt[-(a*C
os[c + d*x])] + Sqrt[a + b]*Sqrt[b + a*Cos[c + d*x]]]))/(2*(-a)^(3/2)*Sqrt[
-a + b]*Sqrt[a + b]*Sqrt[-(a*Cos[c + d*x])]*Sqrt[Sec[c + d*x]]) - ((2*a^4 -
 6*a^2*b^2 - 2*b^4)*(Sqrt[a - b]*(a + b)*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c +
 d*x]])/(Sqrt[a - b]*Sqrt[-(a*Cos[c + d*x])])] + (a - b)*Sqrt[a + b]*ArcTan
[(Sqrt[a]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a + b]*Sqrt[-(a*Cos[c + d*x])])])
*Sqrt[-(a*Cos[c + d*x])]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*(a - b)*(a + b)) - (a
*(2*a^4 - 4*a^2*b^2 + 2*b^4)*(4*Sqrt[a - b]*Sqrt[a + b]*ArcTan[Sqrt[b + a*C
os[c + d*x]]/Sqrt[-(a*Cos[c + d*x])]] - Sqrt[a]*(Sqrt[a + b]*ArcTan[(Sqrt[a
]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a - b]*Sqrt[-(a*Cos[c + d*x])])] + Sqrt[a
 - b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a + b]*Sqrt[-(a*Cos[c
 + d*x])])]))*Sqrt[-(a*Cos[c + d*x])]*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/
(Sqrt[a - b]*Sqrt[a + b]*(a^2 - 2*b^2 + 4*b*(b + a*Cos[c + d*x]) - 2*(b + a
*Cos[c + d*x])^2)))*Sec[c + d*x]^(3/2))/(4*a*(a - b)^2*(a + b)^2*d*(a + b*S
ec[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])^2*((a^4 + a^2*b^2 + 4*b^4)/(2*a
^2*(-a^2 + b^2)^2) - (2*b^5)/(a^2*(a^2 - b^2)^2*(b + a*Cos[c + d*x])) + ((-
a^2 - b^2 + 2*a*b*Cos[c + d*x])*Csc[c + d*x]^2)/(2*(-a^2 + b^2)^2))*Sec[c +
 d*x]^2)/(d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.76, size = 10977, normalized size = 46.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)**3/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(dx+c)^3}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^3/(b*sec(d*x + c) + a)^(3/2), x)
```

3.340 $\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=530

$$\frac{2(2a+b)\left(4a^2+ab-3b^2\right)\cot(c+dx)\sqrt{-\frac{b(\sec(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3ab^3d\sqrt{a+b}}-\frac{2a^2\tan(}{bd\left(a^2-b\right.}$$

```
[Out] (-2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (2*(8*a^4 - 11*a^2*b^2
+ 3*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[-((b*(-1 + Sec[c + d*x]))/(a + b))]*Sqrt[-((b*(1 +
 Sec[c + d*x]))/(a - b))])/(3*a*b^4*Sqrt[a + b]*d) + (2*(2*a + b)*(4*a^2 +
a*b - 3*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[-((b*(-1 + Sec[c + d*x]))/(a + b))]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(3*a*b^3*Sqrt[a + b]*d) - (4*a*Tan[c + d*x])
/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b^2*Tan[c + d*x])/(a*(a^2 -
b^2)*d*Sqrt[a + b*Sec[c + d*x]]) - (2*a^2*Sec[c + d*x]*Tan[c + d*x])/(b*(a^
2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(4*a^2 - b^2)*Sqrt[a + b*Sec[c +
d*x]]*Tan[c + d*x])/(3*b^2*(a^2 - b^2)*d)
```

Rubi [A] time = 1.29759, antiderivative size = 907, normalized size of antiderivative = 1.71, number of steps used = 17, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3895, 3785, 4058, 3921, 3784, 3832, 4004, 3836, 4005, 3845, 4082}

$$-\frac{2\sec(c+dx)\tan(c+dx)a^2}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}-\frac{4\tan(c+dx)a}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(8a^2-5b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right.}{3b^4\sqrt{a+b}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/(a*Sqrt[a + b]*d) - (4*a*Cot[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*Sqrt[a + b]*d)
 + (2*a*(8*a^2 - 5*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
```

```
t[-((b*(1 + Sec[c + d*x]))/(a - b))]])/(3*b^4*Sqrt[a + b]*d) - (2*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/(a*Sqrt[a + b]*d) - (4*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*Sqrt[a + b]*d) + (2*(2*a + b)*(
4*a + b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/(3*b^3*Sqrt[a + b]*d) - (2*Sqrt[a + b]*Cot[c + d*x]*
EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))])/(a^2*d) - (4*a*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c +
 d*x]]) + (2*b^2*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) -
 (2*a^2*Sec[c + d*x]*Tan[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]
]) + (2*(4*a^2 - b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*b^2*(a^2 -
b^2)*d)
```

Rule 3895

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Csc[c + d
*x]^2)^(m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && I
GtQ[m/2, 0] && IntegerQ[n - 1/2]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
```

```
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 3836

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*
(a^2 - b^2)), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \left(\frac{1}{(a+b\sec(c+dx))^{3/2}} - \frac{2\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}} + \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\right) dx$$

$$= -\left(2\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx\right) + \int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx + \int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/}}$$

$$= -\frac{4a\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2b^2\tan(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{2a^2\sec(c+dx)\tan}{b\,(a^2-b^2)\,d\sqrt{a+b\,\mathrm{s}}}$$

$$= -\frac{4a\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2b^2\tan(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{2a^2\sec(c+dx)\tan}{b\,(a^2-b^2)\,d\sqrt{a+b\,\mathrm{s}}}$$

$$= \frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d} - \frac{4a\cot(c+d}{}$$

$$= \frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d} - \frac{4a\cot(c+d}{}$$

Mathematica [A] time = 17.1393, size = 864, normalized size = 1.63

$$\frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\left(\frac{2(3b^2-8a^2)\sin(c+dx)}{3ab^3}-\frac{2(b^2\sin(c+dx)-a^2\sin(c+dx))}{ab^2(b+a\cos(c+dx))}+\frac{2\tan(c+dx)}{3b^2}\right)}{d(a+b\sec(c+dx))^{3/2}}-\frac{2(b+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2
]^2)^(-1)]*(-8*a^3*Tan[(c + d*x)/2] - 8*a^2*b*Tan[(c + d*x)/2] + 3*a*b^2*Ta
n[(c + d*x)/2] + 3*b^3*Tan[(c + d*x)/2] + 16*a^3*Tan[(c + d*x)/2]^3 - 6*a*b
^2*Tan[(c + d*x)/2]^3 - 8*a^3*Tan[(c + d*x)/2]^5 + 8*a^2*b*Tan[(c + d*x)/2]
^5 + 3*a*b^2*Tan[(c + d*x)/2]^5 - 3*b^3*Tan[(c + d*x)/2]^5 + 6*b^3*Elliptic
Pi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2
]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] +
6*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d
*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
b*Tan[(c + d*x)/2]^2)/(a + b)] - (8*a^3 + 8*a^2*b - 3*a*b^2 - 3*b^3)*Ellipt
icE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]
*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d
*x)/2]^2)/(a + b)] + 2*a*b*(4*a + b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a
 - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(
a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(3*a*b^3*d*
(a + b*Sec[c + d*x])^(3/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*T
an[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]) + ((b
+ a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*(-8*a^2 + 3*b^2)*Sin[c + d*x])/(3*a*
b^3) - (2*(-(a^2*Sin[c + d*x]) + b^2*Sin[c + d*x]))/(a*b^2*(b + a*Cos[c + d
*x])) + (2*Tan[c + d*x])/(3*b^2)))/(d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.455, size = 1545, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/3/d/a/b^3*4^(1/2)*(8*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3+8*cos(d*x+c)^2*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-3*cos(d*x+c
)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*
x+c)*a*b^2-3*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*sin(d*x+c)*b^3+6*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-8*cos(d*x+c)^2*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*
b-2*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*sin(d*x+c)*a*b^2+8*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+8*sin(d*x+c)*cos(d*x+c)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-3*sin(d*x+c)*
cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a*b^2-3*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*b^3+6*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3-8*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-2*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a*b^2-8*a^3*cos(d*x+c)^3+4*cos(d*x+c)^3*a^2*b+3*cos(d*x+c)^3*a*b^2-3*
cos(d*x+c)^3*b^3+8*cos(d*x+c)^2*a^3-8*a^2*b*cos(d*x+c)^2-2*cos(d*x+c)^2*a*b
^2+3*cos(d*x+c)^2*b^3+4*cos(d*x+c)*a^2*b-a*b^2)*((b+a*cos(d*x+c))/cos(d*x+c
))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\tan(dx+c)^4}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^4/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**4/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan(dx+c)^4}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^4/(b*sec(d*x + c) + a)^(3/2), x)
```

3.341 $\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=344

$$\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{abd}+\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\text{se}}{a}}}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*b^2*d) + (2*Sqrt[a + b]*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(a*b*d) + (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (2*Tan[c +
d*x])/(a*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.403709, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3894, 4061, 4059, 3921, 3784, 3832, 4004}

$$\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}+\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1}{}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*b^2*d) + (2*Sqrt[a + b]*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(a*b*d) + (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (2*Tan[c +
d*x])/(a*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 3894

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4061

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[
e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 4059

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f
*x]], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{-1+\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{2\tan(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}\left(a^2-b^2\right)+\frac{1}{2}\left(a^2-b^2\right)\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{2\tan(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} - \frac{\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx}{a} - \frac{2\int \frac{\frac{1}{2}\left(a^2-b^2\right)-\frac{1}{2}\left(a^2-b^2\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ab^2d} +$$

$$= \frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ab^2d} +$$

Mathematica [C] time = 23.6942, size = 5162, normalized size = 15.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.298, size = 633, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/d/a/b*4^(1/2)*(cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a+cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*cos(d*x+c)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b+(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*sin(d*x+c)+(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)-2*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/
2))*b-a*cos(d*x+c)^2+b*cos(d*x+c)^2+a*cos(d*x+c)-b*cos(d*x+c))*((b+a*cos(d*
x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\tan(dx+c)^2}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^2/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^{2}(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**2/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(dx+c)^{2}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

3.342 $\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=347

$$-\frac{2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}}+\frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\ldots$$

```
[Out] (2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/(a*Sqrt[a + b]*d) - (2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) - (
2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (2*b^2*Tan[c + d*x])/(a*(
a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.319209, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3785, 4058, 3921, 3784, 3832, 4004}

$$\frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(-3/2),x]
```


```
[Out] (2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/(a*Sqrt[a + b]*d) - (2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) - (
2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (2*b^2*Tan[c + d*x])/(a*(
a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
```

```
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2b^2\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}\left(-a^2+b^2\right)+\frac{1}{2}ab\sec(c+dx)+\frac{1}{2}b^2\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}\left(-a^2+b^2\right)+\left(\frac{ab}{2}-\frac{b^2}{2}\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)} - \frac{b^2\int \frac{\sec(c+dx)(1+\sec(c+d}{\sqrt{a+b\sec(c+dx)}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d} + \frac{2b^2\mathrm{ta}}{a\left(a^2-b^2\right)d\sqrt{}}$$

$$= \frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d} - \frac{2\cot(c+dx)F}{}$$

Mathematica [C] time = 6.14897, size = 1249, normalized size = 3.6

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*b*Sin[c + d*x])/(a*(-a^2 + b^2))
 + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/(d*(a + b*Se
c[c + d*x])^(3/2)) + (2*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[
(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]
^2)]*(a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + b^2*Sqrt[(-a + b)/(a +
b)]*Tan[(c + d*x)/2] - 2*a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + a*
b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - b^2*Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]^5 - (2*I)*a^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[
(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
 + (2*I)*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)
]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
+ b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*a^2*Ell
ipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/
```

```
2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[
(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*b^2*
EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*
x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a -
b)*b*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/
(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a^2 + a*b - 2*
b^2)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/
(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(a*Sqrt[(-a + b)/
(a + b)]*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)*(-1 + Tan[(c + d*x)/2]^2)
*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c +
d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.287, size = 1209, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d/a/(a+b)/(a-b)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*((cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)*cos(d*x
+c)+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a*b-sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)*cos(d*x+c)-2*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1
+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)*cos(d*x+c)+2
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*s
in(d*x+c)*cos(d*x+c)+EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)+EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)*a*b-EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
```

```
(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b-EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)^2
*a*b-cos(d*x+c)^2*b^2-a*b*cos(d*x+c)+cos(d*x+c)*b^2)/(b+a*cos(d*x+c))/sin(d
*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c)
+ a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-3/2), x)
```

3.343 $\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=449

$$-\frac{\left(a^2-ab+2b^2\right)\cot(c+dx)\sqrt{-\frac{b(\sec(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad(a-b)(a+b)^{3/2}}+\frac{2b^2\left(a^2+b^2\right)\text{ta}}{ad\left(a^2-b^2\right)^2\sqrt{a+}}$$

```
[Out] (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (2*(a^2 + b^2)*Cot[c + d
*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)
]*Sqrt[-((b*(-1 + Sec[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/(a*(a - b)*(a + b)^(3/2)*d) - ((a^2 - a*b + 2*b^2)*Cot[c + d*x]*El
lipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt
[-((b*(-1 + Sec[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(a*(a - b)*(a + b)^(3/2)*d) - Cot[c + d*x]/(d*(a + b*Sec[c + d*x])^(3/2)
) + (b^2*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*b^2*
(a^2 + b^2)*Tan[c + d*x])/(a*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.953306, antiderivative size = 664, normalized size of antiderivative = 1.48, number of steps used = 14, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3896, 3785, 4058, 3921, 3784, 3832, 4004, 3875, 3833, 4003, 4005}

$$-\frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{4ab^2\tan(c+dx)}{d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{b^2\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2\sqrt{a+b}\cot(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (4*a*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
 d*x]))/(a - b))])/((a - b)*(a + b)^(3/2)*d) - (2*Cot[c + d*x]*EllipticE[Ar
cSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*Sqrt[a +
 b]*d) - ((3*a - b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/
Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-(
(b*(1 + Sec[c + d*x]))/(a - b))])/((a - b)*(a + b)^(3/2)*d) + (2*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
```

```
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/(a*Sqrt[a + b]*d) + (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, A
rcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) -
Cot[c + d*x]/(d*(a + b*Sec[c + d*x])^(3/2)) + (b^2*Tan[c + d*x])/((a^2 - b^
2)*d*(a + b*Sec[c + d*x])^(3/2)) + (4*a*b^2*Tan[c + d*x])/((a^2 - b^2)^2*d*
Sqrt[a + b*Sec[c + d*x]]) - (2*b^2*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a +
b*Sec[c + d*x]])
```

Rule 3896

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-(m/2)), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &
& ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
```

```
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 3875

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(Tan[e + f*x]*(a + b*Csc[e + f*x])^m)/f, x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3833

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \left(-\frac{1}{(a+b\sec(c+dx))^{3/2}}+\frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\right)dx$$

$$= -\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx+\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{\cot(c+dx)}{d(a+b\sec(c+dx))^{3/2}}-\frac{2b^2\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}-\frac{1}{2}(3b)\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))}$$

$$= -\frac{\cot(c+dx)}{d(a+b\sec(c+dx))^{3/2}}+\frac{b^2\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}}-\frac{2b^2\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+}}$$

$$= -\frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d}-\frac{\cot(c}{d(a+b\sec}$$

$$= -\frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d}+\frac{2\cot(c+d}{}$$

$$= \frac{4a\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{(a-b)(a+b)^{3/2}d}-\frac{2\cot(c+dx}{}$$

Mathematica [C] time = 13.4321, size = 663, normalized size = 1.48

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)^2\left(-\frac{2b\left(a^2+b^2\right)\sin(c+dx)}{a\left(a^2-b^2\right)^2}+\frac{2b^4\sin(c+dx)}{a\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}+\frac{\csc(c+dx)\left(a^2(-\cos(c+dx))+2ab-b^2\cos(c+dx)\right)}{\left(b^2-a^2\right)^2}\right)}{d(a+b\sec(c+dx))^{3/2}}-\frac{2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*(((2*a*b - a^2*Cos[c + d*x] - b^2*Co
s[c + d*x])*Csc[c + d*x])/(-a^2 + b^2)^2 - (2*b*(a^2 + b^2)*Sin[c + d*x])/(
a*(a^2 - b^2)^2) + (2*b^4*Sin[c + d*x])/(a*(a^2 - b^2)^2*(b + a*Cos[c + d*x
]))))/(d*(a + b*Sec[c + d*x])^(3/2)) - (2*Cos[(c + d*x)/2]^2*(b + a*Cos[c +
 d*x])*Sec[c + d*x]^2*((-2*I)*b*(-a^3 + a^2*b - a*b^2 + b^3)*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/
(a - b)] + I*(2*a^4 - a^3*b - 2*a^2*b^2 - 3*a*b^3 + 4*b^4)*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a
 - b)] - (4*I)*(a^2 - b^2)^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-((a + b)/(a - b
)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] -
b*Sqrt[(-a + b)/(a + b)]*(a^2 + b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[
(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(a*Sqrt[(-a + b)/(a + b)]*(a^2 - b^2)^2*d
*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.3, size = 2238, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/2/d/a/(a+b)^2/(a-b)^2*(2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*b^4-4*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
,((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4-4*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4+2*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4+2*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*b^4+8*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*
x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*a^2*b^2-2*cos(d*x+c)^2*a^3*b+2*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))
```

```
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b+2*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
a^2*b^2+2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x
+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*a*b^3+8*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-
b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2-EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b-6*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x
+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*a^2*b^2-3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3+cos(d*x+c)^2*a^4+2*cos(d*x+c)^2*b^4-
2*cos(d*x+c)*b^4+cos(d*x+c)^2*a^2*b^2-2*cos(d*x+c)^2*a*b^3+cos(d*x+c)*a^3*b
-2*cos(d*x+c)*a^2*b^2+3*cos(d*x+c)*a*b^3-EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b-6*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2-3*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3-4*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x
+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*a^4-4*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/
2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4+2*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4+2*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b+2*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2+
2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*a*b^3)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*4^(1/2)/(b+a*cos(d*x+c))/sin(
d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)**2/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

3.344 $\int (a+b\sec(e+fx))^3(d\tan(e+fx))^n\,dx$

Optimal. Leaf size=245

$$\frac{3a^2b\sec(e+fx)\cos^2(e+fx)^{\frac{n+2}{2}}(d\tan(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+2}{2},\frac{n+3}{2},\sin^2(e+fx)\right)}{df(n+1)}+\frac{a^3(d\tan(e+$$

```
[Out] (3*a*b^2*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n)) + (a^3*Hypergeometric2F1[1
, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(1 + n))/(d*f*(1
+ n)) + (3*a^2*b*(Cos[e + f*x]^2)^((2 + n)/2)*Hypergeometric2F1[(1 + n)/2,
(2 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(d*Tan[e + f*x])^(1 + n)
)/(d*f*(1 + n)) + (b^3*(Cos[e + f*x]^2)^((4 + n)/2)*Hypergeometric2F1[(1 +
n)/2, (4 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*Sec[e + f*x]^3*(d*Tan[e + f*x])
^(1 + n))/(d*f*(1 + n))
```

Rubi [A] time = 0.271727, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3886, 3476, 364, 2617, 2607, 32}

$$\frac{3a^2b\sec(e+fx)\cos^2(e+fx)^{\frac{n+2}{2}}(d\tan(e+fx))^{n+1}{}_2F_1\left(\frac{n+1}{2},\frac{n+2}{2};\frac{n+3}{2};\sin^2(e+fx)\right)}{df(n+1)}+\frac{a^3(d\tan(e+fx))^{n+1}{}_2F_1\left(1,\frac{n+1}{2}\right.}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^3*(d*Tan[e + f*x])^n,x]
```


```
[Out] (3*a*b^2*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n)) + (a^3*Hypergeometric2F1[1
, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(1 + n))/(d*f*(1
+ n)) + (3*a^2*b*(Cos[e + f*x]^2)^((2 + n)/2)*Hypergeometric2F1[(1 + n)/2,
(2 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(d*Tan[e + f*x])^(1 + n)
)/(d*f*(1 + n)) + (b^3*(Cos[e + f*x]^2)^((4 + n)/2)*Hypergeometric2F1[(1 +
n)/2, (4 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*Sec[e + f*x]^3*(d*Tan[e + f*x])
^(1 + n))/(d*f*(1 + n))
```


Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+b\sec(e+fx))^3(d\tan(e+fx))^n\,dx = \int \left(a^3(d\tan(e+fx))^n + 3a^2b\sec(e+fx)(d\tan(e+fx))^n + 3ab^2\sec^2(e+$$

$$= a^3\int (d\tan(e+fx))^n\,dx + \left(3a^2b\right)\int \sec(e+fx)(d\tan(e+fx))^n\,dx + \left(3ab\right.$$

$$= \frac{3a^2b\cos^2(e+fx)^{\frac{2+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{2+n}{2};\frac{3+n}{2};\sin^2(e+fx)\right)\sec(e+fx)(d\tan(e}{df(1+n)}$$

$$= \frac{3ab^2(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{a^3\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+f}{df(1+n)}$$

Mathematica [A] time = 3.36643, size = 238, normalized size = 0.97

$$\frac{d\left(-\tan^2(e+fx)\right)^{-n/2}(d\tan(e+fx))^{n-1}\left(9a^2b(n+1)\sqrt{-\tan^2(e+fx)}\sec(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3}{2},\sec\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])^3*(d*Tan[e + f*x])^n,x]
```

```
[Out] (d*(d*Tan[e + f*x])^(-1 + n)*(9*a^2*b*(1 + n)*Hypergeometric2F1[1/2, (1 - n
)/2, 3/2, Sec[e + f*x]^2]*Sec[e + f*x]*Sqrt[-Tan[e + f*x]^2] + b^3*(1 + n)*
Hypergeometric2F1[3/2, (1 - n)/2, 5/2, Sec[e + f*x]^2]*Sec[e + f*x]^3*Sqrt[
-Tan[e + f*x]^2] - 3*a^3*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e
+ f*x]^2]*(-Tan[e + f*x]^2)^((2 + n)/2) + 9*a*b^2*(Sqrt[-Tan[e + f*x]^2] -
(-Tan[e + f*x]^2)^((2 + n)/2))))/(3*f*(1 + n)*(-Tan[e + f*x]^2)^(n/2))
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(a+b\sec\left(fx+e\right)\right)^3\left(d\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^3*(d*tan(f*x+e))^n,x)
```

```
[Out] int((a+b*sec(f*x+e))^3*(d*tan(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^3 \left(d\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3*(d*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^3*(d*tan(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^3\sec\left(fx+e\right)^3+3\,ab^2\sec\left(fx+e\right)^2+3\,a^2b\sec\left(fx+e\right)+a^3\right)\left(d\tan\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3*(d*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(f*x + e)^3 + 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x + e
) + a^3)*(d*tan(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\tan\left(e+fx\right)\right)^n \left(a+b\sec\left(e+fx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**3*(d*tan(f*x+e))**n,x)
```

```
[Out] Integral((d*tan(e + f*x))**n*(a + b*sec(e + f*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^3 \left(d\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3*(d*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^3*(d*tan(f*x + e))^n, x)
```

3.345 $\int (a+b\sec(e+fx))^2(d\tan(e+fx))^n\,dx$

Optimal. Leaf size=160

$$\frac{a^2(d\tan(e+fx))^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(e+fx)\right)}{df(n+1)}+\frac{2ab\sec(e+fx)\cos^2(e+fx)^{\frac{n+2}{2}}(d\tan(e+fx$$

```
[Out] (b^2*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n)) + (a^2*Hypergeometric2F1[1, (1
 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n)
) + (2*a*b*(Cos[e + f*x]^2)^((2 + n)/2)*Hypergeometric2F1[(1 + n)/2, (2 + n
)/2, (3 + n)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(d*Tan[e + f*x])^(1 + n))/(d*f
*(1 + n))
```

Rubi [A] time = 0.182092, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3886, 3476, 364, 2617, 2607, 32}

$$\frac{a^2(d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(e+fx)\right)}{df(n+1)}+\frac{2ab\sec(e+fx)\cos^2(e+fx)^{\frac{n+2}{2}}(d\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n+2}{2}\right.}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^2*(d*Tan[e + f*x])^n,x]
```

```
[Out] (b^2*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n)) + (a^2*Hypergeometric2F1[1, (1
 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n)
) + (2*a*b*(Cos[e + f*x]^2)^((2 + n)/2)*Hypergeometric2F1[(1 + n)/2, (2 + n
)/2, (3 + n)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(d*Tan[e + f*x])^(1 + n))/(d*f
*(1 + n))
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(e+fx))^2(d\tan(e+fx))^n\,dx &= \int \left(a^2(d\tan(e+fx))^n+2ab\sec(e+fx)(d\tan(e+fx))^n+b^2\sec^2(e+fx)\right.\\
&= a^2\int (d\tan(e+fx))^n\,dx+(2ab)\int \sec(e+fx)(d\tan(e+fx))^n\,dx+b^2\int \\
&= \frac{2ab\cos^2(e+fx)^{\frac{2+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{2+n}{2};\frac{3+n}{2};\sin^2(e+fx)\right)\sec(e+fx)(d\tan(e+}{df(1+n)}\\
&= \frac{b^2(d\tan(e+fx))^{1+n}}{df(1+n)}+\frac{a^2\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+fx))}{df(1+n)}\end{aligned}$$

Mathematica [A] time = 1.35733, size = 178, normalized size = 1.11

$$d\left(-\tan^2(e+fx)\right)^{-n/2}(d\tan(e+fx))^{n-1}\left(-a^2\left(-\tan^2(e+fx)\right)^{\frac{n+2}{2}}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(e+fx)\right)+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])^2*(d*Tan[e + f*x])^n,x]
```

```
[Out] (d*(d*Tan[e + f*x])^(-1 + n)*(2*a*b*(1 + n)*Hypergeometric2F1[1/2, (1 - n)/
2, 3/2, Sec[e + f*x]^2]*Sec[e + f*x]*Sqrt[-Tan[e + f*x]^2] - a^2*Hypergeome
tric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(-Tan[e + f*x]^2)^((2 + n
)/2) + b^2*(Sqrt[-Tan[e + f*x]^2] - (-Tan[e + f*x]^2)^((2 + n)/2))))/(f*(1
+ n)*(-Tan[e + f*x]^2)^(n/2))
```

Maple [F] time = 0.946, size = 0, normalized size = 0.

$$\int \left(a+b\sec\left(fx+e\right)\right)^{2}\left(d\tan\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^2*(d*tan(f*x+e))^n,x)
```

```
[Out] int((a+b*sec(f*x+e))^2*(d*tan(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^{2}\left(d\tan\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2*(d*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^2*(d*tan(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\sec\left(fx+e\right)^2+2\,ab\sec\left(fx+e\right)+a^2\right)\left(d\tan\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2*(d*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)*(d*tan(f*x + e))^n
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\tan\left(e+fx\right)\right)^n\left(a+b\sec\left(e+fx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**2*(d*tan(f*x+e))**n,x)
```

```
[Out] Integral((d*tan(e + f*x))**n*(a + b*sec(e + f*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)+a\right)^2\left(d\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2*(d*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^2*(d*tan(f*x + e))^n, x)
```

3.346 $\int (a+b\sec(e+fx))(d\tan(e+fx))^n\,dx$

Optimal. Leaf size=129

$$\frac{a(d\tan(e+fx))^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(e+fx)\right)}{df(n+1)}+\frac{b\sec(e+fx)\cos^2(e+fx)^{\frac{n+2}{2}}(d\tan(e+fx))^{n+1}}{d}$$

```
[Out] (a*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f
*x])^(1 + n))/(d*f*(1 + n)) + (b*(Cos[e + f*x]^2)^((2 + n)/2)*Hypergeometri
c2F1[(1 + n)/2, (2 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(d*Tan[e
 + f*x])^(1 + n))/(d*f*(1 + n))
```

Rubi [A] time = 0.0852123, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3884, 3476, 364, 2617}

$$\frac{a(d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(e+fx)\right)}{df(n+1)}+\frac{b\sec(e+fx)\cos^2(e+fx)^{\frac{n+2}{2}}(d\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n+2}{2};\frac{n+}{2}\right.}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])*(d*Tan[e + f*x])^n,x]
```

```
[Out] (a*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f
*x])^(1 + n))/(d*f*(1 + n)) + (b*(Cos[e + f*x]^2)^((2 + n)/2)*Hypergeometri
c2F1[(1 + n)/2, (2 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(d*Tan[e
 + f*x])^(1 + n))/(d*f*(1 + n))
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (a+b\sec(e+fx))(d\tan(e+fx))^n\,dx = a\int (d\tan(e+fx))^n\,dx + b\int \sec(e+fx)(d\tan(e+fx))^n\,dx$$
$$= \frac{b\cos^2(e+fx)^{\frac{2+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{2+n}{2};\frac{3+n}{2};\sin^2(e+fx)\right)\sec(e+fx)(d\tan(e+fx$$
$$df(1+n)$$
$$= \frac{a\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{b\cos^2(e+fx)^{\frac{2+n}{2}}\,{}_2F$$

Mathematica [A] time = 0.675377, size = 106, normalized size = 0.82

$$\frac{(d\tan(e+fx))^n\left(\frac{a\tan(e+fx)\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(e+fx)\right)}{n+1} + b\csc(e+fx)\left(-\tan^2(e+fx)\right)^{\frac{1-n}{2}}\text{Hypergeometric}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])*(d*Tan[e + f*x])^n,x]
```

```
[Out] ((d*Tan[e + f*x])^n*((a*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e +
 f*x]^2]*Tan[e + f*x])/(1 + n) + b*Csc[e + f*x]*Hypergeometric2F1[1/2, (1 -
 n)/2, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^((1 - n)/2)))/f
```

Maple [F] time = 0.663, size = 0, normalized size = 0.

$$\int \left(a+b\sec \left(fx+e\right)\right) \left(d\tan \left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))*(d*tan(f*x+e))^n,x)
```

```
[Out] int((a+b*sec(f*x+e))*(d*tan(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(fx+e\right)+a\right) \left(d\tan \left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))*(d*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)*(d*tan(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec \left(fx+e\right)+a\right) \left(d\tan \left(fx+e\right)\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))*(d*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e) + a)*(d*tan(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\tan \left(e+fx\right)\right)^{n} \left(a+b\sec \left(e+fx\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))*(d*tan(f*x+e))**n,x)
```

```
[Out] Integral((d*tan(e + f*x))**n*(a + b*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)\left(d\tan\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))*(d*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)*(d*tan(f*x + e))^n, x)
```

3.347 $\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=266

$$\frac{d\left(-\tan^2(e+fx)\right)^{\frac{1-n}{2}+\frac{n-1}{2}}(d\tan(e+fx))^{n-1}\left(-\frac{b(1-\sec(e+fx))}{a+b\sec(e+fx)}\right)^{\frac{1-n}{2}}\left(\frac{b(\sec(e+fx)+1)}{a+b\sec(e+fx)}\right)^{\frac{1-n}{2}}F_1\left(1-n;\frac{1-n}{2},\frac{1-n}{2};2-n;\frac{a+b}{a+b\sec(e+fx)},\right.}{af(1-n)}$$

```
[Out] (d*AppellF1[1 - n, (1 - n)/2, (1 - n)/2, 2 - n, (a + b)/(a + b*Sec[e + f*x]
), (a - b)/(a + b*Sec[e + f*x])]*(-((b*(1 - Sec[e + f*x]))/(a + b*Sec[e + f
*x])))^((1 - n)/2)*((b*(1 + Sec[e + f*x]))/(a + b*Sec[e + f*x]))^((1 - n)/2
)*(d*Tan[e + f*x])^(-1 + n)*(-Tan[e + f*x]^2)^((1 - n)/2 + (-1 + n)/2))/(a*
f*(1 - n)) - (d*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]
*(d*Tan[e + f*x])^(-1 + n)*(-Tan[e + f*x]^2)^((1 - n)/2 + (1 + n)/2))/(a*f*
(1 + n))
```

Rubi [F] time = 0.0498594, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*Tan[e + f*x])^n/(a + b*Sec[e + f*x]),x]
```

```
[Out] Defer[Int][(d*Tan[e + f*x])^n/(a + b*Sec[e + f*x]), x]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx = \int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx$$

Mathematica [B] time = 4.5795, size = 786, normalized size = 2.95

$$\frac{f(a+b\sec(e+fx))\left(\sec^2\left(\frac{1}{2}(e+fx)\right)\left((a+b)F_1\left(\frac{n+1}{2};n,1;\frac{n+3}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-bF_1\left(\frac{n+1}{2};n,1;\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Tan[e + f*x])^n/(a + b*Sec[e + f*x]),x]
```

```
[Out] (2*((a + b)*AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] - b*AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(e + f*x)/2]^2,
 ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])*Tan[(e + f*x)/2]*(d*Tan[e + f*x])^n
)/(f*(a + b*Sec[e + f*x])*(((a + b)*AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - b*AppellF1[(1 + n)/2, n, 1, (3 + n
)/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])*Sec[(e + f*
x)/2]^2 - 16*n*((a + b)*AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] - b*AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(e
+ f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])*Cos[(e + f*x)/2]*Csc[e
+ f*x]^3*Sec[e + f*x]*Sin[(e + f*x)/2]^5 + 2*n*((a + b)*AppellF1[(1 + n)/2,
 n, 1, (3 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - b*AppellF1[(1
+ n)/2, n, 1, (3 + n)/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(
a + b)])*Csc[e + f*x]*Sec[e + f*x]*Tan[(e + f*x)/2] - (2*(1 + n)*((a - b)*b
*AppellF1[(3 + n)/2, n, 2, (5 + n)/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e +
 f*x)/2]^2)/(a + b)] + (a + b)^2*(AppellF1[(3 + n)/2, n, 2, (5 + n)/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - n*AppellF1[(3 + n)/2, 1 + n, 1, (5 +
 n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]) + b*(a + b)*n*AppellF1[(3
+ n)/2, 1 + n, 1, (5 + n)/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^
2)/(a + b)])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]^2)/((a + b)*(3 + n))))
```

Maple [F] time = 0.668, size = 0, normalized size = 0.

$$\int \frac{\left(d\tan\left(fx+e\right)\right)^{n}}{a+b\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

```
[Out] int((d*tan(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\tan\left(fx+e\right)\right)^{n}}{b\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d\tan\left(fx+e\right)\right)^{n}}{b\sec\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*tan(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\tan\left(e+fx\right)\right)^{n}}{a+b\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**n/(a+b*sec(f*x+e)),x)
```

```
[Out] Integral((d*tan(e + f*x))**n/(a + b*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\tan\left(fx+e\right)\right)^{n}}{b\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*tan(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

3.348 $\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left((a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m,x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m, x]
```

Rubi [A] time = 0.068786, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m,x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx=\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$$

Mathematica [A] time = 7.954, size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m,x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m, x]
```

Maple [A] time = 0.281, size = 0, normalized size = 0.

$$\int (a + b\sec(dx + c))^{\frac{3}{2}} (e\tan(dx + c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x)
```

```
[Out] int((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx + c) + a)^{\frac{3}{2}} (e\tan(dx + c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx + c) + a)^{\frac{3}{2}} (e\tan(dx + c))^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(e*tan(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}(e\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

3.349 $\int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m,x\right)$$

```
[Out] Unintegrable[Sqrt[a + b*Sec[c + d*x]]*(e*Tan[c + d*x])^m, x]
```

Rubi [A] time = 0.0610669, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*(e*Tan[c + d*x])^m,x]
```

```
[Out] Defer[Int][Sqrt[a + b*Sec[c + d*x]]*(e*Tan[c + d*x])^m, x]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx = \int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx$$

Mathematica [A] time = 0.700808, size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*(e*Tan[c + d*x])^m,x]
```

```
[Out] Integrate[Sqrt[a + b*Sec[c + d*x]]*(e*Tan[c + d*x])^m, x]
```

Maple [A] time = 0.3, size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec \left(dx+c\right)}\left(e\tan \left(dx+c\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x)
```

```
[Out] int((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec \left(dx+c\right)+a}\left(e\tan \left(dx+c\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sec \left(dx+c\right)+a}\left(e\tan \left(dx+c\right)\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \left(e\tan \left(c+dx\right)\right)^{m}\sqrt{a+b\sec \left(c+dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)*(e*tan(d*x+c))**m,x)
```

```
[Out] Integral((e*tan(c + d*x))**m*sqrt(a + b*sec(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)+a}\,(e\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

3.350 $\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable[(e*Tan[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi [A] time = 0.0624351, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*Tan[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Defer[Int][(e*Tan[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A] time = 2.98802, size = 0, normalized size = 0.

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Integrate[(e*Tan[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [A] time = 0.299, size = 0, normalized size = 0.

$$\int (e\tan (dx+c))^{m} \frac{1}{\sqrt{a+b\sec (dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan (dx+c))^{m}}{\sqrt{b\sec (dx+c)+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*tan(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e\tan (dx+c))^{m}}{\sqrt{b\sec (dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*tan(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan (c+dx))^{m}}{\sqrt{a+b\sec (c+dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**m/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*tan(c + d*x))**m/sqrt(a + b*sec(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^m}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

3.351 $\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable[(e*Tan[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi [A] time = 0.0700814, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*Tan[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Defer[Int][(e*Tan[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A] time = 3.70481, size = 0, normalized size = 0.

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Integrate[(e*Tan[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2), x]
```

Maple [A] time = 0.268, size = 0, normalized size = 0.

$$\int \left(e\tan\left(dx+c\right)\right)^{m}\left(a+b\sec\left(dx+c\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e\tan\left(dx+c\right)\right)^{m}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*tan(d*x + c))^m/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\left(e\tan\left(dx+c\right)\right)^{m}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*(e*tan(d*x + c))^m/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**m/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((e*tan(c + d*x))**m/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\tan(dx+c))^m}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^m/(b*sec(d*x + c) + a)^(3/2), x)
```

3.352 $\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left((e\tan(c+dx))^m(a+b\sec(c+dx))^n,x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^n*(e*Tan[c + d*x])^m, x]
```

Rubi [A] time = 0.0450369, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*(e*Tan[c + d*x])^m,x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*(e*Tan[c + d*x])^m, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx=\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx$$

Mathematica [A] time = 3.16305, size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*(e*Tan[c + d*x])^m,x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^n*(e*Tan[c + d*x])^m, x]
```

Maple [A] time = 0.574, size = 0, normalized size = 0.

$$\int \left(a+b\sec \left(dx+c\right)\right)^{n} \left(e\tan \left(dx+c\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x)
```

```
[Out] int((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(dx+c\right)+a\right)^{n} \left(e\tan \left(dx+c\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec \left(dx+c\right)+a\right)^{n} \left(e\tan \left(dx+c\right)\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*(e*tan(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n (e\tan(dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

3.353 $\int (a+b\sec(c+dx))^n \tan^5(c+dx)\,dx$

Optimal. Leaf size=177

$$-\frac{(a+b\sec(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b\sec(c+dx)}{a}+1\right)}{ad(n+1)}-\frac{a\left(a^2-2b^2\right)(a+b\sec(c+dx))^{n+1}}{b^4d(n+1)}+\frac{(3\ldots}{}$$

```
[Out] -((a*(a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(1 + n))/(b^4*d*(1 + n))) - (Hyperg
eometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c + d*x])^(
1 + n))/(a*d*(1 + n)) + ((3*a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(2 + n))/(b^4
*d*(2 + n)) - (3*a*(a + b*Sec[c + d*x])^(3 + n))/(b^4*d*(3 + n)) + (a + b*S
ec[c + d*x])^(4 + n)/(b^4*d*(4 + n))
```

Rubi [A] time = 0.201419, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3885, 952, 1620, 65}

$$-\frac{a\left(a^2-2b^2\right)(a+b\sec(c+dx))^{n+1}}{b^4d(n+1)}+\frac{\left(3a^2-2b^2\right)(a+b\sec(c+dx))^{n+2}}{b^4d(n+2)}-\frac{3a(a+b\sec(c+dx))^{n+3}}{b^4d(n+3)}+\frac{(a+b\sec(c+d\ldots}{b^4d(n+4)\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^5,x]
```

```
[Out] -((a*(a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(1 + n))/(b^4*d*(1 + n))) - (Hyperg
eometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c + d*x])^(
1 + n))/(a*d*(1 + n)) + ((3*a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(2 + n))/(b^4
*d*(2 + n)) - (3*a*(a + b*Sec[c + d*x])^(3 + n))/(b^4*d*(3 + n)) + (a + b*S
ec[c + d*x])^(4 + n)/(b^4*d*(4 + n))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 952

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x)^(n + 1))/(g*
```

```
e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (a+b\sec(c+dx))^n\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^n\left(b^2-x^2\right)^2}{x}\,dx,x,b\sec(c+dx)\right)}{b^4d}$$

$$= \frac{(a+b\sec(c+dx))^{4+n}}{b^4d(4+n)} + \frac{\text{Subst}\left(\int \frac{(a+x)^n\left(b^4(4+n)-a^3(4+n)x-\left(3a^2+2b^2\right)(4+n)x^2-3a(4+n)x^3}{x}\right.}{b^4d(4+n)}$$

$$= \frac{(a+b\sec(c+dx))^{4+n}}{b^4d(4+n)} + \frac{\text{Subst}\left(\int \left(-a\left(a^2-2b^2\right)(4+n)(a+x)^n + \frac{\left(4b^4+b^4n\right)(a+x}{x}\right.\right.}{}$$

$$= -\frac{a\left(a^2-2b^2\right)(a+b\sec(c+dx))^{1+n}}{b^4d(1+n)} + \frac{\left(3a^2-2b^2\right)(a+b\sec(c+dx))^{2+n}}{b^4d(2+n)} - \frac{3a(a}{}$$

$$= -\frac{a\left(a^2-2b^2\right)(a+b\sec(c+dx))^{1+n}}{b^4d(1+n)} - \frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b}{ad(1+n)}$$

Mathematica [A] time = 2.92646, size = 298, normalized size = 1.68

$$-\frac{\sec^8\left(\frac{1}{2}(c+dx)\right)(a+b\sec(c+dx))^n\left(n(a\cos(c+dx)+b)\left(3a\left(3a^2+b^2\left(n^2-n-8\right)\right)\cos(c+dx)+2b(n+1)\left(b^2\left(n^2\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^5,x]
```

```
[Out] -((n*(b + a*Cos[c + d*x])*(-6*a^2*b + 12*b^3 - 6*a^2*b*n + 16*b^3*n + 4*b^3
*n^2 + 3*a*(3*a^2 + b^2*(-8 - n + n^2))*Cos[c + d*x] + 2*b*(1 + n)*(-3*a^2
+ b^2*(12 + 7*n + n^2))*Cos[2*(c + d*x)] + 3*a^3*Cos[3*(c + d*x)] - 12*a*b^
2*Cos[3*(c + d*x)] - 7*a*b^2*n*Cos[3*(c + d*x)] - a*b^2*n^2*Cos[3*(c + d*x)
]) - 2*b^4*(24 + 50*n + 35*n^2 + 10*n^3 + n^4)*Cos[c + d*x]^4*Hypergeometri
c2F1[1, -n, 1 - n, (a*Cos[c + d*x])/(b + a*Cos[c + d*x])])*Sec[(c + d*x)/2]
^8*(a + b*Sec[c + d*x])^n)/(2*b^4*d*n*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(-1 +
 Tan[(c + d*x)/2]^2)^4)
```

Maple [F] time = 0.402, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))^n (\tan(dx+c))^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c)^5,x)
```

```
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c)^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n \tan(dx+c)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="maxima")
```

[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c)+a)^n\tan(dx+c)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="fricas")

[Out] integral((b*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**5,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n\tan(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="giac")

[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)

3.354 $\int (a+b\sec(c+dx))^n \tan^3(c+dx)\,dx$

Optimal. Leaf size=102

$$\frac{(a+b\sec(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b\sec(c+dx)}{a}+1\right)}{ad(n+1)}-\frac{a(a+b\sec(c+dx))^{n+1}}{b^2d(n+1)}+\frac{(a+b\sec(c+\ldots}{b^2d(n+2\ldots}$$

```
[Out] -((a*(a + b*Sec[c + d*x])^(1 + n))/(b^2*d*(1 + n))) + (Hypergeometric2F1[1,
 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c + d*x])^(1 + n))/(a*d*(
1 + n)) + (a + b*Sec[c + d*x])^(2 + n)/(b^2*d*(2 + n))
```

Rubi [A] time = 0.0923938, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3885, 952, 80, 65}

$$-\frac{a(a+b\sec(c+dx))^{n+1}}{b^2d(n+1)}+\frac{(a+b\sec(c+dx))^{n+2}}{b^2d(n+2)}+\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b\sec(c+dx)}{a}+1\right)}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^3,x]
```

```
[Out] -((a*(a + b*Sec[c + d*x])^(1 + n))/(b^2*d*(1 + n))) + (Hypergeometric2F1[1,
 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c + d*x])^(1 + n))/(a*d*(
1 + n)) + (a + b*Sec[c + d*x])^(2 + n)/(b^2*d*(2 + n))
```

<u>Rule 3885</u>

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

<u>Rule 952</u>

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x)^(n + 1))/(g*
e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
```

```
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (a+b\sec(c+dx))^n\tan^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a+x)^n\left(b^2-x^2\right)}{x}\,dx,x,b\sec(c+dx)\right)}{b^2d}$$

$$= \frac{(a+b\sec(c+dx))^{2+n}}{b^2d(2+n)} - \frac{\text{Subst}\left(\int \frac{(a+x)^n\left(b^2(2+n)+a(2+n)x\right)}{x}\,dx,x,b\sec(c+dx)\right)}{b^2d(2+n)}$$

$$= -\frac{a(a+b\sec(c+dx))^{1+n}}{b^2d(1+n)} + \frac{(a+b\sec(c+dx))^{2+n}}{b^2d(2+n)} - \frac{\text{Subst}\left(\int \frac{(a+x)^n}{x}\,dx,x,b\sec\right.}{d}$$

$$= -\frac{a(a+b\sec(c+dx))^{1+n}}{b^2d(1+n)} + \frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx}{ad(1+n)}$$

Mathematica [A] time = 1.23514, size = 118, normalized size = 1.16

$$\frac{\sec^2(c+dx)(a+b\sec(c+dx))^n\left(n(a\cos(c+dx)+b)(-a\cos(c+dx)+bn+b)-b^2\left(n^2+3n+2\right)\cos^2(c+dx)\text{Hyperge}\right.}{b^2dn(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^3,x]
```

```
[Out] ((n*(b + b*n - a*Cos[c + d*x])*(b + a*Cos[c + d*x]) - b^2*(2 + 3*n + n^2)*C
os[c + d*x]^2*Hypergeometric2F1[1, -n, 1 - n, (a*Cos[c + d*x])/(b + a*Cos[c
 + d*x])])*Sec[c + d*x]^2*(a + b*Sec[c + d*x])^n)/(b^2*d*n*(1 + n)*(2 + n))
```

Maple [F] time = 0.297, size = 0, normalized size = 0.

$$\int (a + b \sec(dx + c))^n (\tan(dx + c))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c)^3,x)
```

```
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c) + a)^n \tan(dx + c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \sec(dx + c) + a)^n \tan(dx + c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sec (c + dx))^n \tan^3 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n*tan(c + d*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx + c) + a)^n \tan (dx + c)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

3.355 $\int (a+b\sec(c+dx))^n \tan(c+dx)\,dx$

Optimal. Leaf size=48

$$-\frac{(a+b\sec(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b\sec(c+dx)}{a}+1\right)}{ad(n+1)}$$

```
[Out] -((Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c
+ d*x])^(1 + n))/(a*d*(1 + n)))
```

Rubi [A] time = 0.0371419, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3885, 65}

$$-\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b\sec(c+dx)}{a}+1\right)}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x],x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c
+ d*x])^(1 + n))/(a*d*(1 + n)))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (a+b\sec(c+dx))^n\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^n}{x}\,dx, x, b\sec(c+dx)\right)}{d}$$
$$= -\frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)}$$

Mathematica [A] time = 0.398119, size = 49, normalized size = 1.02

$$\frac{(a+b\sec(c+dx))^n\text{Hypergeometric2F1}\left(1,-n,1-n,\frac{a\cos(c+dx)}{a\cos(c+dx)+b}\right)}{dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x],x]
```

```
[Out] (Hypergeometric2F1[1, -n, 1 - n, (a*Cos[c + d*x])/(b + a*Cos[c + d*x])]*(a
+ b*Sec[c + d*x])^n)/(d*n)
```

Maple [F] time = 0.318, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))^n\tan(dx+c)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c),x)
```

```
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n\tan(dx+c)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c)+a)^n\tan(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^n\tan(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n*tan(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n\tan(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

3.356 $\int \cot(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal. Leaf size=162

$$-\frac{(a+b\sec(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\sec(c+dx)}{a-b}\right)}{2d(n+1)(a-b)}-\frac{(a+b\sec(c+dx))^{n+1}\text{Hypergeometric2F}}{2d(n+1)(a+}$$

```
[Out] -(Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a + b*S
ec[c + d*x])^(1 + n))/(2*(a - b)*d*(1 + n)) - (Hypergeometric2F1[1, 1 + n,
2 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(2*(a +
b)*d*(1 + n)) + (Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]
*(a + b*Sec[c + d*x])^(1 + n))/(a*d*(1 + n))
```

Rubi [A] time = 0.177171, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3885, 961, 65, 831, 68}

$$-\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a-b}\right)}{2d(n+1)(a-b)}-\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a+b}\right)}{2d(n+1)(a+b)}+\frac{(a+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] -(Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a + b*S
ec[c + d*x])^(1 + n))/(2*(a - b)*d*(1 + n)) - (Hypergeometric2F1[1, 1 + n,
2 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(2*(a +
b)*d*(1 + n)) + (Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]
*(a + b*Sec[c + d*x])^(1 + n))/(a*d*(1 + n))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

```
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \cot(c+dx)(a+b\sec(c+dx))^n\,dx = -\frac{b^2\,\mathrm{Subst}\left(\int \frac{(a+x)^n}{x(b^2-x^2)}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= -\frac{b^2\,\mathrm{Subst}\left(\int \left(\frac{(a+x)^n}{b^2x} - \frac{x(a+x)^n}{b^2(-b^2+x^2)}\right)dx, x, b\sec(c+dx)\right)}{d}$$

$$= -\frac{\mathrm{Subst}\left(\int \frac{(a+x)^n}{x}\,dx, x, b\sec(c+dx)\right)}{d} + \frac{\mathrm{Subst}\left(\int \frac{x(a+x)^n}{-b^2+x^2}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= \frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)} + \frac{\mathrm{Subst}\left(\int \left(-\frac{(a+x)^n}{2(b-x)} + \right.\right.}{}$$

$$= \frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)} - \frac{\mathrm{Subst}\left(\int \frac{(a+x)^n}{b-x}\,dx, x\right.}{2d}$$

$$= -\frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{2(a-b)d(1+n)} - \frac{{}_2F_1\left(1,1+n;2+n;\frac{a}{}\right.}{2(}$$

Mathematica [A] time = 1.42253, size = 163, normalized size = 1.01

$$\frac{(a+b\sec(c+dx))^n\left(-2\mathrm{Hypergeometric2F1}\left(1,-n,1-n,\frac{a\cos(c+dx)}{a\cos(c+dx)+b}\right)+\mathrm{Hypergeometric2F1}\left(1,-n,1-n,\frac{(a+b)\cos(c}{a\cos(c+dx}\right.\right.}{2dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] ((-2*Hypergeometric2F1[1, -n, 1 - n, (a*Cos[c + d*x])/(b + a*Cos[c + d*x])]
 + Hypergeometric2F1[1, -n, 1 - n, ((a + b)*Cos[c + d*x])/(b + a*Cos[c + d*
x])] + (2^n*Hypergeometric2F1[-n, -n, 1 - n, ((-a + b)*Cos[c + d*x]*Sec[(c
+ d*x)/2]^2)/(2*b)])/(((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/b)^n)*(a +
b*Sec[c + d*x])^n)/(2*d*n)
```

Maple [F] time = 0.306, size = 0, normalized size = 0.

$$\int \cot(dx+c)(a+b\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sec(d*x+c))^n,x)
```

```
[Out] int(cot(d*x+c)*(a+b*sec(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx+c)+a)^n \cot (dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\sec (dx+c)+a)^n \cot (dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sec (c+dx))^n \cot (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))**n,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n*cot(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n \cot(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

3.357 $\int \cot^3(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal. Leaf size=279

$$\frac{(a+b\sec(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\sec(c+dx)}{a-b}\right)}{2d(n+1)(a-b)}+\frac{(a+b\sec(c+dx))^{n+1}\text{Hypergeometric2F}}{2d(n+1)(a+}$$

```
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a + b*Se
c[c + d*x])^(1 + n))/(2*(a - b)*d*(1 + n)) + (Hypergeometric2F1[1, 1 + n, 2
 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(2*(a + b
)*d*(1 + n)) - (Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*
(a + b*Sec[c + d*x])^(1 + n))/(a*d*(1 + n)) - (b*Hypergeometric2F1[2, 1 + n
, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a + b*Sec[c + d*x])^(1 + n))/(4*(a
- b)^2*d*(1 + n)) + (b*Hypergeometric2F1[2, 1 + n, 2 + n, (a + b*Sec[c + d*
x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(4*(a + b)^2*d*(1 + n))
```

Rubi [A] time = 0.230759, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3885, 961, 68, 65, 831}

$$\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a-b}\right)}{2d(n+1)(a-b)}+\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a+b}\right)}{2d(n+1)(a+b)}-\frac{(a+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a + b*Se
c[c + d*x])^(1 + n))/(2*(a - b)*d*(1 + n)) + (Hypergeometric2F1[1, 1 + n, 2
 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(2*(a + b
)*d*(1 + n)) - (Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*
(a + b*Sec[c + d*x])^(1 + n))/(a*d*(1 + n)) - (b*Hypergeometric2F1[2, 1 + n
, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a + b*Sec[c + d*x])^(1 + n))/(4*(a
- b)^2*d*(1 + n)) + (b*Hypergeometric2F1[2, 1 + n, 2 + n, (a + b*Sec[c + d*
x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(4*(a + b)^2*d*(1 + n))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
```

```
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^n\,dx = \frac{b^4\,\text{Subst}\left(\int \frac{(a+x)^n}{x\left(b^2-x^2\right)^2}\,dx, x, b\sec(c+dx)\right)}{d}$$
$$= \frac{b^4\,\text{Subst}\left(\int \left(\frac{(a+x)^n}{4b^3(b-x)^2}+\frac{(a+x)^n}{b^4x}-\frac{(a+x)^n}{4b^3(b+x)^2}-\frac{x(a+x)^n}{b^4\left(-b^2+x^2\right)}\right)dx, x, b\sec(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+x)^n}{x}\,dx, x, b\sec(c+dx)\right)}{d} - \frac{\text{Subst}\left(\int \frac{x(a+x)^n}{-b^2+x^2}\,dx, x, b\sec(c+dx)\right)}{d} +$$
$$= -\frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)} - \frac{b\,{}_2F_1\left(2,1+n;2\right.}{}$$
$$= -\frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)} - \frac{b\,{}_2F_1\left(2,1+n;2\right.}{}$$
$$= \frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{2(a-b)d(1+n)} + \frac{{}_2F_1\left(1,1+n;2+n;\right.}{2}$$

Mathematica [A] time = 6.28407, size = 256, normalized size = 0.92

$$\frac{\cot^2(c+dx)\left(a+b\sqrt{\sec^2(c+dx)}\right)(a+b\sec(c+dx))^n\left((a-b)\left(a(a-b)(2a-b(n-2))\tan^2(c+dx)\text{Hypergeometric2}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] (Cot[c + d*x]^2*(a + b*Sec[c + d*x])^n*(a + b*Sqrt[Sec[c + d*x]^2])*(a*(a +
 b)^2*(2*a + b*(-2 + n))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sqrt[Sec
[c + d*x]^2])/(a - b)]*Tan[c + d*x]^2 + (a - b)*(a*(a - b)*(2*a - b*(-2 + n
))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sqrt[Sec[c + d*x]^2])/(a + b)]
*Tan[c + d*x]^2 - 2*(a + b)*(a*(1 + n)*(a - b*Sqrt[Sec[c + d*x]^2]) + 2*(a^
2 - b^2)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sqrt[Sec[c + d*x]^2])/a]
*Tan[c + d*x]^2))))/(4*a*(a - b)^2*(a + b)^2*d*(1 + n))
```

Maple [F] time = 0.268, size = 0, normalized size = 0.

$$\int (\cot(dx+c))^3 (a+b\sec(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sec(d*x+c))^n,x)
```

```
[Out] int(cot(d*x+c)^3*(a+b*sec(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec (dx + c) + a)^n \cot (dx + c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \sec (dx + c) + a)^n \cot (dx + c)^3 , x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sec(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n\cot(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

3.358 $\int (a+b\sec(c+dx))^n \tan^4(c+dx)\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\tan^4(c+dx)(a+b\sec(c+dx))^n, x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^4, x]
```

Rubi [A] time = 0.0392137, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (a+b\sec(c+dx))^n \tan^4(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^4,x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Tan[c + d*x]^4, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n \tan^4(c+dx)\,dx = \int (a+b\sec(c+dx))^n \tan^4(c+dx)\,dx$$

Mathematica [A] time = 6.25644, size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^n \tan^4(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^4,x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^4, x]
```

Maple [A] time = 0.343, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))^n (\tan(dx+c))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x)
```

```
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n \tan(dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c)+a)^n \tan(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n\tan(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

3.359 $\int (a+b\sec(c+dx))^n \tan^2(c+dx)\,dx$

Optimal. Leaf size=236

$$-\text{Unintegrable}\left((a+b\sec(c+dx))^n,x\right)+\frac{\sqrt{2}(a+b)\tan(c+dx)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n-1;\frac{3}{2}\right.}{bd\sqrt{\sec(c+dx)+1}}$$

```
[Out] (Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -1 - n, 3/2, (1 - Sec[c + d*x])/2, (b*(
1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^n*Tan[c + d*x])/(b*d*Sqrt[
1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^n) - (Sqrt[2]*a*AppellF1[1
/2, 1/2, -n, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a
+ b*Sec[c + d*x])^n*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c
 + d*x])/(a + b))^n) - Unintegrable[(a + b*Sec[c + d*x])^n, x]
```

Rubi [A] time = 0.329608, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (a+b\sec(c+dx))^n \tan^2(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^2,x]
```

```
[Out] (Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -1 - n, 3/2, (1 - Sec[c + d*x])/2, (b*(
1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^n*Tan[c + d*x])/(b*d*Sqrt[
1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^n) - (Sqrt[2]*a*AppellF1[1
/2, 1/2, -n, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a
+ b*Sec[c + d*x])^n*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c
 + d*x])/(a + b))^n) - Defer[Int][(a + b*Sec[c + d*x])^n, x]
```

Rubi steps

$$
\begin{aligned}
\int (a+b\sec(c+dx))^n \tan^2(c+dx)\,dx &= \int (a+b\sec(c+dx))^n \left(-1+\sec^2(c+dx)\right)\,dx \\
&= \frac{\int (-b-a\sec(c+dx))(a+b\sec(c+dx))^n\,dx}{b} + \frac{\int \sec(c+dx)(a+b\sec(c+dx))}{b} \\
&= -\frac{a\int \sec(c+dx)(a+b\sec(c+dx))^n\,dx}{b} - \frac{\tan(c+dx)\,\mathrm{Subst}\left(\int \frac{(a+bx)^{1+n}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x,\right.}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c}} \\
&= \frac{(a\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{(a+bx)^n}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec(c+dx)\right)}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}} + \frac{\left((-a-b)(a+b\sec(c+\right.}{} \\
&= \frac{\sqrt{2}(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-n;\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx)}{bd\sqrt{1+\sec(c+dx)}} \\
&= \frac{\sqrt{2}(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-n;\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx)}{bd\sqrt{1+\sec(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 3.3591, size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^n \tan^2(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^2,x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^2, x]
```

Maple [A] time = 0.263, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))^n (\tan(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c)^2,x)
```

```
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n\tan(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c)+a)^n\tan(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^n\tan^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n*tan(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n\tan(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

3.360 $\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\cot^2(c+dx)(a+b\sec(c+dx))^n,x\right)$$

```
[Out] Unintegrable[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^n, x]
```

Rubi [A] time = 0.0405327, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] Defer[Int][Cot[c + d*x]^2*(a + b*Sec[c + d*x])^n, x]
```

Rubi steps

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx = \int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx$$

Mathematica [A] time = 3.56074, size = 0, normalized size = 0.

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] Integrate[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^n, x]
```

Maple [A] time = 0.25, size = 0, normalized size = 0.

$$\int \left(\cot \left(dx + c\right)\right)^{2} \left(a + b\sec \left(dx + c\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x)
```

```
[Out] int(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(dx + c\right) + a\right)^{n} \cot \left(dx + c\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec \left(dx + c\right) + a\right)^{n} \cot \left(dx + c\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sec(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n\cot(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

3.361 $\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\cot^4(c+dx)(a+b\sec(c+dx))^n,x\right)$$

```
[Out] Unintegrable[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^n, x]
```

Rubi [A] time = 0.0407908, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] Defer[Int][Cot[c + d*x]^4*(a + b*Sec[c + d*x])^n, x]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx = \int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx$$

Mathematica [A] time = 5.71424, size = 0, normalized size = 0.

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] Integrate[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^n, x]
```

Maple [A] time = 0.322, size = 0, normalized size = 0.

$$\int (\cot (dx+c))^4 (a+b\sec (dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x)
```

```
[Out] int(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx+c)+a)^n \cot (dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec (dx+c)+a)^n \cot (dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sec(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n \cot(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

3.362 $\int (a+b\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\tan^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^n, x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^(3/2), x]
```

Rubi [A] time = 0.0459208, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (a+b\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^(3/2),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Tan[c + d*x]^(3/2), x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx = \int (a+b\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx$$

Mathematica [A] time = 4.16643, size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^(3/2),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^(3/2), x]
```

Maple [A] time = 0.303, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))^n (\tan(dx+c))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x)
```

```
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n \tan(dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c)+a)^n \tan(dx+c)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n \tan(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

3.363 $\int (a+b\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\sqrt{\tan(c+dx)}(a+b\sec(c+dx))^n,x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]], x]
```

Rubi [A] time = 0.0428503, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (a+b\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]],x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]], x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx=\int (a+b\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx$$

Mathematica [A] time = 5.28421, size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]],x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]], x]
```

Maple [A] time = 0.318, size = 0, normalized size = 0.

$$\int (a+b\sec (dx+c))^{n} \sqrt{\tan (dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x)
```

```
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx+c)+a)^{n} \sqrt{\tan (dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec (dx+c)+a)^{n} \sqrt{\tan (dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sec (c+dx))^{n} \sqrt{\tan (c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n*sqrt(tan(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n\sqrt{\tan(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

3.364 $\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]], x]
```

Rubi [A] time = 0.0440214, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]],x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]], x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx=\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$$

Mathematica [A] time = 4.8888, size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]],x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]], x]
```

Maple [A] time = 0.297, size = 0, normalized size = 0.

$$\int (a + b\sec(dx + c))^n \frac{1}{\sqrt{\tan(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x)
```

```
[Out] int((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx + c) + a)^n}{\sqrt{\tan(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sec(dx + c) + a)^n}{\sqrt{\tan(dx + c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\sec(c + dx))^n}{\sqrt{\tan(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n/tan(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n/sqrt(tan(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^n}{\sqrt{\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

3.365 $\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^n/Tan[c + d*x]^(3/2), x]
```

Rubi [A] time = 0.0463841, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n/Tan[c + d*x]^(3/2),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n/Tan[c + d*x]^(3/2), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

Mathematica [A] time = 5.84766, size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n/Tan[c + d*x]^(3/2),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^n/Tan[c + d*x]^(3/2), x]
```

Maple [A] time = 0.268, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))^{n} (\tan(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x)
```

```
[Out] int((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{n}}{\tan(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(b\sec(dx+c)+a)^{n}}{\tan(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n/tan(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^n}{\tan(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
    func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```